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M. Fawad Zazai, Armin Fügenschuh

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Herausgegeben von der
Professur für Angewandte Mathematik
Professor Dr. rer. nat. Armin Fügenschuh

Helmut-Schmidt-Universität / Universität der Bundeswehr Hamburg
Fachbereich Maschinenbau
Holstenhofweg 85
D-22043 Hamburg

Telefon: +49 (0)40 6541 3540
Fax: +49 (0)40 6541 3672

e-mail: appliedmath@hsu-hh.de
URL: http://www.hsu-hh.de/am

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Computing Pareto-Optimal Transit Routes Through Mathematical Algorithms

M. Fawad Zazai and Armin Fügenschuh

Helmut Schmidt University/University of the Federal Armed Forces, Hamburg, Germany
{zazaif,fuegenschuh}@hsu-hh.de

Abstract. Afghanistan is geo-strategically in an important transit zone in South and Central Asia, but currently lacks of modern infrastructure. We present the construction of optimal transit routes in Afghanistan through mathematical optimization. Basically there are three different optimization goals a) the shortest route w.r.t. the distance, b) the cheapest route w.r.t. the construction cost, and c) the most convenient route w.r.t. the elevation change. It is possible to combine two objectives by considering the Pareto front. For the design and modeling of the routes, a computer program named "Contra" (**C**omputing an **O**ptimal **N**etwork of **T**ransit **R**outes through mathematical **A**lgorithms) was developed. As a demonstrator example, we compute Pareto-optimal routes between two Afghan cities.

Keywords: Afghanistan, transit routes, shortest path problem, graph theory, Dijkstra's algorithm, computational geometry.

1 Aim and Idea

Afghanistan is located geographically in the center of Asia. The country has great potential to transform in South and Central Asia into a "logistical crossroad". The aim of this study is to develop trajectories for optimal transit routes in Afghanistan by mathematical optimization methods. In the present research phase, the focus is to apply algorithms for the shortest path problem, which compute point-to-point connections between two cities. The shortest path problem belongs to the class of graph problem and deals with the issue of how to find an optimal route between two nodes or points (start and end point) within a graph $G = (V, E, w)$ with respect to a cost function that is the sum of non-negative weights $w_{i,j}$ of each edge $\{i, j\} \in E$ that is used in the route. The edge weight can represent a) its length, b) its construction cost, or c) the height variation. In order to estimate the construction cost of an edge, that may become part of a route, several factors are taken into account, in particular the national land use of Afghanistan and the elevation profile of the terrain (topography). These data are taken from publicly available sources. For the design and modeling of the routes, a computer program named "Contra" (**C**omputing an **O**ptimal **N**etwork of **T**ransit **R**outes through mathematical **A**lgorithms) was developed.

Contra transforms the input data (land use, terrain) into a weighted graph and applies Dijkstra's shortest path algorithm [1] to find an optimal routes between any two given nodes. Details of this are given in the next section.

2 Input Data

To determine a route automatically with the help of mathematical optimization, the following input data has to be provided to the program Contra:

1. Coordinates of start and destination points (given as latitude and longitude).
2. National land cover of Afghanistan in shapefiles (currently from date 1997) of the organization AIMS, originally of the Afghan Geodesy and Cartography Head Office [2]. The whole country is separated in polygonal shapes that describe the respective type of land, see Fig. 1.
3. Topographic representation of Afghanistan, the SRTM data of USGS / NASA [3]. The resolution of the SRTM data for Afghanistan is 14000×18001 pixels. As an example, in Fig.2 the area of the Uruzgan province in Afghanistan is depicted.
4. The costs of construction and maintenance of a road. According to reports of the Asian Development Bank the construction of a (two-lane) road in Afghanistan on average is approximately 1 Mio. USD per km[4]. The cost amount can be dependent on the height and the land surface on which it is built [5]. For this research the assumption is an estimated cost ratio among the different land covers, and thus determine the construction costs of the routes. In general, it is possible to modify the estimated cost ratio.

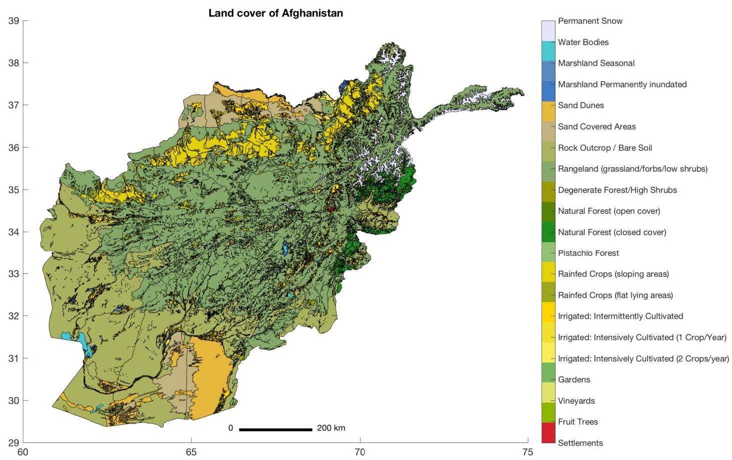

Fig. 1. A map of Afghanistan with different land cover. Each color represents a specific land cover (e.g., deserts or forests).

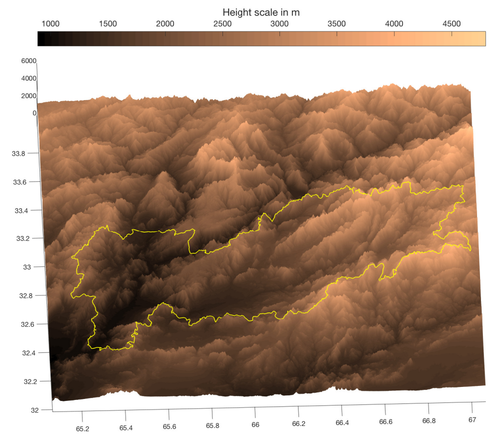

Fig. 2. The topography of the Uruzgan province in Afghanistan. The color spectrum from dark to light indicates the height of the terrain.

3 Creating a Graph and a Shortest-Path-Problem

A weighted graph $G = (V, E, w)$ is created as follows. A regular mesh grid Γ is spanned over the terrain of a selected geographical area A, so that $\Gamma(A)$ defines a geographical mesh grid of A. A grid consists of rectangles, which has corner points in $\mathbb{R}^3$. These points are added as nodes to the set V. The four side lines of each grid rectangle is equidistantly subdivided into $m \in \mathbb{N}$ segments. The end points of these segments are also added as nodes to V. Note that each grid rectangle has $4m$ associated nodes (see Fig. 3). All pairs of these nodes (but excluding those on the same side of the rectangle's boundary) are now connected by edges and added to the edge set E of G, which gives $6m^2$ edges for each rectangle of the grid. The problem of the construction of optimal routes in Afghanistan leads to the shortest path problem. A shortest path is a route that is minimal with respect to the sum of all costs of all segments that are used in the entire route. The non-negative cost per segment (edge weight) $w_{i,j}$ are determined from the construction cost, the length or the height variation (depending on the desired goal of the optimization). To solve the shortest path problem Dijkstra's algorithm [1] is used. The number of rectangular subdivisions of the grid Γ as well as the value of m are set by the user. Clearly, the finer the resolution, the more properly the route can follow the topography of the area. With modern computers, the solution time is not so much a bottleneck, however, the memory consumption is very high and can easily touch the limits also of modern workstations (64 GByte), even when special programming techniques (such as sparse data structures for storing all edges) were applied. Besides focusing on a single optimization goal, there are several conflicting objectives to consider. This could be, for instance, minimizing the total length of the route as well as the construction cost. These two can be in conflict, because a shorter route may go through more difficult

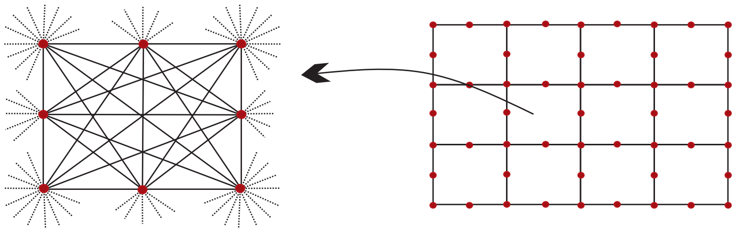

Fig. 3. A single rectangle with $m = 2$ subdivision, hence $4m = 8$ nodes and $6m^2 = 24$ connecting edges. The edges represent the basic building segments of a route in CONTRA.

terrain that a slightly longer route would have avoided, and thus turns out to be lest costly. In general, we consider three objectives "route length", "construction cost" and "elevation variation" that are in conflict with each other. That means, there is no route that is simultaneously optimal for all three. Multi-objective optimization (Pareto optimization) is an area of multi-criteria decision making, that is concerned with mathematical optimization problems involving more than one objective function, that have to be optimized simultaneously [6]. Here one seeks for a Pareto optimum of a route, which is a route that cannot be improved with respect to one criterion without worsening at least another. Using this concept, one can analyze the trade-off between them.

4 Results

As an example, we compute routes between the city of *Khas Uruzgan* and the city of *Kabul*. We first calculate optimal solutions for the three different single objectives, e.g., the shortest route (red), the cost-minimal or cheapest route (blue), and the most convenient route w.r.t. the elevation change (black), see Fig. 4. The columns in Table 1 describe a) the construction costs of the routes in million USD, b) the lengths of the routes in km and c) the absolute elevation changes of the routes, i.e., the sum of all height changes from the starting point to the ending point along the route. Each of these three routes has a certain length and elevation profile. Fig. 5 shows the height profile along these three routes and Fig. 6 shows the Pareto front of the routes from Khas Uruzgan to Kabul regarding the two objectives "route length" and "construction cost". This chart

Route	Cost in million USD	Length in km	Absolute elevation change in m
red	312	**310**	18,570
blue	**291**	318	20,652
black	426	388	**10,494**

Table 1. The result of the three routes regarding the cost, length and absolute elevation change.

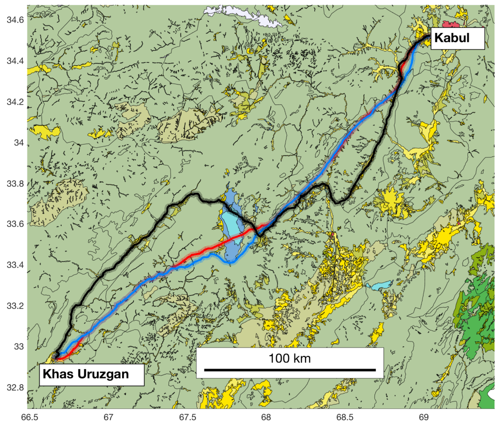

Fig. 4. Three single-objective optimal routes: shortest (red), cheapest (blue), and most convenient (black).

shows on the horizontal axis the length of the routes in km and on the vertical axis the construction cost of the routes in million USD. The small red circles represent specific routes between these two cities. The leftmost circle represents the shortest route and the rightmost circle represents the cheapest route. The circles that lie between these two extremal circles, are other optimal routes that are a combination of the shortest and cheapest route from Khas Uruzgan to Kabul. As the project acronym CONTRA indicates, our future work is to extend these point-to-point connections to automatically design large *networks* that connect several cities with optimally located transit routes.

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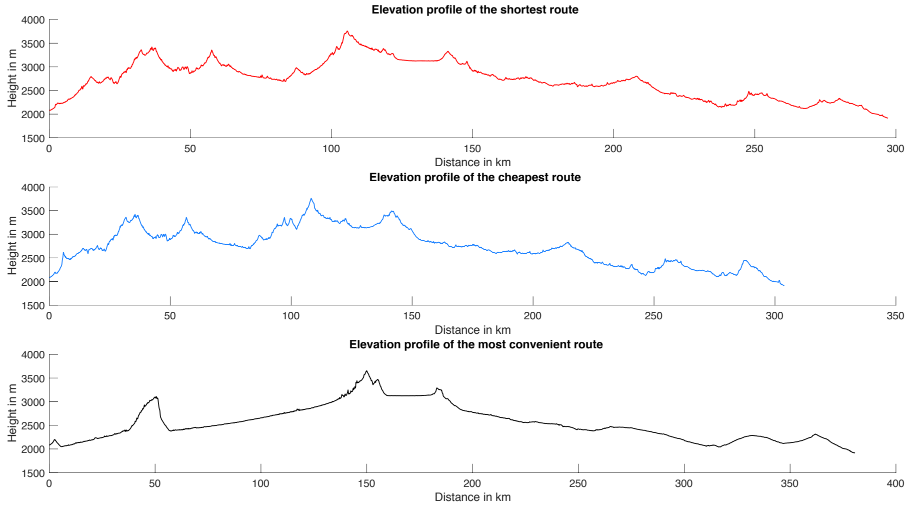

Fig. 5. The height and length diagram of the shortest, the cheapest and the most convenient w.r.t. the elevation route. The line in the middle of each chart is the average elevation.

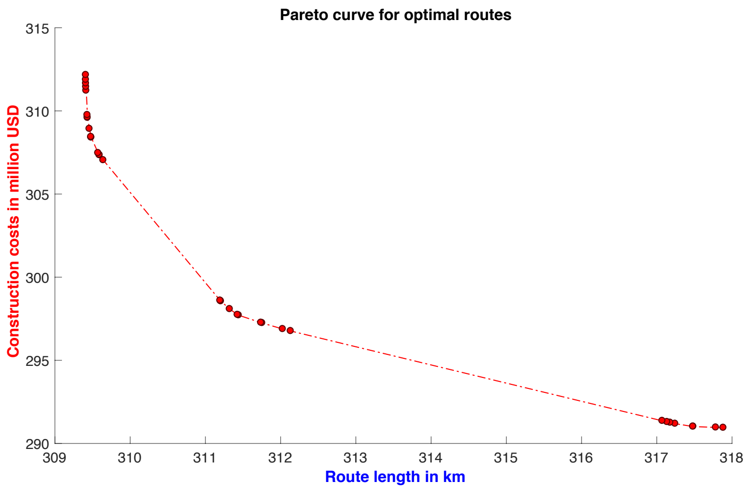

Fig. 6. The Pareto front of the shortest and cheapest routes regarding length and construction cost from Khas Uruzgan to Kabul. The shortest route costs 312.5 Mio. USD and has a length of 309.5 km, whereas the cost for a slightly longer route of 317 km already drops to 291 Mio. USD, and even longer routes do not save much further cost anymore.