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Interpretation of the Narrow $J/\psi\, p$ Peaks in $\Lambda_b \to J/\psi\, p\, K^-$ Decay in the Compact Diquark Model

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Abstract

Recently, the LHCb Collaboration have updated their analysis of the resonant $J/\psi\, p$ mass spectrum in the decay $\Lambda_b^0 \to J/\psi\, p\, K^-$. In the combined Run 1 and Run 2 LHCb data, three peaks are observed, with the former $P_c(4450)^+$ state split into two narrow states, $P_c(4440)^+$ and $P_c(4457)^+$, having the masses $M = (4440.3 \pm 1.3^{+4.1}_{-4.7})$ MeV and $M = (4457.3 \pm 0.6^{+4.1}_{-1.7})$ MeV, and decay widths $\Gamma = (20.6 \pm 4.9^{+8.7}_{-10.1})$ MeV and $\Gamma = (6.4 \pm 2.0^{+5.7}_{-1.9})$ MeV, respectively. In addition, a third narrow peak, $P_c(4312)^+$, having the mass $M = (4311.9 \pm 0.7^{+6.8}_{-0.6})$ MeV and decay width $\Gamma = (9.8 \pm 2.7^{+3.7}_{-4.5})$ MeV is also observed. The LHCb analysis is not sensitive to broad $J/\psi\, p$ contributions like the former $P_c(4380)^+$, implying that there could be more states present in the data. Also the spin-parity, J^P, assignments of the states are not yet determined. We interpret these resonances in the compact diquark model as hidden-charm diquark-diquark-antiquark baryons, having the following spin and angular momentum quantum numbers: $P_c(4312)^+ = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 0, J^P = 3/2^-\}$, the S-wave state, and the other two as P-wave states, with $P_c(4440)^+ = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^P = 3/2^+\}$ and $P_c(4457)^+ = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^P = 5/2^+\}$. The subscripts denote the spins of the diquarks and $L_{\mathcal{P}} = 0, 1$ is the orbital angular momentum quantum number of the pentaquark. These assignments are in accord with the heavy-quark-symmetry selection rules for Λ_b-baryon decays, in which the spin $S = 0$ of the light diquark $[ud]_{s=0}$ is conserved. The masses of observed states can be accommodated in this framework and the two heaviest states have the positive parities as opposed to the molecular-like interpretations. In addition, we predict several more states in the $J/\psi\, p$ mass spectrum, and urge the LHCb Collaboration to search for them in their data.

I. INTRODUCTION

In 2015, the LHCb Collaboration reported the first observation of two hidden-charm pentaquark states $P_c(4380)^+$ and $P_c(4450)^+$ in the decay $\Lambda_b^0 \to J/\psi\, p\, K^-$ [1], having the masses $M = (4380 \pm 8 \pm 29)$ MeV and $M = (4449.8 \pm 1.7 \pm 2.5)$ MeV, and decay widths $\Gamma = (205 \pm 18 \pm 86)$ MeV and $\Gamma = (39 \pm 5 \pm 19)$ MeV, with the preferred spin-parity assignments $J^P = 3/2^-$ and $J^P = 5/2^+$, respectively, but the reversed spin-parity assignments were also tenable. This was followed in a subsequent paper [2], in which evidence was presented for the Cabibbo-suppressed decay $\Lambda_b^0 \to J/\psi\, p\, \pi^-$, found consistent with the production of the $P_c(4380)^+$ and $P_c(4450)^+$ pentaquark states. These states have the quark content ($c\bar{c}uud$) and, like their tetraquark counterparts X, Y, and Z, they lie close in mass to several (charmed meson-baryon) thresholds [3]. This has led to a number of theoretical proposals for their interpretation, which include rescattering-induced kinematical effects [4–7], open charm-baryon and charm-meson bound states [8–12], and baryocharmonia [13]. They have also been interpreted as compact pentaquark hadrons with the internal structure organised as diquark-diquark-antiquark [14–23] or as diquark-triquark [24, 25].

Very recently, the LHCb Collaboration has updated their analysis of the $\Lambda_b \to J/\psi\, p\, K^-$ decay, making use of 9 times more data based on Run 1 and Run 2 than in the Run 1 data alone [26]. Thanks to the impressive number (246 K) of Λ_b-baryon signal events, improved $J/\psi\, p$ mass and momentum resolution, and excellent vertexing, narrow $J/\psi\, p$ structures have been observed, which were insignificant in the older Run 1 data [1]. Nominal fits of the data have been performed with an incoherent sum of Breit-Wigner amplitudes, which have resulted in the observation of three peaks, whose masses, decay widths (with 95% C.L. upper limits) and the ratio $\mathcal{R}$, defined as

$$\mathcal{R} \equiv \frac{\mathcal{B}(\Lambda_b \to P_c^+ K^-)\, \mathcal{B}(P_c^+ \to J/\psi\, p)}{\mathcal{B}(\Lambda_b \to J/\psi\, p\, K^-)}, \tag{1}$$

are given in Table I. The two narrow resonances, hence-after called $P_c(4440)^+$ and $P_c(4457)^+$, replace the older resonance, $P_c(4450)^+$, which is now found split, and the third narrow resonance, called $P_c(4312)^+$, is a new addition to the pentaquark spectrum. The situation with the broader $P_c(4380)^+$ state, observed in the 2015 data, having the large width, $\Gamma = (205 \pm 18 \pm 86)$ MeV, is not clear, as the current analysis is not sensitive to broad structures. Hence, a dedicated analysis of the LHCb data may turn out to have more narrow and broad resonances.

In this work we follow the compact pentaquark interpretation. The basic idea of this approach is that highly correlated diquarks play a key role in the physics of multiquark states [27–29]. Since quarks transform as a triplet $\mathsf{3}$ of the color $SU(3)$-group, the diquarks resulting from the direct product $\mathsf{3} \otimes \mathsf{3} = \bar{\mathsf{3}} \oplus \mathsf{6}$, are thus either a color anti-triplet $\bar{\mathsf{3}}$ or a color sextet $\mathsf{6}$. Of these only the color $\bar{\mathsf{3}}$ configuration is kept, as suggested by perturbative arguments. Both spin-1 and spin-0 diquarks are, however, allowed. In the case of a diquark $[qq']$ consisting of two light quarks, the spin-0 diquark is believed to be more tightly bound than the spin-1, and this hyperfine splitting has implications for the spectroscopy. For the heavy-light diquarks, such as $[cq]$ or $[bq]$, this splitting is suppressed by $1/m_c$ for the $[cq]$ or by $1/m_b$ for the $[bq]$ diquark, and hence both spin configurations are treated at par. For pentaquarks, the mass spectrum depends upon how the five quarks, i. e., the 4 quarks and an antiquark, are dynamically structured. A diquark-triquark picture, in which the two observed pentaquarks consist of a rapidly separating pair of a color-$\bar{\mathsf{3}}$ $[cu]$-diquark and a color-$\mathsf{3}$ triquark $\bar{\theta} = \bar{c}[ud]$, has been presented in [24, 25]. A "Cornell"-type non-relativistic linear-plus-Coulomb potential [30] is used to determine the diquark-triquark separation R and the ensuring phenomenology is worked out.

In this paper, we employ the diquark-triquark picture shown in Fig. 1. Here, the nucleus consists of the doubly-heavy triquark, with the $[cq]$-diquark and $\bar{c}$-antiquark, and a light diquark $[q'q'']$, where q, q', and q'' are u-, d-, and s-quarks, which is in an orbit for the P-wave (and higher orbitally-excited) states, as it is easier to excite a light

TABLE I: Masses, decay widths (with 95% C.L. upper limits), and the ratio $\mathcal{R}$, of the three narrow $J/\psi\, p$ resonances observed by the LHCb Collaboration in the decay $\Lambda_b \to J/\psi\, p\, K^-$ [26].

State	Mass [MeV]	Width [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

diquark to form an orbitally-excited pentaquark. All three constituents are color anti-triplets, $\bar{\mathbf{3}}$, making an overall color-singlet — the pentaquark. Such a description is closer to the doubly-heavy tetraquarks and doubly-heavy baryons, in which the double-heavy diquark may be considered as the static color source, which has received a lot of theoretical attention lately [31–41]. The resulting pentaquark spectrum in the diquark model is very rich. However, imposing the spin conservation in the heavy-quark symmetry limit, which was already advocated in the analysis of the older LHCb data [22], we argue that only that part of the pentaquark spectrum is reachable in Λ_b-decays, in which the pentaquarks have a "good" light diquark, i. e., having the spin $S_{ld} = 0$, in their Fock space. This reduces the number of observable pentaquark states greatly. We show that the newly observed spectrum shown in Table I can be accommodated in the diquark picture, though the spin-parity, J^P, assignment of the observed states is tentative, as these quantum numbers are not yet measured. There are yet more states present in the $J/\psi\, p$ mass spectrum in the compact diquark model, whose masses and J^P quantum numbers are given here, and we urge the LHCb Collaboration to search for them in their data.

This paper is organised as follows. In Section II, we introduce the doubly-heavy triquark — light diquark model of pentaquarks, and define the state vectors having the total angular momentum quantum number J by $|S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J$, where S_{hd}, S_t, and S_{ld} are the spins of the heavy diquark, doubly-heavy triquark and light diquark, respectively, and L_t and L_{ld} are the orbital quantum numbers of the triquark and light-diquark, which are combined into the total orbital angular momentum L of the pentaquark. The corresponding set of the S-wave state vectors (with $L_t = L_{ld} = L = 0$) with the "good" ($S_{ld} = 0$) light diquark is presented in Table II. State vectors of the P-wave pentaquarks with the ground-state triquark ($L_t = 0$, $L_{ld} = L = 1$) and "good" light diquark with the spin $S_{ld} = 0$ are given in Table III. In Section III, we give the effective Hamiltonian used to work out the pentaquark mass spectrum. In Section IV, the analytical expressions for the matrix elements of the effective Hamiltonian are presented taking into account the dominant spin-spin interactions in the heavy and light diquarks and in the hidden-charm triquark. For the P-wave states, additional contributions and mixings due to the orbital and spin-orbit interactions are included. In all the cases, we diagonalise the mass matrices, and analytical equations for all the masses of the S- and P-wave pentaquarks, which can be reached in Λ_b-decays, are presented. Section V contains the values of various input parameters and predictions for the pentaquark masses. Possible assignments of the newly observed pentaquark states are also discussed here. We conclude in Section VI.

II. DOUBLY-HEAVY TRIQUARK — LIGHT DIQUARK MODEL OF PENTAQUARKS

In the pentaquark picture considered here, there are two flux tubes, with the first stretched between the charmed diquark and charmed antiquark from which the diquark is in the color-antitriplet state $\bar{3}$. With the antiquark being

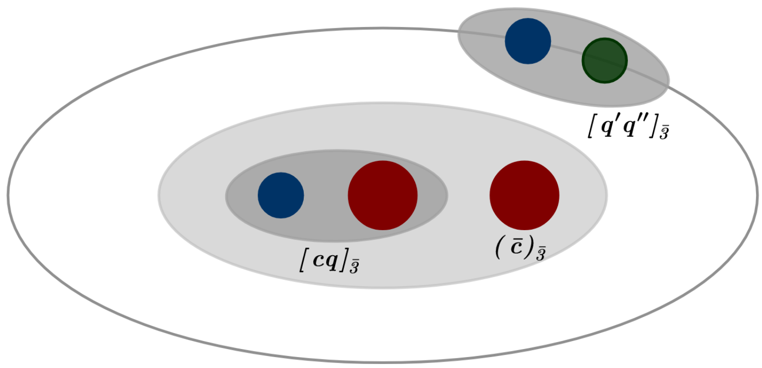

FIG. 1: A picture of pentaquarks in the diquark model involving the heavy diquark $[cq]_{\bar{3}}$ and charmed antiquark $(\bar{c})_{\bar{3}}$, which form the triquark system, which combines with the light diquark $[q'q'']_{\bar{3}}$ to make a color singlet pentaquark. The subscripts indicate that all three constituents are color anti-triplets; q, q', and q'' are light quarks each of which can be u-, d-, or s-quark.

also a color anti-triplet state $\bar{3}$, their product is decomposed into two irreducible representations, $\bar{3} \times \bar{3} = 3 + \bar{6}$, from which the color triplet 3 is kept. This color-triplet triquark makes a color-singlet bound state with the light diquark through the second flux tube in much the same way as the quark and diquark bind in an ordinary baryon. We modify the effective Hamiltonian for the S-wave pentaquarks [22, 23] by keeping the most relevant terms for the mass determination, and then extend it for the P-states by including the orbital and spin-orbit interactions between the hidden-charm triquark and light diquark. Tensor interactions effect the P-wave pentaquark spectrum in which light diquarks have the spin $S_{ld} = 1$, and not shown here.

The effective Hamiltonian for the ground-state pentaquarks is described in terms of two constituent masses of the heavy diquark $m_{[cq]} \equiv m_{hd}$ and the light diquark $m_{[q'q'']} \equiv m_{ld}$, where q, q', and q'' are light u-, d- and s-quarks, the spin-spin interactions between the quarks in each diquark shell, and spin-spin interactions between the diquarks. To these are added the constituent mass m_c of the charmed antiquark and its spin-spin interactions with each of the diquarks.

In the case of the orbitally excited pentaquarks, the orbital angular momentum L of the pentaquark is the sum of two terms, L_t, arising from the triquark system consisting of the heavy diquark and the charmed antiquark, and L_{ld}, which determines the relative motion of the light diquark around the doubly-heavy triquark. The total orbital angular momentum L of the pentaquark is then obtained with the help of the momentum sum rules from quantum mechanics, i. e., it takes a value from the range $L = |L_t - L_{ld}|, \dots, L_t + L_{ld}$. The orbital angular momentum L is combined with the spin S to get the total angular momentum J of the pentaquark using the $L - S$ coupling scheme. With the quantum numbers introduced above, we specify the complete orthogonal set of basis vectors for the hidden-charm pentaquark states. As already stated, we define the state vectors having a total angular momentum quantum number J by $|S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J$.

For the orbitally excited states, one needs to specify which part of the pentaquark wave function is excited. In the triquark-diquark template used here, and shown in Fig. 1, the heavy triquark state consists of the charmed diquark and charmed antiquark. Since both are heavy, the triquark is an (almost) static system. Hence, $L_t = 0$ is the most probable quantum state of the triquark. Thus, the orbital excitation is generated by the light diquark (i. e., $L = L_{ld}$). With this, we list the lowest-lying orbitally excited states ($L = 1$) with the "good" ($S_{ld} = 0$) light diquark in Table III.

TABLE II: Spin-parity J^P and state vectors of the S-wave pentaquarks containing the "good" light diquark with the spin $S_{ld} = 0$. The horizontal line demarcates the spin S_{hd} of the heavy diquark.

J^P	$\|S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J$
$1/2^-$	$\|0, 1/2, 0; 0, 0; 1/2, 0\rangle_{1/2}$
$1/2^-$	$\|1, 1/2, 0; 0, 0; 1/2, 0\rangle_{1/2}$
$3/2^-$	$\|1, 3/2, 0; 0, 0; 3/2, 0\rangle_{3/2}$

TABLE III: Spin-parity J^P and state vectors of the P-wave pentaquarks with the ground-state triquark ($L_t = 0$) and "good" light diquark with the spin $S_{ld} = 0$. The horizontal line demarcates the spin S_{hd} of the heavy diquark.

J^P	$\|S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J$
$1/2^+$	$\|0, 1/2, 0; 0, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$\|0, 1/2, 0; 0, 1; 1/2, 1\rangle_{3/2}$
$1/2^+$	$\|1, 1/2, 0; 0, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$\|1, 1/2, 0; 0, 1; 1/2, 1\rangle_{3/2}$
$1/2^+$	$\|1, 3/2, 0; 0, 1; 3/2, 1\rangle_{1/2}$
$3/2^+$	$\|1, 3/2, 0; 0, 1; 3/2, 1\rangle_{3/2}$
$5/2^+$	$\|1, 3/2, 0; 0, 1; 3/2, 1\rangle_{5/2}$

III. EFFECTIVE HAMILTONIAN FOR PENTAQUARK SPECTRUM

We calculate the mass spectrum of pentaquarks under the assumption that their underlying structure is given by $\bar{c}$, $[cq]$, and $[q'q'']$, with q, q', and q'' being any of the light u-, d-, and s-quarks. For this, we extend the effective Hamiltonian proposed for the tetraquark spectroscopy [42]. The effective Hamiltonian for the S-wave pentaquark mass spectrum can be written as follows:

$$H^{(L=0)} = H_t + H_{ld}. \tag{2}$$

The first term in the Hamiltonian (2) is related with the colored triquark:

$$H_t = m_c + m_{hd} + 2\,(\mathcal{K}_{cq})_{\bar{3}}\,(\mathbf{S}_c \cdot \mathbf{S}_q) + 2\,\mathcal{K}_{\bar{c}q}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\,\mathcal{K}_{\bar{c}c}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_c)\,, \tag{3}$$

where m_c and m_{hd} are the constituent masses of the charmed antiquark and charmed diquark, respectively. The last three terms describe the spin-spin interactions in the charmed diquark and between the diquark constituents and the charmed antiquark, quantified by the couplings $(\mathcal{K}_{cq})_{\bar{3}}$, $\mathcal{K}_{\bar{c}q}$, and $\mathcal{K}_{\bar{c}c}$.

The second term H_{ld} in the Hamiltonian (2) contains the operators responsible for the spin-spin interaction in the light diquark and its interaction with the triquark:

$$H_{ld} = m_{ld} + 2\,(\mathcal{K}_{q'q''})_{\bar{3}}\,(\mathbf{S}_{q'} \cdot \mathbf{S}_{q''}) + H_{SS}^{t-ld}, \tag{4}$$

where m_{ld} is the constituent mass of the light diquark, consisting of the light quarks q' and q'', and the contribution of the spin-spin interaction in this diquark to the pentaquark mass is determined by the coupling $(\mathcal{K}_{q'q''})_{\bar{3}}$. The last term in H_{ld} is responsible for all possible spin-spin interactions between the constituents of the light diquark and heavy triquark:

$$\begin{aligned} H_{SS}^{t-ld} &= 2\,(\tilde{\mathcal{K}}_{cq'})_{\bar{3}}\,(\mathbf{S}_c \cdot \mathbf{S}_{q'}) + 2\,(\tilde{\mathcal{K}}_{qq'})_{\bar{3}}\,(\mathbf{S}_q \cdot \mathbf{S}_{q'}) + 2\,\tilde{\mathcal{K}}_{\bar{c}q'}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{q'}) \\ &+ 2\,(\tilde{\mathcal{K}}_{cq''})_{\bar{3}}\,(\mathbf{S}_c \cdot \mathbf{S}_{q''}) + 2\,(\tilde{\mathcal{K}}_{qq''})_{\bar{3}}\,(\mathbf{S}_q \cdot \mathbf{S}_{q''}) + 2\,\tilde{\mathcal{K}}_{\bar{c}q''}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{q''}). \end{aligned} \tag{5}$$

Here, the coefficients with the tilde differ from the ones introduced earlier in Eq. (3), as they are the couplings of the spin-spin interactions inside the compact objects, like diquarks and triquark, while the ones in Eq. (5) denote the couplings between the constituents of the two objects, the heavy triquark and light diquark, and are anticipated to be strongly suppressed. This then accounts for all possible spin-spin interactions and completes the content of the effective Hamiltonian (2) for the masses of the ground-state pentaquarks.

The general form of the effective Hamiltonian for the orbitally-excited pentaquark mass spectrum can be written as follows:

$$H = H^{(L=0)} + H_L + H_T. \tag{6}$$

In addition to the spin-spin interactions introduced for the ground-state pentaquarks described above, the extended effective Hamiltonian (6) includes terms explicitly dependent on the internal orbital angular momentum L_t of the hidden-charm triquark and orbital momentum L_{ld} of the light diquark relative to the triquark system. The corresponding Hamiltonian is called H_L in (6). The tensor interactions are subsumed in H_T in (6). They are relevant only for the pentaquarks in which the light-diquarks have the spin $S_{ld} = 1$. Since they are not anticipated to be produced in the Λ_b-baryons decays, due to the heavy-quark symmetry constraints, we set $H_T = 0$ in this paper. They have to be included in the extended pentaquark systems, detailed in a forthcoming paper [44]. As already stated, we assume that the triquark state is an S-wave (i. e., $L_t = 0$). Thus, the total angular momentum of the pentaquark is determined by the orbital excitation of the light diquark, $L = L_{ld}$. The terms in H_L which contain the orbital angular momentum operator $\mathbf{L}$ are as follows:

$$H_L = 2A_t\,(\mathbf{S}_t \cdot \mathbf{L}) + 2A_{ld}\,(\mathbf{S}_{ld} \cdot \mathbf{L}) + \frac{1}{2}\,B\,\mathbf{L}^2, \tag{7}$$

where A_t, A_{ld}, and B parametrise the strengths of the triquark spin-orbit, light-diquark spin-orbit and orbital momentum couplings, respectively.

IV. MASS FORMULAS FOR PENTAQUARK SPECTRUM

A. S-wave pentaquarks

With the basis vectors of the pentaquark states chosen, one can derive analytical expressions for calculating the pentaquark spectrum [44]. They are the matrix elements of the effective Hamiltonian presented in Sec. III. Note that this is simpler for the Model II by Maiani *et al.* [42], but becomes more involved in the Model I [27], where additional interactions (5) between the spins of (anti)quarks in compact shells are included. As the later couplings are suppressed in comparison with the spin-spin ones inside the shells, we neglect the contributions from the term (5) and restrict ourselves with the Model II [42].

The universal contribution entering all the pentaquark states is defined as M_0, which is the sum of the constituent masses of the heavy and light diquarks and charmed antiquark:

$$M_0 \equiv m_{hd} + m_{ld} + m_c. \tag{8}$$

Apart from this, there are two terms in the effective Hamiltonian explicitly related with the spins of the diquarks [see Eqs. (3) and (4)]:

$$\begin{aligned} &{}_J\langle S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L|\, 2\,(\mathcal{K}_{cq})_{\bar{3}}\,(\mathbf{S}_c \cdot \mathbf{S}_q)\,|S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J \\ &= (\mathcal{K}_{cq})_{\bar{3}} \left[S_{hd}\,(S_{hd}+1) - \frac{3}{2}\right] = \frac{1}{2}\,(\mathcal{K}_{cq})_{\bar{3}} \times \begin{cases} -3, & S_{hd} = 0, \\ 1, & S_{hd} = 1, \end{cases} \end{aligned} \tag{9}$$

$$
{}_J\langle S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L|\, 2\, (\mathcal{K}_{q'q''})_{\bar{3}}\, (\mathbf{S}_{q'} \cdot \mathbf{S}_{q''})\, |S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J \tag{10}
$$
$$
= (\mathcal{K}_{q'q''})_{\bar{3}} \left[S_{ld}\,(S_{ld}+1) - \frac{3}{2} \right] = \frac{1}{2}\,(\mathcal{K}_{q'q''})_{\bar{3}} \times \begin{cases} -3, & S_{ld}=0, \\ 1, & S_{ld}=1. \end{cases}
$$

The remaining terms in Eq. (3) are responsible for the contributions of the spin-spin interactions between the charmed antiquark and the quarks inside the heavy diquark, which together form the triquark state. To calculate their impact, it is convenient to use the Wigner $6j$-symbols, which allows us to describe a recoupling between three spins inside the triquark. The details of calculations are given in [44] and we present the results here.

From the three S-wave states presented in Table II, the two states with $J^P = 1/2^-$ mix due to the spin-spin interaction of the charmed antiquark and the heavy diquark, and the third one with $J^P = 3/2^-$ remains unmixed.

The two states with $J^P = 1/2^-$, after sandwiching the effective Hamiltonian, yield the following (2×2) mass matrix:

$$
M^{S0}_{J=1/2} = M_0 - \frac{1}{2}\,(\mathcal{K}_{cq})_{\bar{3}} - \frac{3}{2}\,(\mathcal{K}_{q'q''})_{\bar{3}} - (\mathcal{K}_{cq})_{\bar{3}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}\,(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}) \begin{pmatrix} 0 & \sqrt{3} \\ \sqrt{3} & -2 \end{pmatrix}. \tag{11}
$$

Diagonalising this matrix yields the masses of these two states:

$$
m_1^{S0} = M_0 - \frac{1}{4}\,(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}) \left[2 + r_{hd} + 3 r_{ld} + 2\sqrt{3 + (1 - r_{hd})^2} \right], \tag{12}
$$
$$
m_2^{S0} = M_0 - \frac{1}{4}\,(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}) \left[2 + r_{hd} + 3 r_{ld} - 2\sqrt{3 + (1 - r_{hd})^2} \right], \tag{13}
$$

where the superscript $S0$ denotes the S-wave pentaquark with the "good" light diquark, $S_{ld} = 0$, and the two ratios r_{hd} and r_{ld} of the couplings are defined as:

$$
r_{hd} \equiv \frac{2(\mathcal{K}_{cq})_{\bar{3}}}{\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}}, \qquad r_{ld} \equiv \frac{2(\mathcal{K}_{q'q''})_{\bar{3}}}{\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}}. \tag{14}
$$

For the state with $J^P = 3/2^-$, the mass m_3^{S0} is the average of the effective Hamiltonian over this state:

$$
m_3^{S0} = M_0 + \frac{1}{4}\,(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c})\,(2 + r_{hd} - 3 r_{ld}). \tag{15}
$$

The case of S-wave pentaquark states with the "bad" light diquark is worked out in detail elsewhere [44].

B. *P*-wave pentaquarks

As discussed earlier in Sec. III, one needs to include the terms dependent on the internal angular momenta of the pentaquark and, in particular, the term (7) from the pentaquark effective Hamiltonian.

From the seven P-wave pentaquark states with the "good" light diquark shown in Table III, two states with $J^P = 1/2^+$ and the other two with $J^P = 3/2^+$, both pairs having the triquark spin $S_t = 1/2$, mix due to the spin-spin interaction of the charmed antiquark and heavy diquark. The other three states with the triquark spin $S_t = 3/2$ remain unmixed due to this interaction but can mix through the spin-spin interactions between the (anti)quarks entering two separated shells — the heavy triquark and light diquark, as also discussed earlier for the S-wave states. As mentioned earlier, such types of spin-spin interactions are suppressed and neglected in this analysis. From the angular-momentum dependent term H_L of the effective Hamiltonian, one obtains the spin-orbit and orbital contributions to matrix elements.

The pair of states with $J^P = 1/2^+$, after sandwiching the effective Hamiltonian, yields the following (2×2) mass matrix:

$$M^{P0}_{J=1/2} = M_0 - \frac{1}{2}(\mathcal{K}_{cq})_{\bar{3}} - \frac{3}{2}(\mathcal{K}_{q'q''})_{\bar{3}} - (\mathcal{K}_{cq})_{\bar{3}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}) \begin{pmatrix} 0 & \sqrt{3} \\ \sqrt{3} & -2 \end{pmatrix} + B - 2A_t, \quad (16)$$

while the masses of the pair with $J^P = 3/2^+$ are determined by the matrix:

$$M^{P0}_{J=3/2} = M_0 - \frac{1}{2}(\mathcal{K}_{cq})_{\bar{3}} - \frac{3}{2}(\mathcal{K}_{q'q''})_{\bar{3}} - (\mathcal{K}_{cq})_{\bar{3}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}) \begin{pmatrix} 0 & \sqrt{3} \\ \sqrt{3} & -2 \end{pmatrix} + B + A_t. \quad (17)$$

Diagonalising these matrices, we get the masses:

$$m^{P0}_1 = M_0 - \frac{1}{4}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c})\left[2 + r_{hd} + 3r_{ld} + 2\sqrt{3 + (1 - r_{hd})^2}\right] + B - 2A_t, \quad (18)$$

$$m^{P0}_2 = m^{P0}_1 + (\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c})\sqrt{3 + (1 - r_{hd})^2}, \quad (19)$$

$$m^{P0}_{3,4} = m^{P0}_{1,2} + 3A_t, \quad (20)$$

where r_{hd} and r_{ld} are defined in Eq. (14) and the superscript $P0$ means the P-wave pentaquark with the "good" light diquark, $S_{ld} = 0$.

For the three states, enumerated as the fifth, sixth and seventh according to the entries in Table III, the masses $m^{P0}_{5,6,7}$ are simply the average values of the effective Hamiltonian over these states:

$$m^{P0}_5 = M_0 + \frac{1}{4}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c})(2 + r_{hd} - 3r_{ld}) + B - 5A_t, \quad (21)$$
$$m^{P0}_6 = m^{P0}_5 + 3A_t, \qquad m^{P0}_7 = m^{P0}_5 + 8A_t.$$

These states are non-degenerate due to the triquark spin-orbit interaction, i. e., $A_t \neq 0$. Hence, the mass gap between them is a measure of the coupling A_t, and the following relations hold:

$$m^{P0}_6 - m^{P0}_5 = \frac{3}{8}\left(m^{P0}_7 - m^{P0}_5\right) = \frac{3}{5}\left(m^{P0}_7 - m^{P0}_6\right). \quad (22)$$

If more states are experimentally observed and their quantum numbers determined, the validity of these relations will be a strong argument in favour of the compact diquark model.

V. HIDDEN-CHARM PENTAQUARK MASS PREDICTIONS

Working within the constituent quark-diquark model [42], input parameters are the masses of the constituents, charmed quark and diquarks, spin-spin couplings, and other parameters related with the orbital or radial excitations. For the constituent quark masses, there are two possibilities: Either extract them from the masses of the known mesons or from baryons [45]. The charmed quark mass is estimated as $m_c^m = 1667$ MeV from the D-mesons spectrum, and $m_c^b = 1710$ MeV from the charmed baryon masses, yielding a difference of 43 MeV [45]. In particular, with the m_c^b value as an input, predictions for the charmed baryon masses were obtained in [46], which are remarkably accurate and differ from the experimentally measured masses [3] by about 10 MeV. We use m_c^b in the numerical analysis.

In contrast to quarks, which have the fixed spin $S_q = 1/2$, for diquarks being composite objects, there are two possible spin configurations from which the antisymmetric one corresponding to the diquark spin $S = 0$ is energetically more favourable. Both configurations are allowed if the flavors of the quarks are different. As we consider the pentaquarks from the Λ_b-decay in the heavy-quark symmetry limit, we need to specify the mass of the light $[ud]$-diquark, for which we use the value $m_{ld} = m_{[ud]} = 576$ MeV [48]. The charmed diquarks mass $m_{hd} = m_{[cq]} = 1976$ MeV is borrowed from [45]. To estimate the errors on the diquark-masses, one should compare the baryon masses predicted within this (constituent quark-diquark) model and the measured ones. Based on the analysis presented in [46], we infer that the computed masses for the charmed baryons are accurate to within 15 MeV. This value may be taken as indicative of the uncertainty in diquark masses.

In addition, we need the spin-spin couplings, $\mathcal{K}_{\bar{Q}Q'}$ and $(\mathcal{K}_{QQ'})_{\bar{3}}$, which are extracted from the mass spectra of mesons and baryons, respectively. The values of the spin-spin couplings used are: $(\mathcal{K}_{cq})_{\bar{3}} = 67$ MeV, $(\mathcal{K}_{qq'})_{\bar{3}} = 98$ MeV, $\mathcal{K}_{\bar{c}q} = 70$ MeV, and $\mathcal{K}_{\bar{c}c} = 113$ MeV. They are taken from [45], except for $\mathcal{K}_{\bar{c}c}$ which is from [46]. The pentaquark masses involve the ratios of the couplings (14) which for the input values are evaluated as: $r_{hd} = 0.73$ and $r_{ld} = 1.07$. In Model II [42], these ratios are of $O(1)$, which reflects the dominance of the spin-spin interaction inside the diquarks and triquark over other possible spin-spin interactions. These arguments give quantitative support in favour of the Model II [42].

In estimating the P-wave pentaquark mass spectrum, the values of the couplings in the orbital angular momentum term B and the spin-orbit coupling A_t are required. We discuss first the two heaviest pentaquarks $P_c(4440)^+$ and $P_c(4457)^+$. As they replace the former narrow $P_c(4450)^+$ state with the preferred spin-parity $J^P = 5/2^+$, we tentatively assign this spin-parity to one of the observed two states, $P_c(4457)^+$. In this case, the lighter partner, $P_c(4440)^+$, most probably has the spin-parity $J^P = 3/2^+$. In the Model II, used here for the pentaquark spectrum, their mass splitting is related to the spin-orbit coupling A_t of the triquark:

$$M[P_c(4457)^+] - M[P_c(4440)^+] = \left(17^{+6.4}_{-4.5}\right) \text{ MeV} = 5A_t, \tag{23}$$

where the error is obtained by adding the experimental errors on the masses in quadrature. From (23), the value of the coefficient A_t follows immediately:

$$A_t = \left(3.4^{+1.3}_{-0.9}\right) \text{ MeV}. \tag{24}$$

It is not surprising that it is numerically small, as the doubly-heavy triquark is almost static. Of course, as there are more states present in the theoretical spectrum than the number of observed pentaquarks, there are also other assignments possible, but we find them unrealistic and hence not discussed here.

The third narrow state, $P_c(4312)^+$, can have several assignments. Identifying it with the $J^P = 3/2^-$ state, one can work out the mass difference between this state and the heavier pentaquarks, $P_c(4440)^+$ and $P_c(4457)^+$. This is determined by the orbital B and the triquark spin-orbit A_t couplings. The strength of the later coupling is already known from the $P_c(4440)^+$ and $P_c(4457)^+$ mass splitting (24), and the mass difference, say, between $P_c(4312)^+$ and $P_c(4457)^+$, allows us to read off the strength of the orbital interaction:

$$M[P_c(4457)^+] - M[P_c(4312)^+] = \left(145.4^{+4.2}_{-7.1}\right) \text{ MeV} = B + 3A_t. \tag{25}$$

With A_t from (24), we get:

$$B = \left(135.2^{+5.0}_{-8.1}\right) \text{ MeV}. \tag{26}$$

This is too small in comparison with the strengths of the orbital excitations in other hadrons [43], and, in particular, with $B(\Omega_c) = 325$ MeV obtained from Ω_c^*-baryons. Moreover, the theoretically predicted masses of the $P_c(4440)^+$ and $P_c(4457)^+$ states with the value of B from (26) are found to be ~ 70 MeV below the experimental values.

TABLE IV: Masses of the hidden-charm unflavored pentaquarks (in MeV) and their comparison with the results presented in [22, 23]. For the P-wave pentaquarks, following values of the parameters are used: orbital coupling $B = 207$ MeV and spin-orbit coupling of the heavy triquark $A_t = 3$ MeV.

J^P	This work	Refs. [22, 23]
	$S_{ld} = 0,\ L = 0$	
$1/2^-$	3830	4086 ± 42
	4150	4162 ± 38
$3/2^-$	$\boxed{4240}$	4133 ± 55
	$S_{ld} = 0,\ L = 1$	
$1/2^+$	4031	4030 ± 62
	4351	4141 ± 44
	4432	4217 ± 40
$3/2^+$	4040	
	4360	
	$\boxed{4441}$	
$5/2^+$	$\boxed{4456}$	4510 ± 57

Alternatively, assuming that the spin-spin couplings and constituent masses are known, the strength of B can also be determined from the masses of $P_c(4440)^+$ and $P_c(4457)^+$ only, as follows:

$$B = \frac{1}{5}\left\{3M[P_c(4440)^+] + 2M[P_c(4457)^+]\right\} - M_0 - \frac{1}{4}\left(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}\right)\left(2 + r_{hd} - 3r_{ld}\right). \quad (27)$$

With the values of the other parameters already assigned, the value $B = 207$ MeV reproduces the masses of the observed $P_c(4440)$ and $P_c(4457)$ states, as shown in Table IV as the last two entries inserted into the solid boxes. This value of B is closer to the estimates in the hidden and open charm hadrons [43]. With these parameters, the mass of the third pentaquark $M = 4240$ MeV with $J^P = 3/2^-$, also shown in the solid box, is somewhat lower than the mass of the observed $P_c(4312)^+$ peak, but is still in the right ball-park. A correlated error analysis on the pentaquark masses will be undertaken once the spin-parity of the three observed states is determined.

The second possibility is to assign the lowest mass state $P_c(4312)^+$ with the one from the orbitally-excited set of states, in particular, with the one having the spin-parity $J^P = 3/2^+$ and mass $M = 4360$ MeV, or with $J^P = 1/2^+$ and $M = 4351$ MeV. Both predictions are rather close to the observed pentaquark mass. However, in this case, some of the input parameters have unphysical values, and we do not entertain this assignment here.

The threshold for the observed pentaquarks in the $P_c^+ \to J/\psi + p$ decay mode is $M^{\rm thr}_{J/\psi\, p} = M_{J/\psi} + m_p = 4035.17$ MeV [3]. With the masses given in Table IV, there are two states, the $J^P = 1/2^-$ with a mass 3830 MeV, and $J^P = 1/2^+$ with the mass 4031 MeV, which lie below the $M^{\rm thr}_{J/\psi\, p}$ threshold. Also, a third state having $J^P = 3/2^+$, with a mass 4040 MeV, may also lie below $M^{\rm thr}_{J/\psi\, p}$. One of these states is even below the threshold for the decay $P_c^+ \to \eta_c + p$. Hence, it will decay weakly. The others shown in Table IV are reachable in the $\Lambda_b \to J/\psi\, p\, K^-$ decay, and we urge the LHCb Collaboration to search for them.

VI. CONCLUSIONS

We have presented the mass spectrum of the hidden-charm pentaquark states ($c\bar{c}qq'q''$), where q, q', and q'' are light u- and d-quarks, using the isospin symmetry. In doing this, we have used an effective Hamiltonian, based on a doubly-heavy triquark — light diquark picture of a pentaquark, shown in Fig. 1. Apart from the constituent quark and diquark masses, the Hamiltonian incorporates dominant spin-spin, spin-orbit and orbital interactions. Since the narrow $J/\psi\, p$ peaks have been observed in Λ_b-baryon decays, we have imposed the heavy quark symmetry, in which case the light $[ud]$-diquak spin is a good quantum number. Hence, in this limit, only those hadrons (ordinary or exotic) are produced in Λ_b-decays in which the spin of the $[ud]$-diquark is $S_{ld} = 0$. There is an overwhelming support of the heavy-quark symmetry constraints from the available data on Λ_b-decays, which are dominated by the $\Lambda_b \to \Lambda_c + X$ transitions. Here, X stands for a large number of leptonic and hadronic final states, enlisted by Particle Data Group [3]. The decays $\Lambda_b \to \Sigma_c + X$ are sparse, with PDG listing only two [3]: $\Lambda_b \to \Sigma_c(2455)^0\, \pi^+\, \pi^-$ and $\Lambda_b \to \Sigma_c(2455)^{++}\, \pi^-\, \pi^-$, having the branching fractions $\mathcal{B} = (5.7 \pm 2.2) \times 10^{-4}$ and $\mathcal{B} = (3.2 \pm 1.6) \times 10^{-4}$, respectively. Clearly, there is the Σ_c/Λ_c-suppression in the branching ratios of Λ_b-decays. This also restricts the resonant $J/\psi\, p$ spectrum in the $\Lambda_b \to J/\psi\, p\, K^-$ decay. Following this, we interpret the three narrow resonances with the states having the following spin and angular momentum quantum numbers: $P_c(4312)^+ = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 0, J^P = 3/2^-\}$, the S-wave, and the other two as P-wave states, with $P_c(4440)^+ = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^P = 3/2^+\}$, and $P_c(4457)^+ = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^P = 5/2^+\}$. We have presented analytical expressions for the masses of the states having well-defined quantum numbers. However, there are more states present in the spectrum having the quark content ($c\bar{c}uud$). They are listed in Tables II and III. The details of the formalism are given elsewhere [44], where also the spectrum of the hidden-charm pentaquarks, having a non-vanishing strangeness is worked out, as well as the states having spin-1 light diquarks, i. e., with $S_{ld} = 1$. Some of them can be reached in weak decays of bottom-strange baryons, Ξ_b and Ω_b [22].

For the numerical estimates of the pentaquark masses, we infer the input parametric values from the earlier studies of the hidden-charm tetraquarks and charmed baryons. Since not all parameters in the effective Hamiltonian can be uniquely determined from the existing resonances, as there are currently only three such observed states, we use additionally the results of the known heavy baryons, such as the orbitally-excited Ω_c^* states, since $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ are also heavy baryons. It is difficult to quantify the theoretical errors on our mass estimates, as we have dropped sub-dominant spin-spin interactions, but we trust that the masses of the pentaquarks given in Table IV are in the right ball-park. We note that, in addition to the observed ones, there are three states (one with $J^P = 1/2^-$ and two with positive parity, $J^P = 1/2^+$ and $J^P = 3/2^+$) which have $[cu]_{s=0}$-diquark in their Fock space. Their masses are lower than their counterparts with $[cu]_{s=1}$-diquark. Two of these states lie below the $J/\psi\, p$ threshold, probably the third one as well, requiring a different search strategy. We anticipate other pentaquark states waiting to be discovered with yet more data and a detailed study of the $J/\psi\, p$ mass spectrum, including both narrow and broad resonances.

Finally, we stress that it is not enough to claim the observed narrow peaks in the $J/\psi\, p$ mass spectrum as a confirmation of hadronic molecules, just due to their kinematic vicinity to the charmed meson-charmed baryon thresholds, $\Sigma_c\, \bar{D}$ and $\Sigma_c\, \bar{D}^*$, which in this hypothesis are interpreted as loosely-bound S-wave states, and hence have necessarily a negative parity. Crucially, the spin-parities of the $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ remain to be determined. In the compact pentaquark picture presented here, heavy-quark symmetry requirements are built in and the mass estimates are in the right ball-park. The broad resonance $P_c(4380)^+$, present in the older LHCb analysis [1], which was difficult to accommodate in the compact pentaquark picture due to the conflict with the heavy quark symmetry [22], is presumably no longer there. So, the compact pentaquark interpretation of the three P_c-resonances is very much in the game. In particular, and as a crucial test, we anticipate the three narrow resonances to have different parities, i. e., both negative and positive parities are foreseen. Their determination would help greatly in discriminating the compact pentaquark interpretation and the competing ones based on hadronic molecules in various incarnations [49–53].

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