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Mathematical Optimization of a Magnetic Ruler Layout With Rotated Pole Boundaries

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Abstract Magnetic rulers for measuring systems are either based on incremental or absolute measuring methods. Incremental methods need to initialize a measurement cycle at a reference point. From there, the position is determined by counting increments of a periodic graduation. Absolute methods do not need reference points, since the position can be read directly from the ruler. In the state of the art approach the absolute position on the ruler is encoded using two tracks with different graduation. To use only one track for position encoding in absolute measuring a pattern of trapezoidal magnetic areas is considered instead of the common rectangular ones. We present a mixed integer programming model for an optimal placement of the trapezoidal magnetic areas to obtain the longest possible ruler under constraints conditioned by production techniques, physical limits as well as mathematical approximation of the magnetic field.

1 Introduction

As a matter of fact magnetic rulers with incremental position measurement, see Fig. 1, are easy to produce. They consist of equidistant pole rectangles and the position is determined via counting the switches between the north and south pole. The demand of the market focuses however on absolute position measurement systems. In the state of the art approach the production of such magnetic rulers is cost-intensive mainly due to the need of the two magnetic tracks, see Fig. 2. The one track consists of equidistant pole rectangles. The pole rectangles in the second track

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are multiples of one rectangle from the first track. In this way the magnetic ruler can be divided into rectangular regions, each one covering a unique pattern and thus encoding a unique position [2, 3, 4, 5]. The size of such a rectangular region depends on the size of the magnetic reading head.

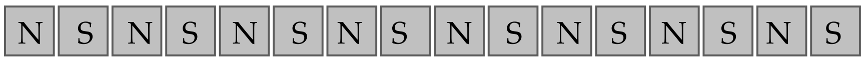

Fig. 1 A magnetic ruler for the incremental position measurement.

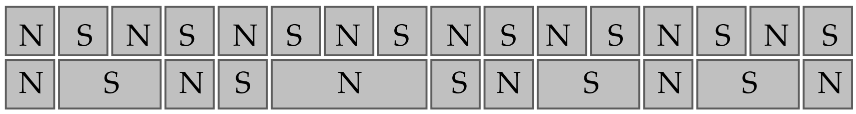

Fig. 2 A magnetic ruler with two tracks for the absolute position measurement.

Fig. 3 A magnetic ruler with rotated pole boundaries.

An approach to reduce the production costs is to elaborate such unique patterns using only one magnetic stripe for the magnetic ruler. This can be achieved, e. g. by the rotation of pole boundaries, see Fig. 3. In this way we obtain a magnetic ruler covered by trapezoid shaped poles. The position is encoded via the signal sequence of the magnetic field, see Fig. 4. The different lengths of the upper and lower sides of the trapezoids consecutively placed give rise to the irregularity of the signal curves providing the uniqueness of the encoded values. However, the longer the magnetic ruler the more probable a repetition of a signal value is.

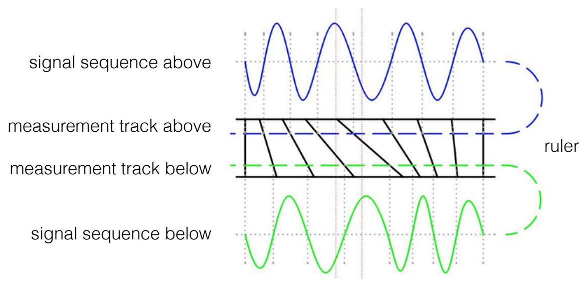

Fig. 4 The signal sequence on a magnetic ruler with rotated pole boundaries.

The fundamental task is to find an appropriate placement of the trapezoids supporting a unique signal sequence on an as long as possible magnetic ruler. The placement underlies further conditions due to production techniques, physical limits as well as mathematical approximation of the magnetic field.

In the approach presented in this paper we aim to construct a magnetic ruler with rotated pole boundaries of the maximal length provided only a few production restriction explained in the next section.

2 Mathematical Optimization Model

To obtain a mathematical model we consider the trapezoids as segments, which are uniquely identified via the lengths of its parallel sides. Having the technical specifications as the minimal, l_{min}, as well as the maximal, l_{max}, length of a parallel side as well as the minimum difference between two adjoining sides s, a list of all possible trapezoid side lengths can be determined a priori:

$$L = \{L_1, \ldots, L_n\}, \quad L_1 = l_{min}, \quad \forall_{1 \le i \le n-1}\, L_i = L_1 + is, \quad L_n = l_{max},$$

where

$$n = \left\lfloor \frac{l_{max} - l_{min}}{s} \right\rfloor + 1.$$

We index the list with the set $I = \{1, \ldots, n\}$. Each item on the list means a length of a parallel trapezoid's side and can be used for the upper as well as the lower side. We obtain the maximum number of $|I \times I| = n^2$ segments, which can be arranged on one magnetic ruler. One obtains the index set $P = \{1, \ldots, n^2\}$ of positions on magnetic ruler at which a segment can be set. Next, we introduce binary variables $x^p_{i,j}$, for all $i, j \in I$ and $p \in P$. If $x^p_{i,j}$ equals 1 a trapezoid with the upper parallel side L_i and the lower parallel side L_j is placed on position p on the magnetic ruler. Furthermore, we introduce the binary variables $y_{i,j}$ for all $i, j \in I$. A $y_{i,j}$ is set to 1, if the corresponding trapezoid is placed on the magnetic ruler at all.

We seek for a magnetic ruler of a maximal length,

$$\max \sum_{i,j \in I} (L_i + L_j) y_{i,j},$$

provided that each trapezoid is assigned to at most one position,

$$\sum_{i,j \in I} x^p_{i,j} \le 1, \qquad \forall p \in P,$$

can be used at most once,

$$\sum_{p \in P} x^p_{i,j} = y_{i,j}, \qquad \forall i, j \in I,$$

and the segments should be placed continuously - without gaps - on the magnetic ruler,

$$\sum_{i,j \in I} x^p_{i,j} \le \sum_{i,j \in I} x^{p-1}_{i,j}, \qquad \forall p \in P \backslash \{1\}.$$

Finally, we formulate the following technical restriction. To magnetize the ruler with rotated pole boundaries, the writing head has to be rotated, but not *too strongly*. The parameter ε determines the maximal possible rotation of the pole boundary. It corresponds to the maximal difference of the cumulated lengths obtained by summing up the upper and the lower parallel sides, respectively, of trapezoids placed upto a given position. We introduce the real variables $\ell_s^p \in \mathbb{R}_+$. Using s we index the sides, $s = 1$ for upper and $s = 2$ for the lower one, and $p \in P$ indicates the position up to which the summation is performed. Thus we obtain the cumulated length on the upper side,

$$\begin{aligned}
\ell_1^1 &= \sum_{i,j \in I} L_i x_{i,j}^1, \\
\ell_1^p &= \ell_1^{p-1} + \sum_{i,j \in I} L_i x_{i,j}^p, \qquad \forall\, p \in P \backslash \{1\},
\end{aligned}$$

and the cumulated length on the lower side,

$$\begin{aligned}
\ell_{2,}^1 &= \sum_{i,j \in I} L_j x_{i,j}^p, \\
\ell_2^p &= \ell_2^{p-1} + \sum_{i,j \in I} L_j x_{i,j}^p, \qquad \forall\, p \in P \backslash \{1\}.
\end{aligned}$$

Having that, we can formulate the condition on the difference of both upper and lower total lengths,

$$|\ell_{1,p}^p - \ell_2^p| \leq \varepsilon, \qquad \forall\, p \in P.$$

2.1 Preprocessing

The parameter ε gives rise to a sort of a preprocessing condition. If the difference of the upper and lower side of a trapezoid exceeds the value 2ε, it can be removed from considerations. Thus for all $i, j \in I$ with $|L_i - L_j| > 2\varepsilon$ we can immediately set $y_{i,j} = 0$.

3 Computational Results

We apply IBM ILOG CPLEX [1] to solve the model presented above. The computations were performed on a MacOS 10.10 with 1.7 GHz Intel Core i7 processor and 8 GB 1600 MHz DDR3 memory. We summarize the computational results in Table 1. The parameter l_{min}, the minimum length of a trapezoid's side, is generally set to 2 mm, as too small trapezoids would not be recognized by the reading magnetic head. The parameter l_{max}, the maximum length of a trapezoid's side, should not be too big, otherwise breaks in the signal sequence may occur. Small values of the step size s, the minimum difference between two adjoining sides, lead to a bigger number of different trapezoids, but then the uniqueness of the encoded signal is vulnerable as the trapezoids are almost equal. The value ε as already mentioned is specified by the production terms.

Table 1 The optimal length of magnetic rulers computed due to given production parameters.

#	l_{min} (mm)	l_{max} (mm)	s (mm)	ε (mm)	Length of an optimal magnetic ruler (mm)	CPU time without preprocessing (s)	gap (%)	CPU time with preprocessing (s)
1.	2.0	4.0	0.5	0.5	57.0	1.04		0.17
2.	2.0	4.0	0.5	1.0	75.0	0.40		0.36
3.	2.0	6.5	0.5	0.5	187.0	267.15		0.88
4.	2.0	6.5	0.5	1.0	297.5	1965.47		2.85
5.	2.0	8.0	0.5	2.0	745.0	*3600.00	18	11.09
6.	2.0	8.0	0.5	3.0	845.0	32.12		31.35
7.	2.0	8.0	0.4	2.0	1130.0	*3600.00	30	56.66
8.	2.0	12.0	0.5	2.0	1809.0	*3600.00	575	1234.79
9.	2.0	12.0	0.4	2.0	2819.0	*3600.00	∞	3041.26

$*$ The computation was interrupted after this time, the optimality gap is given in the next column.

Comparing the computation times with and without the preprocessing the advantage of the preprocessing constraint is obvious. In a very short time we obtain layouts of magnetic rulers with lengths of industrial use. As an example we plot in Fig. 5 the magnetic ruler computed due to parameters given in the first line of Table 1. Additionally, by reflecting the ruler on the upright left side and switching the sensitivity of the start pole one obtains the contrary signal sequence and thus a unique signal sequence on a ruler of a double length.

4 Conclusions

After the production of the magnetic ruler layouts we computed by our industrial collaboration partner it turned out that it is preferred to arrange the trapezoids in a gradually growing way with respect to their lengths. Furthermore, it is desired to have the largest parameter for the stepsize when the length of the ruler is fixed. This cannot be formulated directly as a mixed-integer program, because this requires that the stepsize is a real value. Hence we use our approach with a fixed stepsize, and decrease the stepsize in each consecutive run of the model, until the desired length of the ruler is met.

In the approach presented in this paper a trapezoid with the same upper and lower length can be placed at most once and cannot repeat. This assures the uniqueness of the signal sequence in the upper and lower part of the trapezoids. Since the width of the reading head may exceed the width of a single trapezoid, one may consider to

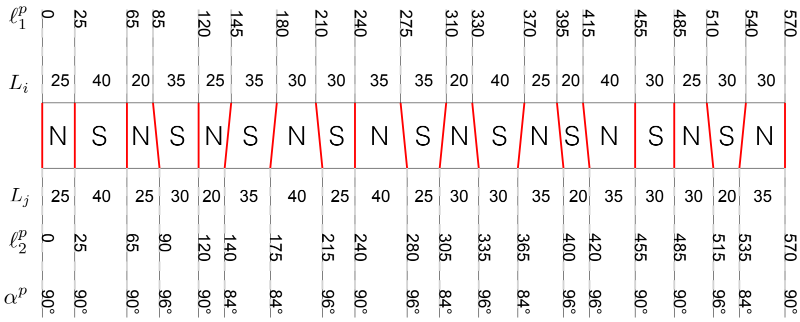

Fig. 5 An optimal magnetic ruler for the parallel side lengths $L := \{20, 25, 30, 35, 40\}$. The unit of the numbers in the figure is $1 \widehat{=} 0.1\,mm$. The numbers in the first line from above are the cumulated upper sides of trapezoids up to a given position. The numbers directly above and directly below a trapezoid are the lengths of its upper and lower side, respectively. In the next line the cumulated lower sides of trapezoids up to a given position are listed. The last line below indicates the measure of the inner left bottom angle of each trapezoid.

cluster two or three adjacent trapezoids, which then cannot repeat on the magnetic ruler. The reformulation of the model in this respect is our ongoing research. We expect to obtain longer rulers with a higher accuracy.

The mathematical optimization model formulated in this paper is a basis to expand, when further production or physical limitations as well as the mathematical approximation by decoding the signal come into play.

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