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WinPos – Magnetic Rulers Designed by Combinatorial Optimization

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1 Executive Summary

Mathematical optimization is used to design magnetic rulers of maximum lengths with trapezoidal shaped ferromagnetic regions on a tape. The non-repetitive pattern is constructed in such way that a one-to-one correspondence to an absolute position can be computed by a fast and compact electronic evaluation unit. Mathematicians and electrical engineers from academia worked together with a medium-size manufacturing enterprise in order to develop an innovative new product. The rulers can be used in various different applications, e.g. manufacturing systems engineering, and are cheaper, more energy efficient and more flexible than existing ones.

2 Challenge Overview

The progress of modern man in the construction of technical apparatus and structures is based on the ability to measure lengths and distances as precisely as possible.

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Ivory rulers were used for this purpose, which were produced over 4000 years ago in the Indus Valley in the north-west of today's India and Pakistan. Indentations on the surface allow measurements with an accuracy of 0.13 mm. In the Han dynasty, rulers made of bronze were used around 200 BC to 200 AD. The folded ruler made of plastic or wood with its millimeter or tenth-inch scale, which can be found in every household, is an invention by A. Ullrich of 1851. The length is read by the eye, and the user (for example, a home handyman, construction worker or architect) can use this information through the work of his hand. For many workpieces of our modern world this precision in the millimeter range is by no means sufficient. In the case of high-volume production, automated work must also be carried out without man being involved in all steps of the production process.

What kind of rulers could be used for this?

3 Problem Description

The company Bogen Electronic GmbH from Berlin produces magnetic rulers and the associated reading heads for such applications. Here, the ruler's scale is no longer visible to the eye because it consists of a sequence of small ferromagnets alternately polarized on a magnetic tape, similar to the tape of an (old) audio or video cassette, see Figure 1. A reading head moves from a certain starting position over this tape and measures the approximately sinusoidal magnetic signal when passing by. The reading head can distinguish more than just a north pole from a south pole. In fact, it is based on the physical principle of the Hall effect, which was discovered in 1879 by Edwin Hall. With the best components such systems can resolve increments of the order of 80 nm. High-resolution and automated measurements are thus possible with such a system. However, it is an "incremental" positioning system, which means that only a relative measurement is returned. For an absolute measurement, e.g. when the measuring system is restarted, the reading head first has to move back to a specially marked starting position in order to start counting the number of northpole-southpole alternations again from there. This restriction is a severe drawback and excludes the measurement method for some applications.

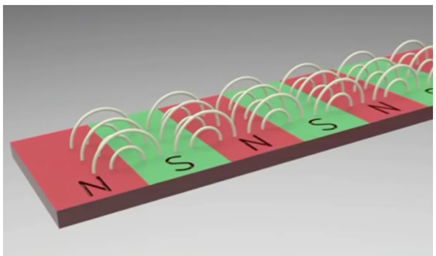

Fig. 1 Incremental ruler with magnetic field lines (image source: Bogen Electronic GmbH, Berlin).

For the construction of magnetic measuring systems, a mathematical description of the magnetic field is an essential aid. James Clerk Maxwell summarized the foundations in 1865 in the equations named after him. This is a system of partial differential equations describing temporally changing electromagnetic fields for every point in space. Such a dynamic field occurs, for example, in the case of alternating current. Magnetic fields of stationary ferromagnets do not change over time, thus simplifying Maxwell's equations. Nevertheless, they are not always analytical solvable, especially in complicated geometries, so one has to use numerical methods, for which there are different methods with individual advantages and disadvantages (for example, finite element or boundary integral methods).

In order to develop magnetic rulers for application areas in which incremental measuring systems can not be used, there are different ways to design absolute systems. In one method, phase shifts between two traces are evaluated using the Nonius principle. In another method, a different pattern, e.g. binary or Gray pattern is written on a second tape parallel to the incremental tape, see Figure 2. While incremental rulers are in principle not limited in their length, such absolute rulers have a limitation by the number of different coding patterns. For binary or Gray patterns there are $2^{1}0 = 1024$ possibilities for S and N at 10 places, which would give a maximum of 512 cm length at 5 mm per magnet. For longer rulers, therefore, more digits are needed. Each additional digit doubles the number of combinations and thus the possible length.

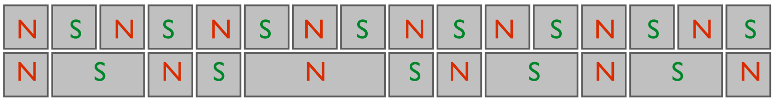

Fig. 2 Absolute ruler with binary pattern.

This method of absolute position determination works and is successfully used in practice. However, there are two main drawbacks: Firstly, two magnetic tracks (instead of one) must be used, doubling production time and costs. Secondly, the reading head with the Hall sensors must be designed in such a way that it can simultaneously measure in two tracks and partly at 10 (or more) measuring points. This makes it quite wide (about 5-7cm) and leads to a higher power consumption. Especially for applications in which the ruler does not lie flat but has to be bent irregularly, a wide reading head is a hindrance.

What could an absolute measuring system look like that circumvents these disadvantages?

4 Implementation

The solution was found (and patented) by Bogen Electronic a few years ago: The magnetic sections do not need to be rectangles, as usual, but can also be trapezoidal. The key question is: how should the trapezoidal magnet pattern be defined, which allows a one-to-one conversion of a sensor signal to an absolute length specification? In order to answer this question, the discrete-combinatorial optimization comes into play: a mathematical description (model) is implemented, whose solution corresponds to a longest possible ruler in which the pattern that is detected by the reading head is unique in every possible position on the entire tape. This model can be formulated as a mixed-integer linear optimization problem. Further conditions resulting from the production of the ruler can simultaneously be considered. For example, the changes in the angles of two adjacent trapezoids must not become too large so that in the production the writing head is able to write the pattern onto the magnetic tape. Interestingly, this optimization problem is purely combinatorial, the inclusion of Maxwell's partial differential equations that describe magnetic fields is not necessary.

5 Results and Achievements

The solution to the optimization problem then provides a suitable ruler of maximum length, which is manufactured by Bogen Electronic. A relatively small read head with a pattern of only 12 Hall sensors (arranged in 2 tracks with 6 sensors each) now measures the magnetic signal from which the angles of the magnetic trapezoids are calculated. This evaluation unit is implemented on DSP and FPGA chips, as a result of which the back-calculation process runs very quickly and can be integrated structurally into the reading head, see Figure 3.

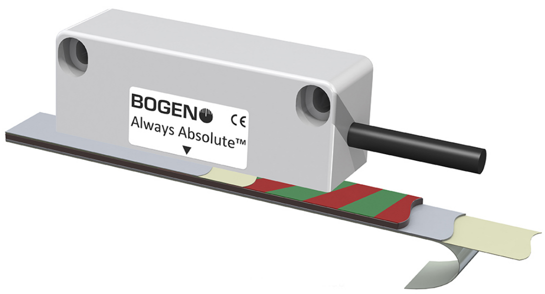

Fig. 3 Absolute ruler with new trapezoidal magnetic pattern and reading head (image source: Bogen Electronic GmbH, Berlin).

Mathematical optimization thus played a decisive role in the fact that a medium-sized Berlin industrial enterprise can continue to persist in the competition by a new, innovative product.

6 Partners in the Project

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