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Strong constraints on clustered primordial black holes as dark matter

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The idea of dark matter in the form of primordial black holes has seen a recent revival triggered by the LIGO detection of gravitational waves from binary black hole mergers. In this context, it has been argued that a large initial clustering of primordial black holes can help alleviate the strong constraints on this scenario. In this letter, we show that on the contrary, with large initial clustering the problem is exacerbated and constraints on primordial black hole dark matter become overwhelmingly strong.

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Introduction.— Soon after realising that black holes (BHs) could form in the early radiation-dominated universe [1–3] from the gravitational collapse of order unity initial density fluctuations, it was pointed out that such objects may even contribute appreciably to the total matter density [4]. An obvious question is therefore whether these primordial black holes (PBHs) could explain *all* of the cosmologically observed dark matter (DM), see Refs. [5, 6] for recent reviews. This idea has seen greatly renewed interest [7–10] after the discovery of binary mergers by the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) [11–14], proving the existence of $\mathcal{O}(10M_\odot)$ BHs with so far unclear origin.

Constraints on the allowed DM fraction $f_{\rm PBH}$ of PBHs derive from a large number of observations and have been explored for a vast range of mass scales, see Refs. [5, 6, 10] for an overview. While there seems to be a broad consensus that $f_{\rm PBH} \sim 1$ is essentially excluded when assuming a homogeneously distributed population of PBHs with a single mass, this general picture changes when either of these conditions is not met. Intriguingly, this also opens the window of PBH masses consistent with the LIGO observations by circumventing the stringent constraints from microlensing and from the cosmic microwave background (CMB) [15] (see however [16, 17]).

In fact, both of these 'exceptions' may actually be the generic expectation of standard production and subsequent merging scenarios for PBHs. The simplest models of inflation generally predict an approximately scale-invariant scalar power spectrum, whereas a monochromatic PBH distribution would require a strongly peaked spectrum. Moreover, since the characteristic time-scale of inflation is the Hubble time, any such peak is expected to be rather broad in terms of the range of enhanced length scales, or PBH masses, since these are exponentially sensitive to this characteristic time-scale [18] (though notable exceptions exist [19–22]).

Realistic PBH distributions have further been argued to be highly clustered rather than homogeneous [23, 24], possibly explaining the existence of super-massive BHs [25, 26]. While this conclusion was recently challenged [27], claiming a random Poisson distribution on small scales, Ref. [28] shortly after argued that, when taking into account the small-scale exclusion volume arising due to the collapse of PBHs, the clustering could indeed be much larger depending on the details of the primordial power spectrum. Highly clustered PBH distributions have also been argued to arise from possible primordial non-Gaussianities [29] and the collapse of domain walls [30].

The initial clustering of PBHs is indeed a key parameter to understanding the phenomenology of PBH DM, affecting merger rates [31, 32], the subsequent structure formation [33], and the interpretation of observational bounds [15]. Here we take a pragmatic and phenomenological approach by parametrising the clustering as a constant, free parameter on the scales of interest. We point out that, for the large PBH clustering discussed in the literature, the expected merger rates easily exceed one per binary and Hubble time. We demonstrate that multiple subsequent mergers severely constrain PBH DM as a possible explanation of the LIGO events because of *i)* the expected (as compared to observed) merger rate, *ii)* the impact of the additional radiation component in gravitational waves (GWs) on both CMB and large-scale structure observations, and *iii)* a present-day stochastic GW background (SGWB) exceeding the sensitivities of current ground- (or future space-) based observatories.

This Letter is organised as follows. We start by describing the GW spectrum and energy density from cosmological PBH mergers, before discussing how the merger rate critically depends on the initial PBH clustering. We then introduce a cascading merger scenario to capture the effects of large clustering, and hence high merger rates. We derive the resulting contributions to the stochastic GW background and the relativistic energy density in GWs, using the cosmological parameters from Ref. [34] whenever relevant. Along with the actual event rate observed by LIGO, we use this to place constraints on $f_{\rm PBH}$.

Gravitational waves from merging black holes.— Coalescing binary BHs emit GWs with a characteristic spectrum $\mathrm{d}E_{\mathrm{GW}}/\,\mathrm{d}\nu$, which integrates to a total energy that makes up a significant fraction of the rest mass [35]. For BHs with identical masses and negligible spins, e.g., one expects $E_{\mathrm{GW}}/M_{2\mathrm{BH}} \simeq 5\%$, where $M_{2\mathrm{BH}}$ is the initial mass of the system. For the five events observed so far by LIGO, this number ranges between 3.9% and 5.2% [35].

In the following we will study the cumulative effect of all mergers throughout the cosmological evolution. The resulting present energy density *per logarithmic frequency interval* is conventionally expressed in units of the critical density, $\rho_c = 3H_0^2/(8\pi G)$, and computed as [36, 37]

$$\Omega_{\mathrm{GW}}(\nu) = \frac{1}{\rho_c}\int_0^\infty \mathrm{d}\tilde{z}\int \mathrm{d}R(\tilde{z})\frac{\nu}{(1+\tilde{z})H(\tilde{z})}\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}\nu_s}\,. \quad (1)$$

Here, the merger rate is denoted as $R(z)$, where z is the cosmological redshift, H is the Hubble rate, and the observed frequency ν corresponds to an emission frequency of $\nu_s = \nu(1+z)$. The *total* energy density in gravitational waves at any given redshift z is therefore

$$\frac{\rho_{\mathrm{GW}}(z)}{(1+z)^4} = \int_z^\infty \mathrm{d}\tilde{z}\int \mathrm{d}\nu\int \mathrm{d}R(\tilde{z})\frac{\mathrm{d}E_{\mathrm{GW}}/\,\mathrm{d}\nu}{(1+\tilde{z})^2 H(\tilde{z})}\,. \quad (2)$$

Covariant conservation of energy implies that the mass density in PBHs must correspondingly decrease as $a^{-3}\,\mathrm{d}(a^3\rho_{\mathrm{PBH}}) = -a^{-4}\,\mathrm{d}(a^4\rho_{\mathrm{GW}})$ [38], which integrates to

$$\frac{\rho_{\mathrm{PBH}}(z)}{(1+z)^3} = C - \int_z^\infty \mathrm{d}\tilde{z}\int \mathrm{d}\nu\int \mathrm{d}R(\tilde{z})\frac{\mathrm{d}E_{\mathrm{GW}}/\,\mathrm{d}\nu}{(1+\tilde{z})H(\tilde{z})}\,. \quad (3)$$

We fix the integration constant C such that $f_{\mathrm{PBH}} \equiv (\rho_{\mathrm{PBH}}/\rho_{\mathrm{DM}})_{z_{\mathrm{CMB}}}$ is the PBH fraction at $z = z_{\mathrm{CMB}} \simeq 1100$. For the GW spectrum $\mathrm{d}E_{\mathrm{GW}}/\,\mathrm{d}\nu$ we follow Ref. [31] in using commonly adopted fitting formulae [39–42].

Merger rates and clustering.— In the early universe, PBH binary formation starts once the Newtonian force between two initial PBHs overcomes the Hubble flow, with a nearby third PBH providing the angular momentum necessary to prevent a head-on collision [9, 43, 44]. Later, peculiar velocities may be too large for this to happen; instead, binary formation can be triggered by the energy loss in GWs during close encounters of two PBHs [45, 46]. For the parameter combinations of interest to us, though, the rate associated to the first formation mechanism largely exceeds that for the second, even for binaries merging only today [31]. Once formed, these binary systems survive until they merge, largely unaffected by the evolution of the surrounding Universe [47].

A crucial input for calculating those merger rates is the initial clustering of PBHs. Phenomenologically, this can be described in terms of an idealised two-point correlation function $\xi_{\mathrm{PBH}}(r)$ that is constant at scales relevant for the formation of PBH binaries [31]:

$$1 + \xi_{\mathrm{PBH}}(r) \approx \delta_{\mathrm{dc}} = const.\,, \quad (4)$$

where δ_{dc} describes the local density contrast, evaluated at the time when the two BHs decouple from the Hubble expansion. A perfectly homogeneous PBH distribution corresponds to $\delta_{\mathrm{dc}} = 1$, while a highly clustered PBH distribution is described by $\delta_{\mathrm{dc}} \gg 1$. Values of $\delta_{\mathrm{dc}} \gtrsim 10^5$ are particularly interesting, as they are required to circumvent the tight constraints on PBH DM from microlensing as well as from the CMB [15]. Furthermore, constraints arising from the conversion of PBH DM into gravitational radiation are alleviated if PBH mergers occur only at high redshift, which was demonstrated to happen for $\delta_{\mathrm{dc}} \gtrsim 10^4$ assuming only a single merger step [31].

We find that a highly clustered initial PBH population not only leads to a *i)* more efficient formation of binaries but also to a *ii)* significantly enhanced merger rate during the whole cosmological evolution until today. This finding crucially extends previous results in the literature (see e.g. [31, 32]), which consider the impact of clustering only on a single merger step. The first aspect implies tightened limits on the allowed PBH fraction resulting from a direct comparison to the rate of the LIGO events, even though those constraints are only proportional to $\delta_{\mathrm{dc}}^{0.3}$ and the sensitivity to clustering has hence been argued to be rather weak [31]. The importance of the second aspect can be best appreciated by noting that the resulting coalescence times are short compared to the Hubble time, making the inclusion of subsequent mergers mandatory. Investigating the consequences of this observation in more detail is the main focus of this work.

Cascading black hole merger events.— To do so, we need to improve the scenario advocated in Ref. [31] by allowing for subsequent merger steps, i.e. binary mergers of systems of previously merged PBHs. For simplicity, we consider an initially monochromatic mass distribution peaked at m_0 and assume that the PBH masses in merger step j are given by

$$m_j = 2m_{j-1} - E_{\mathrm{GW}}(m_{j-1}) \sim 1.9^j\, m_0\,. \quad (5)$$

The average PBH number density $n_j = \rho_{\mathrm{PBH},\infty}/(2^j m_0)$ is locally enhanced by a factor of $1+\xi(r) \approx \delta_{\mathrm{dc},j}$, where $\rho_{\mathrm{PBH},\infty}$ denotes the initial PBH density and the PBH clustering roughly decreases as $\delta_{\mathrm{dc},j} \simeq 2^{-j}\,\delta_{\mathrm{dc},0}$ (with details depending on the exact form of the two-point correlation function).

The merger rate of the jth merger step is given by

$$R_j(t) = \int_0^{\tilde{x}} \mathrm{d}x \int_{\tilde{x}}^\infty \mathrm{d}y\, \frac{\partial^2 n_{3,j}(x,y)}{\partial x \partial y} \delta(t - \tau(x,y,m_j) - \max(t_{\mathrm{dc},j}(x), t_{\mathrm{form}}))\,, \quad (6)$$

where x and y denote the comoving distances from a given PBH to the nearest and next-to-nearest PBH, respectively, and the number density of PBH triples $n_{3,j}(x,y)$ is given by [31]

$$\mathrm{d}n_{3,j}(x,y) = \frac{n_j}{2} \mathrm{e}^{-\frac{4\pi}{3} y^3 n_j \delta_{\mathrm{dc},j}} (4\pi n_j \delta_{\mathrm{dc},j})^2 x^2 y^2 \, \mathrm{d}x \, \mathrm{d}y \, . \tag{7}$$

The delta distribution in Eq. (6) ensures that the coalescence time τ [48],

$$\tau(x,y,m_j) = \tilde{\tau}_j \, (x/\tilde{x}_j)^{37} \, (y/\tilde{x}_j)^{-21} \;, \tag{8}$$

with

$$\tilde{\tau}_j = \frac{3 a_{\mathrm{eq}}^4 \tilde{x}_j^4}{170 (G m_j)^3} \,, \qquad \tilde{x}_j^3 = \frac{3}{4\pi} \frac{2 m_j}{a_{\mathrm{eq}}^3 \rho_{\mathrm{eq}}} \,, \tag{9}$$

is measured from when the PBHs are both formed (t_{form}) and decoupled from the Hubble flow ($t_{\mathrm{dc},j}$). PBHs form almost immediately after the corresponding density perturbations enter the Hubble horizon, with a mass m_0 equalling the total energy within the horizon at that time, and the decoupling from the Hubble flow occurs when the gravitational attraction overcomes the Hubble expansion:

$$t_{\mathrm{form}} = G m_0 \,, \quad t_{\mathrm{dc},j} = \left(\frac{16\pi G}{3} \rho_{\mathrm{eq}} \right)^{-1/2} \left(\frac{x}{\tilde{x}_j} \right)^6 \, . \tag{10}$$

The subscript 'eq' above refers to matter radiation equality. The merger rates R_j in Eq. (6) are connected to the differential one employed in Eqs. (1)-(3) via

$$\mathrm{d}R(\tilde{z}) = \sum_j R_j(t(\tilde{z})) \, \delta(m - m_j) \, \mathrm{d}m \, . \tag{11}$$

To recap, we consider a scenario of subsequent equal-mass mergers with a corresponding shift in the mass distribution and local density contrast in each merger step. Let us stress that even though there are characteristic time-scales implied by the merger rates, we allow PBHs of given mass m_j to merge at any time between the decoupling for the jth step (as long as $t_{\mathrm{dc},j} > t_{\mathrm{form}}$) and today. For rare very early mergers this may lead to situations where coalescence in our model begins when, in reality, instead of the eventually merging two PBHs a preceding set of smaller PBHs would be present; in this case, we slightly underestimate the actual amount of emitted GWs. We further introduce an approximate scenario to better visualise the individual merger steps, adopting rates R_j that are zero before $\max(t_{\mathrm{dc},j}, t_{\mathrm{form}})$, constant until the average coalescence time has passed (with average values for x and y), and zero afterwards.

In Fig. 1 we show the decrease of the PBH energy density in our merger scenario (solid lines) as well as the corresponding increase in gravitational wave radiation (top right inlet). Overall, the agreement between the full rates, as computed from Eq. (6), and the simple approximate scenario mentioned above (dashed lines) is fairly good. In particular we note the sizeable equidistant spacing of the merger steps, justifying the assumption of a hierarchical merging scenario. The dash-dotted curves represent the common assumption of a single merger step. For larger values of $\delta_{\mathrm{dc},0}$, this merger step occurs earlier [31], leading to the tempting conclusion that bounds on the GW production can be evaded since the produced radiation is highly red-shifted. As we discuss below, this conclusion clearly no longer holds once multiple merger steps are taken into account.

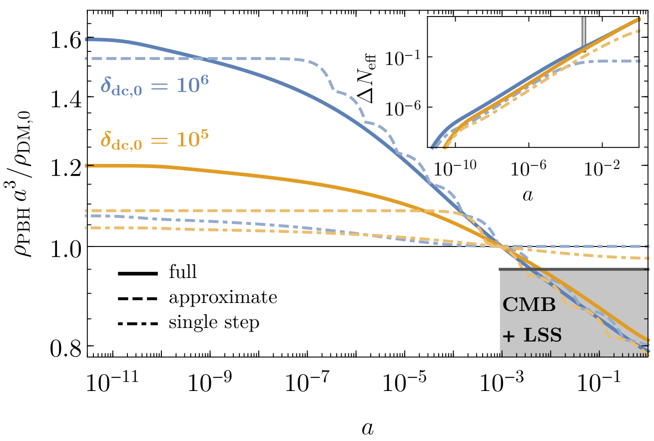

FIG. 1. Conversion of PBH energy density (main panel) into gravitational wave radiation (top right inlet) for $m_0 = 1 M_\odot$, $f_{\mathrm{PBH}} = 1$, and different theoretical treatments. In grey, we indicate regions excluded by cosmology [38].

Cosmological bounds.— The conversion of PBH DM into GW radiation modifies the standard cosmological evolution and is constrained by CMB and large scale structure (LSS) observations [38]. This can be roughly split into *i)* an upper bound on the effective number of neutrino species at the time of the CMB, $\Delta N_{\mathrm{eff}}(z_{\mathrm{CMB}}) \lesssim 0.3$ [34], indicated by the grey area in the inlet of Fig. 1, and *ii)* the amount of DM converted into dark radiation at later times. From Fig. 6 in Ref. [38] one can deduce that not more than $\sim 5\%$ of DM can be converted into dark radiation after the CMB epoch, irrespective of the precise time-dependence of this conversion (and consistent with the 4.2% found for the case of decaying DM [49]). For simplicity, and because other constraints turn out to be stronger, we conservatively adopt this bound of 5% in our analysis (indicated by the grey region in the bottom right of Fig. 1).

The stochastic gravitational wave background.— With the cumulative merger rate described above, it is also straightforward to compute the resulting SGWB as given in Eq. (1). In Fig. 2, we illustrate the predictions for an initial PBH mass of 1 $M_\odot$ and different initial clusterings $\delta_{\mathrm{dc},0} = 10^5$ and $\delta_{\mathrm{dc},0} = 10^6$. The GW spectrum is dominated by late time mergers ($z \lesssim 10$), since earlier GW emission is highly diluted by cosmic expansion. Larger clustering implies that most of these late mergers

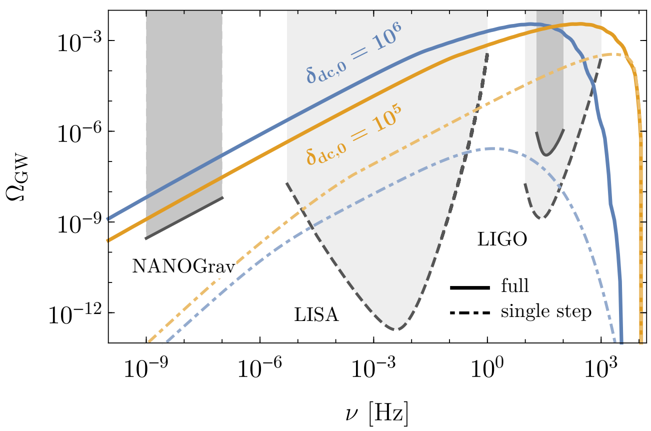

FIG. 2. GW density parameter per logarithmic frequency interval for $m_0 = 1\ M_\odot$, $f_{\rm PBH} = 1$, and different theoretical treatments. In grey, we indicate present (solid lower lines) and projected (dashed lower lines) constraints from NANOGrav [50], LISA [51], and LIGO [52] (present O1 and projected design constraint).

$m/M_\odot$	0.2	1	10	20	40	100	200	300
$R\,{\rm Gpc}^3\,{\rm yr}$	10^6	$1.9\cdot 10^4$	330	77	15	2	5	20

TABLE I. 90% CL upper limits on merger rates in the late universe, taken from Refs. [54–57].

are associated with heavier BHs, which emit GWs with lower frequencies. For the parameter example of Fig. 2 we find that the mergers occurring at $z = 0$ have typically undergone 4 (9) previous mergers for $\delta_{\rm dc,0} = 10^5 (10^6)$, resulting in a PBH mass that is larger by a factor of about 20 in the latter case. Since the frequency of the emitted GWs roughly scales as $\nu \sim 1/M_{2\rm BH}$, cf. [31, 53], this explains the shift between the two solid lines in Fig. 2. Moreover, the merger cascade described above leads to a mild broadening of the high frequency peak. At low frequencies, the $\Omega_{\rm GW} \sim \nu^{2/3}$ scaling indicates the early inspiral phase of the BH binaries [40].

For comparison, the dash-dotted curves show the predictions for a single merger step, where the main effect of large clustering is to shift the merging time to high redshift, strongly suppressing the GW spectrum. However, as Fig. 2 demonstrates, later mergers completely change the picture, leading to a large contribution to the SGWB. The grey contours, finally, indicate the power-law integrated sensitivity curves of LIGO [52] and the pulsar timing array NANOGrav [50], as well as the planned space-based LISA [51] observatory.

Observed merger rate.— The LIGO/VIRGO observations strongly constrain the merger rate of PBHs with masses between 0.2 and 300 $M_\odot$ [54–57]. Interpolating linearly between the limiting rates stated in Tab. I, and comparing this to the calculated $R_j(z = 0)$, allows us to derive an upper bound on $f_{\rm PBH}$. Starting with an initially monochromatic mass function, we would not expect to reproduce the BH mass distribution observed by LIGO. However, requiring to reproduce the total observed merger rate ($12 - 213\ {\rm Gpc}^{-3}{\rm yr}^{-1}$ [14]) with PBH mergers in the sensitivity band of LIGO ($7 - 50 M_\odot$), we obtain a (very conservative) range in m_0 compatible with the total merger rate observed by LIGO.

Results.— In the left panel of Fig. 3, we summarise the resulting constraints on the allowed fraction of DM in PBHs for large initial clustering, $\delta_{\rm dc,0} = 10^6$, as a function of the initial PBH mass m_0 (the shaded regions are excluded). For reference, we also indicate contour lines with the present, rate-averaged PBH mass $m_{\rm avg}$. We depict as blue-green and orange curves, respectively, the cosmological constraints [38] indicated as grey shaded areas in Fig. 1. The blue solid line shows the merger rate constraint; we note that it extends to *average* PBH masses well below the LIGO/VIRGO limit because a small fraction of PBHs will still satisfy $m_{\rm PBH} > 0.2\,M_\odot$ after many merger steps. The remaining lines, finally, correspond to the SGWB constraints from NANOGrav (green), LIGO (purple) and LISA (red, dashed) indicated in Fig. 2. The upcoming space-based LISA experiment may severely tighten these constraints.

In the right panel of Fig. 3, we show our combined results on $f_{\rm PBH}$, illustrating that larger values of the clustering parameter $\delta_{\rm dc,0}$ in fact lead to tighter constraints. For comparison, the dash-dotted lines indicate the much weaker constraints obtained when taking into account only a single merger step. The arrows indicate the range for m_0, where for a suitable $f_{\rm PBH}$ the total present merger rate is consistent with all observed LIGO events being caused by PBH mergers.

Discussion and conclusions.— If PBHs are not homogeneously distributed in the Universe but highly clustered, existing bounds on their abundance must be reinterpreted. In particular, highly clustered PBHs merge earlier, implying that the gravitational radiation produced in these mergers is diluted in the expanding universe. In this letter we have demonstrated that in this case limits on the PBH fraction are not weakened, as claimed previously, but instead strengthened because subsequent merger steps would dominate the SGWB. Taking into account constraints from cosmology and direct GW searches, we find that for $\delta_{\rm dc,0} > 10^4$ the case of pure PBH DM is firmly excluded in the entire range of initial PBH masses between $10^{-5}\ M_\odot$ and $100\ M_\odot$. We note that outside this mass range bounds are also very strong [10], which essentially left this interval as the only realistic option for explaining all DM in terms of PBHs.

For PBH with a final average mass in the LIGO band, $m_{\rm avg} \sim 10 M_\odot$, severe bounds on the maximal fraction of PBH DM arise from the constraints on the merger rate and the SGWB imposed by LIGO and NANOGrav, enforcing $f_{\rm PBH} \lesssim 2\times 10^{-4}$ for $\delta_{\rm dc,0} \geq 10^4$. For a highly clustered PBH mass distribution peaked at smaller val-

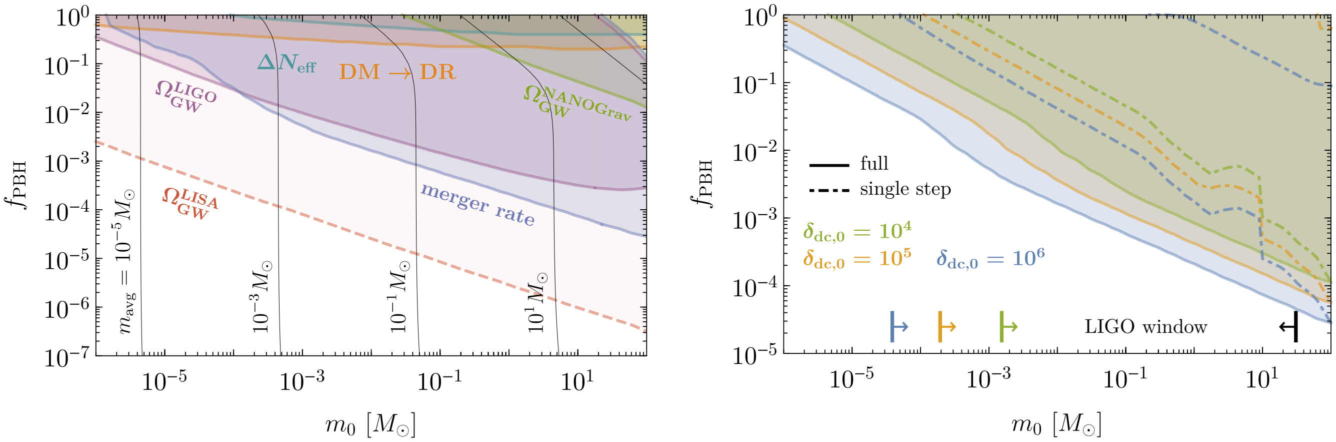

FIG. 3. *Left.* Constraints on the allowed fraction $f_{\rm PBH}$ of PBH DM as a function of the initial PBH mass m_0 for large clustering ($\delta_{\rm dc,0} = 10^6$) in the merger cascade scenario. The thinner black lines indicate contours of the rate-averaged PBH mass $m_{\rm avg} = (\sum_j R_j m_j)/\sum_j R_j$ at $z = 0$. *Right.* Combined constraints on $f_{\rm PBH}$ for different clustering parameters $\delta_{\rm dc,0}$.

ues, but with a high-mass tail extending into the LIGO band yielding the observed merger rate, $f_{\rm PBH}$ is limited to about 5%. Upcoming LIGO observations yielding significant statistical information on the BH mass distribution may be able to distinguish these two possibilities [58].

Throughout this letter we have assumed a monochromatic initial PBH mass distribution. A full investigation of extended mass distributions, as expected from realistic PBH formation scenarios, is beyond the scope of this work. However, we expect merger cascades dominated by late time mergers (as discussed in this letter) to provide the strongest constraints on PBH DM also in this case.

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