

On Moduli Spaces in AdS_4 Supergravity

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ABSTRACT

We study the structure of the supersymmetric moduli spaces of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravity theories in AdS_4 backgrounds. In the $\mathcal{N} = 1$ case, the moduli space cannot be a complex submanifold of the Kähler field space, but is instead real with respect to the inherited complex structure. In $\mathcal{N} = 2$ supergravity the same result holds for the vector multiplet moduli space, while the hypermultiplet moduli space is a Kähler submanifold of the quaternionic-Kähler field space. These findings are in agreement with AdS/CFT considerations.

1 Introduction

Vacua of supersymmetric field theories and supergravities frequently have continuous degeneracies parameterized by the background values of one or more scalar fields. The structure and properties of these moduli spaces depend on the amount of supersymmetry, on the spacetime background, and on whether supersymmetry is a global or local symmetry.

The existence of moduli spaces in supersymmetric compactifications of string theory to four dimensions impedes the construction of realistic models of particle physics and cosmology, and a primary endeavor in string phenomenology is the study of mechanisms that lift the vacuum degeneracy and stabilize the moduli. Moduli spaces of AdS_4 vacua are rather different from the better-understood moduli spaces of supersymmetric Minkowski solutions, and it is worthwhile to discuss their special properties.

In this paper we will focus on the structure of supersymmetric moduli spaces in AdS_4 vacua of general $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravities.¹ Two elementary structural questions are whether a continuous moduli space exists, and when one does, how the moduli space geometry is related to that of the parent configuration space. To frame the question, we compare to the simpler case of a supersymmetric Minkowski background M_4 . Consider an $\mathcal{N} = 1$ theory with global or local supersymmetry, containing n_c complex scalar fields ϕ_i , $i = 1, \dots, n_c$. The scalars parameterize a Kähler manifold \mathcal{C} . The supersymmetric vacua of this theory in a Minkowski background M_4 are determined by the solutions of n_c holomorphic equations $\partial_i W = 0$, where the superpotential W is a holomorphic function of the complex scalars ϕ_i . Generically the solutions are isolated points, but when a continuous moduli space \mathcal{M} exists, it is a complex, and therefore Kähler, submanifold of \mathcal{C} .

The situation is quite different in AdS_4 . For an AdS_4 vacuum of $\mathcal{N} = 1$ supergravity, we will see that \mathcal{M} cannot be a complex submanifold of \mathcal{C} : instead, \mathcal{M} is real with respect to the inherited complex structure, and can at best have real dimension n_c , i.e. half the dimension of the parent configuration space \mathcal{C} . In an AdS_4 vacuum of $\mathcal{N} = 2$ supergravity, \mathcal{C} is the product of a special Kähler manifold and a quaternionic Kähler manifold, and we will show that the moduli space \mathcal{M} is again a submanifold of \mathcal{C} , consisting of a real manifold times a Kähler manifold – also of at most half the parent dimension.

An intuition for the structure of the moduli spaces comes from the AdS/CFT correspondence, which relates an AdS_4 background to a three-dimensional superconformal field theory (SCFT) on the boundary. For $\mathcal{N} = 1$ in the bulk one has four supercharges, leading to a superconformal $N = \frac{1}{2}$ theory on the three-dimensional boundary.² In this case each chiral

¹For early work on AdS supersymmetry, see [1, 2] and the lectures [3]. Recent results in global $\mathcal{N} = 1$ supersymmetry in AdS_4 include [4, 5, 6].

²By $N = \frac{1}{2}$ we mean a three-dimensional theory with two ordinary supercharges, or four supercharges in the superconformal case.

multiplet has only one real scalar, and thus one can find at most a moduli space of half the original dimension. For $\mathcal{N} = 2$ in the bulk one has an $\mathcal{N} = 1$ theory on the three-dimensional boundary which can feature chiral and vector multiplets. In this case \mathcal{M} is a real manifold with a Kähler submanifold. Upon dualizing the three-dimensional vectors, \mathcal{M} becomes Kähler.

This paper is organized as follows: in section 2 we determine the structure of the moduli space in theories with $\mathcal{N} = 1$ supersymmetry, while section 3 extends our analysis to $\mathcal{N} = 2$. In Appendix A we discuss the global limit of $\mathcal{N} = 1$ supergravity in AdS_4 , and Appendix B contains a few illustrative examples. In the main text we set the reduced Planck mass M_{pl} to unity, but we retain explicit factors of M_{pl} in the discussion of decoupling in Appendix A.

2 $\mathcal{N} = 1$ Supergravity in AdS_4

In $\mathcal{N} = 1$ supergravity the scalar fields ϕ^i , $i = 1, \dots, n_c$ are members of chiral multiplets with the two-derivative Lagrangian [7]

$$\mathcal{L} = -\frac{1}{2}R - K_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} - V(\phi, \bar{\phi}) , \quad (2.1)$$

where R is the scalar curvature, $K_{i\bar{j}} = \partial_i\bar{\partial}_{\bar{j}}K$ is a Kähler metric on the scalar field space \mathcal{C} , with Kähler potential K , and the scalar potential V is given by

$$V = e^K(K^{i\bar{j}}D_iW D_{\bar{j}}\bar{W} - 3|W|^2) , \quad \text{with} \quad D_iW = \partial_iW + K_iW . \quad (2.2)$$

Supersymmetric minima occur where

$$D_iW = 0 = \bar{D}_{\bar{i}}\bar{W} \quad \forall i , \quad (2.3)$$

and the moduli space \mathcal{M} is defined as the locus in \mathcal{C} on which (2.3) holds.³ We will use $\langle \rangle$ to denote evaluation on \mathcal{M} , so that $\langle D_iW \rangle = \langle \bar{D}_{\bar{i}}\bar{W} \rangle = 0$ by definition, for all i .

2.1 Structure of the moduli space

From (2.2) one infers that for $\langle D_iW \rangle = \langle W \rangle = 0$ the background is Minkowski space M_4 , while for $\langle W \rangle \neq 0$, $\langle D_iW \rangle = 0$ it is AdS_4 . Therefore the supersymmetric minima in M_4 are determined by the holomorphic equations $\partial_iW = 0$, which are independent of the Kähler potential K . For generic W , these n_c equations determine the n_c complex variables ϕ^i , leaving no continuous moduli space: the vacuum manifold is a set of isolated points. On

³Here we neglect the possibility of having charged scalars and associated D-terms: these do not affect our analysis of the structure of the moduli space, as we will see shortly.

the other hand, for non-generic superpotentials ($W = 0$ being a simple example), there can be a continuous moduli space \mathcal{M} . Because \mathcal{M} is determined by the solution to a set of holomorphic equations, it is a complex submanifold of the (Kähler) field space \mathcal{C} , and so \mathcal{M} is Kähler.

The situation is different in AdS_4 , because for $\langle W \rangle \neq 0$ the F-flatness conditions $D_i W = 0$ depend on the Kähler potential, which is non-holomorphic.⁴ The equation counting is unchanged, so it is still true that for generic K and W , the vacuum solutions are isolated points. However, when a moduli space does arise due to non-generic K and W , its properties are different from the moduli spaces in Minkowski solutions, as we now show.

In order to find the moduli space we infinitesimally vary the equations (2.3) to obtain

$$\begin{aligned} \langle \partial_j D_i W \rangle \delta \phi^j + \langle K_{i\bar{j}} W \rangle \delta \bar{\phi}^{\bar{j}} &= 0, \\ \langle K_{i\bar{j}} \bar{W} \rangle \delta \phi^j + \langle \bar{\partial}_{\bar{j}} \bar{D}_i W \rangle \delta \bar{\phi}^{\bar{j}} &= 0. \end{aligned} \tag{2.4}$$

In matrix form we then have

$$\mathbb{M} \begin{pmatrix} \delta \phi^j \\ \delta \bar{\phi}^{\bar{j}} \end{pmatrix} = 0, \quad \text{with} \quad \mathbb{M} = \begin{pmatrix} m_{ij} & \langle K_{i\bar{j}} W \rangle \\ \langle K_{i\bar{j}} \bar{W} \rangle & \bar{m}_{i\bar{j}} \end{pmatrix}, \tag{2.5}$$

where m_{ij} is proportional to the mass matrix of the fermions in the chiral multiplets, and is given by [7]⁵

$$m_{ij} = \langle \nabla_i D_j W \rangle = \langle \partial_i D_j W \rangle = \langle \partial_i \partial_j W + K_j \partial_i W + K_{ij} W \rangle = \langle \partial_i \partial_j W + (K_{ij} - K_i K_j) W \rangle. \tag{2.6}$$

Note that in an M_4 background m_{ij} is just the second derivative of the superpotential, while in an AdS_4 background $\langle W \rangle \neq 0$ and thus the cosmological constant contributes. Since $\langle K_{i\bar{j}} \rangle$ is necessarily a positive matrix, and $\langle W \rangle \neq 0$ in AdS_4 , we learn that the matrix \mathbb{M} in (2.5) has at least real rank n_c , leaving at most a moduli space of real dimension n_c .⁶

Before we proceed, let us note that including a D-term does not change the analysis. Gauging isometries on a Kähler manifold results in a D-term of the form $D = k^i K_i$ where k^i is an appropriately normalized Killing vector and K_i is the first derivative of the Kähler potential. Gauge invariance of the superpotential further imposes $k^i \partial_i W = 0$, which implies that in AdS_4 the D-term can alternatively be written as [8]

$$D = W^{-1} k^i D_i W. \tag{2.7}$$

⁴Even in *global* AdS_4 supersymmetry, the superpotential transforms under Kähler transformations, as explained in [4, 5].

⁵The proportionality factor is $e^{K/2}$, and it is the matrix $e^{K/2} m_{ij}$ that appears in the Dirac equation. We also have made use of the fact that on \mathcal{M} , partial derivatives and covariant derivatives are equivalent, cf. (2.3).

⁶This follows from the fact that the matrix $\langle K_{i\bar{j}} W \rangle$ is invertible in an AdS vacuum: it therefore contains n_c linearly independent vectors. Thus \mathbb{M} also contains at least n_c linearly independent vectors, so the rank of \mathbb{M} is at least n_c .

This expression shows that on \mathcal{M} the D-term vanishes automatically, and its variation,

$$\delta D = W^{-1} k^i \delta(D_i W), \quad (2.8)$$

is proportional to the variation of $\delta(D_i W)$ which we already analyzed in eq. (2.4). Thus, the D-term imposes no further constraints on the moduli space.

To examine the structure of the moduli space, we rewrite (2.5) in terms of real variations obtained from the decomposition $\phi^i = \frac{1}{\sqrt{2}}(A^i + i B^i)$. In this case, after choosing $\text{Im}(\langle W \rangle) = 0$, we have

$$\mathbb{M}_r \begin{pmatrix} \delta A^j \\ \delta B^j \end{pmatrix} = 0, \quad \mathbb{M}_r = \begin{pmatrix} \langle \text{Re } m_{ij} + K_{i\bar{j}} W \rangle & -\langle \text{Im } m_{ij} \rangle \\ \langle \text{Im } m_{ij} \rangle & \langle \text{Re } m_{ij} - K_{i\bar{j}} W \rangle \end{pmatrix}. \quad (2.9)$$

We now observe that the complex structure on the space of chiral fields in the given basis is

$$J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad J^2 = - \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \quad (2.10)$$

where I is the $n_c \times n_c$ unit matrix. For the non-trivial solution space of (2.9) (i.e. the kernel of the map \mathbb{M}_r) to have the complex structure inherited from \mathcal{M} , the existence of a non-trivial solution to

$$\mathbb{M}_r \begin{pmatrix} \delta A^i \\ \delta B^i \end{pmatrix} = 0 \quad (2.11)$$

must imply that there is a non-trivial solution to

$$\mathbb{M}_r J \begin{pmatrix} \delta A^i \\ \delta B^i \end{pmatrix} = 0 \quad .$$

But this means that

$$(J\mathbb{M}_r - \mathbb{M}_r J) \begin{pmatrix} \delta A^i \\ \delta B^i \end{pmatrix} = 0 \quad . \quad (2.12)$$

However, since

$$J\mathbb{M}_r - \mathbb{M}_r J = 2 \begin{pmatrix} 0 & \langle K_{i\bar{j}} W \rangle \\ \langle K_{i\bar{j}} W \rangle & 0 \end{pmatrix},$$

and $\langle K_{i\bar{j}} W \rangle$ is non-singular, it follows from (2.12) that only the trivial solution exists. Thus, no nontrivial solution space of (2.11) can be complex in the complex structure (2.10).

This result can be seen more explicitly. Suppose there is a complex flat direction, say along the 1 direction, and consider fluctuations along this direction, setting $\delta A^i = \delta B^i = 0$ for $i \neq 1$. From eq. (2.9) we obtain

$$\langle \text{Re } m_{i1} + K_{i\bar{1}} W \rangle \delta A^1 - \langle \text{Im } m_{i1} \rangle \delta B^1 = 0, \quad (2.13)$$

$$\langle \text{Im } m_{i1} \rangle \delta A^1 + \langle \text{Re } m_{i1} - K_{i\bar{1}} W \rangle \delta B^1 = 0, \quad (2.14)$$

which should hold for all i . Since $K_{i\bar{j}}$ has rank n_c , there is at least one index j_\star for which $K_{j_\star\bar{i}} \neq 0$. Taking $i = j_\star$, eq. (2.13) holds for arbitrary $\delta A^1, \delta B^1$ only if $\text{Im } m_{j_\star 1} = 0$ and $K_{j_\star\bar{1}}W = -\text{Re } m_{j_\star 1}$. Then eq. (2.14) would require that $\delta B^1 = 0$, negating the existence of a complex moduli space.

We conclude that the two scalars (A, B) of a chiral multiplet cannot simultaneously be massless moduli. In other words, the moduli space is necessarily real with respect to the original complex structure of the chiral multiplets.⁷

It is instructive to compute the mass matrix of the scalar fields. The first derivative of V reads⁸

$$\partial_k V = \nabla_k V = e^K \left(K^{i\bar{j}} D_k D_i W \bar{D}_{\bar{j}} \bar{W} - 2(D_k W) \bar{W} \right), \quad (2.15)$$

which indeed vanishes at the minimum, where $\langle D_i W \rangle = 0$. From (2.15) we can compute the bosonic mass matrix

$$\begin{aligned} \langle \nabla_k \nabla_{\bar{l}} V \rangle &= e^K \left(K^{i\bar{j}} m_{ki} \bar{m}_{\bar{l}\bar{j}} - 2K_{k\bar{l}} |W|^2 \right), \\ \langle \nabla_k \nabla_l V \rangle &= -e^K m_{kl} \bar{W}, \end{aligned} \quad (2.16)$$

where m_{ki} is defined in (2.6). Decomposing $\phi^i = \frac{1}{\sqrt{2}}(A^i + iB^i)$ one obtains the mass matrices for A^i and B^i ,

$$\begin{aligned} (m_A^2)_{kl} &= e^K \left(K^{i\bar{j}} (m_{ki} - \frac{1}{2} K_{\bar{k}i} W) (\bar{m}_{\bar{l}\bar{j}} - \frac{1}{2} K_{l\bar{j}} \bar{W}) - \frac{9}{4} K_{k\bar{l}} |W|^2 \right), \\ (m_B^2)_{kl} &= e^K \left(K^{i\bar{j}} (m_{ki} + \frac{1}{2} K_{\bar{k}i} W) (\bar{m}_{\bar{l}\bar{j}} + \frac{1}{2} K_{l\bar{j}} \bar{W}) - \frac{9}{4} K_{k\bar{l}} |W|^2 \right), \\ (m_{AB}^2)_{kl} &= 2e^K \text{Im} (m_{kl} \bar{W}). \end{aligned} \quad (2.17)$$

On diagonalizing these equations one finds that only one of the two real scalars in a chiral multiplet can be massless. We relegate the details of this discussion, as well as the relation to the rigid AdS limit and to the formulae of [3], to Appendix A.

2.2 Examples of moduli spaces in AdS_4 supergravity

To illustrate the general results above, we consider a few examples. Take $n_c = 1$ and

$$K = \frac{1}{2}(\phi + \bar{\phi})^2, \quad W = c = \text{constant}. \quad (2.18)$$

The F-term is $D_\phi W = (\phi + \bar{\phi})c$, which vanishes for $\text{Re } \phi = 0$. We see that \mathcal{M} is the locus $\text{Re } \phi = 0$, on which the scalar potential is $\langle V \rangle = -3|c|^2$. Thus, $\text{Im } \phi$ is an (axionic) flat direction parameterizing the moduli space.

⁷Of course it is possible that an even number of real moduli can be combined into complex fields with respect to another complex structure.

⁸We define the Kähler covariant derivative acting on a tensor to be $D_i = \nabla_i + K_i$.

As a slightly more involved example motivated from string theory, consider p chiral fields T and q chiral fields Q (i.e. $n_c = p + q$) with couplings

$$\begin{aligned} K &= K(T, \bar{T}) + Z(T, \bar{T})Q\bar{Q} + \mathcal{O}(Q^3) , \\ W &= c + m(T)Q^2 + \mathcal{O}(Q^3) , \end{aligned} \tag{2.19}$$

where $K(T, \bar{T})$ and $Z(T, \bar{T})$ are for the moment arbitrary real functions of T while $m(T)$ is an arbitrary holomorphic function. The supersymmetry condition for Q reads

$$D_Q W = 2mQ + Z\bar{Q}W + \mathcal{O}(Q^2) \tag{2.20}$$

which is solved by $Q = 0$. On the locus where $Q = 0$, we have $\partial_T W|_{Q=0} = 0$, so that \mathcal{M} is the space of solutions of

$$D_T W|_{Q=0} = K_T c = 0. \tag{2.21}$$

Because the condition (2.21) depends only on $K(T, \bar{T})$, the functions $Z(T, \bar{T})$ and $m(T)$ are unconstrained. For generic K , all T are fixed by (2.21), leaving no moduli space. However, moduli spaces arise in special cases: e.g. for $K = K(T + \bar{T})$ only the $\text{Re} T$ are fixed, leaving the p $\text{Im} T$ directions as axionic moduli. As anticipated, the moduli space is real, with dimension at most half the dimension of the original Kähler manifold. The background value of the potential is again $\langle V \rangle = -3|c|^2$. Note that not every function $K = K(T + \bar{T})$ is compatible with the existence of a moduli space: an additional requirement is that K is non-singular at $K_T = 0$. For example, the Kähler potential $K = -\ln(T + \bar{T})$, which is commonplace in tree-level effective actions of string compactifications, has $K_T = 0$ only at $(T + \bar{T}) \rightarrow \infty$, corresponding to an infinite and thus unacceptable K . On the other hand, a general polynomial $K = \sum_{n=2}^{\infty} a_n (T + \bar{T})^n$ yields a moduli space.

We will next discuss a simple example with a Goldstone-type global $U(1)$ symmetry of the full Lagrangian. Let us choose K and W to be of the form

$$K = \phi_1 \bar{\phi}_1 + \phi_2 \bar{\phi}_2 , \quad W = c + m\phi_1 \phi_2 , \tag{2.22}$$

with m and c being real for simplicity. In Appendix B we show that there are the following two supersymmetric solutions of $D_{\phi_1} W = D_{\phi_2} W = 0$:

A) $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$ and

B) for $|c| > |m|$, non-trivial solutions with $\langle \phi_1 \rangle = \pm \langle \bar{\phi}_2 \rangle \neq 0$.

In both cases the symmetry $\phi_1 \rightarrow e^{i\theta} \phi_1$, $\phi_2 \rightarrow e^{-i\theta} \phi_2$ is unbroken. However, in the first solution no flat direction exists, while if we parameterize $\phi_1 = r_1 e^{i(x+\rho)}$, $\phi_2 = r_2 e^{i(x-\rho)}$, we see that ρ is a flat direction in the second solution.

2.3 Global symmetries and exact moduli spaces

In the examples just discussed, translation along the moduli space corresponds to a continuous shift symmetry. However, well-known arguments exclude exact continuous global symmetries in string theory, and in quantum gravity more generally (see e.g. [9, 10, 11], and the recent discussion in [12]), and one might ask whether these no-go results constrain the existence of exact quantum moduli spaces in quantum gravity theories. To explain why there is no associated constraint, we begin by briefly recalling two of the standard arguments.

Black holes and global symmetries.—Consider a continuous global internal symmetry \mathcal{G} under which one or more species of particles, all with nonzero mass, are charged. Denote by λ_{max} the maximum ratio of \mathcal{G} -charge q to mass m , across all species in the spectrum (not only the lightest species). Form a macroscopic Schwarzschild black hole from constituents of total \mathcal{G} -charge Q and mass M_0 . Then once Hawking radiation causes the black hole to decay to mass $M < M_\star \equiv Q/\lambda_{max}$, it is not possible for any subsequent decay process to release a total charge Q while remaining consistent with conservation of energy. So \mathcal{G} -charge is not conserved. Note that one can make the initial black hole as large as necessary in order to ensure that the Hawking temperature remains as small as desired when the black hole has mass M_\star , so that semiclassical reasoning remains valid.

The possibility of this process implies that in an effective theory derived from a consistent quantum gravity theory with standard black hole thermodynamics, there must be operators violating every⁹ continuous global internal symmetry.

String theory and global symmetries.—Banks and Dixon showed in [9] that for any exactly conserved non-axionic global internal symmetry, one can construct a vertex operator for a gauge boson from the conserved global symmetry current. This implies that any exact non-axionic global internal¹⁰ symmetry must be gauged in string theory. Axionic shift symmetries of the form

$$a \mapsto a + \text{const.} \tag{2.23}$$

are not constrained by this argument. At zero momentum, the vertex operator for an axion a is a worldsheet total derivative, and the worldsheet fields do not transform under (2.23). Thus, the logic of [9] does not apply to axionic symmetries, including translations along the flat directions in the first two examples above.

Accidental symmetries.—In view of the above arguments, moduli spaces protected by exact, non-axionic global symmetries are incompatible with general reasoning about quantum

⁹Violation of axionic symmetries by black holes (and by wormholes [10]) is somewhat subtle, in part because of the possibility of axionic hair: see e.g. [13, 14].

¹⁰The qualifier ‘internal’ is necessary because Lorentz symmetry of noncompact spacetime is an exception to the argument of [9], but this will not be relevant for our discussion.

gravity and string theory. However, it is crucial to recognize that the presence of an exact moduli space *does not* imply the existence of any exact symmetry of the full Lagrangian: it is consistent for the symmetry of translations along the moduli space to be an accidental symmetry that is preserved along some locus. Such a symmetry does not correspond to a current that is conserved at all points in the configuration space, and so is not constrained by either of the arguments above.

Accidental symmetries that hold only on a special locus in the configuration space can be broken by non-derivative couplings or by derivative couplings. To give two examples, the symmetry of translations $\phi \mapsto \phi + \text{const.}$ could be broken by

$$\Delta\mathcal{L} = \phi^2\chi^2, \quad (2.24)$$

where χ is another scalar field, or by

$$\Delta\mathcal{L} = \phi^2 R^2, \quad (2.25)$$

where R is the scalar curvature of spacetime. The latter case is particularly relevant: there is no conserved current in the full theory, but in a Minkowski solution ϕ enjoys the accidental shift symmetry $\phi \mapsto \phi + \text{const.}$, and hence an exact moduli space, while in AdS solutions the coupling (2.25) gives ϕ a mass and lifts the corresponding moduli space.

A particular form of symmetry breaking generalizing (2.25) arises in certain extended supergravities and in string theory compactifications with extended supersymmetry: the quantum gravity effects that destroy global charges and hence prevent the associated global symmetries from being exact only appear beyond the level of the two-derivative action. Examples include compactifications with $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supersymmetry, such as compactifications on T^6 of heterotic string theory and type II string theory, respectively. In these compactifications there is an exact continuous $SO(6, 22)$ or $E_{7(7)}$ global symmetry group at the level of ungauged supergravity, but the continuous symmetries are broken by instantons (e.g. D-instantons), i.e. by the charge lattice of the theory. These instantons contribute only at four and more derivatives, due to supersymmetry, and break the continuous symmetry group to a lattice corresponding to the monodromies of the charges of the theory. In theories of this sort where higher-derivative couplings are the only effect spoiling a symmetry, translation along an exact quantum moduli space can then correspond to a genuine exact symmetry of the two-derivative theory, which is only an accidental symmetry of the full theory incorporating higher derivatives.

To summarize, we inquired whether the presence of an exact moduli space implies the existence of a symmetry that is forbidden by quantum gravity arguments. It does not: the symmetry of translations along the moduli space might be an accidental symmetry of the full theory, preserved on some special locus in the configuration space, hence avoiding no-go results from quantum gravity.

Existence and genericity of moduli spaces.—Let us briefly indicate when exact quantum moduli spaces are generic or non-generic.

In $\mathcal{N} = 1$ supersymmetry in Minkowski space, the moduli space is entirely determined by W , which is not renormalized in perturbation theory. However, nonperturbative effects can contribute corrections to W , lifting the continuous moduli space and leaving only discrete points as quantum vacua. In $\mathcal{N} = 1$ supergravity theories arising from compactifications of string theory to Minkowski space, the possible nonperturbative effects (from strong gauge dynamics and from Euclidean branes) are numerous, and the quantum moduli space is expected to be a set of points in generic cases. On the other hand, in *global* $\mathcal{N} = 1$ supersymmetry in M_4 , there are celebrated examples of supersymmetric gauge theories with exact quantum moduli spaces, e.g. the theory with gauge group $SU(N_c)$ and $N_f = N_c$ families of quarks and anti-quarks in the fundamental representation [15].

In AdS_4 the situation changes due to the presence of the Kähler potential K in the condition for a supersymmetric minimum. The Kähler potential is renormalized at all orders in perturbation theory, and thus even perturbative moduli spaces are non-generic. This intuition is supported by the AdS/CFT correspondence, since the three-dimensional SCFT on the boundary of AdS_4 only has two supercharges (or four superconformal charges), and thus no BPS representations protected by non-renormalization theorems exist.

Exact moduli spaces of Minkowski solutions are more common in theories with extended supersymmetry: for example, in global $\mathcal{N} = 2$ supersymmetry in Minkowski space, the vector multiplet sector generically has a quantum moduli space [16]. More generally, even in local supersymmetry, ungauged $\mathcal{N} = 2$ supergravities, for example those arising in the low-energy limit of Calabi-Yau compactifications of type II string theory, generically have an exact moduli space in the Minkowski vacuum. The reason for this is that there is no superpotential that can get corrected: only kinetic terms receive quantum corrections. When there are no gaugings, there are no prepotentials, and therefore no potential. On the other hand, when there are gaugings, quantum corrections will correct the potential (since they correct the special Kähler and quaternionic-Kähler metrics). For AdS vacua, there must be gaugings, and one expects corrections to the potential.

3 $\mathcal{N} = 2$ Supergravity in AdS_4

3.1 Preliminaries

Let us start with a brief summary of $\mathcal{N} = 2$ supergravity in four space-time dimensions.¹¹ Apart from the gravitational multiplet, a generic $\mathcal{N} = 2$ spectrum contains n_v vector multi-

¹¹For a more comprehensive review see e.g. [17].

plets and n_h hypermultiplets with the following field content. A vector multiplet contains a vector A_μ , two gaugini $\lambda^{\mathcal{A}}$, $\mathcal{A} = 1, 2$ and a complex scalar t , while a hypermultiplet contains two hyperini ζ_α and four real scalars q^u . Finally, the gravitational multiplet contains the spacetime metric $g_{\mu\nu}$, two gravitini $\Psi_{\mu\mathcal{A}}$ and the graviphoton A_μ^0 .¹²

The scalar field space splits into the product

$$M = M_v \times M_h , \quad (3.1)$$

where the first component M_v is a special Kähler manifold of complex dimension n_v spanned by the scalars $t^i, i = 1, \dots, n_v$ in the vector multiplets. This implies that the metric obeys

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K^v , \quad \text{with} \quad K^v = -\ln i (\bar{X}^\Lambda \Omega_{\Lambda\Sigma} X^\Sigma) , \quad (3.2)$$

where $X^\Lambda = (X^I, \mathcal{F}_I), I = 0, \dots, n_v$ is a $2(n_v+1)$ -dimensional symplectic vector that depends holomorphically on the t^i . $\mathcal{F}_I = \partial\mathcal{F}/\partial X^I$ is the derivative of a holomorphic prepotential \mathcal{F} which is homogeneous of degree 2 and $\Omega_{\Lambda\Sigma}$ is the standard symplectic metric.

The second factor of the field space, M_h , is spanned by the real scalars $q^u, u = 1, \dots, 4n_h$ in the hypermultiplets, and is quaternionic Kähler and of real dimension $4n_h$. Such a manifold admits a triplet of almost complex structures $I^x, x = 1, 2, 3$ satisfying $I^x I^y = -\delta^{xy} \mathbf{1} + \epsilon^{xyz} I^z$, with the metric h_{uv} being Hermitian with respect to all three I^x . The associated two-forms K^x are the field strengths of the $SU(2)$ connection ω^x , i.e.

$$K^x = d\omega^x + \frac{1}{2} \epsilon^{xyz} \omega^y \wedge \omega^z , \quad (3.3)$$

and thus are covariantly closed, $\nabla K^x = 0$.

One of the differences compared to $\mathcal{N} = 1$ supergravity is that no superpotential is allowed in $\mathcal{N} = 2$, and thus for Abelian vectors and neutral hypermultiplets no potential is possible: the entire field space (3.1) is the moduli space of an M_4 background. A potential only appears when some of the hypermultiplets are charged and/or when the gauge group is non-Abelian. Let us first discuss a non-Abelian gauge group G . In this case the scalars t^i are in the adjoint representation of G , and the contribution to the potential is nonnegative and vanishes for $t^i = 0$. Thus a spontaneous breaking of G by a non-trivial $\langle t^i \rangle$ can induce a positive contribution to the cosmological constant, but cannot be responsible for the AdS background in the first place. For that reason we discard non-Abelian gauge groups in the following analysis and only consider hypermultiplets that are charged with respect to some Abelian $G = [U(1)]^{n_v}$. However we do allow for the possibility that the hypermultiplets carry mutually local electric and magnetic charges. This situation is conveniently discussed in the embedding tensor formalism, where the covariant derivatives are given by [18]

$$D_\mu q^u = \partial_\mu q^u - A_\mu^\Lambda \Theta_\Lambda^\lambda k_\lambda^u(q) , \quad (3.4)$$

¹²Strictly speaking, the definition of the graviphoton is $X^I \text{Im} \mathcal{F}_{IJ} A_\mu^J$, which can be read off from the gravitino variation and depends on the scalar fields in the vector multiplets.

with $A_\mu^\Lambda = (A_\mu^I, B_{\mu I})$ being a symplectic vector of electric and magnetic gauge fields. Here $k_\lambda^u(q)$ are the independent Killing vectors on M_h , labeled by the index λ , while Θ_Λ^λ is the (constant) matrix of gauge charges (or the embedding tensor) that parameterizes the isometries that are gauged. Mutual locality additionally imposes the quadratic constraint $\Theta^{\Lambda\lambda}\Theta_\Lambda^\kappa = 0$.

The resulting scalar potential reads

$$V = \frac{1}{2}g_{i\bar{j}}W^{iAB}W_{AB}^{\bar{j}} + N_\alpha^A N_A^\alpha - 6S_{AB}\bar{S}^{AB}, \quad (3.5)$$

where W^{iAB} , N_α^A and S_{AB} arise as the scalar parts of the supersymmetry variations of the gaugino, hyperino and gravitino, respectively

$$\begin{aligned} \delta_\epsilon \lambda^{iA} &= W^{iAB} \epsilon_B + \dots, \\ \delta_\epsilon \zeta_\alpha &= N_\alpha^A \epsilon_A + \dots, \\ \delta_\epsilon \Psi_{\mu A} &= D_\mu \epsilon_{\mathcal{A}}^* - S_{AB} \gamma_\mu \epsilon^B + \dots. \end{aligned} \quad (3.6)$$

Here ϵ^A are the two supersymmetry parameters, and

$$\begin{aligned} S_{AB} &= \frac{1}{2}e^{K^v/2} X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x (\sigma^x)_{AB}, \\ W^{iAB} &= ie^{K^v/2} g^{i\bar{j}} (\nabla_{\bar{j}} \bar{X}^\Lambda) \Theta_\Lambda^\lambda P_\lambda^x (\sigma^x)^{AB}, \\ N_\alpha^A &= 2e^{K^v/2} \bar{X}^\Lambda \Theta_\Lambda^\lambda \mathcal{U}_{\alpha u}^A k_\lambda^u, \end{aligned} \quad (3.7)$$

where in our conventions the Pauli matrices with both indices up (or down) are¹³

$$(\sigma^1)^{AB} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\sigma^2)^{AB} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}, \quad (\sigma^3)^{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (3.8)$$

$\mathcal{U}_u^{A\alpha}$ are the vielbeins on M_h , which are related to the metric h_{uv} and the three curvature two-forms K_{uv}^x defined in (3.3) via

$$C_{\alpha\beta} \mathcal{U}_u^{A\alpha} \mathcal{U}_v^{B\beta} = -\frac{i}{2} K_{uv}^x \sigma^{xAB} - \frac{1}{2} h_{uv} \epsilon^{AB}, \quad (3.9)$$

where $C_{\alpha\beta}$, $\alpha, \beta = 1, \dots, 2n_h$ is the flat $Sp(n_h)$ metric. Furthermore, we abbreviate $\nabla_i X^\Lambda := \partial_i X^\Lambda + (\partial_i K^v) X^\Lambda$, and P_λ^x are the Killing prepotentials defined by

$$-2k_\lambda^u K_{uv}^x = \nabla_v P_\lambda^x, \quad (3.10)$$

where ∇_v is the $SU(2)$ -covariant derivative.

In the following discussion we will also need the fermion mass matrices [17]

$$\begin{aligned} \mathcal{M}_{ijAB} &= \frac{1}{2}e^{K^v/2} (\nabla_i \nabla_j X^\Lambda) \Theta_\Lambda^\lambda P_\lambda^x (\sigma^x)_{AB}, \\ \mathcal{M}_{iA}^\alpha &= -4e^{K^v/2} (\nabla_i X^\Lambda) \Theta_\Lambda^\lambda \mathcal{U}_{A u}^\alpha k_\lambda^u, \\ \mathcal{M}^{\alpha\beta} &= -e^{K^v/2} X^\Lambda \Theta_\Lambda^\lambda \mathcal{U}_u^{\alpha A} \mathcal{U}_v^{\beta B} \epsilon_{AB} \nabla^{[u} k_\lambda^{v]}, \end{aligned} \quad (3.11)$$

¹³The indices are raised and lowered with ϵ_{AB} .

where \mathcal{M}_{ijAB} is the mass matrix of the gauginos, $\mathcal{M}^{\alpha\beta}$ is the mass matrix of the hyperini and \mathcal{M}_{iA}^α is a possible mixing term.¹⁴ Supersymmetry relates the shift matrices in (3.7) and the fermion mass matrices (3.11) by the following “gradient flow” equations [19]

$$\begin{aligned} \nabla_j W^{iAB} &= 2\delta_j^i \bar{S}^{AB}, & \nabla_{\bar{j}} W^{iAB} &= -g^{i\bar{i}} \mathcal{M}_{i\bar{j}}^{AB}, & \nabla_u \bar{W}_{AB}^{\bar{j}} &= -\frac{1}{2} g^{i\bar{j}} \mathcal{M}_{i(A}^\alpha \mathcal{U}_{B)\alpha u}, \\ \nabla_i N_A^\alpha &= \frac{1}{2} \mathcal{M}_{iA}^\alpha, & \mathcal{U}^{u\beta B} \nabla_u N^{\alpha A} &= 4C^{\alpha\beta} S^{AB} + \epsilon^{AB} \mathcal{M}^{\alpha\beta}, \\ \nabla_i S_{AB} &= \frac{1}{2} g_{i\bar{j}} W_{AB}^{\bar{j}}, & \nabla_{\bar{i}} S_{AB} &= 0, & \nabla_u S_{AB} &= -\frac{1}{2} U_{u\alpha(A} N_{B)}^\alpha, \end{aligned} \quad (3.12)$$

where $W_{AB}^{\bar{j}} = (W^{iAB})^*$ and $\mathcal{M}_{i\bar{j}}^{AB} = (\mathcal{M}_{ijAB})^*$.

3.2 Structure of the moduli space

In [20, 21] the conditions for a four-dimensional $\mathcal{N} = 2$ supersymmetric AdS vacuum in $\mathcal{N} = 2$ supergravity were discussed. In terms of the fermionic supersymmetry variations (3.6) one demands

$$\langle W^{iAB} \rangle = 0, \quad \langle N^{\alpha A} \rangle = 0, \quad \langle S_{AB} \rangle \epsilon^B = \frac{1}{2} \Lambda \epsilon_{\mathcal{A}}^*, \quad (3.13)$$

where $|\Lambda|^2$ is related to the cosmological constant of the $\mathcal{N} = 2$ vacuum as in (A.4). Using (3.7) the conditions (3.13) can be explicitly translated into the following conditions on the $\mathcal{N} = 2$ couplings [21]

$$\langle X^\Lambda \Theta_\Lambda^\lambda k_\lambda^u \rangle = 0, \quad \langle \nabla_i X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x \rangle = 0, \quad (3.14)$$

and

$$\Theta_\Lambda^\lambda \langle P_\lambda^x \rangle = -\frac{1}{2} \Omega_{\Lambda\Sigma} \langle e^{K^V/2} \text{Im}(\hat{\Lambda} \bar{X}^\Sigma) \rangle a^x, \quad (3.15)$$

where a is an arbitrary real vector on S^2 and $\hat{\Lambda}$ is related to Λ by a phase. We can use the local $Sp(1)$ symmetry of $\mathcal{N} = 2$ to rotate a^x into a frame where $a^x = a\delta^{x3}$ and hence only the combination $\Theta_\Lambda^\lambda \langle P_\lambda^3 \rangle \neq 0$ in (3.15). (We will frequently use this simplification below.)

By contracting (3.15) with X^Λ it was shown in [21] that the right hand side is proportional to the graviphoton direction in field space, and thus a cosmological constant can only appear if an isometry in this direction is gauged. Let us denote this direction by $\lambda = 0$, so that (3.15) implies

$$\langle X^\Lambda \Theta_\Lambda^0 P_0^x \rangle \neq 0, \quad \langle X^\Lambda \Theta_\Lambda^{\lambda \neq 0} P_{\lambda \neq 0}^x \rangle = 0, \quad (3.16)$$

and thus, inserted into (3.7),

$$\langle S_{AB} \rangle = \frac{1}{2} \langle e^{K^V/2} X^\Lambda \Theta_\Lambda^0 P_0^x \rangle (\sigma^x)_{AB} = \frac{1}{2} \langle e^{K^V/2} X^\Lambda \Theta_\Lambda^0 P_0^3 \rangle (\sigma^3)_{AB} \neq 0. \quad (3.17)$$

¹⁴Strictly speaking the fermion mass matrices are the values of these quantities evaluated in the AdS background.

The first equation in (3.14) combined with the requirement (3.16) has two types of solutions:

$$\begin{aligned} \text{minimal solution :} & \quad \langle k_\lambda^u \rangle = 0 \quad \forall \lambda , \\ \text{non-minimal solution :} & \quad \langle k_0^u \rangle = 0 , \quad \langle k_{\lambda \neq 0}^u \rangle \neq 0 . \end{aligned} \tag{3.18}$$

For the non-minimal solution, (3.14) is satisfied only by imposing $\langle X^\Lambda \Theta_\Lambda^{\lambda \neq 0} \rangle = 0$. In this case the gauge symmetry is spontaneously broken and $n_m := \text{rk}(\Theta_\Lambda^\lambda k_\lambda^u)$ long vector multiplets become massive, each with a total of five massive scalars, two from vector multiplets and three from hypermultiplets [21].¹⁵ Note that consistency imposes

$$n_m \leq n_v \quad \text{and} \quad n_m \leq n_h . \tag{3.19}$$

As in section 2, we now determine properties of the moduli space by varying the conditions (3.14) and (3.16). Let us start with (3.16) and study the variation of $\langle X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x \rangle$ for all λ . This has the two terms

$$\langle \delta(X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x) \rangle = \langle \nabla_i X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x \rangle \delta t^i + 2 \langle X^\Lambda \Theta_\Lambda^\lambda k_\lambda^v K_{uv}^x \rangle \delta q^u = 0 , \tag{3.20}$$

which both vanish for both solutions of (3.18), due to (3.14). Thus no condition is imposed on the moduli space.

Next we consider the variation of $\langle \nabla_i X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x \rangle$ in (3.14), which yields

$$\langle \mathcal{M}_{ij\mathcal{A}\mathcal{B}} \rangle \delta t^j - 2 \langle \bar{S}_{\mathcal{A}\mathcal{B}} g_{i\bar{j}} \rangle \delta \bar{t}^{\bar{j}} + \frac{1}{2} \langle M_{i\mathcal{A}}^\alpha \mathcal{U}_{\mathcal{B}} \rangle_{\alpha u} \delta q^u = 0 , \tag{3.21}$$

where $\mathcal{M}_{ij\mathcal{A}\mathcal{B}}$ and $M_{i\mathcal{A}}^\alpha$ are defined in (3.11) and we used (3.7) and (3.10). For the minimal solution we find from the definition in (3.11) that the mass matrix $\mathcal{M}_{i\mathcal{A}}^\alpha$ vanishes and one is left with only the first two terms in (3.21). As we anticipated the analysis can be further simplified by using the local $Sp(1)$ symmetry of $\mathcal{N} = 2$ to rotate into a frame where among the $\Theta_\Lambda^\lambda P_\lambda^x$ only $\Theta_\Lambda^\lambda P_\lambda^3$ is non-zero. Inserting (3.11) and (3.15) into (3.21) in that frame yields

$$\langle \text{Im}(\hat{\Lambda} \bar{X}^\Sigma) \rangle \Omega_{\Sigma\Lambda} (\langle \nabla_i \nabla_j X^\Lambda \rangle \delta t^j - 2 \langle X^\Lambda g_{i\bar{j}} \rangle \delta \bar{t}^{\bar{j}}) = 0 . \tag{3.22}$$

These are n_v complex equations, and comparing with (2.4) we see that the $\mathcal{N} = 1$ analysis of the previous section applies verbatim. Thus the AdS_4 moduli space of the vector multiplets is again real and at most of (real) dimension n_v .

Let us postpone the discussion of (3.21) for the non-minimal solution where $\langle \mathcal{M}_{i\mathcal{A}}^\beta \rangle \neq 0$ and instead turn to the first condition in (3.14). The variation of $\langle X^\Lambda \Theta_\Lambda^\lambda k_\lambda^u \rangle$ yields

$$\frac{1}{2} C_{\alpha\beta} \langle \mathcal{M}_{i\mathcal{A}}^\beta \rangle \delta t^i + \langle \mathbb{M}_{\mathcal{A}\alpha\mathcal{B}\beta} \mathcal{U}_u^{\mathcal{B}\beta} \rangle \delta q^u = 0 , \tag{3.23}$$

¹⁵The fourth hyper-scalar is the Goldstone boson eaten by the vector.

where we defined

$$\mathbb{M}_{\mathcal{A}\alpha\mathcal{B}\beta} = 4e^{K_v/2} X^\Lambda \Theta_\Lambda^\lambda (\nabla_v k_{\lambda u}) \mathcal{U}_{\mathcal{B}\beta}^v \mathcal{U}_{\mathcal{A}\alpha}^u = 4C_{\alpha\beta} S_{AB} - \epsilon_{AB} M_{\alpha\beta} , \quad (3.24)$$

and the last equality used (3.7) and (3.11). As before let us first analyze the minimal solution with $\langle \mathcal{M}_{i\mathcal{A}}^\beta \rangle = 0$ and the first term in (3.23) vanishing. Due to (3.16) we rotate into the frame where only

$$P_0^3 = 2\langle e^{K_v/2} X^\Lambda \Theta_\Lambda^\lambda P_\lambda^3 \rangle = \langle e^{K_v} \text{Im}(\hat{\Lambda} \bar{X}^\Sigma) \Omega_{\Sigma\Lambda} X^\Lambda \rangle , \quad (3.25)$$

is non-zero, and using (3.8) and (3.17) the $(4n_h \times 4n_h)$ matrix $\mathbb{M}_{\mathcal{A}\alpha\mathcal{B}\beta}$ then takes the form

$$\mathbb{M}_{\mathcal{A}\alpha\mathcal{B}\beta} = \begin{pmatrix} 0 & C_{\alpha\beta} P_0^3 - M_{\alpha\beta} \\ C_{\alpha\beta} P_0^3 + M_{\alpha\beta} & 0 \end{pmatrix} . \quad (3.26)$$

Note that $C_{\alpha\beta}$ is anti-symmetric while $M_{\alpha\beta}$ is symmetric, and as a consequence $\mathbb{M}^\top = -\mathbb{M}$. That is, \mathbb{M} is altogether antisymmetric, and thus its eigenvalues come in pairs. For the case at hand this means that the hypermultiplet scalars become massive pairwise, and similarly the zero modes come in pairs. Furthermore, C is the flat $Sp(n_h)$ metric and thus by appropriately tuning $M_{\alpha\beta}$ one can reduce the rank of both $(2n_h \times 2n_h)$ matrices $C_{\alpha\beta} P_0^3 \pm M_{\alpha\beta}$ to be n_h , but no smaller. In other words $\mathbb{M}_{\mathcal{A}\alpha\mathcal{B}\beta}$ has at least rank $2n_h$ (instead of $4n_h$), and at most $2n_h$ scalars can be massless.

Let us now show that the $\mathcal{N} = 2$ AdS_4 moduli space of the hypermultiplet scalars is Kähler. First of all there is a complex structure given by I^3 that acts on the flat indices as σ^3 . Indeed it is easy to check that

$$(\sigma^3)_A^C \mathbb{M}_{C\alpha\mathcal{B}\beta} = -\mathbb{M}_{A\alpha\mathcal{C}\beta} (\sigma^3)_B^C , \quad (3.27)$$

so that in particular the massless spectrum is invariant, and the moduli space is a complex manifold (with respect to I^3). We will now show that $\langle K^1 \rangle = \langle K^2 \rangle = 0$ which via (3.3) then implies

$$\langle dK^3 \rangle = -\langle \omega^1 \wedge K^2 \rangle + \langle \omega^2 \wedge K^1 \rangle = 0 . \quad (3.28)$$

Thus K^3 is closed on the $\mathcal{N} = 2$ locus, which shows that the AdS_4 moduli space is not only complex but actually Kähler, with K^3 as its Kähler form. Another consequence of $\langle K^1 \rangle = \langle K^2 \rangle = 0$ is that the resulting moduli space is real with respect to the complex structures I^1 and I^2 .

Let us prove $\langle K^1 \rangle = 0$ explicitly — $\langle K^2 \rangle = 0$ then follows by permutations of indices. Using (3.9) and the algebra of the Pauli matrices we can write K^1 as

$$\begin{aligned} \langle K_{uv}^1 \rangle &= ((\sigma^2)_A^D (\sigma^3)_D^C - (\sigma^3)_A^D (\sigma^2)_D^C) \epsilon_{CB} C_{\alpha\beta} \langle U_u^{\alpha A} U_v^{\beta B} \rangle \\ &= ((\sigma^2)_A^C (\sigma^3)_{CB} + (\sigma^3)_{AC} (\sigma^2)_B^C) C_{\alpha\beta} \langle U_u^{\alpha A} U_v^{\beta B} \rangle \\ &= ((\sigma^2)_A^C \epsilon_{CB} + \epsilon_{AC} (\sigma^2)_B^C) \langle M_{\alpha\beta} U_u^{\alpha A} U_v^{\beta B} \rangle = 0 , \end{aligned} \quad (3.29)$$

where in the last step we used (3.23). This completes our proof. We have shown that the AdS_4 moduli space of the scalars in $\mathcal{N} = 2$ hypermultiplets is a Kähler submanifold of the parent quaternionic-Kähler manifold (which has real dimension $4n_h$), and has real dimension at most $2n_h$. In fact this coincides with the mathematical theorem that a Kähler submanifold of a quaternionic-Kähler manifold can have at most half the dimension of the parent [22].

Thus for the minimal solution in (3.18) the moduli space is a direct product of a real manifold spanned by the vector multiplet scalars and a Kähler manifold spanned by the hypermultiplet scalars. This is indeed consistent with the AdS/CFT expectation of a Kähler moduli space for the three-dimensional boundary theory, since in three-dimensional supersymmetric theories with four supercharges the vector multiplet contains as bosonic components a vector and a real scalar. Dualizing the vector to a real scalar, the entire multiplet becomes dual to a chiral multiplet. The associated Kähler moduli space can only appear after dualizing the vector: in the four-dimensional bulk description the Kähler structure is not visible. Furthermore, in the minimal solution we have a direct product of moduli spaces, which is a special case of the generic situation in three-dimensional supersymmetric theories. As we will see shortly, this feature will not hold for the non-minimal solution, and a mixing between vector and hypermultiplet scalars occurs.

As promised let us now discuss the moduli space for the non-minimal solution in (3.18) which has $\langle k_0^u \rangle = 0, \langle k_{\lambda \neq 0}^u \rangle \neq 0, \langle M_{iA}^\alpha \rangle \neq 0$. Thus we have to reconsider the variations (3.21) and (3.23), as both sets of equation have additional terms. Before we plunge into the technical analysis let us sketch the intuition. For $\langle k_{\lambda \neq 0}^u \rangle \neq 0$ the gauge symmetry is spontaneously broken and n_m long vector multiplets become massive. In this case a vector multiplet eats an entire hypermultiplet and thus consists of a Goldstone boson from the hypermultiplet and five massive scalars, two from the vector multiplets and three from the hypermultiplets. As we will see shortly this situation is manifest in (3.21) and (3.23), but the structure of the moduli space is unchanged and only its dimension is reduced as n_m additional vector and hypermultiplets are fixed.

We start by rewriting (3.23) and (3.21) in a more explicit form, using (3.11), and simplify the analysis by rotating into the frame where only P_0^3 is non-zero. This yields

$$-2\Theta_\Lambda^\lambda \langle e^{K_V/2} U_{\alpha A u} k_\lambda^u \nabla_i X^\Lambda \rangle \delta t^i + \langle \mathbb{M}_{A\alpha B\beta} \mathcal{U}_u^{B\beta} \rangle \delta q^u = 0, \quad (3.30)$$

$$\Theta_\Lambda^\lambda \langle k_\lambda^w K_{wu}^{1,2} \rangle \delta q^u = 0, \quad (3.31)$$

$$\langle e^{K_V/2} \text{Im}(\hat{\Lambda} \bar{X}^\Sigma) \rangle \Omega_{\Sigma\Lambda} \langle (\nabla_i \nabla_j X^\Lambda) \delta t^j - 2 \langle X^\Lambda g_{i\bar{j}} \rangle \delta \bar{t}^{\bar{j}} \rangle + 4 \langle \nabla_i X^\Lambda \rangle \Theta_\Lambda^\lambda \langle k_\lambda^w K_{wu}^3 \rangle \delta q^u = 0. \quad (3.32)$$

As we already stated, for an Abelian theory the Goldstone bosons have to be recruited out of hypermultiplets and thus the n_m Goldstone directions drop out of (3.30)-(3.32). This can be seen by explicitly inserting $\delta q^u \sim \Theta_\Sigma^\kappa k_\kappa^u$. For (3.31) and (3.32) we can use the equivariance

condition (see for example [17])

$$K_{uv}^x k_\lambda^u k_\sigma^v - \frac{1}{2} \epsilon^{xyz} P_\lambda^y P_\sigma^z = \frac{1}{2} f_{\lambda\sigma}^\rho P_\rho^x, \quad (3.33)$$

where $f_{\lambda\sigma}^\rho$ are the structure constants of the gauge algebra, i.e.

$$[k_\lambda, k_\sigma] = f_{\lambda\sigma}^\rho k_\rho, \quad k_\lambda \equiv k_\lambda^u \partial_u. \quad (3.34)$$

Since we only consider Abelian gauged isometries we have $f_{\lambda\sigma}^\rho = 0$, and as only P_λ^3 is non-vanishing, we find from (3.33) that

$$K_{uv}^x k_\lambda^u k_\sigma^v = 0. \quad (3.35)$$

This immediately implies that the Goldstone directions $\delta q^u \sim \Theta_\Sigma^\kappa k_\kappa^u$ indeed drop out of (3.31) and (3.32). For (3.30) we use (3.24) to find that the Goldstone directions do not contribute as a consequence of

$$X^\Lambda \Theta_\Lambda^\lambda \Theta_\Sigma^\sigma k_\sigma^v \nabla_v k_\lambda^u = X^\Lambda \Theta_\Lambda^\lambda \Theta_\Sigma^\sigma [k_\sigma, k_\lambda]^u = 0, \quad (3.36)$$

where we used (3.14) in the first equality, and in the second equality we used that the gauged directions are Abelian. Thus, the Goldstone directions also drop out of (3.30). For later use let us note that due to (3.27) and $(\sigma^3)_B^A U_u^{\alpha\beta} = U_v^{\alpha\beta} (I^3)_u^v$, the deformations $\delta q^u \sim \Theta_\Sigma^\sigma (I^3)_v^u k_\sigma^v$ also do not appear in (3.30).

Let us now discuss which scalars are fixed by (3.30)-(3.32). From (3.31) we immediately see that $2n_m$ hypermultiplet scalars become massive. They are related to a Goldstone boson by I^1 or I^2 and therefore reside in the same hypermultiplet. We continue with (3.30) and consider first the indices $(\mathcal{A}\alpha)$ for which $U_{\alpha\mathcal{A}u} k_\lambda^u = 0$ holds. In this case the first term vanishes and the discussion for the minimal solutions applies. For indices which have $U_{\alpha\mathcal{A}u} k_\lambda^u \neq 0$ (3.30) fixes the n_m complex scalars that are in the same multiplets as the massive vectors. Finally let us turn to (3.32). For indices i that have $\mathcal{M}_{i\mathcal{A}}^\alpha = 0$ the last term in (3.32) vanishes and one is left with the minimal case for the vector multiplets that we discussed above. For indices i that have $\mathcal{M}_{i\mathcal{A}}^\alpha \neq 0$ the last term fixes n_m additional scalars which are related by I^3 to the Goldstone direction.

We have thus shown that for any gauged non-vanishing Killing vector, $5n_m$ scalars are massive: these are members of n_m long massive vector multiplets. Therefore, at least $(n_v + n_m)$ vector multiplet scalars become massive, and the moduli space has at most real dimension $(n_v - n_m)$. For the hypermultiplets, at least $2n_h + n_m$ scalars become massive, and n_m scalars are eaten. Thus, there are at most $(n_h - n_m)$ complex directions corresponding to hypermultiplet moduli. However, compared to the minimal solution both sectors mix non-trivially. The massive scalars in the long vector multiplets are actually combinations of vector multiplet and hypermultiplet scalars, due to the fact that the three mass matrices \mathcal{M}_{ijAB} , $\mathcal{M}^{\alpha\beta}$, and $\mathcal{M}_{i\mathcal{A}}^\alpha$ in (3.21) and (3.23) are nonvanishing. This in turn leads to a mixing of vector multiplet and hypermultiplet scalar fields in the kinetic terms, and thus the moduli space is no longer a direct product.

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Appendix

A Relation to Global $\mathcal{N} = 1$ Supersymmetry in AdS_4

In this appendix we recall the global limit of a generic supergravity theory in AdS_4 . For this we need to restore the dependence on the gravitational coupling $\kappa \equiv M_{\text{Pl}}^{-1}$. We take all fields to have mass dimension one (denoted by $[\phi] = 1$), and correspondingly take $[K] = 2$, $[W] = 3$, and $[V] = 4$. In these conventions the potential (2.2) reads

$$V = e^{\kappa^2 K} \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3\kappa^2 |W|^2 \right), \quad \text{with} \quad D_i W = \partial_i W + \kappa^2 K_i W. \quad (\text{A.1})$$

Without loss of generality we can parameterize the superpotential as

$$W = W_0 + W_g, \quad (\text{A.2})$$

with

$$\langle W_g \rangle = 0 \quad \text{and} \quad \kappa^2 W_0 \equiv \Lambda \neq 0, \quad (\text{A.3})$$

where we have taken W_0 to be real. Furthermore, by a choice of Kähler gauge we may set $\langle K \rangle = 0$. The cosmological constant $\kappa^2 \langle V \rangle$ that appears in the Einstein equations is related to Λ by

$$\kappa^2 \langle V \rangle = -3\Lambda^2. \quad (\text{A.4})$$

In order to obtain an AdS_4 background for global supersymmetry from supergravity, one needs to take the limit $\kappa \rightarrow 0$, Λ fixed. Expanding V in this limit one arrives at¹⁶ [4]

$$V = K^{i\bar{j}} \mathcal{D}_i \bar{\mathcal{D}}_{\bar{j}} - 3\Lambda(W_g + \bar{W}_g + \Lambda K) - 3\kappa^{-2} \Lambda^2 + \mathcal{O}(\kappa^2), \quad (\text{A.5})$$

where $\mathcal{D}_i \equiv \partial_i W + K_i \Lambda$ (not to be confused with $D_i \equiv \partial_i + K_i$) vanishes at the supersymmetric minimum, i.e. $\langle \mathcal{D}_i \rangle = 0$. The first derivative of V reads

$$\partial_k V = \nabla_k V = K^{i\bar{j}} (\nabla_k \mathcal{D}_i) \bar{\mathcal{D}}_{\bar{j}} - 2\Lambda \mathcal{D}_k + \mathcal{O}(\kappa^2), \quad (\text{A.6})$$

which indeed vanishes at the minimum, because $\langle \mathcal{D}_i \rangle = 0$.

From (A.6) we can compute the ‘mass matrix’

$$\begin{aligned} \langle \nabla_k \nabla_{\bar{l}} V \rangle &= -2K_{k\bar{l}} \Lambda^2 + K^{i\bar{j}} m_{ki} \bar{m}_{\bar{l}\bar{j}} \\ \langle \nabla_k \nabla_l V \rangle &= -m_{kl} \Lambda \end{aligned} \quad (\text{A.7})$$

where

$$m_{ki} = e^{\kappa^2 K/2} \nabla_k \mathcal{D}_i \quad (\text{A.8})$$

¹⁶Note that V diverges in this limit, but the Einstein equations are finite.

is the fermionic mass matrix. Decomposing $\phi^i = \frac{1}{\sqrt{2}}(A^i + iB^i)$ we obtain the mass matrices for A^i and B^i ,

$$\begin{aligned}(m_A^2)_{kl} &= K^{i\bar{j}}m_{ki}\bar{m}_{i\bar{j}} - \Lambda m_{kl} - 2\Lambda^2 K_{k\bar{l}} , \\ (m_B^2)_{kl} &= K^{i\bar{j}}m_{ki}\bar{m}_{i\bar{j}} + \Lambda m_{kl} - 2\Lambda^2 K_{k\bar{l}} , \\ (m_{AB}^2)_{kl} &= 2 \operatorname{Im} m_{kl} \Lambda .\end{aligned}\tag{A.9}$$

The mass matrices (A.9) agree with [3] when there is only one chiral multiplet and $\operatorname{Im} m$ is taken to be zero. For $\operatorname{Im} m \neq 0$ we can consider for simplicity the case of one multiplet with canonical Kähler metric. In this case the matrices are easily diagonalized, with the mass-squared eigenvalues

$$M_{\pm}^2 = |m|^2 \pm \Lambda|m| - 2\Lambda^2.\tag{A.10}$$

We see that one cannot have $M_+ = M_- = 0$ without setting $\Lambda = 0$.

One might be tempted to think that flat directions in the potential arise when $m_{ij} = 0$, as in flat space. This is incorrect, as we now explain in the simple case of a single chiral multiplet scalar ϕ (so that $m_{ij} \rightarrow m$), with $K = \phi\bar{\phi}$. The equation of motion in Einstein frame reads

$$\nabla^2 \phi = \frac{1}{\sqrt{g}} \partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu \phi = V_{\bar{\phi}} \quad ,\tag{A.11}$$

and for $m = 0$ the right hand side does not vanish, cf. (A.6). In fact, a scalar field ϕ with $m = 0$ is a *conformally coupled* scalar. To see this, we perform a Weyl rescaling to the Jordan frame, so that the Lagrangian reads

$$\mathcal{L} = -\frac{1}{2\kappa^2} R e^{-\frac{\kappa^2}{3}K} - K^J(\phi, \bar{\phi}) \partial_\mu \phi \partial^\mu \bar{\phi} - V^J(\phi, \bar{\phi}) + \dots \quad ,\tag{A.12}$$

where K^J, V^J are the metric and potential in the Jordan frame. In the limit $\kappa \rightarrow 0, \Lambda$ fixed one finds

$$V^J = K^{i\bar{j}} \mathcal{D}_i \bar{\mathcal{D}}_{\bar{j}} - 3\Lambda(W_g + \bar{W}_g) - \Lambda^2 K - 3\kappa^{-2} \Lambda^2 + \mathcal{O}(\kappa^2) \quad ,\tag{A.13}$$

where compared to (A.5) only the coefficient of the $\Lambda^2 K$ term has changed. As a consequence, the mass matrix (A.10) in the Jordan frame takes the form

$$M_{J\pm}^2 = |m|(|m| \pm \Lambda)\tag{A.14}$$

which vanishes for $m = 0$. Moreover, the equation of motion in the Jordan frame becomes

$$\nabla^2 \phi = V_{\bar{\phi}}^J - \frac{R}{6} \phi + \mathcal{O}(\kappa^2) = V_{\bar{\phi}}^J - 2\Lambda^2 \phi + \mathcal{O}(\kappa^2) = V_{\bar{\phi}} \quad ,\tag{A.15}$$

where in the second equality the Einstein equations have been used, and the final relation uses (A.5). Thus, $m = 0$ implies that $V_{\bar{\phi}}^J = 0$, but that (for $\phi \neq 0$) $V_{\bar{\phi}} \neq 0$, and hence $\nabla^2 \phi \neq 0$. In summary, a field with $m = 0$ is conformally coupled, but is not a modulus.

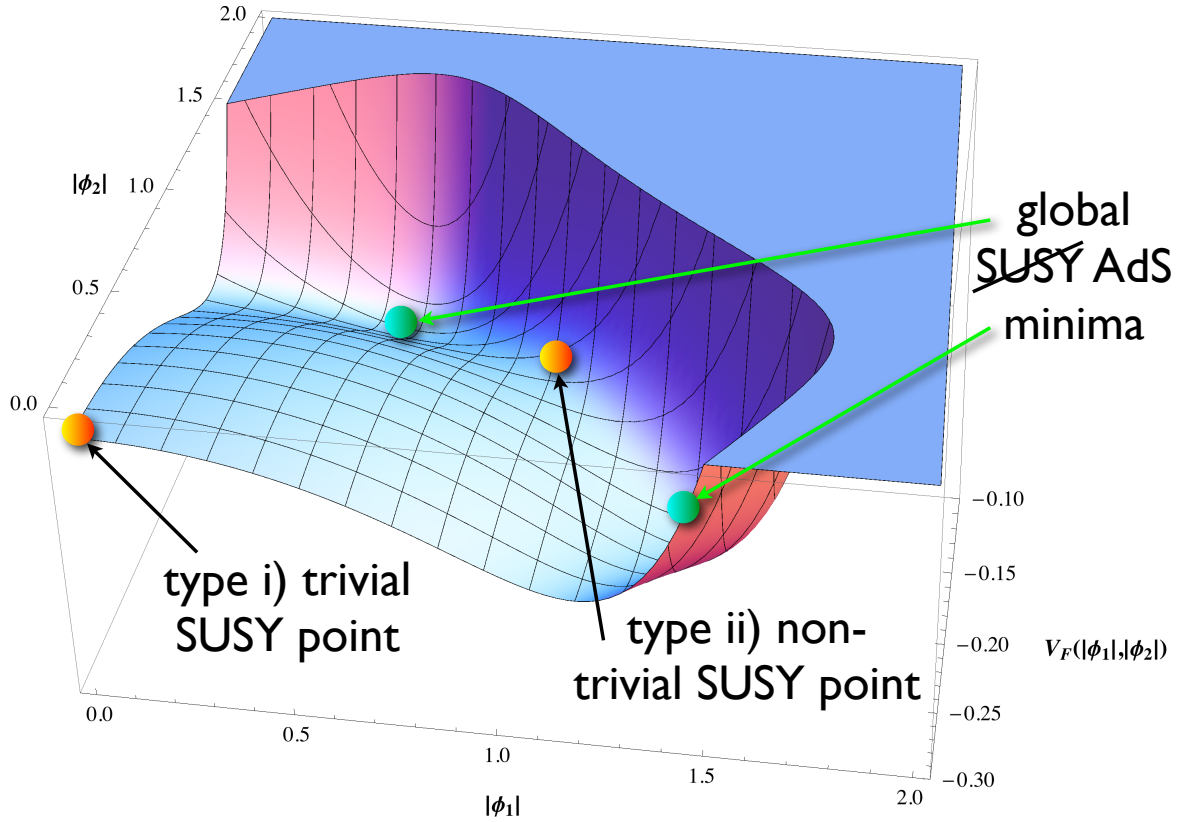


Figure 1: The potential $V(|\phi_1|, |\phi_2|)$ at the axion minimum $\chi = \pi/2$ for $m = c/2 = 1/10$.

B Further Examples of Moduli Spaces in AdS_4

In this appendix we give further explicit examples of $\mathcal{N} = 1$ supergravities with degenerate AdS_4 backgrounds. Let us first supply the details of the third example discussed in section 2.2, where K and W are given by

$$K = \phi_1 \bar{\phi}_1 + \phi_2 \bar{\phi}_2, \quad W = c + m\phi_1 \phi_2, \quad (\text{B.1})$$

with c and m real. One easily computes

$$D_{\phi_1} W = m\phi_2 + \bar{\phi}_1 W, \quad D_{\phi_2} W = m\phi_1 + \bar{\phi}_2 W \quad (\text{B.2})$$

and solves $D_{\phi_1} W = D_{\phi_2} W = 0$ by parameterizing $\phi_1 = r_1 e^{i(\chi+\rho)}$, $\phi_2 = r_2 e^{i(\chi-\rho)}$. The type A trivial solution $\phi_1 = \phi_2 = 0$ is immediately apparent.

If c and m have opposite sign and furthermore $|c| \geq |m|$, one finds one branch of the type

B non-trivial solution,

$$r_1 = r_2 = \sqrt{-\frac{c}{m} - 1}, \quad \chi = 0, \quad \rho \text{ arbitrary.} \quad (\text{B.3})$$

If c and m have the same sign and again $|c| \geq |m|$, one finds instead the other branch of the type B non-trivial solution,

$$r_1 = r_2 = \sqrt{\frac{c}{m} - 1}, \quad \chi = \pi/2, \quad \rho \text{ arbitrary.} \quad (\text{B.4})$$

These solutions can also be expressed as $\phi_1 = \pm \bar{\phi}_2$, as in section 2.2. For $|c| = |m|$ these solutions coincide with the trivial solution $\phi_1 = \phi_2 = 0$. We see that ρ is a flat direction for the type B solution with $|c| > |m|$, which can be seen immediately from the fact that K and W are independent of ρ . Fig. 1 shows an example of V for $m = c/2$, displaying both solution A at the origin and the second type of the solutions B as saddles at $(\phi_1, \phi_2) = (-1, 1)$ and at $(\phi_1, \phi_2) = (1, -1)$.

Let us study the minima of the potential in slightly more detail, as they reveal a somewhat unusual structure. The scalar potential in the variables r_1, r_2, χ reads explicitly

$$V = e^{r_1^2 + r_2^2} (c^2 (r_1^2 + r_2^2 - 3) + 2cmr_1r_2 (r_1^2 + r_2^2 - 1) \cos(2\chi) + m^2 (r_1^4 r_2^2 + r_1^2 (r_2^4 + r_2^2 + 1) + r_2^2)) . \quad (\text{B.5})$$

The eigenvalues of the matrix of second derivatives at $\phi_1 = \phi_2 = 0$ are given by

$$V_{ii} = \{0, 0, -2(2c - m)(c + m), -2(2c + m)(c - m)\} , \quad (\text{B.6})$$

with a cosmological constant $\langle V \rangle = -3c^2$. We see that, as expected, both axions remain flat.

For the non-trivial solutions B the eigenvalues of the matrix of second derivatives read

$$V_B = \left\{ 0, -4m^2 e^{\frac{2c}{m} - 2}, 4e^{\frac{2c}{m} - 2} (c - m)(2c + m), 4cm^{-1} e^{\frac{2c}{m} - 2} (2c - 3m)(c - m) \right\}$$

for $c \geq m$ and

$$V_B = \left\{ 0, -4m^2 e^{-\frac{2(c+m)}{m}}, -4e^{-\frac{2(c+m)}{m}} (m - 2c)(c + m), -4cm^{-1} e^{-\frac{2(c+m)}{m}} (c + m)(2c + 3m) \right\}$$

for $c \leq -m$. The cosmological constant at these extrema turns out to be $\langle V \rangle = -3m^2 e^{2\frac{|c|-m}{m}}$. Hence, all of these solutions contain a scalar with negative mass squared, allowed by the Breitenlohner-Freedman (BF) bound

$$m_{BF}^2 = -\frac{27}{4} e^K |W|^2 = -\frac{27}{4} e^{\frac{2c}{m} - 2} m^2 . \quad (\text{B.7})$$

This universal BF scalar is a linear combination of the two radial modes $r_{1,2}$, while the orthogonal linear combination is massive for $|c| > m$. The ρ axion stays massless as expected.

Note that the last eigenvalue gives the mass squared of the χ axion. We see that for $|c| > \frac{3}{2}m$ this axion is massive, while its mass vanishes for $|c| = \frac{3}{2}m$, and it becomes a BF-stable tachyon for $m < |c| < \frac{3}{2}m$. This behavior is clear from the scalar potential (B.5).

Finally, note the unusual vacuum structure of the model. The supersymmetric critical points of type A and B comprise Breitenlohner-Freedman stable tachyonic maxima or saddle points, respectively. We find that for $|c| > m$ the global AdS minima of the model *break* supersymmetry. Hence, the global minima of the scalar potential break supersymmetry, and they have lower vacuum energy than any of the supersymmetric critical points. This is a feature which contradicts intuition from the global case, but is often realized in the context of racetrack models of nonperturbative moduli stabilization in string theory.

As another example let us consider a supergravity defined by

$$K = -\ln(T + \bar{T}) , \quad W = e^{aT} . \quad (\text{B.8})$$

The supersymmetric minimum $D_T W = W(a - (T + \bar{T})^{-1}) = 0$ is found for $T + \bar{T} = a^{-1}$ with $\langle V \rangle = -3ae$. It only exists if a is real and positive, and then $\text{Im} T$ is the modulus of this AdS_4 background. This can be generalized for a generic $K = K(T + \bar{T})$ with a shift symmetry, as long as $K_T \neq 0, K_{T\bar{T}} > 0$. In this case $\text{Re} T$ is fixed by $K_T = -a$. In string theory $a > 0$ does not easily appear, as W then diverges in the large T limit. However, a superpotential $W = A \exp(-b(S - aT/b))$ with $b, a > 0$ can arise, for example, in heterotic backgrounds, where the second term in the exponent can be a threshold correction and S is the dilaton.

As a further example consider

$$K = \phi\bar{\phi} , \quad W = a\phi^b , \quad (\text{B.9})$$

which has an R-symmetry. The supersymmetric minimum $D_\phi W = a\phi^{b-1}(b + \bar{\phi}\phi) = 0$ is AdS_4 for $b < 0, a \neq 0$ and $\phi\bar{\phi} = |b|$. In this case the phase of ϕ is the modulus and we have $\langle V \rangle = -3e^{|b|}|a|^2|b|^b$. Note however that the superpotential needs to be singular at the origin.

Our final example realizes a compact $U(1)$ moduli space but is a bit more involved. This is due to the fact that this case is set up to avoid singularities in field space as well as the use of an arbitrary constant term in the superpotential. Consider

$$K = \phi\bar{\phi} + \chi\bar{\chi} + X\bar{X} , \quad W = X(\chi\phi - \mu^2) + m\phi\chi , \quad (\text{B.10})$$

which are invariant under $\phi \rightarrow e^{i\alpha}\phi, \chi \rightarrow e^{-i\alpha}\chi$. One easily computes

$$\begin{aligned} D_X W &= \chi\phi - \mu^2 + \bar{X}W , \\ D_\phi W &= (m + X)\chi + \bar{\phi}W , \\ D_\chi W &= (m + X)\phi + \bar{\chi}W . \end{aligned} \quad (\text{B.11})$$

This theory has a supersymmetric AdS_4 background with a flat direction corresponding to the $U(1)$ symmetry. The full solution can be obtained analytically but is not particularly illuminating. Instead we display the solution for $m \ll \mu$, which captures the essential features:

$$\begin{aligned} \langle |\chi| \rangle &= \langle |\phi| \rangle = \mu \cdot \left[1 + \frac{m^2}{2} (1 + \mu^2) \right] + \mathcal{O}(m^3) , \\ \langle X \rangle &= -(1 + \mu^2) m + \mathcal{O}(m^3) , \\ \langle W \rangle &= m\mu^2 \cdot [1 - m^2\mu^2(1 + \mu^2)] + \mathcal{O}(m^4) , \end{aligned} \tag{B.12}$$

where a common phase of ϕ and χ is left undetermined.

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