

Velocity Dependence from Resonant Self-Interacting Dark Matter

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The dark matter density distribution in small-scale astrophysical objects may indicate that dark matter is self-interacting, while observations from clusters of galaxies suggest that the corresponding cross section depends on the velocity. Using a model-independent approach, we show that resonant self-interacting dark matter (RSIDM) can naturally explain such a behavior. In contrast to what is often assumed, this does not require a light mediator. We present explicit realizations of this mechanism and discuss the corresponding astrophysical constraints.

Dark matter (DM) makes up more than 80% of the matter in the Universe today and played a crucial role in forming stars and galaxies, and hence us. Yet its nature is unknown. Currently the best pieces of information come from astrophysical observations. N-body simulations of collisionless DM predict astrophysical halos with DM density following a universal profile that scales as $\rho \propto r^{-3}$ in its outskirts but exhibits a central cusp, $\rho \propto r^{-\beta}$, with $\beta \simeq 1$, referred to as the Navarro-Frenk-White (NFW) profile [1–3]. Nevertheless, many studies show hints of a DM mass deficit in the inner regions of certain halos. Notably, observations indicate that numerous dwarf galaxies [4–6] and some low-surface-brightness spiral galaxies [7–9] have a shallower central DM density, better described by a core of constant density, *i.e.*, by $\beta \simeq 0$. This is known as the core-vs-cusp problem. Although it is more pressing in small-scale objects, shallower DM density profiles –with a slope of $\beta \simeq 0.5$ – have been reported for certain galaxy clusters [10, 11]. Moreover, the DM mass deficit also manifests itself in halos that are less dense than what simulations suggest if they host the galaxies that we observe. This is the too-big-to-fail problem, observed for the subhalos of the Milky Way [12], Andromeda [13] and the Local Group [14].

Several explanations for these discrepancies have been discussed in the literature. The systematic uncertainties introduced in deriving DM distributions from observations of luminous objects are one of them. Most importantly, the motions of HI gas and stars may not be faithful tracers of the DM circular velocity [16–29]. Baryonic processes are another conceivable explanation for the discrepancies, since the aforementioned simulations only include collisionless DM. Solutions along this line include supernova-driven baryonic winds [30–33], DM heating due to star formation [34], infalling baryonic clumps [35–38] as well as active galactic nuclei or black holes [39]. Nonetheless, there is no consensus on why systematic uncertainties or baryonic processes lead to a seemingly universal mass deficit at various scales.

A more exciting possibility consists of considering DM collisions in the inner regions of astrophysical objects [40].

This is known as self-interacting dark matter (SIDM). N-body simulations [41–46] confirm that DM scattering processes indeed reduce the central density of DM halos, providing a solution to both problems. For a recent review see [47].

The observed mass deficit is more appreciable in small-scale halos, where the DM velocity dispersion is relatively low. Therefore, a self-scattering cross section that decreases with the DM velocity can better fit observations [15]. A long-range force induced by a light boson interacting with DM is often invoked to obtain such a cross section [40, 48]. Other possibilities that do not involve a light mediator include exothermic inelastic scatterings [49] and self-heating DM [50].

The essence of this work is to discuss the resonant self-interaction of DM (RSIDM) as another mechanism for achieving the desired velocity dependence of SIDM. Such a resonant behavior was pointed out for DM annihilation in [51, 52], and applied to DM self-scattering [53, 54] in specific scenarios. Nevertheless, the velocity dependence of resonant self-scattering and its general astrophysical consequences have not been explored in detail. In this letter, we do so in a model-independent way, and show that resonant scattering is able to address the observed DM mass deficit at all astrophysical scales. Concrete DM scenarios and indirect searches are discussed later.

Resonant scattering in DM halos. Numerous studies claim that the density distribution of certain DM halos do not follow a NFW profile in the inner region. In the SIDM hypothesis, this is due to DM collisions that thermalize the DM particles in such a region thereby reducing its average density [40]. Hence the inner profile is closely related to the velocity-averaged scattering cross section per unit of DM mass, $\langle\sigma v\rangle/m$, where [97]

$$\langle\sigma v\rangle = \int_0^{v_{\max}} f(v, v_0) \sigma v dv, \quad f(v, v_0) = \frac{4v^2 e^{-v^2/v_0^2}}{\sqrt{\pi} v_0^3}. \quad (1)$$

Here, v is the relative velocity, which we assume to follow a Maxwell-Boltzmann distribution truncated at the

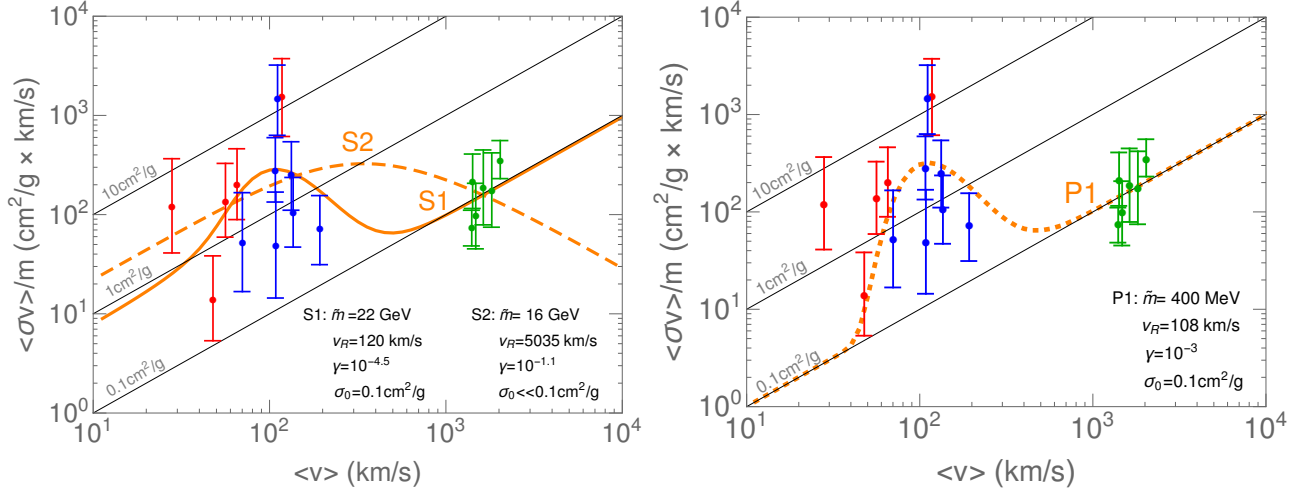


Figure 1: RSIDM cross section per unit of mass as a function of the velocity. Best-fit curves to data [15] for S -wave (left) and P -wave scatterings (right). The latter is also the best-fit curve for $L > 1$ after rescaling the mass with Eq. (8). Here $\tilde{m} = mS^{-1/3}$. See text for details.

escape velocity, v_{\max} , of the corresponding halo. v_0 is a parameter related to the average relative velocity via $\langle v \rangle \simeq 2v_0/\sqrt{\pi}$. Notice that in dwarf galaxies $\langle v \rangle \sim 20$ km/s whereas in clusters of galaxies $\langle v \rangle \sim 2000$ km/s.

A semi-analytical method has been proposed in [15] to infer the value of $\langle\sigma v\rangle/m$ for a given DM halo from observational data. The method was applied to five clusters from [11], seven low-surface-brightness spiral galaxies in [55] and six dwarf galaxies of the THINGS sample [56]. Fig. 1 shows their results in green, blue and red, respectively. At face value, this figure demonstrates that a cross section independent of the velocity –the ones corresponding to the diagonal lines– can hardly accommodate all points. Roughly speaking, $\sigma/m \simeq 2\text{ cm}^2/\text{g}$ is required for small galaxies and $\sigma/m \simeq 0.1\text{ cm}^2/\text{g}$ at cluster scales. Notice that the latter is in agreement with observations from the Bullet Cluster giving $\sigma/m \lesssim 1.3\text{ cm}^2/\text{g}$ [57, 58], which is one of the strongest constraints on DM self-interactions.

The figure suggests that the cross section depends on $\langle v \rangle$. In this letter, we propose that this is due to RSIDM. This takes place when there exists an intermediate particle, denoted as R , so that the total self-scattering cross section can be cast as a sum of a constant piece, σ_0 , plus a Breit-Wigner resonance [98]. More explicitly, for non-relativistic DM,

$$\sigma = \sigma_0 + \frac{4\pi S}{mE(v)} \cdot \frac{\Gamma(v)^2/4}{(E(v) - E(v_R))^2 + \Gamma(v)^2/4}, \quad (2)$$

where the total kinetic energy and symmetry factor read

$$E(v) = \frac{1}{2} m v^2 \quad \text{and} \quad S = \frac{2J_R + 1}{(2J_{\text{DM}} + 1)^2}. \quad (3)$$

Here, J_R and J_{DM} are the spins of the resonance and the DM particles, respectively. $m/2$ is the reduced mass. If DM has internal degrees of freedom other than its spin, they must be accounted for in S . The collision hits the resonance when $v = v_R$ and hence $E(v_R) = m_R/2 - m$.

In addition, the width in Eq. (2) can be calculated in terms of the resonance self-energy by means of $\Gamma(v) = \text{Im} \Sigma(v)/m_R$. This, as well as the denominator in Eq. (2), assumes that the total width is dominated by the process $R \rightarrow \text{DMDM}$. Besides that, Eq. (2) is completely general as it directly follows from unitarity considerations of the scattering matrix [59]. In perturbative theories, the running width can be written as

$$\Gamma(v) = m_R \gamma v^{2L+1}. \quad (4)$$

Here, L is the orbital angular momentum, $\Gamma(v_R)$ is the decay rate, and a constant $\gamma \lesssim O(1)$ characterizes the coupling between the resonance and DM. The factor v^{2L+1} accounts for the phase space and possible angular momentum suppression. Then we find $\langle\sigma v\rangle = \sigma_0 \langle v \rangle + 256\pi S \mathcal{I}_L(\gamma, v_R, v_0)/m^2$, where a dimensionless

$$\mathcal{I}_L(\gamma, v_R, v_0) \equiv \int_0^{v_{\max}} \frac{\gamma^2 f(v, v_0) v^{4L+1} dv}{(v^2 - v_R^2)^2 + 16\gamma^2 v^2 (2L+1)} \quad (5)$$

determines the non-trivial velocity-dependence of the resonant self-scattering. For S -wave and P -wave scatterings, we calculate the best-fit parameter sets S1, S2, and P1 based on the inferred data from Ref. [15] and show them in Fig. 1. σ_0/m is fitted with the other parameters for S1 and P1 while for S2 a negligible $\sigma_0/m \ll 0.1\text{ cm}^2/\text{g}$ is taken as a prior. They all lead to $\chi^2/\text{d.o.f.} \simeq 2$. For S1 and P1, we show the 95% C.L. contours in Fig. 2. Many

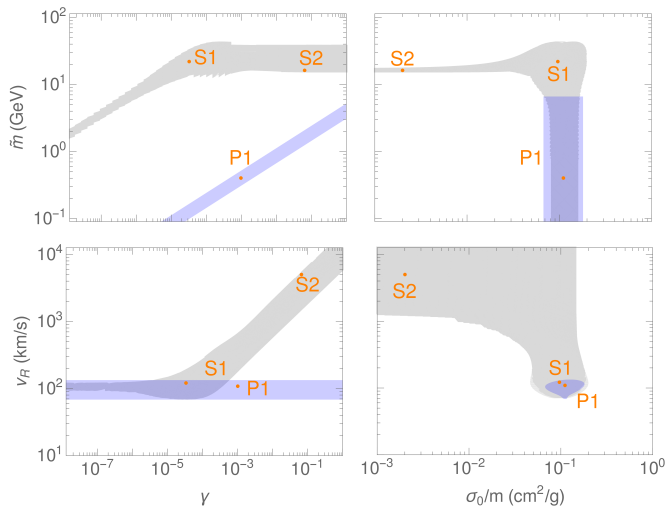


Figure 2: 95% C.L. contours for S -wave (gray) and P -wave (purple) scatterings together with the corresponding parameter sets of Fig. 1.

comments are in order.

First, we have numerically checked that a precise knowledge of the escape velocity is not necessary for calculating \mathcal{I}_L . This is because Eq. (5) converges quite fast due to the Boltzmann factor. In fact, as discussed in the Appendix, exact solutions exist in the limit $v_{\max} \gg v_0$, which will be implicitly applied hereafter for simplicity.

Second, to qualitatively understand Figs. 1 and 2, one can use the narrow-width approximation (NWA). See Eq. (10) of the Appendix. It works very well for $L \geq 1$ because $\gamma^2 v^{2(2L+1)} \ll v^4$. In this case, we find that $\mathcal{I}_L(\gamma, v_R, v_0)$ scales as $\gamma^2 v_0^{4L+1}/v_R^4$ at $v_0 \ll v_R$, and as $\gamma^2 v_0^{4L-3}$ at $v_0 \gg v_R$. In both regions, \mathcal{I}_L can not be much larger than one. Therefore, the resonant effect is negligible except for the intermediate region, where the NWA captures the velocity dependence as

$$\left. \frac{\langle \sigma v \rangle}{m} \right|_{\text{NWA}} = \frac{\sigma_0 \langle v \rangle}{m} + \frac{128S\pi^{\frac{3}{2}}\gamma v_R^{2L+1}}{m^3 v_0^3} e^{-v_R^2/v_0^2}. \quad (6)$$

Notice that the peak lies at $v_0 \sim v_R$ as illustrated by P1 in Fig. 1. The corresponding line actually applies to any $L \geq 1$, because the dependence on L can be absorbed by rescaling m . Using Eq. (6) we find that the best-fit parameters at 95% C.L. for $L \geq 1$ are given by

$$v_R = 108_{-43}^{+28} \text{ km/s}, \quad \sigma_0/m = 0.11_{-0.05}^{+0.10} \text{ cm}^2/\text{g}, \quad (7)$$

$$\tilde{m} = 400_{-90}^{+120} \text{ MeV} \cdot \left(\frac{\gamma}{10^{-3}}\right)^{\frac{1}{3}} \left(\frac{v_R}{3 \cdot 10^3 \text{ km/s}}\right)^{2(L-1)/3}. \quad (8)$$

The regions where this applies are shown in Fig. 2. For P -wave scattering, demanding $\gamma \lesssim 1$ leads to $\tilde{m} \equiv m S^{-1/3} \lesssim 5 \text{ GeV}$. Moreover, a perturbative σ_0/m around $0.1 \text{ cm}^2/\text{g}$ requires sub-GeV DM masses unless

Scenario	Interaction Lagrangian	L	J_{DM}	J_R^P	S	γ
I	$g R \overline{\text{DM}} \gamma^5 \text{DM}$	0	$\frac{1}{2}$	0^-	$\frac{1}{4}$	$\frac{g^2}{32\pi}$
IIa	$g R \text{DM}^i \text{DM}^i$	0	0	0^+	$\frac{1}{3}$	$\frac{g^2}{16\pi m_R^2}$
IIb	$g \epsilon_{ijk} R_\mu^i \text{DM}^j \partial^\mu \text{DM}^k$	1	0	1^-	1	$\frac{g^2}{384\pi}$
III	$\frac{1}{\Lambda} R_{\mu\nu} \mathcal{T}_{\text{DM}}^{\mu\nu}$	2	0	2^+	5	$\frac{m_R^2}{30720\pi\Lambda^2}$

Table I: Benchmark RSIDM models.

$S \gg 1$. Interestingly, P1 predicts $\sigma/m \sim 0.1 \text{ cm}^2/\text{g}$ at $\langle v \rangle \ll 100 \text{ km/s}$. In fact, scatterings with $L \geq 1$ can realize small cross sections at very low velocities. Hence, the recent claim based on Draco observations [29] is consistent with RSIDM.

As long as $v_R \gtrsim 4\gamma$, the NWA also applies for S -wave scattering. For $v_R \ll 4\gamma$, \mathcal{I}_L is proportional to v_0 (to $1/v_0$) below (above) $v_{\text{peak}} \sim v_R^2/(4\gamma) \ll v_R$, because such large values of γ broaden the resonance. S1 and S2 illustrate the narrow and the broad width cases, respectively.

We thus conclude that resonant scattering is able to address the observed DM mass deficit at all astrophysical scales. For this, we need a near-threshold resonance with $(m_R - 2m)/m$ ranging from $\sim 10^{-6}$ for narrow resonances to 10^{-2} for S -wave scattering with broad widths.

RSIDM Models. Below we illustrate the previous model-independent results in concrete RSIDM scenarios. We first introduce a Lagrangian specifying the coupling of the DM to the resonance (see Table I) and calculate the cross section and the self-energy. We subsequently corroborate that they can be cast as Eqs. (2) and (4) show. The scenarios are:

I. *Fermionic DM with a pseudoscalar mediator.* The scattering process is S -wave while $\sigma_0 \simeq 0$. The corresponding best fit is thus S2. Notice that a light pseudoscalar mediator does not lead to SIDM because it induces a suppressed Yukawa potential (see e.g. [60]). Due to this and because it leads to velocity-suppressed direct-detection rates, this candidate is phenomenologically interesting.

II. *Dark mesons.* In QCD-like theories, DM can be a dark pion. Analogous to real pions, it can be a triplet DM^i , with $i = 1, 2, 3$. If R is a dark σ resonance (IIa), the scattering takes place via the S -wave, where we expect GeV DM and $\sigma_0/m \ll 0.1 \text{ cm}^2/\text{g}$. The best fit is thus S2. If R is a dark ρ resonance (IIb), the scattering is P -wave suppressed. The constant piece of the cross section is given by $\sigma_0 \sim \pi\gamma^2/m^2$ in perturbation theory, but it is plausible that there are other contributions. We therefore leave σ_0 as a free parameter. The corresponding best-fit curve is P1. We expect $m \sim 400 \text{ MeV}$ in this case. In the same fashion, minimal QCD-like theories can also lead to spin-1 DM [61]. In all cases, DM can be produced by means of the SIMP [62–83] and the freeze-in [84–86] mechanisms.

III. *Tensor resonances.* They also arise in strongly-

coupled theories. Despite the potential complications of such theories, the generality of our approach allows to describe the scattering induced by a spin-2 resonance $R_{\mu\nu}$ [99]. If this couples to the DM energy-momentum tensor with a cut-off scale Λ , and taking scalar DM as an example, we find that the corresponding Feynman rules [87] indeed lead to a D -wave cross section given by Eq. (2). For $m \sim 10^{-3}\Lambda$, we obtain keV DM with $\gamma \sim 10^{-13}$. The corresponding best fit is given by P1 in Fig. 1 after rescaling the mass by means of Eq. (8).

Annihilation vs. Scattering. If the resonance also decays into a pair of Standard Model particles $f\bar{f}$, in analogy to Eq. (2), the resonant DM annihilation into $f\bar{f}$ has a cross section

$$\sigma_{\text{anni}} \simeq \frac{4\pi S}{mE(v)} \cdot \frac{\Gamma(v) \cdot m_R \gamma_f / 4}{(E(v) - E(v_R))^2 + \Gamma(v)^2 / 4}, \quad (9)$$

where $m_R \gamma_f$ is the decay width for $R \rightarrow f\bar{f}$. As above, we assume that the resonance dominantly decays to a pair of DM particles, and thus that the contribution of f to the imaginary part of the resonance self-energy, $m_R^2 \gamma_f$, is subleading. As expected for annihilations (but not for elastic scatterings), $\sigma_{\text{anni}} v \propto v^{2L}$ as long as $v \ll v_R$. Furthermore, for the cases where NWA applies, $\langle \sigma_{\text{anni}} v \rangle|_{\text{peak}} \sim 32\pi^2 S \gamma_f / (m^2 v_R^3)$. In contrast, for broad S -wave resonances such as S2, where $v_{\text{peak}} \ll v_R$, $\langle \sigma_{\text{anni}} v \rangle|_{\text{peak}}$ gets enhanced by another factor $(v_R/v_{\text{peak}})^{2L+1}$.

The coupling to light charged particles is mostly constrained by Fermi-LAT observations of local satellites [88, 89] and the Planck data on CMB [90, 91]. For instance, the corresponding Fermi-LAT upper limit on $\langle \sigma v_{\text{anni}} \rangle$ for GeV DM is of the order of $10^{-26} \text{ cm}^3/\text{s}$. For S2, this leads to an upper limit on the branching ratio, $\gamma_f/(\gamma v_R)$, of about 10^{-13} – 10^{-12} . This bound is much stronger than that of S1 and P1, due to the enhancement factor mentioned above. Motivated by this, we conservatively fix $\gamma_f/(\gamma v_R^{2L+1}) = 10^{-13}$ and calculate the annihilation cross section as a function of $\langle v \rangle$ for the same parameter sets of Fig. 1. The result is shown in Fig. 3. Therefore, the resonance can only couple feebly to light charged particles, which is why the SIDM candidates with thermal freeze-out from [53] are excluded. Of course this is model-dependent. For instance, if the resonance only couples to neutrinos, the bound on $\langle \sigma_{\text{anni}} v \rangle$ becomes much weaker, and larger γ_f/γ are thus allowed.

Furthermore, as for SIDM with light mediators [92–95], the strong velocity dependence of $\langle \sigma v_{\text{anni}} \rangle$ suggests that the usual freeze-out can hardly work. Nevertheless, the DM abundance might arise from other SIDM production mechanisms [86]. Indeed, for the S -wave case, producing the observed DM abundance with small couplings is possible via freeze-in [84, 85] or 4-to-2 annihilations [72].

Discussion. We advocate the resonant scattering as

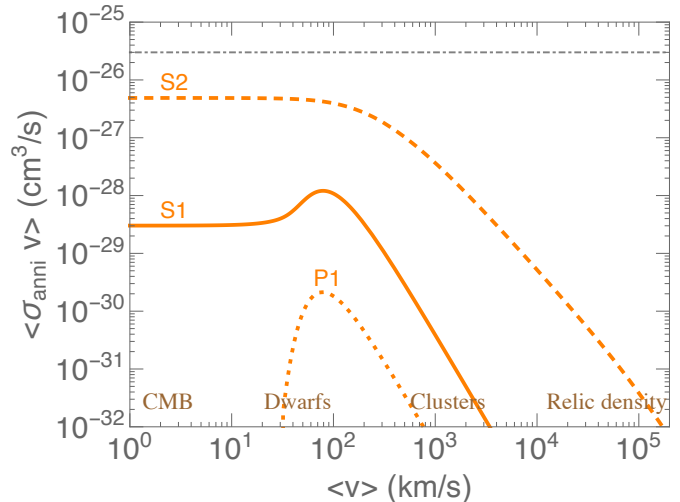


Figure 3: Annihilation cross section into a pair of charged fermions for the parameter sets of Fig. 1, assuming a branching ratio $\gamma_f/(\gamma v_R^{2L+1}) = 10^{-13}$. The horizontal line gives the standard freeze-out benchmark.

a possible SIDM realization with a velocity-dependent scattering cross section. This RSIDM scenario requires a near-threshold resonance, instead of a light mediator. Such resonances exist in Nature. As an example, α particles resonantly scatter by means of ${}^8\text{Be}$ in exactly the same way as described above. In fact, these processes were the main subject of the original article by Breit and Wigner [59] and they may as well occur in the DM sector. Actually, dark nucleons as SIDM have been studied in [54] [100]. Furthermore, lattice studies suggest that QCD-like theories of DM might possess such states [96].

Conclusions. We find that this RSIDM hypothesis can certainly address the core-vs-cusp and the too-to-big-fail problems while still being in agreement with cluster observations. We have also discussed indirect detection signatures, which are nevertheless model-dependent. Additionally, we would like to emphasize that usual SIMPs –which are often said to be disfavored because their scattering cross section does not vary with velocity– can easily accommodate the mechanism proposed here.

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Useful Integrals

In this appendix we solve Eq. (5) for $v_{\max} \rightarrow \infty$. In the narrow width approximation, i.e. when the second term in the denominator is much smaller than the first one, we can do the replacement

$$\frac{1}{(v^2 - v_R^2)^2 + 16\gamma^2 v^{2(2L+1)}} \rightarrow \frac{\pi\delta(v - v_R)}{8\gamma v_R^{2(L+1)}}. \quad (10)$$

This leads to

$$\mathcal{I}_L(\gamma, v_R, v_0) \simeq \frac{\pi\gamma}{8} f(v_R, v_0) v_R^{2L-1}, \quad (11)$$

which is valid for $\gamma \ll v_R^{1-2L}$. If $\gamma \lesssim \mathcal{O}(1)$, which is the region of interest in this work, this approximation is very good for any $L \neq 0$. We have corroborated this numerically. For $L = 0$, such an approximation does not always work. That is however not a problem because there is an exact formula in terms of the exponential integral function, $\text{Ei}(z)$, defined by the principal value of $-\int_{-z}^{\infty} e^{-t} dt/t$.

For this, let us first notice that when $L = 0$ one can rearrange the denominator of Eq. (5) in terms of the integrals

$$\mathcal{I}_{\pm}(\gamma, v_R, v_0) \equiv \int_0^{\infty} \frac{\gamma^2 f(v, v_0) v dv}{(v^2 \mp v_R^2)^2 \pm 16\gamma^2}, \quad (12)$$

which are defined for γ and v_R real. Specifically

$$\mathcal{I}_{L=0}(\gamma, v_R, v_0) = \frac{1}{\pm(v_R^2 - 4\gamma)} \mathcal{I}_{\pm} \left(\gamma \sqrt{\pm(v_R^2 - 4\gamma^2)}, \sqrt{\pm(v_R^2 - 8\gamma^2)}, v_0 \right), \quad (13)$$

where the plus sign applies for $\gamma^2 < v_R^2/8$ and the negative one for $\gamma^2 > v_R^2/4$. (For simplicity, we do not report the expression for the narrow range $v_R^2/8 < \gamma^2 < v_R^2/4$). Finally, \mathcal{I}_- can be calculated by means of

$$\mathcal{I}_-(\gamma, v_R, v_0) = \frac{\gamma}{16} f \left(i \sqrt{v_R^2 + 4\gamma}, v_0 \right) \text{Ei} \left(-\frac{v_R^2 + 4\gamma}{v_0^2} \right) + (\gamma \rightarrow -\gamma). \quad (14)$$

Moreover, by analytically extending the previous expression, we can calculate the other integral. This is

$$\mathcal{I}_+(\gamma, v_R, v_0) = -\mathcal{I}_-(i\gamma, iv_R, v_0) + \frac{\pi\gamma}{8v_R^2} f(v_R, v_0) \left(v_R^2 \cos \left(\frac{4\gamma}{v_0^2} \right) + 4\gamma \sin \left(\frac{4\gamma}{v_0^2} \right) \right). \quad (15)$$

As a check, one can take the limit $\gamma \rightarrow 0$ and recover the narrow width approximation of Eq. (11).

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- [98] Note that the interference term only exists for *S*-wave scattering. For the cases discussed here, that term was found to be negligible with respect to the second term of Eq. (2). Furthermore, it changes its sign from below to above resonance and hence nearly cancels out upon integration over the velocity profile.
- [99] Not to be confused with the Ricci tensor.
- [100] Their findings suggest that their model is described by S2.