

Angewandte Mathematik und Optimierung Schriftenreihe  
Applied Mathematics and Optimization Series  
AMOS # 31(2015)

Armin Fügenschuh, Marzena Fügenschuh, Marina Ludszuweit,  
Aleksandar Mojsic, and Joanna Sokół

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Measuring Systems in Industrial Applications

Herausgegeben von der  
Professur für Angewandte Mathematik  
Professor Dr. rer. nat. Armin Fügenschuh

Helmut-Schmidt-Universität / Universität der Bundeswehr Hamburg  
Fachbereich Maschinenbau  
Holstenhofweg 85  
D-22043 Hamburg

Telefon: +49 (0)40 6541 3540  
Fax: +49 (0)40 6541 3672

e-mail: [appliedmath@hsu-hh.de](mailto:appliedmath@hsu-hh.de)  
URL: <http://www.hsu-hh.de/am>

Angewandte Mathematik und Optimierung Schriftenreihe (AMOS), ISSN-Print 2199-1928  
Angewandte Mathematik und Optimierung Schriftenreihe (AMOS), ISSN-Internet 2199-1936

# Mathematical Model for Absolute Magnetic Measuring Systems in Industrial Applications

Armin Fügenschuh<sup>1</sup>, Marzena Fügenschuh<sup>2</sup>, Marina Ludszuweit<sup>1</sup>, Aleksandar Mojsic<sup>1</sup> and Joanna Sokół<sup>1</sup>

<sup>1</sup>Helmut Schmidt University University of the Federal Armed Forces Hamburg, Department of Mechanical Engineering, Holstenhofweg 85, 22043 Hamburg, Germany

<sup>2</sup>Beuth Hochschule für Technik Berlin, Luxemburger Straße 10, 13353 Berlin, Germany

E-mail: [marina.ludszuweit@hsu-hh.de](mailto:marina.ludszuweit@hsu-hh.de)

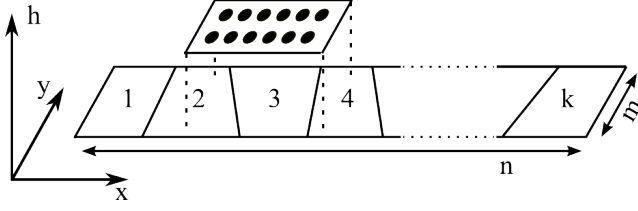
**Abstract.** Scales for measuring systems are either based on incremental or absolute measuring methods. Incremental scales need to initialize a measurement cycle at a reference point. From there, the position is computed by counting increments of a periodic graduation. Absolute methods do not need reference points, since the position can be read directly from the scale. The positions on the complete scales are encoded using two incremental tracks with different graduation. We present a new method for absolute measuring using only one track for position encoding up to micrometre range. Instead of the common perpendicular magnetic areas, we use a pattern of trapezoidal magnetic areas, to store more complex information. For positioning, we use the magnetic field where very position is characterized by a set of values measured by a hall sensor array. We implement a method for reconstruction of absolute positions from the set of unique measured values. We compare two patterns with respect to uniqueness, accuracy, stability and robustness of positioning. We discuss how stability and robustness are influenced by different errors during the measurement in real applications and how those errors can be compensated.

## 1. Introduction

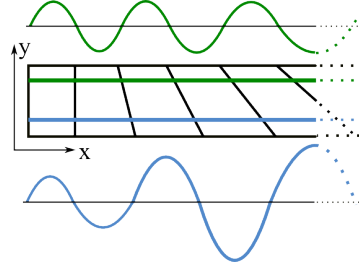
In many areas of industrial processing, length measurement systems are important. To determine the distance between two objects along a linear distance, these systems consist of a sensor device, a measurement scale and a processing unit. Possible applications are positioning control of the milling head in CNC machines or position control of industrial robots.

In length measurement two methods are common. In incremental measuring, the sensor reads increments of periodic graduation from the scale. The measured distance is computed by counting the read increments, so only relative distances are measurable. Absolute measuring methods allow the sensor to be placed at any point on a scale. The absolute position is computed directly from a uniquely encoded scale. Both measurement methods are used in several modalities of systems, e.g., magnetic and optical systems.

The unique encoding in absolute positioning is realized using properties such as patterns of light wavelength for optical scales or magnetic field information for magnetic scales [1, 2]. To ensure uniqueness more than one pattern track may be necessary. The different magnetic poles induce magnetic fields according to the pattern. This allows reading the information using hall sensor devices, which convert magnetic field information into signals. Measuring values with a



**Figure 1.** Illustration of the measurement setup.



**Figure 2.** Schematic of a pattern with related magnetic signals.

hall cell along a magnetic pattern results in a function of magnetic signals with zeros at the boundaries between two poles [3].

We present a method to be used in magnetic systems which requires only one magnetic track for unique encoding and resolution up to micrometer range. This saves overall production costs and allows for smaller, less energy consuming systems to be used in applications where size is important.

## 2. Experimental setup

We define a three-dimensional, right-handed coordinate system with the  $x$ -axis parallel and the  $y$ -axis orthogonal to the measuring direction of the magnetic scale and the  $h$ -axis pointing away from the scale. See also Figure 1.

The sensor containing the hall sensors is placed above the scale and moved in  $x$ -direction. For our experiments, we consider a hall cell array with a  $2 \times 6$  sensor placement (see Figure 1). The distance between two adjacent cells is 1.5mm in  $x$ - and 5mm in  $y$ -direction.

The measurement stripe with its scale is described as an area of width  $m \in \mathbb{R}^+$  and length  $n \in \mathbb{R}^+$  divided into  $k \in \mathbb{N}$  consecutive trapezoidal magnetic poles. The initial value  $n = 0$  is equal to the left pole boundary of the first magnet. Considering the distance between two hall sensors in  $y$ -direction,  $m \geq 5\text{mm}$  is required for our setup; usually 10mm in practice.

In order to encode for an absolute position, each measurement on the scale must be described by a set of unique values of orthogonal magnetic field components as measured by the hall cell array. Following, the magnetic field information from the hall sensor is denoted as magnetic signal. Our sensor is build to read two different tracks of magnetic signals at once. The measured values of magnetic signals are influenced by the length  $l_i$  of magnetic pole  $i$ ,  $i = 1, \dots, k$ . Constructing a magnetic pattern with different lengths between the pole boundaries on each track results in trapezoidal magnets along the scale. By  $\alpha_i \in (-90^\circ, 90^\circ)$ ,  $i = 1, \dots, k$ , we denote the angle of the pole boundary between two magnets  $i$  and  $i + 1$ .

We used two different magnetic patterns  $A$  and  $B$  for computations. For pattern  $A$ , the pole boundaries in the middle of the reading lines have a constant length  $d_i^A$  and continuously increased angles  $\alpha_i^A$  from  $\alpha_{min}^A$  to  $\alpha_{max}^A$  in steps of  $\alpha_{step}^A$ . For Pattern  $B$  we define a set of lengths  $d_i^B$  from  $d_{min}^B \leq d_i^B \leq d_{max}^B$  with steps of  $d_{step}^B$  for available trapezoids. The difference for total lengths on the boundaries of the trapezoids is limited to  $d_\varepsilon^B$ . The construction of pattern  $B$  is based on a mixed-integer linear method programming model, where the constraints of the pattern construction were formulated by algebraic inequalities. The resulting optimization problem is solved by a linear programming based branch-and-bound procedure. An example of a pattern and its schematic magnetic signals are presented in Figure 2. The scales  $A$  and  $B$  were produced by our project partner Bogen Electronic GmbH, Berlin

### 3. Method

In order to do absolute positioning, the actual scale needs to be known. Hence we measure the function  $f$  of the magnetic signal from the manufactured tape. Computing the absolute position is done by finding the best fit between the set of measured values and functions  $f_i : [0, l_i] \rightarrow \mathbb{R}$  of approximatives to the magnetic signals on each reading track. Here the  $f_i$  are piecewise functions of  $f$ . For each pole  $i$ ,  $i = 1, \dots, k$  with length  $l_i$  we use a linear combination  $g_i : [0, 1] \rightarrow \mathbb{R}$ ,  $g_i(x) = \lambda_{i,1}p_1(x) + \lambda_{i,2}p_2(x) + \dots + \lambda_{i,5}p_5(x)$  of 5 polynomials  $p_1, p_2, \dots, p_5$  of 5<sup>th</sup> degree and coefficients  $\lambda_{i,1}, \lambda_{i,2}, \dots, \lambda_{i,5}$  to construct  $f_i(x) \approx g_i(x/l_i)$ . For every pole  $i$ , we save the 5 coefficients of the linear combination on the processing unit. Due to memory limitations on the processing unit, we did not implement higher polynomial approximation methods.

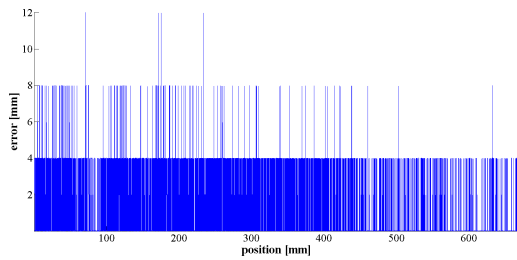
To find the best fit of measured data on  $f$ , we use a two-step algorithm. The first step evaluates the sequence of signs of values measured to find all sets of possible subsequences of magnets from the scale. In the second step the set of measured values are fitted to  $f$  for all found subsequences from the first step by calculating the best fit according to the smallest residual  $\varepsilon_f = \min(\|f_i - \mathbf{v}\|)$  for the measured values  $\mathbf{v} = v_1, \dots, v_6$  each reading line where  $\varepsilon_f$  is assumed to give the correct position on the scale. Our method of reconstructing the correct position from measured data was implemented on a DSP and on a FPGA by our project partners at the Technical University Berlin.

We evaluate our results with respect to accuracy and stability. Let the error of a measurement  $\varepsilon_p = |P_a - P_r|$  be the distance between actual position  $P_a \in [0, n]$  and reconstructed position  $P_r \in [0, n]$ . As a score for the accuracy we define the length  $l_a$  of the longest interval with an error below a threshold  $t_\varepsilon$ . Due to the way our algorithm works, we have additional potential positions for every reconstructed one. For small differences between the potential positions it becomes more likely, that the position chosen by our algorithm may be wrong. Hence, we measure stability as the mean distance  $l_s$  between the two best candidates. The larger  $l_s$ , the better the scale, because it is more robust against small measuring errors.

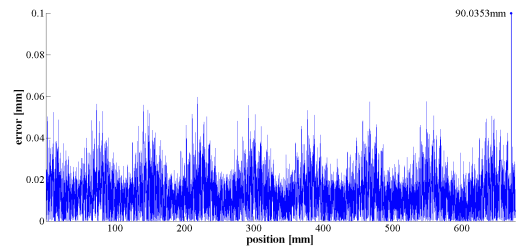
### 4. Results

For testing we constructed scales for patterns  $A$  and  $B$  with lengths of 675mm( $A$ ) and 672mm( $B$ ). For Pattern  $A$  we used  $d_i^A = 2\text{mm}$  and increased the angles from  $\alpha_{min}^A = 0^\circ$  to  $\alpha_{max}^A = 32^\circ$  in steps of  $\alpha_{step}^A = 0.095^\circ$ . In pattern  $B$  we used  $d_{min}^B = 2\text{mm}$  and  $d_{max}^B = 6.5\text{mm}$  with stepsize  $d_{step}^B = 0.35\text{mm}$  and  $d_\varepsilon^B = 3.4\text{mm}$ .

We recorded  $f$  with 1.5kHz sampling rate at a movement speed of 20mm/s using a  $2 \times 6$  hall cell array device. The polynomials and coefficients for  $f$  are stored on the processing unit. The same scale is measured again with same sampling rate and speed and to recover the positions for each set of measured values from the approximated functions stored on the processing unit. The results in figure 3 and 4 show the error for each position. A value of less than 0.1mm is



**Figure 3.** Error  $\varepsilon_p$  of computed positions for magnetic pattern  $A$ .



**Figure 4.** Error  $\varepsilon_p$  of computed positions for magnetic pattern  $B$ .

**Table 1.** Accuracy  $l_a$  of  $A$  and  $B$ .

$t_\epsilon$ [mm]	0.01	0.04	0.06	4.00	4.02	8.01	12.01
$A$ [mm]	2.02	6.26	6.26	8.67	130.20	436.70	675.00
$B$ [mm]	1.89	64.10	671,10	671,10	...	...	671,10

acceptable. For our scales we measured stabilities  $l_s = 14$  for  $A$  and  $l_s = 305$  for  $B$ .

## 5. Discussion

Our results show huge differences in both accuracy and stability as defined between both scales, with scale  $B$  delivering far better results than scale  $A$ . Good accuracy for scale  $A$  is only achievable at very small intervals with the error being in the range of several centimeters for most reconstructions while a value of 0.1mm would be acceptable. The main reason for this is the high regularity in the construction of the pattern leading to overall many candidates after the first step of our algorithm. Additionally, the small changes in the angles between the magnetic poles result in locally very similar magnetic signals and pole lengths. This corresponds to the low stability of the scale. Hence, for most of the positions the candidates for reconstruction have similar high errors and therefore only small changes in the signal (e.g., due to noise) can lead to different results in the reconstruction. Such effects do not occur regularly on scale  $B$ , leading to a far better accuracy which also corresponds to the better stability. Still due to similar magnetic areas that occur in the construction of  $B$ , we observed some spikes which are also the result of too small changes in pole lengths.

## 6. Conclusions

We were able to show that for appropriate scales our algorithm delivers precise results, sufficient for practical applications. Still, local but high errors arise and should be corrected by manually adjusting the scale or introducing additional rules in construction to avoid these patterns.

For real world industrial applications, we have to take into account additional errors like waves on the stripe, shaking of the machines, shifts in the track, turns and twists. Additionally the distance between sensor and stripe, either in  $h$ - or  $y$ -direction influences the amplitude. Hence, the stability decreases. We are currently working on data from different sensor-stripe shifts to improve stability. Further misplacements of the sensor like twists and turns will need to be addressed in the future, too.

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## Acknowledgments

This work is part of a joint project funded by the Central Innovation Program SME supported by the German Federal Ministry for Economic Affairs and Energy. The results were achieved in cooperation with BOGEN Electronic GmbH and the Technical University of Berlin, chair of Electronics and Medical Signal Processing.



