# Determining CP violation angle $\gamma$ with $B$ decays into a scalar/tensor meson 

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#### Abstract

We propose a new way for determining the CP violation angle $\gamma$ without any hadronic uncertainty. The suggested method is to use the two triangles formed by the decay amplitudes of $B^{ \pm} \rightarrow\left(D^{0}, \bar{D}^{0}, D_{C P}^{0}\right) K_{0(2)}^{* \pm}(1430)$. The advantages are that large CP asymmetries are expected in these processes and only singly Cabibbo-suppressed $D$ decay modes are involved. Measurements of the branching fractions of the neutral $B_{d}$ decays into $D K_{0(2)}^{*}(1430)$ and the time-dependent CP asymmetries in $B_{s} \rightarrow\left(D^{0}, \bar{D}^{0}\right) M\left(M=f_{0}(980), f_{0}(1370), f_{2}^{\prime}(1525), f_{1}(1285), f_{1}(1420), h_{1}(1180)\right)$ provide an alternative way to extract the angle $\gamma$, which will increase the statistical significance.


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CP violation in the standard model (SM) originates from a single, irreducible phase in the $3 \times 3$ quark mixing matrix called the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Precision test of its unitarity allows us to explore the SM description of the CP violation and reveal any physics beyond the SM. One of the foremost tasks during the past decades has been to study the so-called (bd) unitarity triangle, the graphical representation of the condition stemming from the unitarity of the CKM matrix: $V_{u d} V_{u b}^{*}+V_{c d} V_{c b}^{*}+V_{t d} V_{t b}^{*}=0$. Its sides can be measured by leptonic and semileptonic meson decays, while the determinations of the angles ( $\alpha, \beta, \gamma$ ), satisfying $\alpha+\beta+\gamma=180^{\circ}$, rely mostly on nonleptonic $B$ decays.

Our knowledge of the angle $\beta$ to a large extent benefits from the gold-plated channel $B \rightarrow J / \psi K_{S}$ and the current results already have a precision better than $1^{\circ}[1,2]$. The accuracy on the angle $\alpha$ is around $4^{\circ}$, thanks to the measurements of charmless tree dominated processes $B \rightarrow \pi^{+} \pi^{-}, B \rightarrow \rho^{ \pm} \pi^{\mp}, B \rightarrow \rho^{+} \rho^{-}$and $B \rightarrow a_{1}^{ \pm} \pi^{\mp}$. In contrast, results for the angle $\gamma$ are less accurate, with a precision of roughly $10^{\circ}$, which is one of the main sources of the current uncertainties on the apex of the unitary triangle.
Since the angle $\gamma \equiv \arg \left(-V_{u d} V_{u b}^{*} /\left(V_{c d} V_{c b}^{*}\right)\right)$ is the relative weak phase involving the decays $b \rightarrow c \bar{u} s$ and $b \rightarrow u \bar{c} s$, several methods on the basis of the decays $B^{ \pm} \rightarrow D K^{ \pm}$, with $D$ being any admixture of $D^{0}$ and $\bar{D}^{0}$, have been proposed (for a review, see Ref. [3]). The most productive ones are the Gronau-London-Wyler (GLW) method [4-6], with $D$ decaying into the CP eigenstates including $\pi^{0} K_{S}, \pi^{+} \pi^{-} K_{S}, K^{+} K^{-}, \pi^{+} \pi^{-}$, the Atwood-Dunietz-Soni (ADS) method [7, 8], using the Cabibbo-favored and doubly Cabibbo-suppressed $D$ decay modes, and the Giri-Grossman-Soffer-Zupan (GGSZ) method [9], which makes use of a Dalitz-plot distribution of the products of the multi-body $D$ decays. All three methods are theoretically clean and do not require any time-dependent measurement.

In the GLW method, the sensitivity of the CP asymme-

[^0]tries to $\gamma$ is proportional to the ratio of the two interfering amplitudes, which is of the order $10 \%$. The ADS method demands a detailed knowledge of the doubly Cabibbosuppressed $D$ decays, while the GGSZ method requires a Dalitz-plot analysis of multibody $D$ decays. In this work, we propose a new method which is based on $B \rightarrow D M$ decays with $M$ being a light scalar/tensor meson. The proposed method has both advantages, namely on the one hand the interference and the CP violation in the chosen decay modes are sizable and on the other hand neither doubly Cabibbo-suppressed $D$ decays nor the Dalitz plot are needed. Among the various $B$ decays into a p-wave scalar/tensor meson to be discussed, of particular interest are the $B^{ \pm} \rightarrow\left(D^{0}, \bar{D}^{0}, D_{C P}^{0}\right) K_{0(2)}^{* \pm}(1430)$ modes, where $K_{0(2)}^{*}(1430)$ is a scalar (tensor) meson with $J^{P}=0^{+}\left(2^{+}\right)$. The small (zero) decay constant of $K_{0}^{*}(1430)\left(K_{2}^{*}(1430)\right)$ compensates the large Wilson coefficient in the color-allowed amplitude, resulting in similar sizes for the decay amplitudes of $B^{ \pm} \rightarrow D^{0} K_{0(2)}^{* \pm}(1430)$ and $B^{ \pm} \rightarrow \bar{D}^{0} K_{0(2)}^{* \pm}(1430)$. As a consequence, there are large CP asymmetries. Measurements of branching ratios (BRs) of the neutral $B_{d}$ decays into $D K_{0(2)}^{*}(1430)$ and time-dependent CP asymmetries in $B_{s} \rightarrow D M(M=$ $\left.f_{0}(980), f_{0}(1370), f_{2}^{\prime}(1525), f_{1}(1285), f_{1}(1420), h_{1}(1180)\right)$ provide an alternative way to extract the angle $\gamma$. For the sake of brevity, hereafter we use $K_{0,2}^{*}$ and $f_{0}, f_{2}^{\prime}$ to abbreviate $K_{0,2}^{*}(1430)$ and $f_{0}(980), f_{2}^{\prime}(1525)$, respectively.

All three methods [4-9] to extract $\gamma$ based on $B^{ \pm} \rightarrow$ $\left(D^{0}, \bar{D}^{0}, D_{C P}^{0}\right) K^{ \pm}$use the information that the six decay amplitudes form two triangles in the complex plane, graphically representing the following identities

$$
\begin{align*}
\sqrt{2} A\left(B^{+} \rightarrow D_{ \pm}^{0} K^{+}\right) & =A\left(B^{+} \rightarrow D^{0} K^{+}\right) \\
& \pm A\left(B^{+} \rightarrow \bar{D}^{0} K^{+}\right) \\
\sqrt{2} A\left(B^{-} \rightarrow D_{ \pm}^{0} K^{-}\right) & =A\left(B^{-} \rightarrow D^{0} K^{-}\right) \\
& \pm A\left(B^{-} \rightarrow \bar{D}^{0} K^{-}\right) \tag{1}
\end{align*}
$$

where the convention $C P\left|D^{0}\right\rangle=\left|\bar{D}^{0}\right\rangle$ has been adopted and $D_{+}^{0}\left(D_{-}^{0}\right)$ denotes the CP even (odd) eigenstate. The corresponding Feynman diagrams for these processes are given in Fig. 1 Measurements of the decay rates of the six processes completely determine the sides and apexes


FIG. 1: Feynman diagrams for the color-suppressed contributions in the process $B^{-} \rightarrow D^{0} K_{0(2)}^{*-}(1430)$ (a), $B^{-} \rightarrow$ $\bar{D}^{0} K_{0(2)}^{*-}(1430)(\mathrm{b})$, and the color-allowed contributions in the $B^{-} \rightarrow D^{0} K_{0(2)}^{*-}(1430)(\mathrm{c})$. In the diagrams ( $\mathrm{a}, \mathrm{b}$ ), the spectator quark can also be a $\bar{d}$ or $\bar{s}$ quark, in which the light hadron consists of $K_{0}^{*}(1430), f_{0}(980)$ and $f_{2}^{\prime}(1525)$.
of the two triangles, in particular the relative phase between $A\left(B^{-} \rightarrow \bar{D}^{0} K^{-}\right)$and $A\left(B^{+} \rightarrow D^{0} K^{+}\right)$is $2 \gamma$.
The shape of the two triangles is governed by two quantities

$$
\begin{aligned}
& r_{B}^{K_{J}} \equiv \mid A\left(B^{-}\right.\left.\rightarrow \bar{D}^{0} K_{J}^{-}\right) / A\left(B^{-} \rightarrow D^{0} K_{J}^{-}\right) \mid, \\
& \delta_{B}^{K_{J}} \equiv \arg \left[e^{i \gamma} A\left(B^{-} \rightarrow \bar{D}^{0} K_{J}^{-}\right) / A\left(B^{-} \rightarrow D^{0} K_{J}^{-}\right)\right],
\end{aligned}
$$

with $K_{J}=K, K_{0,2}^{*}$. The $B^{-} \rightarrow \bar{D}^{0} K^{-}$is both Cabibbosuppressed and color suppressed. Thus the ratio $r_{B}^{K} \sim$ $\left|V_{u b} V_{c s}^{*} /\left(V_{c b} V_{u s}^{*}\right) a_{2} / a_{1}\right| \sim 0.1$ is small and in fact the world averages for the parameters [2]

$$
r_{B}^{K}=0.107 \pm 0.010, \delta_{B}^{K}=\left(112_{-13}^{+12}\right)^{\circ}
$$

indicate that the two triangles are squashed. Physical observables to be experimentally measured, defined as

$$
\begin{aligned}
R_{C P \pm}^{K} & =2 \frac{\mathcal{B}\left(B^{-} \rightarrow D_{C P \pm} K^{-}\right)+\mathcal{B}\left(B^{+} \rightarrow D_{C P \pm} K^{+}\right)}{\mathcal{B}\left(B^{-} \rightarrow D^{0} K^{-}\right)+\mathcal{B}\left(B^{+} \rightarrow \bar{D}^{0} K^{+}\right)} \\
& =1+\left(r_{B}^{K}\right)^{2} \pm 2 r_{B}^{K} \cos \delta_{B}^{K} \cos \gamma, \\
A_{C P \pm}^{K} & =\frac{\mathcal{B}\left(B^{-} \rightarrow D_{C P \pm} K^{-}\right)-\mathcal{B}\left(B^{+} \rightarrow D_{C P \pm} K^{+}\right)}{\mathcal{B}\left(B^{-} \rightarrow D_{C P \pm} K^{-}\right)+\mathcal{B}\left(B^{+} \rightarrow D_{C P \pm} K^{+}\right)} \\
& = \pm 2 r_{B}^{K} \sin \delta_{B}^{K} \sin \gamma / R_{C P \pm}^{K},
\end{aligned}
$$

have a mild sensitivity to the angle $\gamma$, and their values are expected to be $R_{C P \pm}^{K} \sim 1$ and $A_{C P \pm}^{K} \sim 0$.

Since the $K_{0(2)}^{*}$ have the same flavor structure as the $K$ meson, the relations given in Eq. (11) also apply to $B^{ \pm} \rightarrow\left(D^{0}, \bar{D}^{0}, D_{C P}^{0}\right) K_{0(2)}^{* \pm}(1430)$. We wish to point out that, because of the suppression of the color-allowed decay amplitudes, the low sensitivity problem is highly improved and in particular large CP asymmetries are expected. Although the $K_{0(2)}^{*}$-emission diagram, as depicted in Fig. [1(c), has a large Wilson coefficient $a_{1} \sim 1$, the emitted meson is generated by a local vector or axialvector current (at the lowest order in $\alpha_{s}$ ), whose matrix element between the QCD vacuum and the $K_{0}^{*}\left(K_{2}^{*}\right)$ state is small (identically zero).

A crude and model-dependent estimate of the amplitudes can be made with the help of the factorization hy-
pothesis

$$
\begin{align*}
A\left(B^{-} \rightarrow \bar{D}^{0} K_{0}^{*-}\right) & =-V_{u b} V_{c s}^{*} C \\
A\left(B^{-} \rightarrow D^{0} K_{0}^{*-}\right) & =-V_{c b} V_{u s}^{*}(C-T), \tag{2}
\end{align*}
$$

where $C=G_{F} f_{D} a_{2}\left(m_{B}^{2}-m_{K_{0}^{*}}^{2}\right) F_{0}^{B K_{0}^{*}}\left(m_{D}^{2}\right) / \sqrt{2}, T=$ $G_{F} f_{K_{0}^{*}} a_{1}\left(m_{B}^{2}-m_{D}^{2}\right) F_{0}^{B D}\left(m_{K_{0}^{*}}^{2}\right) / \sqrt{2}$, and $G_{F}$ is the Fermi constant. The decay constant, defined via

$$
\left\langle K_{0}^{*-}(1430)\right| \bar{s} \gamma^{\mu} u|0\rangle=f_{K_{0}^{*}} p_{K_{0}^{*}}^{\mu},
$$

vanishes in the $\mathrm{SU}(3)$ symmetry limit and may get a nonzero but small value due to the symmetry breaking effects. The current experimental data on $\tau \rightarrow$ $K_{0}^{*-}(1430) \bar{\nu}_{\tau}$ places an upper bound [10]

$$
\left|f_{K_{0}^{*}}\right|<107 \mathrm{MeV}
$$

which is not very stringent. Adopting an estimate based on QCD sum rules [11]

$$
f_{K_{0}^{*}}=-24 \mathrm{MeV}, \quad \text { or } \quad f_{K_{0}^{*}}=36 \mathrm{MeV}
$$

which contains a sign ambiguity, we find the relation $2 a_{1}\left|f_{K_{0}^{*}}\right| \sim a_{2} f_{D}$, with the $D$ meson decay constant extracted from $D^{-} \rightarrow \mu \bar{\nu}_{\mu}: f_{D}=(221 \pm 18) \mathrm{MeV}[10]$. Using one set of results for the $B \rightarrow K_{0}^{*}$ form factors calculated in the perturbative QCD approach [12] (corresponding to $f_{K_{0}^{*}}=36 \mathrm{MeV}$ ), the $B \rightarrow D$ form factors from Ref. [13] and $a_{2}=0.2, a_{1}=1$ we estimate $C / T \sim 1.2$ and

$$
\begin{equation*}
r_{B}^{K_{0}^{*}}=\left|C V_{u b} V_{c s}^{*} /\left[V_{c b} V_{u s}^{*}(C-T)\right]\right| \sim 2 \tag{3}
\end{equation*}
$$

The corresponding BRs are roughly

$$
\begin{equation*}
\mathcal{B}\left(B^{-} \rightarrow \bar{D}^{0} K_{0}^{*-}\right) \sim 4 \times 10^{-6} . \tag{4}
\end{equation*}
$$

Since the strong phase can not be computed at present, we take several benchmark values to illustrate the dependence of $R_{C P+}^{K_{0}^{*}}$ and $A_{C P+}^{K_{0}^{*}}$ in Fig. 2, In panels (a,b), $r_{B}^{K_{0}^{*}}=2$ is employed, and in panels (c,d) $r_{B}^{K_{0}^{*}}=1$. In the last two panels (e,f), we consider the case in which the ratio is not enhanced too much $r_{B}^{K_{0}^{*}}=0.3$. The solid (green), dashed (black), dotted (blue) and dotdashed (orange) lines in diagrams (a,c,e) are obtained with $\delta_{B}^{K_{0}^{*}}=(30,60,120,150)^{\circ}$ respectively, while the corresponding lines in diagrams (b,d,f) correspond to $\delta_{B}^{K_{0}^{*}}=(30,60,-30,-60)^{\circ}$. The shadowed (light-green) region denotes the current bounds on $\gamma=\left(68_{-11}^{+10}\right)^{\circ}$ from a combined analysis of $B^{ \pm} \rightarrow D K^{ \pm}$[2], in which the vertical (red) line corresponds to the central value. The CP odd quantities can be obtained similarly, for instance $R_{C P-}^{K_{0}^{*}}=\left(R_{C P+}^{K_{0}^{*}}\right)_{\delta_{B}^{K_{0}^{*}} \rightarrow 180^{\circ}-\delta_{B}^{K_{0}^{*}}}$.

Turning to the $B^{ \pm} \rightarrow D K_{2}^{* \pm}$ mode in which the matrix element of the vector and the axial-vector current between the QCD vacuum and the $K_{2}^{*}$ state is zero, we find


FIG. 2: The dependence of $R_{C P+}^{K_{0}^{*}}$ and $A_{C P+}^{K_{0}^{*}}$ on $\gamma$. In panels (a,b), $r_{B}^{K_{0}^{*}}=2$ is employed, in panels (c,d) $r_{B}^{K_{0}^{*}}=1$ and in panels (e,f) $r_{B}^{K_{0}^{*}}=0.3$. The solid (green), dashed (black), dotted (blue) and dot-dashed (orange) lines in diagrams (a,c,e) correspond to $\delta_{B}^{K_{0}^{*}}=(30,60,120,150)^{\circ}$ respectively, while the corresponding lines in diagrams (b,d,f) correspond to $\delta_{B}^{K_{0}^{*}}=(30,60,-30,-60)^{\circ}$. The shadowed (lightgreen) region denotes the current bounds on $\gamma=\left(68_{-11}^{+10}\right)^{\circ}$ from a combined analysis of $B^{ \pm} \rightarrow D K^{ \pm}[2]$, in which the vertical (red) line corresponds to the central value.
a vanishing color-allowed amplitude $T$. Accordingly, the ratio $r_{B}^{K_{2}^{*}}$ is from the product of CKM matrix elements which is roughly 0.5 . An estimate of the branching ratios can be made by using the data on the $B \rightarrow J / \psi K_{2}^{*}$

$$
\frac{\mathcal{B}\left(B^{-} \rightarrow D^{0} K_{2}^{*-}\right)}{\mathcal{B}\left(B \rightarrow J / \psi K_{2}^{* 0}\right)} \simeq x_{K_{2}^{*}}\left|\frac{V_{c b} V_{u s}^{*}}{V_{c b} V_{c s}^{*}} \frac{f_{D}}{f_{J / \psi}}\right|^{2} \sim 0.8 \%,(5)
$$

with $x_{K_{2}^{*}}$ being the ratio of the form factor products which is evaluated from a recent calculation of $B \rightarrow K_{2}^{*}$ form factors [14]: $x_{K_{2}^{*}} \simeq 0.5$. The branching ratio $\mathcal{B}\left(B \rightarrow J / \psi K_{2}^{* 0}\right)=(4.0 \pm 2.4) \times 10^{-4}$ 15] extracted from the data on $B^{-} \rightarrow J / \psi K^{-} \pi^{+} \pi^{-}$[16] gives

$$
\begin{equation*}
\mathcal{B}\left(B^{-} \rightarrow D^{0} K_{2}^{*-}\right) \simeq 3 \times 10^{-6} \tag{6}
\end{equation*}
$$

The method to use the two triangles formed by the six decay amplitudes for determining $\gamma$ is also valid in the neutral $B_{d}$ decays into $D K_{0,2}^{* 0}$, in which the tree amplitude $T$ is identically zero. The $K_{0,2}^{*}$ is self-tagging, thus no time-dependent measurement is required. Since the amplitudes involving $D^{0}$ and $\bar{D}^{0}$ arise from the same type of diagram, one expects that $\delta_{B}^{K_{0}^{*}} \sim 0$. If true, the CP asymmetries $A_{C P \pm}^{K_{0}^{*}}$ would be still close to 0 but $R_{C P \pm}^{K_{0}^{*}}$ can largely deviate from 1 .

The long-distance contributions in the form of final state interactions (FSI) might change the factorization analysis in at least two aspects. First, FSI can give nontrivial strong phases to $C$ and $T$ which are zero in the factorization approach. Second, FSI might also modify the size of the amplitudes and the $r_{B}^{K_{0,2}^{*}}$. Despite these changes, no hadronic uncertainties will be introduced as the CKM matrix elements in the final state interactions are the same as the ones in Eq. (21). To account for such effects, we also show in Fig. 2 the dependence of $R_{C P+}^{K_{0}^{*}}$ and $A_{C P+}^{K_{0}^{*}}$ on $\gamma$ with different ratios of amplitudes: $r_{B}^{K_{0}^{*}}=1$ and $r_{B}^{K_{0}^{*}}=0.3$. The latter corresponds to the sign of Wilson coefficient $a_{2}$ reversed namely $a_{2}=-0.2$. In this case, despite a small ratio $r_{B}^{K_{0}^{*}}=0.3$ the branching fractions $\mathcal{B}\left(B^{-} \rightarrow D^{0} K_{0,2}^{*-}\right)$ can reach $10^{-5}$.
The $D_{C P}^{0}$ meson in the final state can be reconstructed in the CP eigenstates, including the modes $\pi^{0} K_{S}, \pi^{+} \pi^{-} K_{S}, K^{+} K^{-}, \pi^{+} \pi^{-}$. These modes have quite large BRs, for instance, $\mathcal{B}\left(D^{0} \rightarrow \pi^{+} \pi^{-} K_{S}\right) \simeq 3 \%$ [10]. The $K_{0,2}^{*}$ have significant decay rates into $K \pi$, with $\mathcal{B}\left(K_{0}^{*} \rightarrow K \pi\right)=(93 \pm 10) \%$ and $\mathcal{B}\left(K_{2}^{*} \rightarrow K \pi\right)=$ $(49.9 \pm 1.2) \%$, and the final mesons are also easy to detect in experiments at hadron colliders. Moreover, since the CKM matrix elements for the $K_{0}^{*}$ and $K_{2}^{*}$ are the same, no knowledge of the resonance structure in this method is required and therefore the angle $\gamma$ can be extracted without any hadronic uncertainty. Compared with the BR of $B^{-} \rightarrow \bar{D}^{0} K^{-}$, of order $10^{-6}$, which is an unavoidable entry in the currently-adopted methods to determine $\gamma$, the summed BRs for the channels involving $K_{0}^{*}$ and $K_{2}^{*}$, of order $10^{-5}$, are comparable or even larger, and hence their measurements will not be statistically limited. The large amount of data accumulated by LHCb recently and in future will lead to a promising prospect of the proposed method.

In the above discussion, we have neglected effects caused by the CP violation in $D$ decays which is anticipated to be small in the standard model. Based on the $0.62 \mathrm{fb}^{-1}$ of data collected in 2011, the LHCb collaboration [17] has measured the difference between CP asymmetries in singly Cabibbo-suppressed decays $D^{0} \rightarrow K^{+} K^{-}$and $D^{0} \rightarrow \pi^{+} \pi^{-}, \Delta A_{C P} \equiv A_{C P}\left(D^{0} \rightarrow\right.$ $\left.K^{+} K^{-}\right)-A_{C P}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)$, given by

$$
\begin{equation*}
\Delta A_{C P}=(-0.82 \pm 0.21(\text { stat. }) \pm 0.11(\text { sys. })) \% \tag{7}
\end{equation*}
$$

where the first uncertainty is statistical and the second is systematic. Together with the CDF results [18] and previous world average from Heavy Flavor Averaging Group [19], the new world average for $\Delta A_{C P}$ is found to be [20]

$$
\begin{equation*}
\Delta A_{C P}=-(0.645 \pm 0.180) \% \tag{8}
\end{equation*}
$$

Although the new world-averaged $\Delta A_{C P}$ is about $3.6 \sigma$ away from zero, its magnitude is smaller than 1 percent. As a consequence, the CP violation effects in charm decays are less important in our method to determine $\gamma$, especially when compared to large uncertainties in current
knowledge of $\gamma$. Moreover, since the direct CP violation in $D^{0} \rightarrow K^{+} K^{-}$and $D^{0} \rightarrow \pi^{+} \pi^{-}$modes is expected to have opposite signs, part of the CP violation effects will cancel when both decay modes are used in the reconstruction of $D$ meson.
Now we turn to the $B_{s} \rightarrow D M$ decays, whose Feynman diagrams are depicted in Fig. 1 with $\bar{q}=\bar{s}$. It is proposed in Ref. [5, 21] that the time-dependent CP asymmetries in $B_{s} \rightarrow D \phi$ can be used to extract $\gamma$ and this method is applied to a pure annihilation mode $B_{s} \rightarrow D^{ \pm} \pi^{\mp}$ in Ref. [22] and modes like $B_{s} \rightarrow D \eta\left(\eta^{\prime}\right)$ in Ref. [23]. In the example of $B_{s} \rightarrow D f_{0}$, there are four decay modes having the amplitudes

$$
\begin{aligned}
& A\left(\bar{B}_{s} \rightarrow \bar{D}^{0} f_{0}\right)=V_{u b} V_{c s}^{*} A_{1}, A\left(B_{s} \rightarrow D^{0} f_{0}\right)=V_{u b}^{*} V_{c s} A_{1} \\
& A\left(\bar{B}_{s} \rightarrow D^{0} f_{0}\right)=V_{c b} V_{u s}^{*} A_{2}, A\left(B_{s} \rightarrow \bar{D}^{0} f_{0}\right)=V_{c b}^{*} V_{u s} A_{2} .(9)
\end{aligned}
$$

For each amplitude, there is only one weak phase in the SM, and therefore no direct CP asymmetry is expected. Any nonzero value from the experiment would be a signal for new physics. We define the relative size and strong phase of the two amplitudes as

$$
\begin{equation*}
r_{B_{s}}^{f_{0}}=\left|V_{u b} V_{c s}^{*} A_{1} /\left(V_{c b} V_{u s}^{*} A_{2}\right)\right|, \quad \delta_{B_{s}}^{f_{0}}=\arg \left(A_{1} / A_{2}\right) \tag{10}
\end{equation*}
$$

Since both $A_{1}$ and $A_{2}$ are from the same Feynman diagrams, it is likely that $A_{1} \simeq A_{2}$, which implies $r_{B_{s}}^{f_{0}} \sim 0.5$ and $\delta_{B_{s}}^{f_{0}} \sim 0$.

The neutral $B_{s}$ system is described by the mixing

$$
\left|B_{L}\right\rangle=p\left|B_{s}^{0}\right\rangle+q\left|\bar{B}_{s}^{0}\right\rangle,\left|B_{H}\right\rangle=p\left|B_{s}^{0}\right\rangle-q\left|\bar{B}_{s}^{0}\right\rangle,
$$

with $|p|^{2}+|q|^{2}=1$, and $q / p$ denotes the weak phase in the $B_{s}-\bar{B}_{s}$ mixing $q / p=V_{t b}^{*} V_{t s} /\left(V_{t b} V_{t s}^{*}\right)=e^{-2 i \beta_{s}}$. In the SM , this ratio is close to unity and the phase $\beta_{s}$ is negligibly small $\beta_{s} \simeq-0.019 \mathrm{rad}$. The normalized timedependent decay widths are [24, 25]:

$$
\begin{align*}
& \Gamma\left(\bar{B}_{s}^{0}(t) \rightarrow D^{0}\left(\bar{D}^{0}\right) f_{0}\right)=e^{-t / \tau_{B_{s}}}[1 \\
& \left.+\cos (\Delta m t) C_{D^{0}\left(\bar{D}^{0}\right) f_{0}}+\sin (\Delta m t) S_{D^{0}\left(\bar{D}^{0}\right) f_{0}}\right] \tag{11}
\end{align*}
$$

where $\bar{\Gamma}$ is the averaged decay width. For the corresponding $B_{s}^{0}$ decays, the plus signs in front of cosine and sine terms should be replaced by minus signs. Substituting the amplitudes defined in Eq. (9), we have

$$
\begin{array}{r}
C_{D^{0} f_{0}}=C_{\bar{D}^{0} f_{0}}=\left[1-\left(r_{B_{s}}^{f_{0}}\right)^{2}\right] /\left[1+\left(r_{B_{s}}^{f_{0}}\right)^{2}\right] \\
S_{D^{0} f_{0}}=-2 r_{B_{s}}^{f_{0}} \sin \left(\gamma+\delta_{B_{s}}^{f_{0}}+2 \beta_{s}\right) /\left[1+\left(r_{B_{s}}^{f_{0}}\right)^{2}\right] \\
S_{\bar{D}^{0} f_{0}}=-2 r_{B_{s}}^{f_{0}} \sin \left(-\gamma+\delta_{B_{s}}^{f_{0}}+2 \beta_{s}\right) /\left[1+\left(r_{B_{s}}^{f_{0}}\right)^{2}\right] . \tag{12}
\end{array}
$$

The equality $C_{D^{0} f_{0}}=C_{\bar{D}^{0} f_{0}}$ is a consequence of the uniqueness of the weak phase in decay amplitudes. Since both the strong phase difference $\delta_{B_{s}}^{f_{0}}$ and the $B_{s}-\bar{B}_{s}$ mixing phase are expected small, $S_{D^{0} f_{0}}$ and $S_{\bar{D}^{0} f_{0}}$ will have similar magnitudes but differ in sign.

The BRs of $\bar{B}_{s} \rightarrow D f_{0}\left(f_{2}^{\prime}\right)$ can be estimated by using the experimental data on $B_{s} \rightarrow J / \psi f_{0}\left(f_{2}^{\prime}\right)$ together with the ratio of the BRs

$$
\begin{aligned}
& \frac{\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{0} f_{0}\right)}{\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi f_{0}\right)} \simeq x_{B_{s}}^{f_{0}}\left|\frac{V_{c b} V_{u s}^{*}}{V_{c b} V_{c s}^{*}} \frac{f_{D}}{f_{J / \psi}}\right|^{2} \sim(1.3-1.5) \%, \\
& \frac{\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow D^{0} f_{2}^{\prime}\right)}{\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi f_{2}^{\prime}\right)} \simeq x_{B_{s}}^{f_{2}^{\prime}}\left|\frac{V_{c b} V_{u s}^{*}}{V_{c b} V_{c s}^{*}} \frac{f_{D}}{f_{J / \psi}}\right|^{2} \sim 0.8 \%
\end{aligned}
$$

with the product of the form factors $x_{B_{s}}^{f_{0}}=(0.8-1.0)[12$, [26] and $x_{B_{s}}^{f_{2}^{\prime}}=0.50$ [14]. A recent measurement [27]

$$
\begin{aligned}
& \mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi f_{0}\right) \sim \mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi f_{2}^{\prime}\right) \\
& \quad \sim 0.2 \mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow J / \psi \phi\right) \sim 2 \times 10^{-4}
\end{aligned}
$$

shows that the $B_{s} \rightarrow D^{0} f_{0}\left(f_{2}\right)$ decays have a BR of order $10^{-6}$. In this estimate the decays $f_{0}$ into $\pi^{+} \pi^{-}$and $f_{2}^{\prime}$ into $K^{+} K^{-}$have been taken into account.
It is straightforward to incorporate the $B_{s}$ decays into other light p-wave mesons, like $f_{0}(1370)$, $h_{1}(1170), h_{1}(1380), f_{1}(1285)$ and $f_{1}(1420)$. But they require high statistics to have an impact on $\gamma$, due to either the suppressed production rates in $B_{s}$ decays [28] or the difficulty in the reconstruction of the decay modes [10].

Finally, we remark on the BR estimate, which is obtained under the factorization approach in conjunction with the experimental data. The validity of this method can be tested by considering the ratios in the processes $\bar{B}^{0} \rightarrow\left(D^{0}, J / \psi\right)\left(\bar{K}^{0}, \bar{K}^{* 0}\right):$

$$
\begin{gathered}
y_{K} \equiv \mathcal{B}\left(\bar{B}^{0} \rightarrow D^{0} \bar{K}^{0}\right) / \mathcal{B}\left(\bar{B}^{0} \rightarrow J / \psi \bar{K}^{0}\right) \sim 1.4 \% \\
y_{K^{*}} \equiv \mathcal{B}\left(\bar{B}^{0} \rightarrow D^{0} \bar{K}^{* 0}\right) / \mathcal{B}\left(\bar{B}^{0} \rightarrow J / \psi \bar{K}^{* 0}\right) \sim 0.5 \%,(13)
\end{gathered}
$$

where the form factor products are used from Ref. [13]. Compared with the data $y_{K} \sim 6.0 \%$ and $y_{K^{*}} \simeq$ $3.2 \%$ [10], these ratios are theoretically undershot. If it is the same in $B / B_{s}$ decays into $K_{0,2}^{*} /\left(f_{0}, f_{2}^{\prime}\right)$, the estimated BRs will be enhanced roughly by a factor of (4-6), which makes the proposed method more appealing.
In summary, we have explored the possibility to extract the CP violation angle $\gamma$ with $B \rightarrow$ $\left(D^{0}, \bar{D}^{0}, D_{C P}^{0}\right) K_{0(2)}^{*}(1430)$ and $B_{s} \rightarrow\left(D^{0}, \bar{D}^{0}\right) M(M=$ $\left.f_{0}(980), f_{0}(1370), f_{2}^{\prime}(1525), f_{1}(1285), f_{1}(1420), h_{1}(1180)\right)$. A clean method is to use the two triangles formed by the decay amplitudes of $B^{ \pm} \rightarrow\left(D^{0}, \bar{D}^{0}, D_{C P}^{0}\right) K_{0(2)}^{* \pm}(1430)$. We expect that $B^{ \pm} \quad \rightarrow \quad D^{0} K_{0(2)}^{* \pm}(1430)$ and $B^{ \pm} \rightarrow \bar{D}^{0} K_{0(2)}^{* \pm}(1430)$ have similar decay rates and the CP asymmetries have a strong correlation with $\gamma$. Our method does not require the separation of the Cabibbo-suppressed $D$ decays, which are usually buried under the combinatorial background. With the help of the factorization approach and the relevant experimental data we estimate the branching ratios of these modes to be of order $10^{-5}-10^{-6}$. Measurements of the branching fractions of $B_{d} \rightarrow D K_{0(2)}^{*}(1430)$ and time-dependent CP asymmetries in $B_{s} \rightarrow D M$ provide
an alternative way to extract the angle $\gamma$. No knowledge of the resonance structure in this method is required and therefore the angle $\gamma$ can be extracted without any hadronic uncertainty.

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