

Drell-Yan lepton pair production at high energies in the k_T -factorization approach

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Abstract

In the framework of the k_T -factorization approach, the production of unpolarized Drell-Yan lepton pair at high energies is studied. The consideration is based on the $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ off-shell partonic matrix elements with virtual photon γ^* and Z boson exchange. The calculations include leptonic decays of Z bosons with full spin correlations as well as $\gamma^* - Z$ interference. The unintegrated parton densities in a proton are determined by the Kimber-Martin-Ryskin prescription. Our numerical predictions are compared with the data taken by the DØ, CDF and CMS collaborations at the Tevatron and LHC energies. Special attention is put on the specific angular distributions measured very recently by the CDF collaboration for the first time.

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1 Introduction

With the start of the LHC experiments, high energy particle physics entered a new era. The LHC opens a new kinematic regime where the number of novel physical phenomena can occur. One of the most useful tools to study hadronic interactions at high energies is the Drell-Yan dilepton production, in which quark-antiquark annihilation form intermediate virtual photon or Z bosons decaying to lepton pairs. This process is presently of considerable interest from both experimental and theoretical points of view. In particular, Drell-Yan pair production is a unique process which offers high sensitivity to the parton (quark and gluon) distributions in a proton. It provides a major source of background to a number of processes such as Higgs, $t\bar{t}$ pair, di-boson or W' and Z' bosons production (and other processes beyond

the SM) studied at hadron colliders. Dilepton production has a large cross section and clean signature in the detectors and therefore it is used for monitoring of the collider luminosity and calibration of detectors. Moreover, it is an important reference process for measurements of electroweak boson properties at hadron colliders. Therefore it is essential to have an accurate QCD predictions for corresponding cross sections and related kinematical distributions.

Theoretical investigations of Drell-Yan pair production have own long story. It is one of the few processes in hadron-hadron collisions where the collinear QCD factorization has been rigorously proven [1–4]. Within this framework, the NLO pQCD calculations of inclusive cross sections have been performed [5–7], and later it was done on up to NNLO accuracy [8,9]. Recently fully exclusive NNLO pQCD calculations became available, including the leptonic decay of intermediate Z boson [10–13]. The results of these calculations agree with the Tevatron and LHC data within the theoretical and experimental uncertainties. Of course, typically for collinear QCD factorization, perturbative calculations diverge at small dilepton transverse momenta $p_T \ll M$ (where M is the invariant mass of produced lepton pair) with terms proportional to $\ln M/p_T$ appearing due to soft and collinear gluon emission. Therefore special soft gluon resummation technique [14–20] should be used to make QCD predictions at low p_T . Such soft gluon resummation can be performed either in the transverse momentum space [21] or in the Fourier conjugate impact parameter space [22]. Differences between the two formalisms are discussed in [23]. The traditional calculations combine fixed-order perturbation theory with analytic resummation and some matching criterion.

An alternative description can be achieved within the framework of the k_T -factorization approach of QCD [24]. This approach is based on the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [25] or Ciafaloni-Catani-Fiorani-Marchesini (CCFM) [26] equations and provides solid theoretical grounds for the effects of initial gluon radiation and intrinsic parton transverse momentum k_T . A detailed description and discussion of the k_T -factorization formalism can be found, for example, in reviews [27]. Here we only mention that, in contrast with the collinear approximation of QCD, the initial gluon emissions in the k_T -factorization approach generate the finite dilepton transverse momentum p_T already at Born level. Moreover, the soft gluon resummation formulas are the result of the approximate treatment of the solutions of CCFM equation, as it was shown in [28].

In the present note we apply the k_T -factorization approach to unpolarized Drell-Yan pair production in $p\bar{p}$ and pp collisions. A non-collinear factorization theorem for this process has been proven [29] for $p_T \ll M$. Below we assume it in a wide range of p_T for phenomenological purposes¹. First application of k_T -factorization approach to lepton pair production has been performed in [31], where authors have considered only diagrams with virtual photon exchange and concentrated mostly on the rather low energies covered by the RHIC and UA1 experiments. More general consideration of high energy resummation for Drell-Yan processes was done in [32]. Our main goal is to give a systematic analysis of the Tevatron data [33–38] and first LHC measurements [39] performed by the CMS collaboration. The consideration is based on the $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ off-shell (depending on the non-zero transverse momenta of incoming partons) production amplitudes where we take into account both γ^* and Z boson exchange. Our calculations include also leptonic decays of Z bosons with full spin correlations. Thus we easily can produce various kinematical distribution and apply cuts,

¹For discussion of the k_T -factorization for high energy processes see, for example, [30].

analogous to experimental ones. Specially we study the angular distributions of produced lepton pair measured very recently [37] by the CDF collaboration for the first time and investigate the different sources of theoretical uncertainties.

The outline of our paper is following. In Section 2 we recall shortly the basic formulas of the k_T -factorization approach with a brief review of calculation steps. In Section 3 we present the numerical results of our calculations and a discussion. Section 4 contains our conclusions.

2 Theoretical framework

There are three subprocesses which describe Drell-Yan pair production at order of $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$:

$$q + \bar{q} \rightarrow \gamma^*/Z \rightarrow l^+ + l^- \quad (1)$$

$$q + g^* \rightarrow \gamma^*/Z + q \rightarrow l^+ + l^- + q \quad (2)$$

$$q + \bar{q} \rightarrow \gamma^*/Z + g \rightarrow l^+ + l^- + g \quad (3)$$

The corresponding Feynman graphs are shown on the Fig. 1. Note that in the framework of k_T -factorization approach contribution from the subprocess (3) is already taken into account by the quark-antiquark annihilation (1) due to the initial state gluon radiation. Therefore below to avoid the double counting we consider only the subprocesses (1) and (2). It is in a contrast with the collinear QCD factorization where contributions from the subprocesses (1) — (3) should be taken into account separately.

Let us start from the kinematics. We denote the 4-momenta of incoming partons and outgoing leptons by k_1, k_2, p_1 and p_2 . The initial hadrons have the 4-momenta $p^{(1)}$ and $p^{(2)}$, and the final state quark in (2) has 4-momentum p_3 . In the center-of-mass frame of colliding particles we can write:

$$p^{(1)} = \frac{\sqrt{s}}{2}(1, 0, 0, 1), \quad p^{(2)} = \frac{\sqrt{s}}{2}(1, 0, 0, -1) \quad (4)$$

where \sqrt{s} is the total energy of the process under consideration and we neglect the masses of the incoming protons. The initial parton four-momenta in the high energy limit can be written as follows:

$$k_1 = x_1 p^{(1)} + k_{1T}, \quad k_2 = x_2 p^{(2)} + k_{2T}, \quad (5)$$

where k_{1T} and k_{2T} are the corresponding transverse 4-momenta. It is important that $\mathbf{k}_{1T}^2 = -k_{1T}^2 \neq 0$, $\mathbf{k}_{2T}^2 = -k_{2T}^2 \neq 0$. From the conservation laws we can easily obtain the following relations for annihilation subprocess (1):

$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_{1T} + \mathbf{p}_{2T}, \quad (6)$$

$$x_1 \sqrt{s} = m_{1T} e^{y_1} + m_{2T} e^{y_2}, \quad (7)$$

$$x_2 \sqrt{s} = m_{1T} e^{-y_1} + m_{2T} e^{-y_2}, \quad (8)$$

and the similar ones for QCD Compton subprocess (2):

$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_{1T} + \mathbf{p}_{2T} + \mathbf{p}_{3T}, \quad (9)$$

$$x_1\sqrt{s} = m_{1T}e^{y_1} + m_{2T}e^{y_2} + m_{3T}e^{y_3}, \quad (10)$$

$$x_2\sqrt{s} = m_{1T}e^{-y_1} + m_{2T}e^{-y_2} + m_{3T}e^{-y_3}, \quad (11)$$

where p_{1T} , p_{2T} and p_{3T} are the transverse momenta of produced particles, y_1 , y_2 and y_3 are their center-of-mass rapidities and m_{1T} , m_{2T} and m_{3T} are the corresponding transverse masses, i.e. $m_{iT}^2 = m_i^2 + \mathbf{p}_{iT}^2$. The matrix elements of (1) and (2) can be presented as follows:

$$\mathcal{M}_1^\gamma = ie^2 e_q \bar{v}_{s_1}(k_2) \gamma^\mu u_{s_2}(k_1) \frac{g_{\mu\nu}}{s} \bar{u}_{r_1}(p_1) \gamma^\nu v_{r_2}(p_2), \quad (12)$$

$$\begin{aligned} \mathcal{M}_1^Z &= i \frac{g_w^2}{4 \cos^2 \theta_W} \bar{v}_{s_1}(k_2) \gamma^\mu (C_V^q - C_A^q \gamma^5) u_{s_2}(k_1) \times \\ &\times \left(g_{\mu\nu} - \frac{(k_1 + k_2)_\mu (k_1 + k_2)_\nu}{m_Z^2} \right) \frac{\bar{u}_{r_1}(p_1) \gamma^\nu (C_V^e - C_A^e \gamma^5) v_{r_2}(p_2)}{(s - m_Z^2 - im_Z \Gamma_Z)}, \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{M}_2^\gamma &= -e^2 e_q g_s t^a \epsilon_\mu(k_2) \bar{u}_{s_1}(k_1) \left(\gamma^\nu \frac{\hat{k}_1 + \hat{k}_2}{s} \gamma^\mu + \gamma^\mu \frac{-\hat{k}_2 + \hat{p}_3}{(-k_2 + p_3)^2} \gamma^\nu \right) u_{s_2}(p_3) \times \\ &\times \frac{g_{\nu\rho}}{(p_1 + p_2)^2} \bar{u}_{r_1}(p_1) \gamma^\rho v_{r_2}(p_2), \end{aligned} \quad (14)$$

$$\begin{aligned} \mathcal{M}_2^Z &= -\frac{g_w^2 g_s}{4 \cos^2 \theta_W} t^a \epsilon_\mu(k_2) \times \\ &\times \bar{u}_{s_1}(k_1) \left(\gamma^\nu (C_V^q - C_A^q \gamma^5) \frac{\hat{k}_1 + \hat{k}_2}{s} \gamma^\mu + \gamma^\mu \frac{-\hat{k}_2 + \hat{p}_3}{(-k_2 + p_3)^2} \gamma^\nu (C_V^q - C_A^q \gamma^5) \right) u_{s_2}(p_3) \times \\ &\times \left(g_{\rho\nu} - \frac{(p_1 + p_2)_\rho (p_1 + p_2)_\nu}{m_Z^2} \right) \frac{\bar{u}_{r_1}(p_1) \gamma^\rho (C_V^e - C_A^e \gamma^5) v_{r_2}(p_2)}{(p_1 + p_2)^2 - m_Z^2 - im_Z \Gamma_Z}, \end{aligned} \quad (15)$$

where e and e_q are the electron and quark (fractional) electric charges, $s = (k_1 + k_2)^2$, g_w and g_s are the weak and strong charges, m_Z and Γ_Z are the mass and full decay width of Z boson, θ_W is the Weinberg mixing angle, ϵ^μ and a are the polarization 4-vector and eight-fold color index of incoming off-shell gluon, C_V and C_A are the vector and axial constants. Here we neglected the masses and virtualities of incoming quarks and took propagator of intermediate Z boson in a Breit-Wigner form to avoid an artificial singularities in numerical calculations. When we calculate the matrix elements squared, the summation over the incoming off-shell gluon polarizations is carried out in according to the k_T -factorization prescription [24]:

$$\sum \epsilon^\mu \epsilon^{*\nu} = \mathbf{k}_{2T}^\mu \mathbf{k}_{2T}^\nu / \mathbf{k}_{2T}^2. \quad (16)$$

In the collinear limit, where $|\mathbf{k}_{2T}| \rightarrow 0$, this expression converges to the ordinary $\sum \epsilon^\mu \epsilon^{*\nu} = -g^{\mu\nu}/2$ after averaging on the azimuthal angle. In all other respects the evaluation follows the standard QCD Feynman rules. The calculation of traces in (12) — (15) is straightforward and was done using the algebraic manipulation systems Form [40]. We do not list here

the obvious expressions because of lack of space. The obtained expression for Compton subprocess (2) coincides with the result [32].

To calculate the cross section of Drell-Yan lepton pair production, in according to the k_T -factorization theorem, one should convolute off-shell partonic cross sections with the relevant unintegrated quark and/or gluon distributions in a proton:

$$\sigma = \sum_{i,j=q,g} \int \hat{\sigma}_{ij}^*(x_1, x_2, \mathbf{k}_{1T}^2, \mathbf{k}_{2T}^2) f_i(x_1, \mathbf{k}_{1T}^2, \mu^2) f_j(x_2, \mathbf{k}_{2T}^2, \mu^2) dx_1 dx_2 d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2, \quad (17)$$

where $\hat{\sigma}_{ij}^*(x_1, x_2, \mathbf{k}_{1T}^2, \mathbf{k}_{2T}^2)$ is the off-shell partonic cross section and $f_i(x, \mathbf{k}_T^2, \mu^2)$ is the unintegrated parton densities in a proton. The contributions to the total Drell-yan cross section from quark-antiquark annihilation and QCD Compton subprocesses can be easily rewritten as follows:

$$\sigma = \sum_q \int \frac{1}{16\pi(x_1 x_2 s)^2} |\bar{\mathcal{M}}_1^{\gamma, Z}|^2 \times \quad (18)$$

$$\times f_q(x_1, \mathbf{k}_{1T}^2, \mu^2) f_q(x_2, \mathbf{k}_{2T}^2, \mu^2) d\mathbf{p}_{1T}^2 d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy_1 dy_2 \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi},$$

$$\sigma = \sum_q \int \frac{1}{256\pi^3(x_1 x_2 s)^2} |\bar{\mathcal{M}}_2^{\gamma, Z}|^2 \times \quad (19)$$

$$\times f_q(x_1, \mathbf{k}_{1T}^2, \mu^2) f_g(x_2, \mathbf{k}_{2T}^2, \mu^2) d\mathbf{p}_{1T}^2 d\mathbf{p}_{2T}^2 d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy_1 dy_2 dy_3 \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\psi_1}{2\pi} \frac{d\psi_2}{2\pi},$$

where ϕ_1, ϕ_2, ψ_1 and ψ_2 are the azimuthal angles of initial partons and produced leptons, respectively. If we average these expressions over ϕ_1 and ϕ_2 and take the limit $|\mathbf{k}_{1T}| \rightarrow 0$ and $|\mathbf{k}_{2T}| \rightarrow 0$, then we recover the corresponding formulas in the collinear QCD factorization.

Concerning the unintegrated quark and gluon densities in a proton, we apply the Kimber-Martin-Ryskin (KMR) approach [41] to calculate them. The KMR approach is the formalism to construct the unintegrated parton distributions from the known conventional ones. In this approximation the unintegrated quark and gluon distributions are given by

$$f_q(x, \mathbf{k}_T^2, \mu^2) = T_q(\mathbf{k}_T^2, \mu^2) \frac{\alpha_s(\mathbf{k}_T^2)}{2\pi} \times \quad (20)$$

$$\times \int_x^1 dz \left[P_{qq}(z) \frac{x}{z} q\left(\frac{x}{z}, \mathbf{k}_T^2\right) \Theta(\Delta - z) + P_{qg}(z) \frac{x}{z} g\left(\frac{x}{z}, \mathbf{k}_T^2\right) \right],$$

$$f_g(x, \mathbf{k}_T^2, \mu^2) = T_g(\mathbf{k}_T^2, \mu^2) \frac{\alpha_s(\mathbf{k}_T^2)}{2\pi} \times \quad (21)$$

$$\times \int_x^1 dz \left[\sum_q P_{gq}(z) \frac{x}{z} q\left(\frac{x}{z}, \mathbf{k}_T^2\right) + P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, \mathbf{k}_T^2\right) \Theta(\Delta - z) \right],$$

where $P_{ab}(z)$ are the usual unregulated LO DGLAP splitting functions. The theta functions which appears in (20) and (21) imply the angular-ordering constraint $\Delta = \mu/(\mu + |\mathbf{k}_T|)$ specifically to the last evolution step to regulate the soft gluon singularities. Numerically, for the input we have used leading-order parton densities $xq(x, \mu^2)$ and $xg(x, \mu^2)$ from recent

MSTW'2008 set [42]. The Sudakov form factors $T_q(\mathbf{k}_T^2, \mu^2)$ and $T_g(\mathbf{k}_T^2, \mu^2)$ enable us to include logarithmic loop corrections to the calculated cross sections. To take into account the non-logarithmic loop corrections we use the approach proposed in [43]. It was demonstrated that main part of the non-logarithmic loop corrections to the quark-antiquark annihilation cross section (1) can be absorbed in the effective K -factor:

$$K = \exp \left[C_F \frac{\alpha_s(\mu^2)}{2\pi} \pi^2 \right], \quad (22)$$

where color factor $C_F = 4/3$. A particular choice $\mu^2 = \mathbf{p}_T^{4/3} M^{2/3}$ has been proposed [23, 43] to eliminate sub-leading logarithmic terms. We choose this scale to evaluate the strong coupling constant in (22).

The multidimensional integrations in (18) and (19) have been performed by the means of Monte Carlo technique, using the routine VEGAS [44]. The full C++ code is available from the author on request².

3 Numerical results

We now are in a position to present our numerical results. First we describe our input and the kinematic conditions. After we fixed the unintegrated gluon distributions, the cross sections (18) and (19) depend on the renormalization and factorization scales μ_R and μ_F . Numerically, we set them to be equal to $\mu_R = \mu_F = \xi M$. To estimate the scale uncertainties of our calculations we vary the parameter ξ between 1/2 and 2 about the default value $\xi = 1$. Following to [45], we set $m_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV, $\sin^2 \theta_W = 0.23122$ and use the LO formula for the strong coupling constant $\alpha_s(\mu^2)$ with $n_f = 4$ active quark flavors at $\Lambda_{\text{QCD}} = 200$ MeV, so that $\alpha_s(M_Z^2) = 0.1232$.

The results of our calculations are presented in Figs. 2 — 4 in comparison with the DØ [38], CDF [33–37] and CMS data [39]. Solid histograms are obtained by fixing both the factorization and renormalization scales at the default value $\mu = M$, whereas the upper and lower dashed histograms correspond to the scale variation as it was described above. The predicted total cross sections are listed in Table 1. One can see that the Tevatron and LHC experimental data are reasonable well described by the k_T -factorization approach in the whole range of invariant masses. Our predictions tend to only slightly overestimate the rapidity distribution of dilepton pair in the region of Z boson peak $66 < M < 116$ GeV, but agree with the data within the uncertainties. Specially we point out a good description of dilepton transverse momentum distributions measured by the CDF collaboration since this observable strongly depends on the unintegrated parton density used.

The relative contributions of quark-antiquark annihilation and QCD Compton subprocesses to the Drell-Yan cross sections at the Tevatron and LHC energies are shown in Fig. 5 as a function of azimuthal angle difference between the transverse momenta of produced leptons. Note that this observable is singular in the collinear QCD approximation at LO due to back-to-back kinematics. It is in a contrast with the k_T -factorization approach, where, as it

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Source	$\sigma(66 < M < 116 \text{ GeV}), \text{ pb}$	$\sigma(M > 116 \text{ GeV}), \text{ pb}$
k_T -factorization (KMR)	285 ± 31	3.7 ± 0.4
NNLO pQCD [36]	227 ± 9	3.3
CDF data [36]	$250 \pm 4 \text{ (stat.)} \pm 10 \text{ (syst.)}$	$4.0 \pm 0.4 \text{ (stat.+syst.)} \pm 0.2 \text{ (lumi.)}$

Table 1: Total cross sections of Drell-Yan pair production in $p\bar{p}$ collisions at $\sqrt{s} = 1800 \text{ GeV}$.

was mentioned above, the finite transverse momentum of dilepton pair is generated already in LO quark-antiquark annihilation (1). We find that latter dominates at high $\Delta\phi \sim \pi$ for both the Tevatron and LHC energies, whereas at $\Delta\phi < \pi/2$ quark-antiquark annihilation and QCD Compton subprocesses contribute equally. Note that here we applied no cuts on the final-state phase space.

Now we turn to more detailed analysis of angular distributions in dilepton production. The general expression can be described by the polar θ and azimuthal ϕ angles of produced particles in the dilepton rest frame. When integrated over $\cos\theta$ or ϕ , respectively, the angular distribution can be presented as follows:

$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) + \frac{1}{2}A_0(1 - 3\cos^2\theta) + A_4\cos\theta, \quad (23)$$

$$\frac{d\sigma}{d\phi} \sim 1 + \beta_3\cos\phi + \beta_2\cos 2\phi, \quad (24)$$

where $\beta_3 = 3\pi A_3/16$ and $\beta_2 = A_2/4$. Note that the angular coefficients A_0 and A_2 are the same for γ^* or Z boson exchange, and A_3 and A_4 originate from the $\gamma^* - Z$ interference. The Lam-Tung relation [46] $A_0 = A_2$ is valid for both quark-antiquark annihilation and QCD Compton subprocesses at $\mathcal{O}(\alpha\alpha_s)$ order. Higher-order QCD calculations [47, 48] as well as QCD resummation up to all orders [49] indicate that violations of the Lam-Tung relation are small. Very recently the CDF collaboration reported [37] the first measurement of the angular coefficients A_0, A_2, A_3 and A_4 in the Z peak region ($66 < M < 116 \text{ GeV}$) at $\sqrt{s} = 1960 \text{ GeV}$. Below we estimate these coefficients regarding the CDF measurements. Our evaluation generally followed the experimental procedure. We have collected the simulated events in the specified bins of dilepton transverse momentum, generated the decay lepton angular distributions according to the matrix elements (12) — (15) and then applied a two-parametric fit based on (23) and (24). The estimated values of angular coefficients in the Collins-Soper frame are shown in Fig. 6. We find that our predictions agree well with the CDF data as well as collinear QCD predictions listed in [37]. We would like to only remark that the latter predict a flat behaviour of A_3 in a whole p_T range whereas CDF data tends to support our predictions (slight decreasing of A_3 when we move to large p_T values).

Finally, we can conclude that k_T -factorization predictions in general are rather similar to ones based on the collinear QCD factorization with the NNLO accuracy. It demonstrates again that the k_T -factorization approach at LO level automatically incorporates a large piece of the standard (collinear) high-order corrections [27]. It is important for further studies of small- x physics at hadron colliders, and, in particular, for searches of effects of new physics beyond the SM at the LHC.

4 Conclusions

We have investigated unpolarized Drell-Yan lepton pair production in $p\bar{p}$ and pp collisions at the Tevatron and LHC energies within the framework of the k_T -factorization approach. Our consideration is based on the $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ off-shell production amplitudes where γ^* and Z boson exchange is taken into account. The calculations include leptonic decays of Z bosons with full spin correlations and $\gamma^* - Z$ interference. The unintegrated parton densities in a proton are determined by the Kimber-Martin-Ryskin prescription. We obtained a reasonable well agreement (at a similar level as in the NNLO pQCD) between our predictions and the available data taken by the DØ, CDF and CMS collaborations. Specially we studied the specific angular distributions measured very recently by the CDF collaboration for the first time.

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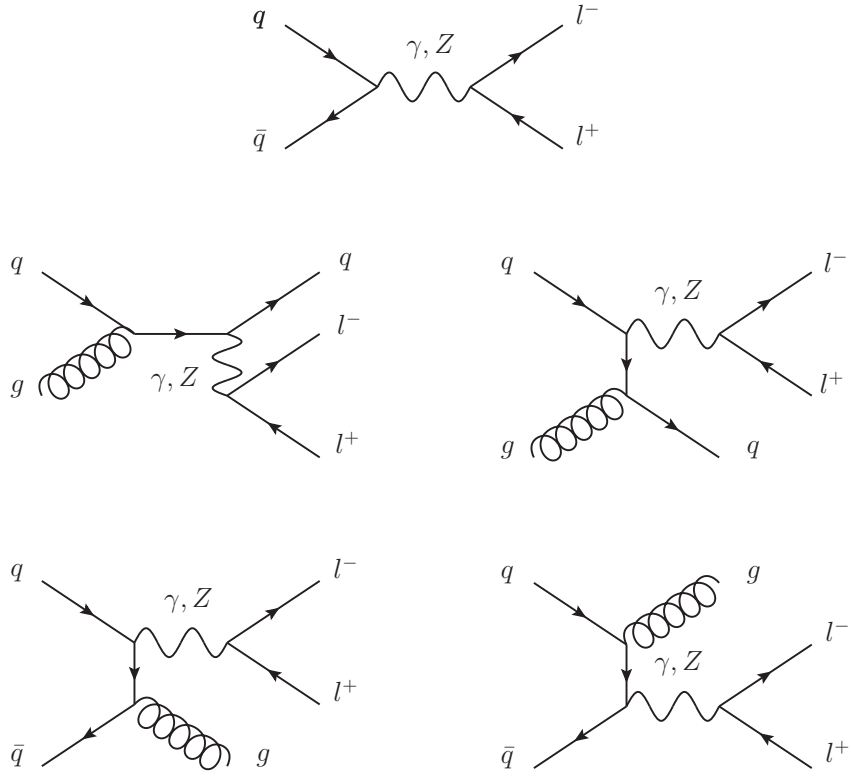


Figure 1: The Feynman graphs for Drell-Yan pair production at the $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ orders.

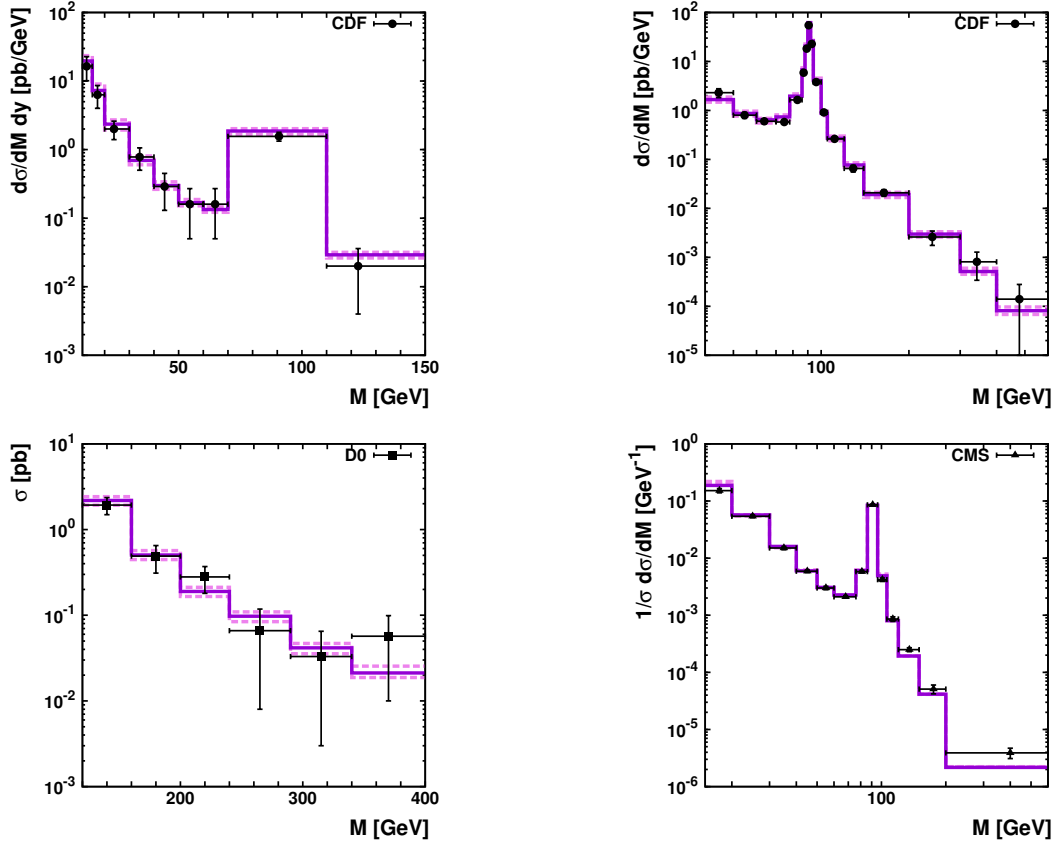


Figure 2: The total and differential cross sections of the Drell-Yan pair production in $p\bar{p}$ and pp collisions at the Tevatron and LHC as a function of dilepton invariant mass M . The solid histograms correspond to the results obtained with the KMR parton densities. The upper and lower dashed histograms correspond to scale variations, as it is described in the text. The experimental data are from $D\bar{0}$ [38], CDF [33, 35] and CMS [39].

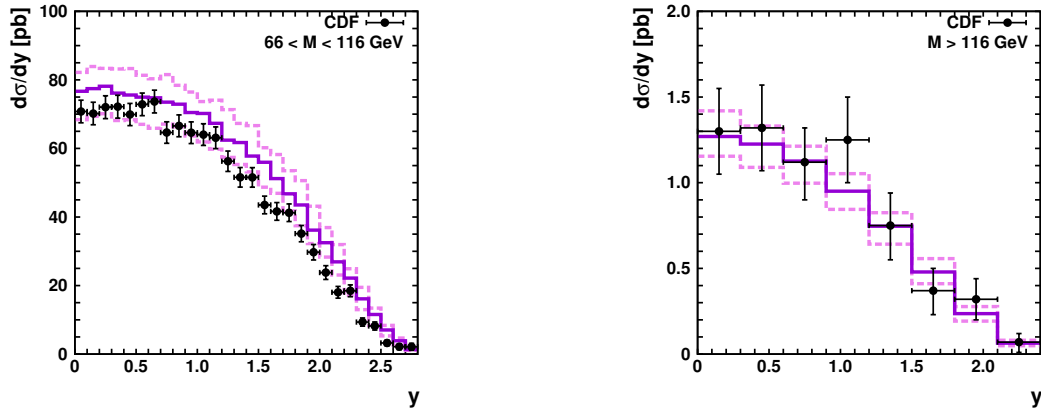


Figure 3: The differential cross sections $d\sigma/dy$ of dilepton production at $\sqrt{s} = 1800$ TeV compared to the CDF data [36]. Notation of all histograms is the same as in Fig. 1.

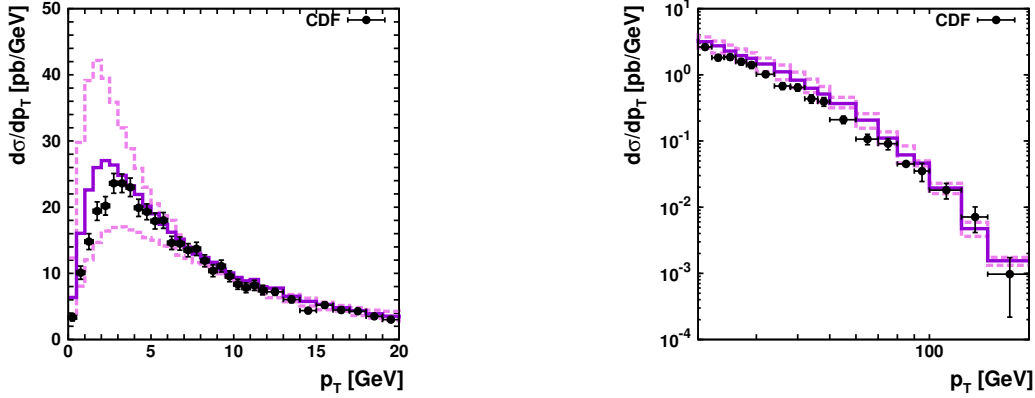


Figure 4: The differential cross sections $d\sigma/dp_T$ of dilepton production at $\sqrt{s} = 1800$ TeV compared to the CDF data [34]. Notation of all histograms is the same as in Fig. 1.

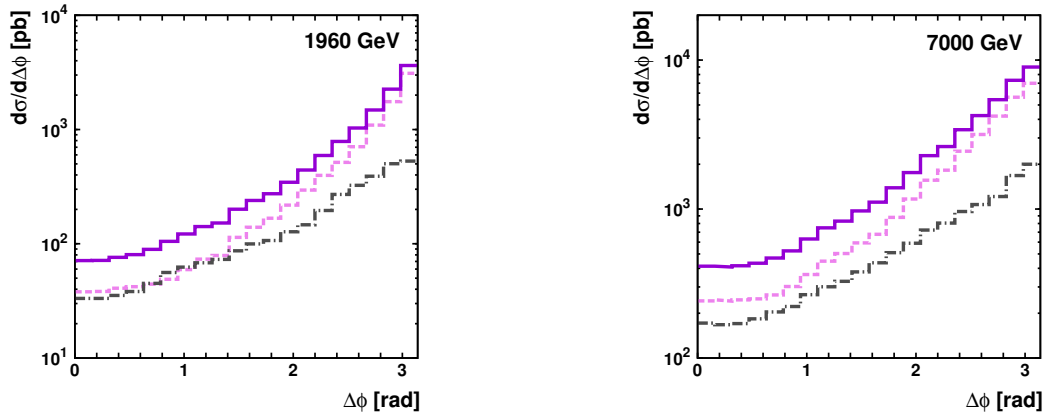


Figure 5: Different contributions to the Drell-Yan pair cross section in $p\bar{p}$ and pp collisions at the Tevatron and LHC energies as a function of produced lepton azimuthal angle difference. The dashed and dash-dotted histograms correspond to the contributions from quark-antiquark annihilation and QCD Compton subprocesses, respectively. The solid histograms represent the sum of these components.

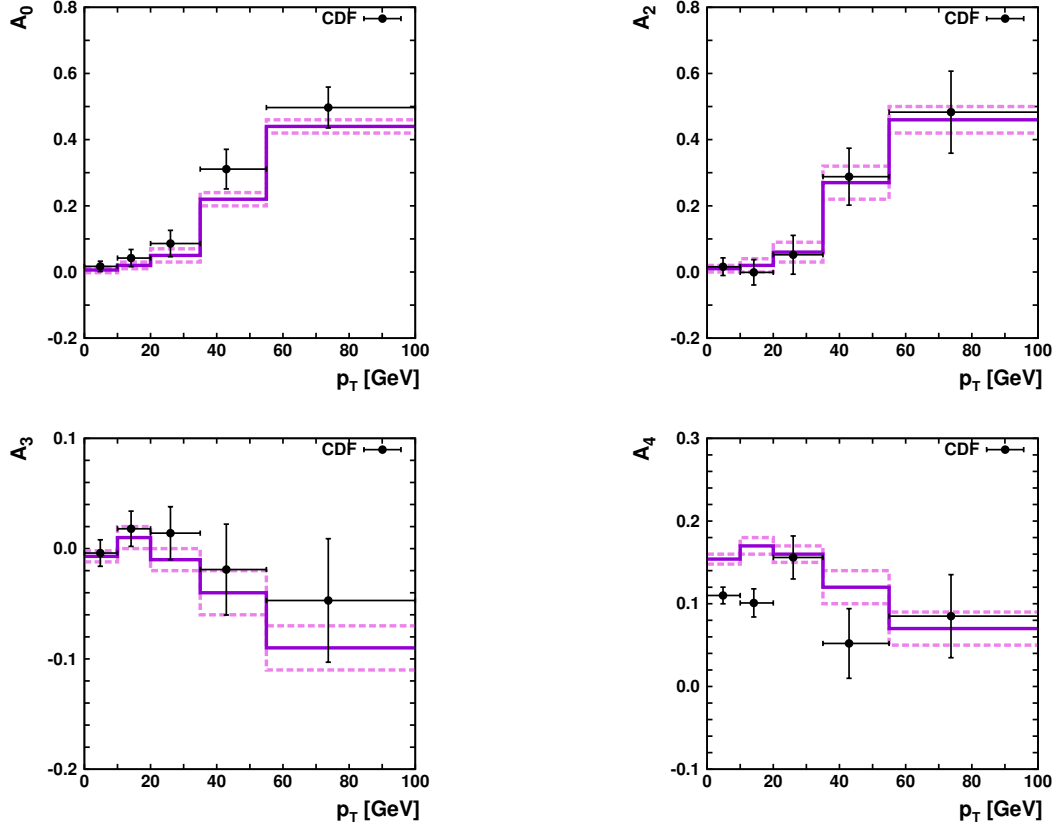


Figure 6: Angular coefficients A_0 , A_2 , A_3 and A_4 of dilepton production as a function of p_T compared to the CDF data [37]. Solid and two dashed histograms represent fitted values of angular coefficients and corresponding uncertainties of fitting procedure. The default scale $\mu = M$ has been applied.