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Measuring the Boiling Point of the Vacuum of Quantum Electrodynamics

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It is a long-standing non-trivial prediction of quantum electrodynamics that its vacuum is unstable in the background of a static, spatially uniform electric field and, in principle, sparks with spontaneous emission of electron-positron pairs. However, an experimental verification of this prediction seems out of reach because a sizeable rate for spontaneous pair production requires an extraordinarily strong electric field strength $|\mathbf{E}|$ of order the Schwinger critical field, $\mathrm{E}_c = m_e^2/e \simeq 1.3\times 10^{18}$ V/m, where m_e is the electron mass and e is its charge. Here, we show that the measurement of the rate of pair production due to the decays of high-energy bremsstrahlung photons in a high-intensity laser field allows for the experimental determination of the Schwinger critical field and thus the boiling point of the vacuum of quantum electrodynamics.

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Quantum Electrodynamics (QED) is one of the most successful theories in physics. Its predictions for observables accessible by an ordinary perturbative expansion in the electromagnetic coupling e, such as for example for the anomalous magnetic moment of the electron, have been verified experimentally to a very high accuracy.

There are, however, also observables which are inaccessible by ordinary perturbation theory and whose prediction lacks an experimental verification. Among them, the most famous is the rate (per unit volume V) of spontaneous electron-positron pair production (SPP) in a strong static electric field $\mathbf{E}$ [1–3] ,

$$\frac{\Gamma_{\rm SPP}}{V} = \frac{m_e^4}{(2\pi)^3}\left(\frac{|\mathbf{E}|}{\mathrm{E_c}}\right)^2 \sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-n\pi\frac{\mathrm{E_c}}{|\mathbf{E}|}\right), \quad (1)$$

where

$$\mathrm{E}_c \equiv \frac{m_e^2}{e} \simeq 1.3\times 10^{18}\ \mathrm{V/m} \quad (2)$$

is the so-called Schwinger critical field. Clearly, this rate is non-perturbative in e,

$$\Gamma_{\rm SPP} \propto \exp\left(-\pi\frac{m_e^2}{e|\mathbf{E}|}\right), \quad (3)$$

as typical for a process which can occur, for $|\mathbf{E}| \lesssim \mathrm{E}_c$, only via quantum tunnelling. This so-called Schwinger effect and its analogues have been suggested to play a role in many problems of phenomenological and cosmological interest, ranging from black hole quantum evaporation [4–7] to particle production in hadronic collisions [8–10] and in the early universe [11–13], to mention only a few. Unfortunately, there is no practical way to produce a static electric field of this strength in the foreseeable future[1]. Therefore, a direct laboratory test of prediction (1) seems utopic.

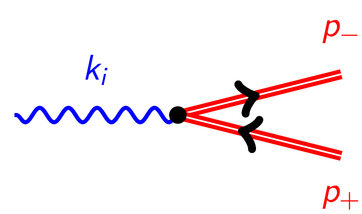

FIG. 1. Leading order Furry picture [19] Feynman diagram for OPPP. The double line pointing forward (backward) in time represents an electron (a positron) in the background of the electromagnetic field of the laser.

As an alternative to spontaneous pair production in a strong static electric field, we consider here laser-assisted one photon pair production (OPPP) – the decay of a high energy photon in the overlap with an intense optical laser beam into an electron-positron pair, cf. Fig. 1. This process is kinematically possible because the electron-positron pair can pick up momentum from the laser photons. Already in the 1960‘s, when first lasers where developed, this process has been identified as an opportunity to study the transition from stimulated to spontaneous pair production in an external electromag-

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[1] One possibility considered was the field at the crossing of two intense laser beams [14–17]. However, the required laser peak power is in the hundreds of exawatt range (for a laser operating in the optical range, focussed to the diffraction limit) [18] and thus still far beyond the present technology.

netic field [20, 21].[2] We will show in this Letter that it offers a timely way to probe the so far elusive boiling of the vacuum of QED and to determine the Schwinger critical field experimentally.

To leading order in Furry picture [19] perturbation theory (for a recent review, see Ref. [27]), the rate of laser-assisted OPPP can be written in the form

$$\Gamma_{\rm OPPP} = \frac{\alpha m_e^2}{4\,\omega_{\rm i}}\, F_\gamma(\xi,\chi_\gamma)\,, \tag{4}$$

where $\alpha = e^2/(4\pi)$ is the fine structure constant, $k_i = (\omega_{\rm i}, \mathbf{k}_{\rm i})$, with $\omega_i^2 = \mathbf{k}_{\rm i}^2$, is the four-momentum of the initial state photon, and ξ and χ_γ are the laser intensity parameter and the photon recoil parameter, respectively,

$$\xi \equiv \frac{e\,|\mathbf{E}|}{\omega m_e} = \frac{m_e}{\omega}\frac{|\mathbf{E}|}{\mathrm{E}_c}\,, \qquad \chi_\gamma \equiv \frac{k\cdot k_i}{m_e^2}\,\xi = (1+\cos\theta)\,\frac{\omega_i}{m_e}\frac{|\mathbf{E}|}{\mathrm{E}_c}\,, \tag{5}$$

in terms of the electric field $|\mathbf{E}|$ of the laser beam, its frequency ω, and its angle θ with respect to the direction of the incident photon. The dimensionless function $F_\gamma(\xi,\chi_\gamma)$, for the idealized case that the electromagnetic field of the laser beam can be described as a circularly polarized infinite plane wave, is given by a sum over the effective number of laser photons n absorbed by the electron-positron pair [21],

$$F_\gamma(\xi,\chi_\gamma) = \sum_{n>n_0}^{\infty}\int_1^{v_n}\frac{\mathrm{d}v}{v\sqrt{v(v-1)}}\left[2\,J_n^2(z_v) + \xi^2(2v-1)\left(J_{n+1}^2(z_v)+J_{n\text{-}1}^2(z_v)-2J_n^2(z_v)\right)\right], \tag{6}$$

with Bessel functions J_n and

$$n_0 \equiv \frac{2\xi\left(1+\xi^2\right)}{\chi_\gamma}, \qquad z_v \equiv \frac{4\xi^2\sqrt{1+\xi^2}}{\chi_\gamma}\left[v\left(v_n - v\right)\right]^{1/2}, \qquad v_n \equiv \frac{\chi_\gamma\, n}{2\xi(1+\xi^2)}. \tag{7}$$

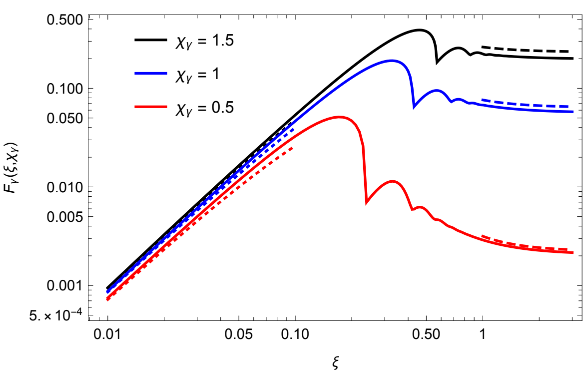

FIG. 2. The dimensionless function $F_\gamma(\xi,\chi_\gamma)$, Eq. (6), describing the probability of laser-assisted OPPP, as a function of the laser intensity parameter ξ, for different values of the photon recoil parameter χ_γ (solid lines). The dotted (dashed) line shows the analytic result valid at small (large) values of the intensity parameter, Eq. (8) (Eq. (9)).

In Fig. 2, we display $F_\gamma(\xi,\chi_\gamma)$ as a function of ξ, for three values of χ_γ. Clearly, at low laser intensities, $\xi \ll 1$, laser-assisted OPPP appears to proceed perturbatively, $F_\gamma \propto \xi^2 \propto \alpha$, as expected from the necessity to absorb at least one laser photon to allow for photon decay kinematically. In fact, expanding F_γ for small ξ yields

$$F_\gamma(\xi,\chi_\gamma) = 2\,\xi^2\left[\log\left(\frac{2\chi_\gamma}{\xi}\right) - 1\right] + \mathcal{O}\left(\xi^3\log\xi\right). \tag{8}$$

This behaviour reproduces the full result for laser-assisted OPPP up to values of $\xi \sim 0.1$, cf. Fig. 2. As the laser intensity ξ increases, the threshold number of absorbed photons n_0 to produce an electron-positron pair increases, and more and more terms in the summation over the number of absorbed laser photons in Eq. (6) drop out of the probability, resulting in the appearance of less and less pronounced maxima in F_γ, see Fig. 2. At large ξ, finally, the probability of laser-assisted OPPP approaches a finite value, the latter growing with increasing χ_γ. Indeed, for $\xi \gtrsim 1/\sqrt{\chi_\gamma} \gg 1$, F_γ behaves as [28]

$$F_\gamma(\xi,\chi_\gamma) = \frac{3}{4}\sqrt{\frac{3}{2}}\,\chi_\gamma\,\mathrm{e}^{\left[-\frac{8}{3\chi_\gamma}\left(1-\frac{1}{15}\xi^{-2}+\mathcal{O}(\xi^{-4})\right)\right]}. \tag{9}$$

This behaviour applies to a very good accuracy already for $\xi \gtrsim 1$ and $\chi_\gamma \lesssim 1$, cf. Fig. 2. Importantly, we infer from Eq. (9) that the asymptotic value of F_γ is non-perturbative in the electromagnetic coupling e and that the rate of laser-assisted OPPP asymptotes to

[2] The OPPP process continues to attract much modern interest with new theoretical approaches [22], analyses which take into account real interacting laser pulses [23–25] and experimental schemes to realise the process [26].

$$\Gamma_{\rm OPPP} \to \frac{3}{16}\sqrt{\frac{3}{2}}\,\alpha\, m_e\,(1+\cos\theta)\;\frac{|\mathbf{E}|}{\rm E_c}\exp\left[-\frac{8}{3}\frac{1}{1+\cos\theta}\frac{m_e}{\omega_i}\frac{\rm E_c}{|\mathbf{E}|}\right], \tag{10}$$

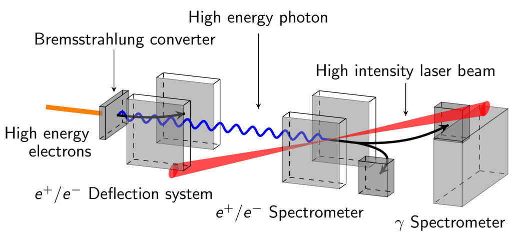

FIG. 3. Sketch of an experiment to produce high energy photons by bremsstrahlung conversion in a high-Z thin target and to cross them with a laser beam to let them decay into electron-positron pairs. Switching off the laser allows for a determination of the bremsstrahlung spectrum. Removing the target allows in addition for the study of HICS, followed by OPPP, and of the one-step trident process.

ressembling the rate (1) of SPP in a constant electric field[3]. This has to be expected, since large intensity parameter, $\xi \gg 1$, corresponds to a quasi-static electric field of the laser, $\omega \ll e\,|\mathbf{E}|\,/m_e$, cf. Eq. (5). However, in contrast to SPP, in laser-assisted OPPP the produced electron-positron pair, in its rest frame, experiences an electric field enhanced by the relativistic boost factor ω_i/m_e. This enhanced electric field is of order the Schwinger critical value $\rm E_c$, if the photon recoil parameter is $\chi_\gamma \sim 1$, cf. Eq. (5). Hence, the Schwinger critical field – the boiling point of the QED vacuum – can be determined in principle experimentally from the measurement of the rate of laser-assisted OPPP at $\xi \gtrsim 1/\sqrt{\chi_\gamma} \gg 1$.

Note, that for a laser of frequency $\omega = 1.053\,{\rm eV}$, focussed to an intensity I, corresponding to[4]

$$\xi = 3.2\left(\frac{I}{10^{20}\,{\rm W/cm^2}}\right)^{1/2}\left(\frac{1.053\,{\rm eV}}{\omega}\right), \tag{11}$$

the condition $\xi \gtrsim 1/\sqrt{\chi_\gamma}$ leads to a lower bound on the energy of the high energy photon, $\omega_i \gtrsim 7.6\,{\rm GeV}\left(\frac{1.053\,{\rm eV}}{\omega}\right)\frac{(3.2/\xi)^3}{(1+\cos\theta)}$. Unfortunately, there are no mono-energetic photon beams with energies in the $\mathcal{O}(10)$ GeV range available. On the other hand, there are $\mathcal{O}(10)$ GeV electron beams, notably the ones exploited by X-ray free electron lasers, such as LCLS [29] in Stanford or the European XFEL [30] in Hamburg. Such an electron beam can be sent to a high-Z target in which it is converted by bremsstrahlung into a collimated high energy photon beam, which can then be crossed with a high-intensity laser beam, cf. Fig. 3. Such an experiment to study laser-assisted bremsstrahlung photon pair production (BPPP) has been envisaged long time ago in Ref. [31] and has recently been discussed in Refs. [32, 33]. Here, we show that even after integration over the bremsstrahlung spectrum, the Schwinger critical field can be determined experimentally from the measurement of the total rate of electron-positron pair production at large laser intensity.

Given the energy spectrum ${\rm d}N_\gamma/{\rm d}\omega_i$ of photons generated by an electron impinging on the foil, the rate of laser-assisted BPPP is given by

$$\begin{aligned}\Gamma_{\rm BPPP} &= \frac{\alpha\, m_e^2}{4}\int_0^{E_e}\frac{{\rm d}\omega_i}{\omega_{\rm i}}\frac{{\rm d}N_\gamma}{{\rm d}\omega_i}F_\gamma(\xi,\chi_\gamma(\omega_i))\\ &= \frac{\alpha\, m_e^2}{4}\frac{\chi_e}{E_e}\int_0^{\chi_e}\frac{{\rm d}\chi_\gamma}{\chi_\gamma}\frac{{\rm d}N_\gamma}{{\rm d}\chi_\gamma}F_\gamma(\xi,\chi_\gamma)\,,\end{aligned} \tag{12}$$

where E_e is the energy of the incident electrons and $\chi_e \equiv k\cdot k_e\,\xi/m_e^2 = (1+\cos\theta)\omega E_e\xi/m_e^2$ is the electron recoil parameter. For a target of thickness $X \ll X_0$, where X_0 is the radiation length, the bremsstrahlung spectrum can be approximated by [34]

$$\omega_i\frac{{\rm d}N_\gamma}{{\rm d}\omega_i} \approx \left[\frac{4}{3}-\frac{4}{3}\left(\frac{\omega_i}{E_e}\right)+\left(\frac{\omega_i}{E_e}\right)^2\right]\frac{X}{X_0}\,, \tag{13}$$

if one assumes complete screening.[5] This results, at high laser intensities, $\xi \gtrsim 1/\sqrt{\chi_e} \gg 1$, in the non-perturbative, $e^{-8/(3\chi_e)}$ dependence of the laser-assisted BPPP rate,

$$\Gamma_{\rm BPPP} \to \frac{\alpha\, m_e^2}{E_e}\frac{9}{128}\sqrt{\frac{3}{2}}\,\chi_e^2\, e^{-\frac{8}{3\chi_e}\left(1-\frac{1}{15\xi^2}\right)}\frac{X}{X_0}\,, \tag{14}$$

ressembling the behavior of the laser-assisted OPPP rate, Eqs. (4) and (9), if one replaces in the latter expression χ_γ by χ_e. Therefore, the Schwinger critical field can be inferred from the asymptotic behavior of laser-assisted BPPP for high laser intensities,

[3] The leading term in the exponent in Eq. 10 is independent of the laser polarisation, while the pre-factor depends on it [28]

[4] This relation assumes that the intensity is given by the modulus of the Pointing vector, i.e. $I = |\mathbf{E}|^2$ for a plane wave.

[5] We have checked via Monte Carlo simulations with GEANT [35] that (13) is valid in the parameter range we use it in e.g. Figs. 4 and (5). For the interpretation of the experiment itself one does not have to rely on a theoretical prediction, since the bremsstrahlung spectrum can be measured by switching off the laser, cf. Fig. 3.

$$\Gamma_{\rm BPPP} \to \frac{9}{128}\sqrt{\frac{3}{2}}\,\alpha\, E_e\,(1+\cos\theta)^2 \left(\frac{|\mathbf{E}|}{\mathrm{E}_c}\right)^2 \exp\left[-\frac{8}{3}\frac{1}{1+\cos\theta}\frac{m_e}{\omega_i}\frac{\mathrm{E}_c}{|\mathbf{E}|}\right]. \tag{15}$$

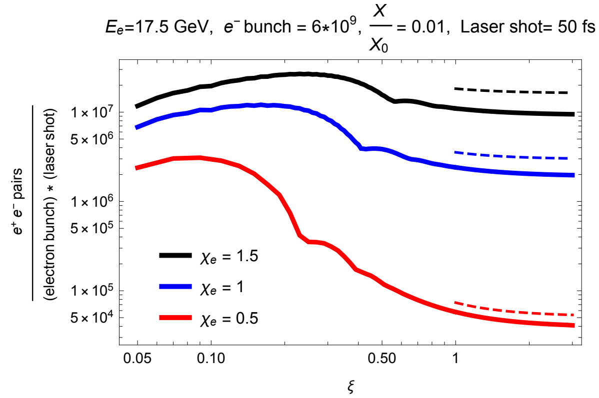

FIG. 4. Number of e^+e^- pairs produced per electron bunch (6×10^9 electrons of energy $E_e = 17.5\,$GeV) impinging on the converter target (thickness $X/X_0 = 0.01$) and per laser shot (duration 50 fs) crossed with the bremsstrahlung photons, as a function of the laser intensity parameter ξ, for different values of χ_e. The dashed line shows the analytic prediction resulting from (14), valid at $\xi \gtrsim 1/\sqrt{\chi_e} \gg 1$.

The high energy electrons will impinge in bunches onto the target. The electron beam of the European XFEL, for example, contains 6×10^9 electrons of energy $E_e = 17.5\,$GeV, with small energy spread and a good emittance [30]. The high intensities of the laser are reached conceivably in laser pulses of duration around 50 fs. In Fig. 4, we show the number of pairs produced per electron bunch and per laser shot expected in this case. The solid lines are obtained from the numerical solution of Eqs. (6) and (12), while the dashed lines exploit the analytic asymptotics 14. Importantly, the latter approaches the former already at $\xi \gtrsim 1$ and $\chi_e \lesssim 1$. Moreover, the number of produced pairs is favorably high, even for the most interesting parameter range of large ξ and small χ_e. From this we conclude that it should be easy to measure the Schwinger critical field in this type of experiment.

In practice, in an experiment as sketched in Fig. 3, it will be easiest to change the intensity of the laser and the energy E_e of the electron beam. In this case, the electron recoil parameter can be expressed as

$$\chi_e = 0.22\,(1+\cos\theta)\left(\frac{E_e}{17.5\,\mathrm{GeV}}\right)\left(\frac{I}{10^{20}\,\mathrm{W/cm^2}}\right)^{1/2}. \tag{16}$$

Figure 5 shows that the number of produced pairs per electron bunch and laser shot rapidly grows with increasing intensity to values above one already at laser intensities $\sim 2\times10^{19}\,\mathrm{W/cm^2}$. Therefore, with relative modest parameters for a focussed intense laser to ensure stable

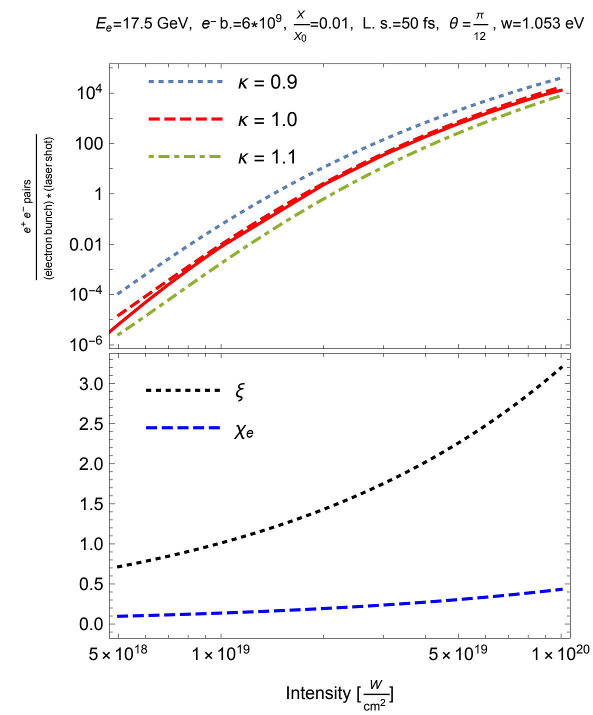

FIG. 5. *Top panel:* Number of e^+e^- pairs produced per electron bunch (6×10^9 electrons of energy $E_e = 17.5\,$GeV) impinging on the bremsstrahlung target (thickness $X/X_0 =$ 0.01) and per laser shot (duration 50 fs, laser frequency $\omega = 1.053\,$eV) crossed with the bremsstrahlung photons at an angle of $\theta = \pi/12$, as a function of the laser intensity. The dashed line shows the analytic prediction resulting from (14), exploiting the relations (11) and (16). The dotted (dot-dashed) line shows the same analytic prediction, but for the case where the value of the Schwinger critical field E_c deviates by a multiplicative factor of $\kappa = 0.9$ ($\kappa = 1.1$) from its nominal value (2). *Bottom panel:* The laser intensity parameter ξ (dotted) and the electron recoil parameter (dashed), as a function of the intensity, cf. Eqs. (11) and (16).

operation at a strong field experimental interaction point, the asymptotic regime for the BPPP process should be experimentally accessible with reasonable accuracy. This will allow a precision comparison with the asymptotic result according to Eq. (14), which sensitively depends on the value of E_c, cf. the top panel in Fig. 5: A variation of E_c around its nominal value (2) by 10 % results in a change in the predicted rate by nearly an order of magnitude, in the intensity range of interest.

Design studies are under way to plan for such an experiment [36]. By removing the target in the experimental setup of Fig. 3, the strong-field trident process can be studied in addition. In its two-step variant, it occurs via high intensity Compton scattering (HICS), followed by OPPP. Exploiting in this way the energetic photons from HICS as an alternative source of high energy photons, the asymptotic regime of the OPPP process can again be in principle measured. However, the rate for HICS is considerably lower and is cut off by the stepped Compton edge, compared to that of bremsstrahlung. Nevertheless, trident pair production is of significant interest [37–41] since its measurement in the 1990s [42] and is an important additional strong field process that can be measured by the experiment described in this Letter.

In order to quantify the expected experimental accuracy, detailed simulations will have to be carried out. These will include a GEANT [35] model of the converter target to produce a large flux of energetic photons. Macro-particles, representing a train of electron bunches varying randomly within known beam conditions, will also be required. An accurate representation of laser pulse shape and jitter will be needed, as well as a full accounting for crossing angles and beam overlap. Along with GEANT, a full strong field QED particle-in-cell code including higher order processes has also to be developed.

One may ask what happens if the quantum recoil parameters continue to increase in value with either increasing gamma energy and/or electromagnetic field intensity. In such a case, our strong field experiment would probe both smaller distances and the quantum vacuum would be increasingly polarised. In such circumstances, there is good reason to think that higher order, strong field processes may increasingly play a role. There is not only the opportunity to study and search for such higher order processes in dedicated strong field experiments, there is also the ability to question non-perturbative quantum field theory itself. The Furry picture [19], which is the semi non-perturbative theory from which most strong field QED predictions are made, includes the quantum recoil parameters in its effective coupling constant. Theoretical estimates [28, 43] put the effective coupling constant at $\alpha\chi^{1/3}$, meaning that the theory breaks down at a high enough value of the recoil parameters. Whereas this breakdown regime is experimentally some distance away, the value of the effective coupling constant through the higher order terms is possibly in reach.

The experiment and theory study, presented here, already promises us the first ever measurement of the Schwinger critical field value, through an asymptotic limit. Many additional, important strong field effects can be experimentally tested through further theoretical and phenomenological studies.

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