High-precision predictions for the light CP-even Higgs Boson Mass of the MSSM

T. Hahn[1], S. Heinemeyer[2], W. Hollik[1], H. Rzehak[3], G. Weiglein[4]

[1]*Max-Planck-Institut für Physik, Föhringer Ring 6, D–80805 München, Germany*
[2]*Instituto de Física de Cantabria (CSIC-UC), Santander, Spain*
[3]*Albert-Ludwigs-Universität Freiburg, Physikalisches Institut, D–79104 Freiburg, Germany*
[4]*DESY, Notkestraße 85, D–22607 Hamburg, Germany*

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For the interpretation of the signal discovered in the Higgs searches at the LHC it will be crucial in particular to discriminate between the minimal Higgs sector realised in the Standard Model (SM) and its most commonly studied extension, the Minimal Supersymmetric SM (MSSM). The measured mass value, having already reached the level of a precision observable with an experimental accuracy of about 500 MeV, plays an important role in this context. In the MSSM the mass of the light CP-even Higgs boson, M_h, can directly be predicted from the other parameters of the model. The accuracy of this prediction should at least match the one of the experimental result. The relatively high mass value of about 126 GeV has led to many investigations where the scalar top quarks are in the multi-TeV range. We improve the prediction for M_h in the MSSM by combining the existing fixed-order result, comprising the full one-loop and leading and subleading two-loop corrections, with a resummation of the leading and subleading logarithmic contributions from the scalar top sector to all orders. In this way for the first time a high-precision prediction for the mass of the light CP-even Higgs boson in the MSSM is possible all the way up to the multi-TeV region of the relevant supersymmetric particles. The results are included in the code `FeynHiggs`.

I. INTRODUCTION

After the spectacular discovery of a signal in the Higgs-boson searches at the LHC by ATLAS and CMS [1, 2], the exploration of the properties of the observed particle is meanwhile in full swing. In particular, the observation in the $\gamma\gamma$ and the $ZZ^{(*)} \to 4\ell$ channels made it possible to determine its mass with already a remarkable precision. Currently, the combined mass measurement from ATLAS is $125.5 \pm 0.2 \pm 0.6$ GeV [3], and the one from CMS is $125.7 \pm 0.3 \pm 0.3$ GeV [4]. The other properties that have been determined so far (with significantly lower accuracy) are compatible with the minimal realisation of the Higgs sector within the Standard Model (SM) [5], but a large variety of other interpretations is possible as well, corresponding to very different underlying physics. While within the SM the Higgs-boson mass is just a free parameter, in theories beyond the SM (BSM) the mass of the particle that is identified with the signal at about 126 GeV can often be directly predicted, providing an important test of the model. The most popular BSM model is the Minimal Supersymmetric Standard Model (MSSM) [6], whose Higgs sector consists of two scalar doublets accommodating five physical Higgs bosons. In lowest order these are the light and heavy CP-even h and H, the CP-odd A, and the charged Higgs bosons $H^\pm$.

The parameters characterising the MSSM Higgs sector at lowest order are the gauge couplings, the mass of the CP-odd Higgs boson, M_A, and $\tan\beta \equiv v_2/v_1$, the ratio of the two vacuum expectation values. Accordingly, all other masses and mixing angles can be predicted in terms of those parameters, leading to the famous tree-level upper bound for the mass of the light CP-even Higgs boson, $M_h \leq M_Z$, determined by the mass M_Z of the Z boson. This tree-level upper bound, which arises from the gauge

sector, receives large corrections from the Yukawa sector of the theory, which can amount up to $\mathcal{O}(50\%)$ (depending on the model parameters) upon incorporating the full one-loop and the dominant two-loop contributions [7]. The prediction for the light CP-even Higgs-boson mass in the MSSM is affected by two kinds of theoretical uncertainties, namely parametric uncertainties induced by the experimental errors of the input parameters, and intrinsic theoretical uncertainties that are due to unknown higher-order corrections. Concerning the SM input parameters, the dominant source of parametric uncertainty is the experimental error on the top-quark mass, m_t. Very roughly, the impact of the experimental error on m_t on the prediction for M_h scales like $\delta M_h^{\mathrm{para},m_t}/\delta m_t^{\mathrm{exp}} \sim 1$ [8]. As a consequence, high-precision top-physics providing an accuracy on m_t much below the GeV-level is a crucial ingredient for precision physics in the Higgs sector [8]. Concerning the intrinsic theoretical uncertainties caused by unknown higher-order corrections, an overall estimate of $\delta M_h^{\mathrm{intr}} \sim 3$ GeV has been given in Refs. [7, 9] (the more recent inclusion of the leading $\mathcal{O}(\alpha_t \alpha_s^2)$ 3-loop corrections [10] has slightly reduced this estimated uncertainty by few $\mathcal{O}(100$ MeV)), while it was pointed out that a more detailed estimate needs to take into account the dependence on the considered parameter region of the model. In particular, the uncertainty of this fixed-order prediction is expected to be much larger for scalar top masses in the multi-TeV range. This region of the parameter space has received considerable attention recently, partly because of the relatively high value of $M_h \approx 126$ GeV, which generically requires either large stop masses or large mixing in the scalar top sector, and partly because of the limits from searches for supersymmetric (SUSY) particles at the LHC. While within the general MSSM the lighter

scalar superpartner of the top quark is allowed to be relatively light (down to values even as low as m_t), both with respect to the direct searches and with respect to the prediction for M_h (see e.g. Ref. [11]), the situation is different in more constrained models. For instance, global fits in the Constrained MSSM (CMSSM) prefer scalar top masses in the multi-TeV range [12, 13].

Here we present a significantly improved prediction for the mass of the light $\mathcal{CP}$-even Higgs boson in the MSSM, which is expected to have an important impact on the phenomenology in the region of large squark masses and on its confrontation with the experimental results.

II. IMPROVED PREDICTION FOR M_h

In the MSSM with real parameters (we restrict to this case for simplicity; for the treatment of complex parameters see Refs. [14, 15] and references therein), using the Feynman diagrammatic (FD) approach, the higher-order corrected $\mathcal{CP}$-even Higgs boson masses are derived by finding the poles of the (h, H)-propagator matrix. The inverse of this matrix is given by $-i\times$

$$\begin{pmatrix} p^2 - m_{h,\mathrm{tr}}^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_{H,\mathrm{tr}}^2 + \hat{\Sigma}_{HH}(p^2) \end{pmatrix}, \quad (1)$$

where $m_{h,H,\mathrm{tr}}$ denote the tree-level masses, and $\hat{\Sigma}_{hh,HH,hH}(p^2)$ are the renormalized Higgs boson self-energies evaluated at the squared external momentum p^2 (for the computation of the leading contributions to those self-energies it is convenient to use the basis of the fields ϕ_1, ϕ_2, which are related to h, H via the (tree-level) mixing angle α: $h = -\sin\alpha\,\phi_1 + \cos\alpha\,\phi_2$, $H = \cos\alpha\,\phi_1 + \sin\alpha\,\phi_2$). The status of higher-order corrections to these self-energies is quite advanced. The complete one-loop result within the MSSM is known [16, 17]. The by far dominant one-loop contribution is the $\mathcal{O}(\alpha_t)$ term due to top and stop loops ($\alpha_t \equiv h_t^2/(4\pi)$, h_t being the top-quark Yukawa coupling). The computation of the two-loop corrections has meanwhile reached a stage where all the presumably (sub)dominant contributions are available, see Ref. [7] and references therein. The public code FeynHiggs [7, 14, 18, 19] includes all of the above corrections, where the on-shell (OS) scheme for the renormalization of the scalar quark sector has been used (another public code, based on the Renormalization Group (RG) improved Effective Potential, is CPsuperH [20]). A full 2-loop effective potential calculation (supplemented by the momentum dependence for the leading pieces and the leading 3-loop corrections) has been published [21]. However, no computer code is publicly available. Most recently another leading 3-loop calculation at $\mathcal{O}(\alpha_t\alpha_s^2)$ became available (based on a $\overline{\mathrm{DR}}$ or a "hybrid" renormalisation scheme for the scalar top sector), where the numerical evaluation depends on the various SUSY mass hierarchies [10], resulting in the code H3m (which adds the 3-loop corrections to the FeynHiggs result).

We report here on an improved prediction for M_h where we combine the fixed-order result obtained in the OS scheme with an all-order resummation of the leading and subleading contributions from the scalar top sector. We have obtained the latter from an analysis of the RG Equations (RGEs) at the two-loop level [22]. Assuming a common mass scale $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ ($m_{\tilde{t}_{1,2}}$ denote the two scalar top masses, and $M_S \gg M_Z$) for all relevant SUSY mass parameters, the quartic Higgs coupling λ can be evolved via SM RGEs from M_S to the scale Q (we choose $Q = m_t$ in the following) where M_h^2 is to be evaluated (see, for instance, Ref. [23] and references therein),

$$M_h^2 = 2\lambda(m_t)v^2 \ . \quad (2)$$

Here $v \sim 174$ GeV denotes the vacuum expectation value of the SM. Three coupled RGEs are relevant for this evolution, the ones for λ, h_t and g_s (the strong coupling constant, $\alpha_s = g_s^2/(4\pi)$). Since SM RGEs are used, the relevant parameters are given in the $\overline{\mathrm{MS}}$ scheme. We incorporate the one-loop threshold corrections to $\lambda(M_S)$ as given in Ref. [23], with $x_t = X_t/M_S$, $h_t = h_t(M_S)$,

$$\lambda(M_S) = (3\,h_t^4)/(8\,\pi^2)x_t^2\left[1 - 1/12\,x_t^2\right] \ , \quad (3)$$

where as mentioned above X_t is an $\overline{\mathrm{MS}}$ parameter. Eq. (3) ensures that Eq. (2) consists of the "pure loop correction" that will be denoted $(\Delta M_h^2)^{\mathrm{RGE}}$ below. Using RGEs at two-loop order [22], including fermionic contributions from the top sector only, leads to a prediction for the corrections to M_h^2 including leading and subleading logarithmic contributions L^n and $L^{(n-1)}$ at n-loop order ($L \equiv \ln(M_S/m_t)$), originating from the top/stop sector of the MSSM. We have obtained both analytic solutions of the RGEs up to the 7-loop level as well as a numerical solution incorporating the leading and subleading logarithmic contributions up to all orders. In a similar way in Ref. [24] the leading logarithms at 3- and 4-loop order have been evaluated analytically.

Concerning the combination of the higher-order logarithmic contributions obtained from solving the RGEs with the fixed-order FD result implemented in FeynHiggs comprising corrections up to the two-loop level in the OS scheme, we have used the parametrisation of the FD result in terms of the running top-quark mass at the scale m_t, $\overline{m_t} = m_t^{\mathrm{pole}}/(1+4/(3\pi)\alpha_s(m_t^{\mathrm{pole}})-1/(2\pi)\alpha_t(m_t^{\mathrm{pole}}))$, where m_t^{pole} denotes the top-quark pole mass. Avoiding double counting of the logarithmic contributions up to the two-loop level and consistently taking into account the different schemes employed in the FD and the RGE approach, the correction ΔM_h^2 takes the form

$$\Delta M_h^2 = (\Delta M_h^2)^{\mathrm{RGE}}(X_t^{\overline{\mathrm{MS}}}) - (\Delta M_h^2)^{\mathrm{FD,LL1,LL2}}(X_t^{\mathrm{OS}})$$
$$M_h^2 = (M_h^2)^{\mathrm{FD}} + \Delta M_h^2 \ . \quad (4)$$

Here $(M_h^2)^{\mathrm{FD}}$ denotes the fixed-order FD result, $(\Delta M_h^2)^{\mathrm{FD,LL1,LL2}}$ are the logarithmic contributions up to

the two-loop level obtained with the FD approach in the OS scheme, while $(\Delta M_h^2)^{\mathrm{RGE}}$ are the leading and sub-leading logarithmic contributions (either up to a certain loop order or summed to all orders) obtained in the RGE approach, as evaluated via Eq. (2). In all terms of Eq. (4) the top-quark mass is parametrised in terms of $\overline{m_t}$; the relation between $X_t^{\overline{\mathrm{MS}}}$ and X_t^{OS} is given by

$$X_t^{\overline{\mathrm{MS}}} = X_t^{\mathrm{OS}} \left[1 + 2L \left(\alpha_s/\pi - (3\alpha_t)/(16\pi) \right) \right] \qquad (5)$$

up to non-logarithmic terms, and there are no logarithmic contributions in the relation between $M_S^{\overline{\mathrm{MS}}}$ and M_S^{OS}.

Since the higher-order corrections beyond 2-loop order have been derived under the assumption $M_A \gg M_Z$, to a good approximation these corrections can be incorporated as a shift in the prediction for the $\phi_2\phi_2$ self-energy (where ΔM_h^2 enters with a coefficient $1/\sin^2\beta$). In this way the new higher-order contributions enter not only the prediction for M_h, but also all other Higgs sector observables that are evaluated in `FeynHiggs`. The latest version of the code, `FeynHiggs 2.10.0`, which is available at `feynhiggs.de`, contains those improved predictions as well as a refined estimate of the theoretical uncertainties from unknown higher-order corrections. Taking into account the leading and subleading logarithmic contributions in higher orders reduces the uncertainty of the remaining unknown higher-order corrections. Accordingly, the estimate of the uncertainties arising from corrections beyond two-loop order in the top/stop sector is adjusted such that the impact of replacing the running top-quark mass by the pole mass (see Ref. [7]) is evaluated only for the non-logarithmic corrections rather than for the full two-loop contributions implemented in `FeynHiggs`. Further refinements of the RGE resummed result are possible, in particular extending the result to the case of a large splitting between the left- and right-handed soft SUSY-breaking terms in the scalar top sector [25] and to the region of small values of M_A (close to M_Z) as well as including the corresponding contributions from the (s)bottom sector. We leave those refinements for future work.

III. NUMERICAL ANALYSIS

In this section we briefly analyze the phenomenological implications of the improved M_h prediction for large stop mass scales, as evaluated with `FeynHiggs 2.10.0`. The upper plot of Fig. 1 shows M_h as a function of M_S for $X_t = 0$ and $X_t/M_S = 2$ (which corresponds to the minimum and the maximum value of M_h as a function of X_t/M_S, respectively; here and in the following X_t denotes X_t^{OS}). The other parameters are $M_A = M_2 = \mu = 1000$ GeV, $m_{\tilde{g}} = 1600$ GeV (M_2 is the SU(2) gaugino mass term, μ the Higgsino mass parameter and $m_{\tilde{g}}$ the gluino mass) and $\tan\beta = 10$. The plot shows for the two values of X_t/M_S the fixed-order FD result containing corrections up to the two-loop level (labelled as "FH295", which refers to the previous version of the
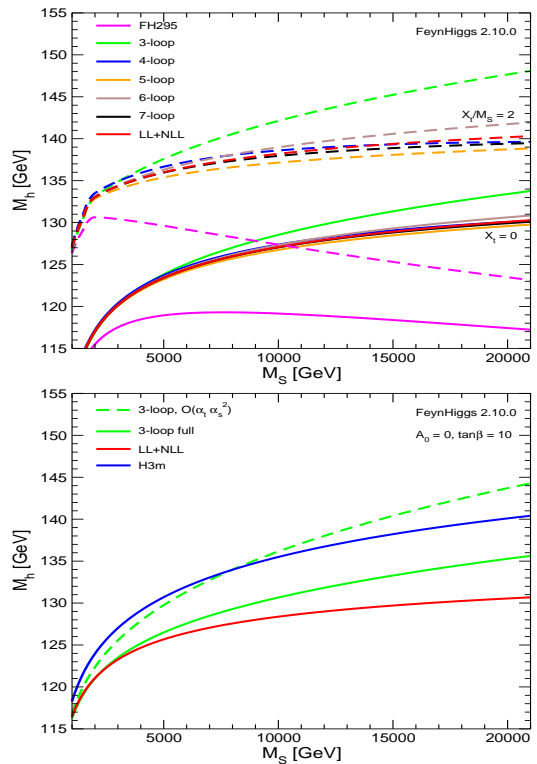

FIG. 1. Upper plot: M_h as a function of M_S for $X_t = 0$ (solid) and $X_t/M_S = 2$ (dashed). The full result ("LL+NLL") is compared with results containing the logarithmic contributions up to the 3-loop, … 7-loop level and with the fixed-order FD result ("FH295"). Lower plot: comparison of `FeynHiggs` (red) with `H3m` (blue). In green we show the `FeynHiggs` 3-loop result at $\mathcal{O}(\alpha_t\alpha_s^2)$ (full) as dashed (solid) line.

code `FeynHiggs`) as well as the latter result supplemented with the analytic solution of the RGEs up to the 3-loop, … 7-loop level (labelled as "3-loop" … "7-loop"). The curve labelled as "LL+NLL" represents our full result where the FD contribution is supplemented by the leading and next-to-leading logarithms summed to all orders. One can see that the impact of the higher-order logarithmic contributions is relatively small for $M_S = \mathcal{O}(1$ TeV), while large differences between the fixed-order result and the improved results occur for large values of M_S. The 3-loop logarithmic contribution is found to have the largest impact in this context, but for $M_S \gtrsim 2500(6000)$ GeV for $X_t/M_S = 2(0)$ also contributions beyond 3-loop are important. A convergence of the higher-order logarithmic contributions towards the full resummed result is visible. At $M_S = 20$ TeV the difference between the 7-loop result and the full resummed result is around 900(200) MeV for $X_t/M_S = 2(0)$. The corresponding deviations stay below 100 MeV for $M_S \lesssim 10$ TeV. The plot furthermore shows that for $M_S \approx 10$ TeV (and the value of $\tan\beta = 10$ chosen here) a predicted value of M_h of about 126 GeV is obtained even for the case of vanishing mixing in the scalar top sector ($X_t = 0$). Since the predicted value of M_h grows further with increasing M_S it becomes appar-

ent that the measured mass of the observed signal, when interpreted as M_h, can be used (within the current experimental and theoretical uncertaintes) to derive an *upper bound* on the mass scale M_S in the scalar top sector, see also Ref. [26].

In the lower plot of Fig. 1 we compare our result with the one based on the code H3m [10] using a CMSSM scenario with $m_0 = m_{1/2} = 200$ GeV ... 15000 GeV, $A_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. The spectra were generated with SoftSusy 3.3.10 [27]. The H3m result (blue line) is based on the FeynHiggs result up to the two-loop order and incorporates the $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections containing also non-logarithmic contributions. Besides our result where FeynHiggs is supplemented by the leading and subleading logarithmic corrections to all orders (red line) we also show the expansion of our result up to the 3-loop level (green solid line), containing at this level the L^3 and L^2 terms, and the result restricting the contributions at the 3-loop level to the ones of $\mathcal{O}(\alpha_t \alpha_s^2)$ (green dashed). We find that the latter result agrees rather well with H3m, with maximal deviations of $\mathcal{O}(1\text{ GeV})$ for $M_S \lesssim 10$ TeV. The observed deviations can be attributed to the L^1 and L^0 terms contained in H3m, to the various SUSY mass hierarchies taken into account in H3m, and to the different renormalization schemes employed. However, one can see that the 3-loop contributions beyond the $\mathcal{O}(\alpha_t \alpha_s^2)$ terms, i.e. corrections of $\mathcal{O}(\alpha_t^2 \alpha_s, \alpha_t^3)$ that are not contained in H3m, have a sizable effect giving rise to a (downward) shift in M_h by ~ 5 GeV for $M_S = 10$ TeV. The corrections beyond the 3-loop order yield an additional shift of about 2 GeV for $M_S = 10$ TeV, in accordance with our analysis above. Larger changes are found for $M_S > 10$ TeV.

In summary, we have obtained an improved prediction for the light $\mathcal{CP}$-even Higgs boson mass in the MSSM by combining the FD result at the one- and two-loop level with an all-order resummation of the leading and subleading logarithmic contributions from the top/stop sector obtained from solving the two-loop RGEs. Particular care has been taken to consistently match these two different types of corrections. The result, providing the most precise prediction for M_h in the presence of large masses of the scalar partners of the top quark, has been implemented into the public code FeynHiggs. We have found a sizable effect of the higher-order logarithmic contributions for $M_S \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gtrsim 2$ TeV which grows with increasing M_S. In comparison with H3m, which contains the $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections to M_h, we find that additional 3-loop corrections of $\mathcal{O}(\alpha_t^2 \alpha_s, \alpha_t^2)$ and also higher-loop corrections are both important for a precise M_h prediction, amounting to effects of ~ 7 GeV for $M_S = 10$ TeV in our example. Finally, we have shown that for sufficiently high M_S the predicted values of M_h reach about 126 GeV even for vanishing mixing in the scalar top sector. As a consequence, even higher M_S values are disfavoured by the measured mass value of the Higgs signal.

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