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Rolling Schedules with Capacitated Lot-Sizing and Service Level Constraints

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In practice deterministic multi-period planning models are implemented in rolling schedules since they allow to revise decisions beyond the frozen horizon and thus to take into account realizations and updated forecasts of uncertain data (e.g., customer demands). For this it is common to hold additional safety stocks to ensure given service levels. As we will show, this approach, implemented in rolling schedules, often results in increased setup and holding costs while (over-)accomplishing given service levels. A well-known alternative to deterministic planning models are stochastic, static, multi-period planning models, which results in feasible and stable plans. However, one problem of these models is the missing flexibility to react to realization of uncertain data. Consequently, the variance of costs and service levels is very high. We propose a new strategy, named stabilized cycle, which combines and enlarges ideas from literature for minimizing setup and holding costs in rolling schedules while controlling actual product-specific service levels by given upper and lower control limits for a given evaluation interval. A computational study with the big bucket production planning model *CLSP* demonstrates that this new strategy yields a good compromise between costs and downside deviations from given β service levels.

Key words: rolling schedules, capacitated lot-sizing, demand uncertainty, service level, stabilized cycle

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1. Introduction

A major issue for manufacturing companies in a make-to-stock production environment are demand uncertainties (e.g., forecast errors), even when the distribution and its parameters are known. It becomes even more challenging, if production plans have to be made for multi-items and limited capacities. In situations of uncertain demand actual demand will be lower than expected with a probability of 0.5. As a result inventory increases, which leads to higher inventory holding costs. If demand is higher than expected, it might not be fulfilled directly from stock and the unfulfilled demand is assumed to be backordered. Backorders have to be fulfilled with the next goods receipt.

Since backorder costs are hard to define a more common approach to handle demand uncertainty is the use of service level constraints. In practice service levels are usually controlled over an evaluation interval, which consists of a pre-specified number of periods and is in general defined either internally (e.g., reviewed managerial performance) or externally (e.g., key customer). However, actual service levels are random variables and it is almost impossible to reach exactly the given service levels (Thomas 2005). Therefore, the given service levels are usually interpreted as a lower bound, which have at least be reached per product by the end of the evaluation interval. In case of lot size production, costs for setting-up the resource(s) have to be considered by production planning as well. Thus, the aim is minimizing setup and inventory holding costs while simultaneously ensuring given product-specific service levels at the end of the evaluation interval. Practice-oriented strategies are needed to solve such production planning problems with suitable trade-offs between inventory holding costs, setup costs, schedule stability, and service levels.

A common approach for creating Master Production Schedules (MPS) in light of demand uncertainty is to use a deterministic, capacitated lot-sizing model with additional safety stocks in *rolling schedules*. This strategy usually has the benefit to (over-)accomplish given service levels. However, it often leads to high setup and inventory holding costs as well as high schedule instability. This is caused by frequently revised setup decisions in the first period of the planning interval.

An alternative is the *static uncertainty* strategy of Bookbinder and Tan (1988). Within this strategy stochastic, capacitated lot-sizing models are used to determine setup and lot size decisions. In this strategy setup and lot size decisions are fixed in advance for all periods of the planning interval without the possibility of modification at a later time. Obviously, this strategy leads to total schedule stability. On the downside, it needs relatively high average inventories - especially in later periods of the planning interval - in order to ensure given service levels. Therefore, this approach often comes along with high inventory holding costs and high deviations of the actual service levels from the given service levels by the end of the evaluation interval.

To the authors best knowledge there does not exist a production planning strategy for an evaluation interval with demand uncertainty and limited capacities, which provides a practical trade-off between inventory holding costs, setup costs, schedule stability, and service levels. To generate a low-cost stable production plan while simultaneously yielding rather small product-specific β_j service level (fill rate) variations, we propose the *stabilized cycle* strategy. This strategy is designed for big bucket deterministic lot-sizing models with service level constraints embedded in rolling schedules. It will be demonstrated by an extended capacitated lot-sizing problem ($CLSP$).

The paper is structured as follows. Section 2 contains a literature review of production planning with demand uncertainty. In Section 3 a model formulation of the deterministic $CLSP$ as well as three new model extensions to cope with demand uncertainty in rolling schedules are proposed. The *stabilized cycle* strategy is introduced in Section 4. A computational study of rolling schedules is presented in Section 5. Finally, Section 6 summarizes the findings and presents an outlook on future research.

2. Literature review

According to Bookbinder and Tan (1988) there exist three production planning strategies to deal with demand uncertainty. First, the *static uncertainty* strategy, in which setup and lot size decisions are made in advance for a planning interval. Meaning, once the decisions are made they cannot be revised during the planning interval regardless of the actual demands. This strategy uses a stochastic version of the $CLSP$ (S-$CLSP$) to consider demand uncertainty. In contrast to the $CLSP$ the S-$CLSP$ tries to control the demand uncertainty either by backorder costs or by service level constraints. To control backorder costs or to satisfy the service level constraints safety stocks are used. One heuristic approach to solve an uncapacitated version of the S-$CLSP$ (S-LSP) was developed by Silver (1978). The heuristic starts with the determination of reorder periods. Next, production cycle lengths, which are subject to the reorder points, are obtained by the Silver-Meal heuristic. Finally, lot sizes are calculated by considering the reorder periods, the production cycle lengths and the uncertain demands within the production cycles. Askin (1981) improved the heuristic of Silver (1978) by a simultaneous determination of production cycle length and lot size.

Second, Bookbinder and Tan (1988) introduced the *static-dynamic uncertainty* strategy, in which only setup periods are fixed in advance. Hence, lot sizes are adjusted by considering the actual inventory level and the period-specific *order-up-to-level* at the beginning of the setup periods. Furthermore, Bookbinder and Tan (1988) propose a solution approach for the *static-dynamic uncertainty* strategy. The approach starts with the determination of setup periods within the planning interval by using a deterministic or stochastic lot-sizing model formulation with a period orientated α^p service level constraints. In a further step the *order-up-to-level* is calculated for every setup period. The *order-up-to-level* calculation takes the demand uncertainty of the periods between two consecutive setup periods - time-between-order (TBO) - into consideration and uses safety stocks to satisfy the service level constraints. Since setup and lot size decisions are made consecutively the approach of Bookbinder and Tan (1988) is a heuristic (Tarim and Kingsman 2004). For the optimal solution of the *static-dynamic uncertainty* strategy Tarim and Kingsman (2004) introduce a mixed integer programming (MIP) model formulation for the S-LSP with α^p service level constraints. By using the MIP model formulation the consecutive procedure is replaced by a simultaneous determination of TBO and lot size. In Tarim and Kingsman (2006) the MIP model formulation of Tarim and Kingsman (2004) is enhanced by a backorder cost term in the objective function replacing the α^p service level constraints. Furthermore, Tempelmeier (2007) varies the MIP model formulation of Tarim and Kingsman (2004) such that the long-run average of either an α^p service level, a production cycle orientated α^{pc} service level or a β^{pc} service level can be ensured. Note, Tarim and Kingsman (2004) assume that expected average backorders are extremely low for a high α^{pc} service level. Under this assumption the determination of inventory holding costs is sufficiently accurate. However, for a rather lower service level (e.g., $\beta^{pc} = 0.8$), backorders cannot be neglected any more and the MIP model formulation of Tarim and Kingsman (2004) might lead to suboptimal solutions (Tempelmeier 2007). Consequently, the MIP model formulation of Tempelmeier (2007) contains the expected inventory level for every period, making it possible to determine setup and lot size decisions while considering expected inventory holding costs.

In Tempelmeier and Herpers (2011) the MIP model formulation of Tempelmeier (2007) for the S-LSP is adapted to the *static uncertainty* strategy. Moreover, they present an exact solution algorithm for the *Shortest-Path-Formulation* of the $S\text{-}LSP$. In Tempelmeier and Herpers (2010) the authors present a MIP model formulation of the multi-item $S\text{-}CLSP$ for the application in the *static uncertainty* strategy. Since setup and lot size decisions for the planning interval are fixed in advance, the *static uncertainty* strategy is suitable for capacitated production planning problems. Such problems are normally used in a MPS-driven production planning (Tempelmeier 2011). In the absence of an exact solution algorithm, Tempelmeier and Herpers (2010) propose an ABC_β heuristic, which is an extension of the ABC heuristic of Maes and van Wassenhove (1986). A further extension of the ABC_β heuristic is proposed in Tempelmeier (2011). Here, the author uses a *Set-Partitioning-Model* formulation of the $S\text{-}CLSP$ as well as a *Column Generation* (CG) method combined with the ABC_β heuristic. In a computational study Tempelmeier (2011) shows that on average, the CG ABC_β heuristic dominates the ABC_β heuristic. Another solution approach for the $S\text{-}CLSP$ is presented in Tempelmeier and Hilger (2015). There the authors introduce MIP model formulations to approximate the non-linear functions of expected backorders and expected inventories. A variant of the Fix-and-Optimize (F&O) heuristic is used to solve the models. In a computational study Tempelmeier and Hilger (2015) show, that models solved by the F&O heuristic are superior to the CG ABC_β heuristic as well as to the ABC_β heuristic. As an alternative, Helber et al. (2013) propose a MIP model formulation of the $S\text{-}CLSP$ and recommend backorder-oriented δ service levels. Unfortunately, considering δ service levels lead to a non-linear model formulation. To overcome this problem two approximate linear models are presented and evaluated in a computational study.

The *dynamic uncertainty* strategy is the third strategy proposed by Bookbinder and Tan (1988) to deal with demand uncertainty. Here, decisions are only made for the next period once information that become available at a later point (e.g., the actual demand of the prior period) are known. However, according to the authors the *dynamic uncertainty* strategy is unsuitable for practical usage, since the planning interval length of one period would lead to a production in almost every period. Therefore, this strategy would generate production plans which are far away from the optimum in situations with a high ratio of setup and inventory holding costs (Bookbinder and Tan 1988).

When using the *dynamic uncertainty* strategy in rolling schedules, it can be interpreted as a special case of a rolling schedule strategy with a planning interval and a frozen horizon of one period. In contrast, if the *static uncertainty* strategy is applied in rolling schedules, the re-planning interval, the frozen horizon and the planning interval are all of the same length, which is also a special case. However, since the *dynamic uncertainty* strategy is unsuitable for practical usage, we omit it in the further course of the paper. A visualization of a general scheme of rolling schedules is shown in Figure 1. According to Stadtler and Fleischmann (2012) in rolling schedules a plan is

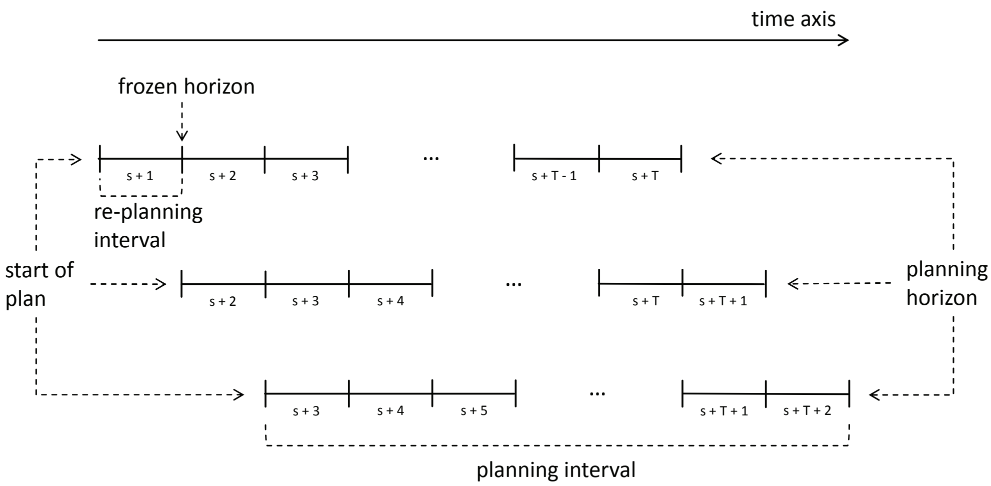


Figure 1 **Rolling schedules with a planning interval of T periods (Stadtler and Fleischmann 2012)**

created for a number of periods (planning interval), but only the first period(s) decision(s) (frozen horizon) are implemented. Whereas, decisions of later periods might be revised in subsequent schedules. After the periods of the re-planning interval have elapsed, all information is updated and a new plan (re-plan) is generated for the upcoming periods of the planning interval. Thus, rolling schedules with a multi-period deterministic MIP model (e.g., $CLSP$) are common to cope with data uncertainties in real world situations and often used within a Material Requirements Planning (MRP) driven production planning (Stadtler and Fleischmann 2012, Fleischmann et al.

2015). In case a deterministic, multi-period model is applied, it is common to make use of safety stocks to meet given service levels. The performance of a planning system using rolling schedules depends on the chosen parameters for the re-planning interval, the frozen horizon as well as the planning interval length. According to Federgruen and Tzur (1994) one of the main problems of rolling schedules is the *truncate horizon effect*, which occurs when the planning interval is too short to obtain optimal decisions for periods within the frozen horizon. To handle this, Stadtler (2000) proposes the *looking beyond the planning horizon* approach for a single-item uncapacitated lot-sizing model and enhances it for a multi-item capacitated lot-sizing model in Stadtler (2003). In a computational study Stadtler (2000) shows that extending a lot-sizing model by the *looking beyond the planning horizon* approach and solving it with an exact algorithm leads to at least the same results as well-known heuristics in rolling schedules. Moreover, lot-sizing models extended by the *looking beyond the planning horizon* approach are insensitive to the chosen length of the planning interval (Stadtler 2000).

Schedule instability is another major issue of rolling schedules and is specified by changes in consecutive production plans in rolling schedules. The schedule instability is mainly caused by the uncertain demand. For a better understanding of schedule instability Sridharan and Berry (1990) study an uncapacitated single-item MPS with demand uncertainty in rolling schedules with different parameter combinations. The parameter combinations of the rolling schedules are generated by different order-based and period-based frozen horizons, different replanning periodicity, and different length of the planning interval. Safety stocks are used to handle demand uncertainty and to meet a given β service level. In a computational study they compare the parameter combinations with respect to costs and schedule instability. The results show that an order-based frozen horizon leads to beneficial effects subject to costs and schedule instability. The use of an order-based frozen horizon can be seen as a special implementation of the *static-dynamic uncertainty* strategy in rolling schedules. Sridharan and LaForge (1994) present more detailed simulations to clarify the impact of different parameter combinations of rolling schedules on the β service level. They show that an increased frozen horizon leads in general to an increasing average inventory level, but not to a major decrease of the β service level. Zhao and Lee (1993) extend the research of Sridharan and Berry (1990) to a multi-level MRP system. They confirm the finding of Sridharan and Berry (1990), that an order-based frozen horizon results in lower costs than a period-based frozen horizon in most cases and observe that less frequent replanning is beneficial regarding costs, schedule instability, and service levels. In Zhao et al. (1995), the study of Zhao and Lee (1993) is extended by analysing the impact of the lot-sizing rule on the costs, schedule instability and the actual service level. In Xie et al. (2003) the research of Sridharan and Berry (1990) is extended to a multi-item single-level MPS with limited capacity. The purpose of this article is to investigate the impact of different parameter combinations on the performance of capacitated production systems. They find out that all parameters have a significant impact on costs, schedule stability, and actual service levels. Moreover, they discover that the impact of the planning interval length and the freezing proportion on the performance is significantly influenced by the capacity constraints. The authors sum up, that a planning strategy is needed, which obtains a trade-off between costs, schedule instability and service levels.

As our literature review shows, the majority of research has been done to solve lot-sizing problems under demand uncertainty either with a static, stochastic lot-sizing model or with a deterministic lot-sizing model and additional *static* safety stocks within rolling schedules. To the authors best knowledge, little research has been done on the intersection of the stochastic lot-sizing in rolling schedules, especially when capacity is constrained. Two articles in which the *static uncertainty* and the *static-dynamic uncertainty* strategy is applied to rolling schedules are Bookbinder and H'ng (1986) and Bookbinder and Tan (1988). In the articles the parameter of the frozen horizon length is dynamically determined subject to an α^{pc} service level. According to Bookbinder and Tan (1988) all advantages the *static-dynamic uncertainty* strategy has over the *static uncertainty* strategy only come into existence in static, uncapacitated planning situations. Moreover, the *static-dynamic uncertainty* strategy might be unsuitable for capacitated lot-sizing problems (Tempelmeier 2013).

This paper fills the gap between static, stochastic lot-sizing models in a *static uncertainty* strategy and deterministic lot-sizing models in rolling schedules by presenting the *stabilized cycle* strategy for rolling schedules. The aim of the *stabilized cycle* strategy is to reach cost minimal and stable production plans by simultaneously taking into account lower and upper control limits for β_j service levels. Therefore, the *stabilized cycle* strategy is a new β service level based method consisting of dynamic product-specific frozen horizons and rules that keep β_j service levels within given control limits at the end of an evaluation interval. Within the *stabilized cycle* strategy a deterministic CLSP extended by dynamic *TBO-dependent safety stocks* (*TBO-SS*) and the *looking beyond the planning horizon* approach is used. The extended MIP formulation is presented in Section 3. The benefit of such a MIP model formulation is that it can easily be implemented in APS or as an add-on for MRP systems, which are often used in practice. The concept of the *stabilized cycle* strategy is described in Section 4.

3. Model formulation for the $CLSP$ and extensions

In this section we present the well-known $CLSP$ model formulation of Billington et al. (1983) as well as stepwise extensions for an implementation in rolling schedules with demand uncertainty.

The following notation is used below:

Sets

$J = \{j | j = 1, .., \bar{j}\}$ set of products

$T = \{t | t = 1, .., \bar{t}\}$ set of periods

Data

b_t production capacity in period t

$d_{j,t}$ expected demand of product j in period t

hc_j inventory holding costs per period t and product unit j

κ_j production coefficient of product j

sc_j setup costs of product j

Variables

$I_{j,t}$ inventory of product j at the end of period t

$X_{j,t}$ lot size of product j in period t

$Y_{j,t}$ 1, if a setup for product j takes place in period t, 0 otherwise

Z^{CLSP} total inventory holding and setup costs

$$\min Z^{CLSP} = \sum_{j=1}^{\bar{j}} \sum_{t=1}^{\bar{t}} hc_j \cdot I_{j,t} + \sum_{j=1}^{\bar{j}} \sum_{t=1}^{\bar{t}} sc_j \cdot Y_{j,t} \tag{1}$$

subject to

$$I_{j,t-1} + X_{j,t} - I_{j,t} = d_{j,t} \quad \forall j \in J, t \in T \tag{2}$$

$$X_{j,t} - M \cdot Y_{j,t} \leq 0 \quad \forall j \in J, t \in T \tag{3}$$

$$\sum_{j=1}^{\bar{j}} \kappa_j \cdot X_{j,t} \leq b_t \quad \forall t \in T \tag{4}$$

$$Y_{j,t} \in \{0,1\} \quad \forall j \in J, t \in T \tag{5}$$

$$X_{j,t} \geq 0 \quad \forall j \in J, t \in T \tag{6}$$

$$I_{j,t} \geq 0 \quad \forall j \in J, t \in T \tag{7}$$

The objective function (1) of the $CLSP$ minimizes setup and inventory holding costs under the assumption of deterministic demands. Equations (2) balance the inventory levels and lot sizes while ensuring demand fulfilment for every product j and period t. Constraints (3) ensure that the machine is setup for product j in period t, if a production is scheduled for product j in period t. Constraints (4) restrict the cumulated production times in each period to the available capacities b_t. Constraints (5)-(7) define the domains of the decision variables $Y_{j,t}$, $X_{j,t}$, and $I_{j,t}$.

To ensure feasibility, the $CLSP$ is further extended by soft capacity constraints (9). Since the use of an additional capacity unit C_t (10) is related to (high) costs (pc_t), the model will only choose this option, if capacity b_t is insufficient. Thus, C_t must be considered in an extended objective function $Z^{CLSP^{soft}}$ (8) as well as in the soft capacity constraints (9), which replace constraints (4).

$$\min Z^{CLSP^{soft}} = Z^{CLSP} + \sum_{t=1}^{\bar{t}} pc_t \cdot C_t \tag{8}$$

$$\sum_{j=1}^{\bar{j}} \kappa_j \cdot X_{j,t} \leq b_t + C_t \quad \forall t \in T \tag{9}$$

$$C_t \in \{0\} \cup [1, \infty] \quad \forall t \in T \tag{10}$$

In the following, three extensions for the $CLSP$ are proposed. One for incorporating *TBO-SS* (Subsection 3.1), a second for overcoming the *truncate horizon effect* (Subsection 3.2), and a third for accelerating the solution process by introducing valid inequalities (Subsection 3.3).

3.1. First extension: TBO-dependent safety stocks

To cope with demand uncertainties, safety stocks are proposed. But in contrast to the commonly used *static* safety stocks (e.g., subject to the demand within the static optimal TBO), we recommend dynamic *TBO-SS*. Thus, the safety stocks are chosen subject to the actual production cycle length, which can vary within the planning interval. We will present a model formulation of *TBO-SS* extending common lot-sizing models, like the $CLSP$ (Billington et al. 1983) or the $CLSP$ with linked lot sizes ($CLSP$-L) (Suerie and Stadtler 2003). In the extended model formulation we introduce a new binary variable $V_{j,t,t+\tau}$. It is set to 1, if a setup for product j is scheduled in period t while the next setup is scheduled in period $t+\tau$. In all other cases $V_{j,t,t+\tau}$ is set to 0. Here, τ represents the TBO. Moreover, the parameter $ss_{j,t,t+\tau}$ indicates the safety stocks for a production cycle which begins in period t and has a length of τ periods. Furthermore, τ_j^{max} is an externally given product-specific upper bound for the TBO. During phases of high machine utilization, production cycles might have non-integer period lengths. Therefore, this model formulation is an integer approximation of *TBO-SS*. In case a TBO has a non-integer length, our model will round the actual TBO down to the next smallest integer. Since a smaller TBO leads to higher safety stocks (Tempelmeier and Herpers 2011), our model formulation might result in (slightly) overestimated safety stocks for non-integer production cycles.

$$I_{j,t-1} \geq \underbrace{ss_{j,l_j,t} \cdot V_{j,l_j,t}}_{\text{if } t \leq l_j + \tau_j^{max} \wedge ss_{j,l_j,t} \geq 0} + \underbrace{\sum_{\tau=1}^{\min(\tau_j^{max}, t-1)} ss_{j,t-\tau,t} \cdot V_{j,t-\tau,t}}_{\text{if } ss_{j,t-\tau,t} \geq 0} \quad \forall j \in J, t = 2, .., \bar{t}+1 \tag{11}$$

$$I_{j,\bar{t}} \geq \underbrace{\sum_{t=1}^{\bar{t}}}_{\text{if } t+\tau_j^{max} \geq \bar{t}+2} \sum_{\tau=\bar{t}-t+2}^{\tau_j^{\max}} ss_{j,t,t+\tau} \cdot V_{j,t,t+\tau} \quad \forall j \in J \tag{12}$$

$$\underbrace{\sum_{\tau=1}^{\tau_j^{\max}}}_{\text{if } (t-\tau) \in T \cup \{l_j\}} V_{j,t-\tau,t} = Y_{j,t} \quad \forall j \in J, t \in T \tag{13}$$

$$\underbrace{\sum_{\tau=1}^{\tau_j^{\max}}}_{\text{if } (t-\tau) \in T \cup \{l_j\}} V_{j,t-\tau,t} = \underbrace{\sum_{\tau=1}^{\bar{t}-t+1}}_{\text{if } \tau \leq \tau_j^{\max}} V_{j,t,t+\tau} \quad \forall j \in J, t \in T \tag{14}$$

$$\sum_{\tau=-l_j+1}^{\tau_j^{\max}} V_{j,l_j,l_j+\tau} = 1 \quad \forall j \in J \tag{15}$$

$$\underbrace{\sum_{t=1}^{\bar{t}}}_{\text{if } t+\tau_j^{\max} \geq \bar{t}+1} V_{j,t,\bar{t}+1} = 1 \quad \forall j \in J \tag{16}$$

$$V_{j,t,t+\tau} \in \{0,1\} \quad \forall j \in J, t \in T \cup \{l_j\}, \tau = 1,..,\tau_j^{max} \tag{17}$$

The intention is to set a lower bound for the inventory level at the end of the last period of a production cycle. The lower bound is defined by the required safety stocks corresponding to the production cycle length. Assuming that t and $t+\tau$ are adjacent setup periods for product j, then $V_{j,t,t+\tau}$ will be set to 1. Hence, $V_{j,t,t+\tau}$ activates the safety stocks $ss_{j,t,t+\tau}$, which become the lower bound for the inventory of product j at the end of period $t+\tau-1$. This is ensured by constraints (11) and (12) in which l_j represents the last setup period of product j prior the planning interval. Equations (13) require that each setup period defines the end of a production cycle. Furthermore, equations (14) are the common flow constraints, meaning that, if a production cycle ends in period $t-1$, the next production cycle will start in period t. Constraints (15) set the start of the first production cycle of every product j to l_j. Equations (16) define that the last setup period of every product j is scheduled in the first period beyond the planning horizon. Since $ss_{j,t-\tau,t}$ might yield negative values, the model formulation has to allow backorders. Therefore, we introduce the variables $BO_{j,t}$, which represent the backorders of product j at the end of period t.

$BO_{j,t}$ are bounded by $ss_{j,t-\tau,t}$ due to constraints (18) and (19).

$$BO_{j,t-1} \leq \underbrace{-ss_{j,l_j,t} \cdot V_{j,l_j,t}}_{\text{if } t \leq l_j + \tau_j^{max} \wedge ss_{j,l_j,t} < 0} + \underbrace{\sum_{\tau=1}^{\min(\tau_j^{max}, t-1)}}_{\text{if } ss_{j,t-\tau,t} < 0} -ss_{j,t-\tau,t} \cdot V_{j,t-\tau,t} \quad \forall j \in J, t = 2, .., \bar{t}+1 \qquad (18)$$

$$BO_{j,t} \geq 0 \quad \forall j \in J, t \in T \qquad (19)$$

$$I_{j,t-1} + X_{j,t} - BO_{j,t-1} - I_{j,t} = d_{j,t} - BO_{j,t} \quad \forall j \in J, t \in T \qquad (20)$$

In case $ss_{j,t-\tau,t}$ yields a positive value, $BO_{j,t-1}$ is bounded to 0 by constraints (18) and (19). If $ss_{j,t-\tau,t}$ is negative, $BO_{j,t-1}$ will be limited to the absolute value of $ss_{j,t-\tau,t}$ by constraints (18). Hence, the original inventory balance constraints of the $CLSP$ (2) have to be extended by $BO_{j,t}$. Therefore, constraints (20) replace constraints (2). By expanding a common lot size model, like the $CLSP$, by constraints (11)-(20) it can simultaneously determine lot sizes and *TBO-SS*.

To provide *TBO-SS* for the model, they have to be calculated in advance for every product $j \in J$, period $t \in T$, and TBO ($\tau = 1, .., \tau_j^{max}$) by the optimization model of Tempelmeier (2011).

$$\min X_{j,t,t+\tau} \qquad (21)$$

$$s.t. \;\; 1 - \frac{E\{\sum_{s=t}^{t+\tau-1} BO_{j,s}(X_{j,t,t+\tau})\}}{E\{\sum_{s=t}^{t+\tau-1} d_{j,s})\}} \geq \beta_j^{ss} \qquad (22)$$

The model yields the minimum lot size (21), which ensures the β_j^{ss} service level constraints (22) for the related production cycle starting in period t and ending in period $t+\tau-1$. The optimization model is solved by the binary search heuristic of Manna and Waldinger (1987). Since we are not interested in the minimum lot size for every product, period, and TBO ($\tau = 1, .., \tau_j^{max}$) but in the safety stocks, we subtract the cumulated expected period demands in the production cycle (23).

$$ss_{j,t,t+\tau} = X_{j,t,t+\tau} - \sum_{s=t}^{t+\tau-1} E\{d_{j,s}\} \qquad (23)$$

3.2. Second extension: looking beyond the planning horizon

In general only periods within the planning interval are considered for decision-making, if rolling schedules are used. However, there exist further periods beyond the planning horizon, which might have an impact on decisions within the planning interval. According to Stadtler (2000), the so called *truncate horizon effect*, even occurs in situations with deterministic demands. This section contains a model formulation extending a $CLSP$ by the *looking beyond the planning horizon* approach of Stadtler (2003) to avoid this effect. Hence, we expand the objective function $Z^{CLSP^{soft}}$ (8) to $Z^{CLSP^{soft}\text{-}PH}$ (24) by adding (negative) bonus payments $bonus_{j,t,t+\tau}$ for the last production cycle of every product j.

$$\min Z^{CLSP^{soft}\text{-}PH} = Z^{CLSP^{soft}} + \sum_{j=1}^{\bar{j}} \underbrace{\sum_{t=1}^{\bar{t}}}_{\text{if } t+\tau^{\mu}_{j,t} \geq \bar{t}+2} \sum_{\tau=\bar{t}-t+2}^{\tau^{\mu}_{j,t}} bonus_{j,t,t+\tau} \cdot V_{j,t,t+\tau} \tag{24}$$

subject to (3),(5)-(7),(9),(10) (11),(13),(15),(17)-(20) and

$$I_{j,\bar{t}} \geq \underbrace{\sum_{t=1}^{\bar{t}}}_{\text{if } t+\tau^{\mu}_{j,t} \geq \bar{t}+2} \sum_{\tau=\bar{t}-t+2}^{\tau^{\mu}_{j,t}} (ss_{j,t,t+\tau} + \sum_{s=\bar{t}+1}^{t+\tau-1} d_{j,s}) \cdot V_{j,t,t+\tau} \quad \forall j \in J \tag{25}$$

$$\underbrace{\sum_{\tau=1}^{\tau^{\max}_j}}_{\text{if } (t-\tau)\in T\cup\{l_j\}} V_{j,t-\tau,t} = \underbrace{\sum_{\tau=1}^{\bar{t}-t+1}}_{\text{if } \tau \leq \tau^{\max}_j} V_{j,t,t+\tau} + \sum_{\tau=\bar{t}-t+2}^{\tau^{\mu}_{j,t}} V_{j,t,t+\tau} \quad \forall j \in J, t \in T \tag{26}$$

$$\underbrace{\sum_{t=1}^{\bar{t}}}_{\text{if } t+\tau^{\max}_j \geq \bar{t}+1} V_{j,t,\bar{t}+1} + \underbrace{\sum_{t=1}^{\bar{t}}}_{\text{if } t+\tau^{\mu}_{j,t} \geq \bar{t}+2} \sum_{\tau=\bar{t}-t+2}^{\tau^{\mu}_{j,t}} V_{j,t,t+\tau} = 1 \quad \forall j \in J \tag{27}$$

To activate the bonus payments $bonus_{j,t,t+\tau}$ we use variables $V_{j,t,t+\tau}$ introduced in Section 3.1. Since the idea of this approach is to produce units within the planning interval intended to satisfy demands beyond the planning horizon, the final inventory level $I_{j,\bar{t}}$ has to be linked to the last scheduled production cycle with a length of τ. Hence, constraints (12) are extended to (25) by adding the expected demands beyond the planning horizon, which belong to the periods of the last production cycle, to the safety stocks. Here, $\tau^{\mu}_{j,t}$ presents the static optimal TBO. The expected demands beyond the planning horizon are calculated by a simple moving average approach. Further, we have to adjust constraints (14) and (16) to allow that production cycles may end in periods beyond the horizon. Therefore, constraints (26) substitute constraints (14) and constraints (27) replace constraints (16), if both extensions are used simultaneously.

$bonus_{j,t,t+\tau}$ are calculated in advance for every product j, period t, and TBO ($\tau = 1, .., \tau^{\mu}_{j,t}$) by equation (28) for every potential production cycle beginning in period t and ending in a period beyond the planning horizon. As proposed in Stadtler (2000) $\tau^{\mu}_{j,t}$ is determined for every product j and period t by *Groff's heuristic.*

$$bonus_{j,t,t+\tau} := (29) - (30) \tag{28}$$

In (29) the partial costs of a potential last production cycle, which begins in period t, for the periods inside the planning interval are considered (Stadtler 2003, p.491).

$$\frac{\bar{t} - t + 1}{\tau} \cdot [sc_j + \sum_{s=t}^{t+\tau-1} hc_j \cdot (s - t) \cdot d_{j,s}] \tag{29}$$

Note, the objective function $Z^{CLSP^{soft}}$ (8) already contains setup costs sc_j and inventory holding costs relating to the last lot size within the planning horizon (30).

$$sc_j + \sum_{s=t}^{\bar{t}} hc_j \cdot (s - t) \cdot d_{j,s} + hc_j \cdot (\bar{t} - t + 1) \cdot \sum_{s=\bar{t}+1}^{t+\tau-1} d_{j,s} \tag{30}$$

Now, $bonus_{j,t,t+\tau}$ are calculated in (28) as the difference of (29) and (30) and added to the objective function $Z^{CLSP^{soft}}$ (8). Thus, the new objective function $Z^{CLSP^{soft}\text{-}PH}$ (24) only considers the term (29), which is the correct proportion of lot size costs for the last production cycle belonging to the planning interval.

3.3. Third extension: valid inequalities

In order to accelerate the solving process we introduce additional valid inequalities (31) to the model formulation. The intention of these valid inequalities is to set lower bounds for the inventory levels at the end of every setup period by adding the period demands beyond the setup period and the safety stocks of the related production cycle. The first term of (31) sets the minimal inventory level at the end of every setup period which production cycle ends within the planning interval. The second term defines the minimal inventory level for the last setup period of the planning interval if the production cycle ends beyond the planning horizon. Hence, the second term of (31) is equal to constraints (25). Thus, constraints (25) are redundant.

$$I_{j,t} \geq \sum_{\tau=1}^{\min\{\bar{t}-t+1, \tau_j^{\max}\}} (ss_{j,t,t+\tau} + \sum_{s=t+1}^{t+\tau-1} d_{j,s}) \cdot V_{j,t,t+\tau} + \sum_{\tau=\bar{t}-t+2}^{\tau^{\mu}_{j,t}} (ss_{j,t,t+\tau} + \sum_{s=t+1}^{t+\tau-1} d_{j,s}) \cdot V_{j,t,t+\tau} \quad \forall j \in J, t \in T \tag{31}$$

To summarize, the extended $CLSP$ creates production plans, which include appropriate $TBO\text{-}SS$ as well as final inventory levels for the expected demands beyond the planning horizon. This should help to deal with demand uncertainty, to reduce the *truncate horizon effect*, and to yield decisions, which are insensitive to the length of the planning interval (Stadtler 2000).

4. *Stabilized cycle* strategy

The *stabilized cycle* strategy is designed to cope with demand uncertainty for deterministic models in rolling schedules. The strategy considers given β_j service levels for an evaluation interval and simultaneously tends to minimize setup and inventory holding costs. Note that optimal solutions in a planning interval become heuristic in rolling schedules. Before every re-plan the values that have been observed in the past are refreshed. In case a current β_j service level is out of given control limits (e.g., given β_j service level (β_j^{tar}) $\pm\ 0.X$), appropriate control mechanisms are used to push the β_j service level back into its control limits. As a result, we "fix" product-specific decisions of the *current* production cycle (i.e., a production cycle started in 'the past' and to be finished in the current planning interval) to decrease plan instability as long as the current β_j service level stays within given control limits. Otherwise, decisions relating to a product's current production cycle might be revised. In this sense the current production cycle is not "frozen" but "stabilized". With the current production cycle being stabilized there is a good chance that also subsequent production cycles in the re-plan will remain as before, i.e. will be stabilized.

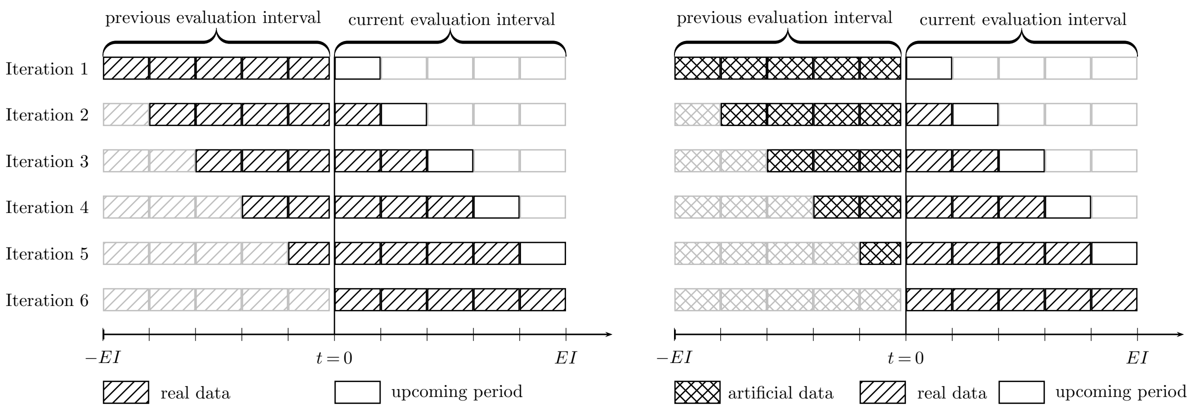
Once the current β_j service level is higher than the product-specific upper control limit (UCL_j) a reduction of safety stocks is used as an upper control mechanism (UCM) (Section 4.2). To counteract β_j service levels falling below product-specific lower control limits (LCL_j) a service level based lower control mechanism (LCM) is applied (Section 4.3). The interaction of the UCM and the LCM as well as a numerical example of the resulting β_j service levels in rolling schedules are examined in Section 4.4. Within the *stabilized cycle* strategy we use the extended deterministic MIP model formulation of the $CLSP$ as presented in Section 3.

4.1. Control parameter: $\beta_j^{\varnothing EI}$

The aim of the extended $CLSP$ is to minimize setup and inventory holding costs while simultaneously meeting β_j service levels for an evaluation interval in rolling schedules. Here, we use the current average β_j service levels ($\beta_j^{\varnothing EI}$) over the past EI periods as control parameters (32). In (32) $\hat{d}_{j,s}$ is the directly fulfilled demand of product j in the past period s, while $d_{j,s}$ represents the actual period demand.

$$\beta_j^{\varnothing EI} = \frac{\sum_{s=t-EI}^{t-1} \hat{d}_{j,s}}{\sum_{s=t-EI}^{t-1} d_{j,s}} \tag{32}$$

Either real historical data or artificial data for the past periods can be used (see Figure 2). Figure 2 shows six iterations and five re-plans of a rolling schedule with a re-planning interval of one period and an evaluation interval of five periods. Assuming that a verification of achieving β_j^{tar} is sufficient at the end of the evaluation interval, artificial data are applied at the start of rolling schedules (e.g., with period demands equal to the mean). Thus, the system is more flexible in the beginning of rolling schedules. This flexibility leads to less interventions of the control mechanisms, which in turn stabilizes the plans and decreases setup and inventory holding costs. Once EI re-plans have been executed, $\beta_j^{\varnothing EI}$ no longer includes artificial demands.



(a) With historical data (b) With artificial data

Figure 2 Chronological sequence of the $\beta_j^{\varnothing EI}$ calculation in rolling schedules with an $EI = 5$

Due to demand uncertainty $\beta_j^{\varnothing EI}$ might deviate from β_j^{tar}. If the actual demand during a production cycle is lower than the expected demand $\beta_j^{\varnothing EI}$ increases, and vice versa. In an evaluation interval with an infinite length the lower and the upper demand deviations offset each other. For the *S-CLSP* Tempelmeier (2007) has shown that the long-run average service level ($\beta_j^{\varnothing\infty}$) will reach β_j^{tar}. However, in reality contracts between partners within a supply chain are not made for infinite evaluation intervals, but for finite ones. Hence, we propose two control mechanisms to keep $\beta_j^{\varnothing EI}$ within given LCL_j and UCL_j in a finite evaluation interval.

4.2. Dynamic *TBO*-dependent safety stocks as an upper control mechanism

To ensure β_j^{tar} service levels, we use *TBO-SS*. According to Tempelmeier (2011) β_j^{tar} has a high influence on the safety stocks calculation (see constraint (22) in Section 3.1 with $\beta_j^{ss} = \beta_j^{tar}$) and is usually fixed for the evaluation interval. But due to demand uncertainty, production cycles might occur with lower demands than expected, resulting in larger production cycle service levels (β_j^{pc}) than β_j^{tar}. Also, a prematurely aborted TBO due to the inability to reach the required β_j^{tar} in a production cycle will lead to a $\beta_j^{pc} > \beta_j^{tar}$ in most cases. In rolling schedules, a prematurely aborted TBO occurs in a situation, in which demand in earlier periods of the current production cycle is higher than expected. However, until that period normally all demands have been fulfilled directly from stock such that $\beta_j^{pc} = 1$. This is due to a violation of the inventory balance constraints ((2) or (20)).

According to Tempelmeier (2011) the calculated safety stocks ensure β_j^{tar} by the end of every production cycle under a given probability assumption. Assuming that β_j^{tar} has to be ensured by the end of the evaluation interval, which normally contains several production cycles, we are not forced to ensure β_j^{tar} in every production cycle.

Due to the fact, that there are production cycles with $\beta_j^{pc} > \beta_j^{tar}$, we can also allow some production cycles with $\beta_j^{pc} < \beta_j^{tar}$ as long as β_j^{tar} can be achieved by the end of the evaluation interval. Therefore, we recommend to use a dynamic calculation of the *TBO-SS* by implementing a modification of β_j^{ss} in constraint (22) in Section 3.1. For the β_j^{ss} modification, we calculate a minimal service level (β_j^{min}), see equation (33). There, $d_{j,t}$ stands for the demand of product j in the past periods ($t = 2 - EI, .., 0$), while $\hat{d}_{j,t}$ is the directly fulfilled demand and $E\{d_{j,1}\}$ the expected demand for the next period. Thus β_j^{min} can be interpreted as the minimal β_j service level for the upcoming period which will ensure β_j^{tar} in the next period, if $d_{j,1} = E\{d_{j,1}\}$. Hence, it should be used to calculate the required safety stocks for the next production cycle.

$$\beta_j^{min} = \frac{\beta_j^{tar} \cdot (\sum_{t=2-EI}^{0} d_{j,t} + E\{d_{j,1}\}) - \sum_{t=2-EI}^{0} \hat{d}_{j,t}}{E\{d_{j,1}\}} \tag{33}$$

In case of $\beta_j^{\varnothing EI} > \beta_j^{tar}$, β_j^{min} decreases. As a result safety stocks decrease, which in turn results in lower inventory holding costs. However, safety stocks will increase in situations of $\beta_j^{\varnothing EI} < \beta_j^{tar}$, resulting in higher inventory holding costs. Therefore, β_j^{ss} will only be set to β_j^{min} in situations in which $\beta_j^{\varnothing EI}$ exceeds β_j^{tar} and a setup for product j is planned in the first period aiming to reduce $\beta_j^{\varnothing EI}$ within the next production cycle, see equation (34).

$$\beta_j^{ss} = \begin{cases} \beta_j^{min} & \text{, if } \beta_j^{\varnothing EI} > \beta_j^{tar} \\ \beta_j^{tar} & \text{, else} \end{cases} \tag{34}$$

4.3. Lower control mechanism

As mentioned before, in principal decisions within the *frozen horizon* are fixed and decisions for periods beyond the *frozen horizon* are subject to revision of subsequent re-plans (Stadtler et al. 2012). Obviously, a short *frozen horizon* enables more flexibility to handle uncertainty than a long one. To yield a high planning flexibility in rolling schedules, the *frozen horizon* is usually determined as short as possible (e.g., just the first period). However, in case of demand uncertainty, demand in the frozen horizon may be higher than expected and no feasible re-plan might exist unless the current production cycle is aborted prematurely. In case of lower demand, previous plans will be adjusted with respect to the lot sizes, while the previous scheduled production cycles stay stable. Therefore, the length of the *frozen horizon* has a direct impact on the performance of the production planning system in rolling schedules. While the static period-based or order-based *frozen horizons* of Sridharan and Berry (1990) omit given service levels, we propose a dynamic β_j service level based method to determine product-specific *frozen horizons*.

This method prematurely aborts a current production cycle of a product j and enforces a setup in period $t = 1$ of the current planning interval, if the expected β_j service level (β_j^{exp}) is smaller than

β_j^{min}. β_j^{exp} is determined according to Tempelmeier (2011). Hence, we calculate β_j^{exp} based on the expected backorders $E\{F_{n,t=1}(I_{j,t=0})\}$ and the expected demand of the next period, see equation (35).

$$\beta_j^{exp} = 1 - \frac{E\{F_{j,1}(I_{j,0})\}}{E\{d_{j,1}\}} \tag{35}$$

The expected backorders for the next period can be calculated by equation (36), in which $G_Y^1(I) = \int_I^\infty (y - I) \cdot f_y(y) \cdot dy$ is the first-order-loss function with the random demand Y and the current inventory I (Tempelmeier 2011). Moreover, $BO_{j,0}$ represents the currently existing backorders of product j.

$$E\{F_{j,1}(I_{j,0})\} = G^1_{Y_j^1}(I_{j,0}) - BO_{j,0} \tag{36}$$

Note that, $\beta_j^{exp} < \beta_j^{min}$ would result in $\beta_j^{\varnothing EI} < \beta_j^{tar}$ at the end of the next period. Therefore, the method can be seen as a service level based LCM. In case $\beta_j^{exp} \geq \beta_j^{min}$, decisions of the previous production plan subject to the current production cycle of product j stay stable (e.g., $Y_{j,t} = 0 \; \forall t = 1,..,PC_j^{End}$). Here, PC_j^{End} represents the last period of the current production cycle of product j. Note that, PC_j^{End} is set to 0 and $Y_{j,1}$ is set to 1, if the current production cycle of product j is prematurely aborted by the LCM. Considering the result of the LCM within the extended MIP model formulation, the formerly defined ranges of constraints (11) and (18) $t = 2,..,\bar{t}+1$ have to be adjusted to $t = PC_j^{End} + 2,..,\bar{t}+1$.

4.4. Simultaneous use of upper and lower control mechanisms

Since demand uncertainty in rolling schedules most often results in $\beta_j^{\varnothing EI}$ deviating from β_j^{tar}, a goal-driven controlling of $\beta_j^{\varnothing EI}$ is needed. However, if β_j^{tar} is used as an aspiration level in the control mechanisms, the mechanisms would be too sensitive. Meaning, one control mechanism - either the lower or the upper - would influence the production plans in almost every period. Hence, we recommend the use of an UCL_j (β_j^{UCL}) and a LCL_j (β_j^{LCL}) (e.g., $\beta_j^{tar} \pm 0.005$) to control $\beta_j^{\varnothing EI}$.

By using β_j^{UCL} and β_j^{LCL} instead of β_j^{tar} as control limits for the mechanisms, a feasible range for $\beta_j^{\varnothing EI}$ is defined and equation (37) replaces (34).

$$\beta_j^{ss} = \begin{cases} \beta_j^{min} & \text{, if } \beta_j^{\varnothing EI} > \beta_j^{\mathbf{UCL}} \\ \beta^{tar} & \text{, else} \end{cases} \tag{37}$$

Similarly, β_j^{min}, which is previously determined by equations (33), is now calculated by (38) for determining *TBO-SS* and by (39) for the decisions whether to revise the current production cycle or not. In equation (38), ϵ represents a small number (e.g., 0.00001), so that $\beta_j^{min} < \beta_j^{UCL}$, which is feasible with respect to the control limits.

$$\beta_j^{min} = \frac{(\beta_j^{\mathbf{UCL}} - \epsilon) \cdot (\sum_{t=2-EI}^{0} d_{j,t} + E\{d_{j,1}\}) - \sum_{t=2-EI}^{0} \hat{d}_{j,t}}{E\{d_{j,1}\}} \tag{38}$$

$$\beta_j^{min} = \frac{\beta_j^{\mathbf{LCL}} \cdot (\sum_{t=2-EI}^{0} d_{j,t} + E\{d_{j,1}\}) - \sum_{t=2-EI}^{0} \hat{d}_{j,t}}{E\{d_{j,1}\}} \tag{39}$$

The procedure for setting up the *stabilized cycle* strategy is shown in Algorithm 1. It is repeated in every periodof the evaluation interval. Subsequent to the initialisation of the common input data of a $CLSP$, the parameters (e.g., β_j^{exp}, β_j^{min},..) for the current planning interval (T) regarding RI_j, the *looking beyond the planning horizon* approach, and the *TBO-SS* have to be calculated. Finally, the extended $CLSP$ is solved by a commercial solver and the first period(s) decisions are implemented. Decisions for later periods are temporary and can be revised at a later time.

The impact of the control mechanisms on rolling schedules is explained by a numerical example, which is visualized in Figure 3. For simplifying we assume uncapacitated rolling schedules for an arbitrary product j with demand uncertainty. The period demand is assumed to be normally distributed and occurs at the end of each period t. Product j is characterized by a optimal TBO_j of 2 periods and a $\beta_j^{tar} = 0.95$. Moreover, the control limits are $\beta_j^{LCL} = 0.94$ and $\beta_j^{UCL} = 0.96$. The evaluation interval is set to 10 periods and the *re-planning interval* equals the length of one period. Furthermore, we use an artificial dataset for the periods of the previous evaluation interval each with a β service level of 95%. Therefore, the control parameter $\beta_j^{\varnothing EI}$ is initialised to β_j^{tar} at the beginning of the current evaluation interval.

Algorithm 1: Rolling schedules with the stabilized cycle strategy for the current period t (re-plan)

```
initialize input data (e.g., number of products, available capacities, inventory levels,...);
forall the j ∈ J do
    calculate β_j^{∅EI} by equation (32), β_j^{exp} by equation (35) and β_j^{min} by equation (39);
    determine whether to fix or to free the product's current production cycle;
    forall the t ∈ T do
        calculate optimal TBO τ^μ_{j,t} (e.g., by Groff's heuristic (Stadtler 2000, p.320));
        for τ = 1,..,τ^μ_{j,t} ,if t + τ − 1 > t̄ do
            calculate bonus payments bonus_{j,t,t+τ} by equation (28);
        end
        recalculate β_j^{min} by equation (38) and calculate β_j^{ss} by equation (37);
        for τ = 1,..,τ_j^{max} do
            calculate ss_{j,t,t+τ} by the optimization model of Tempelmeier (2011) ((21) - (23));
        end
    end
end
solve the extended CLSP: min Z^{CLSP^{soft}-PH} (24) subject to (3),(5)-(7),(9),(10)
(11),(13),(15),(17)-(20),(25)-(27),(31);
```

Assuming, that the current inventory level $I_{j,0}$ at the end of the current period ($t=0$) is insufficient to yield a $\beta_j^{exp} \geq \beta_j^{min}$, RI_j for the first period is set to 1. Now, the first period will be a production period for product j. Since $\beta_j^{\varnothing EI} < \beta_j^{UCL}$, there is no need to adjust the safety stocks calculation for the next production cycle. Regarding the TBO_j, the lot size will cover the expected demand for the next two periods ($t = 1$ and 2). Additionally, it is assumed that the lot size also covers the safety stocks for $TBO_j = 2$. Hence, the next production is then scheduled for the third period.

In the first re-plan - at the end of the first period - $\beta_j^{\varnothing EI}$ is re-calculated for $t = 2$ (see $(2)\beta_j^{\varnothing EI}$ in Figure 3) on the data basis of periods $t = -8, .., 0, 1$. Since the first period is a production period and the produced lot size has been planned to cover the expected demand of two periods, the actual demand of the first period, which is assumed to be lower than expected, is directly fulfilled from stock. This leads to an increase of $(2)\beta_j^{\varnothing EI}$ compared to $(1)\beta_j^{\varnothing EI}$. In a next step RI_j needs to be determined for the second period. Since the demand in the first period is lower than expected, $\beta_j^{exp} > \beta_j^{min}$. Thus, RI_j is set to 0 and the former planned production cycle stays stable.

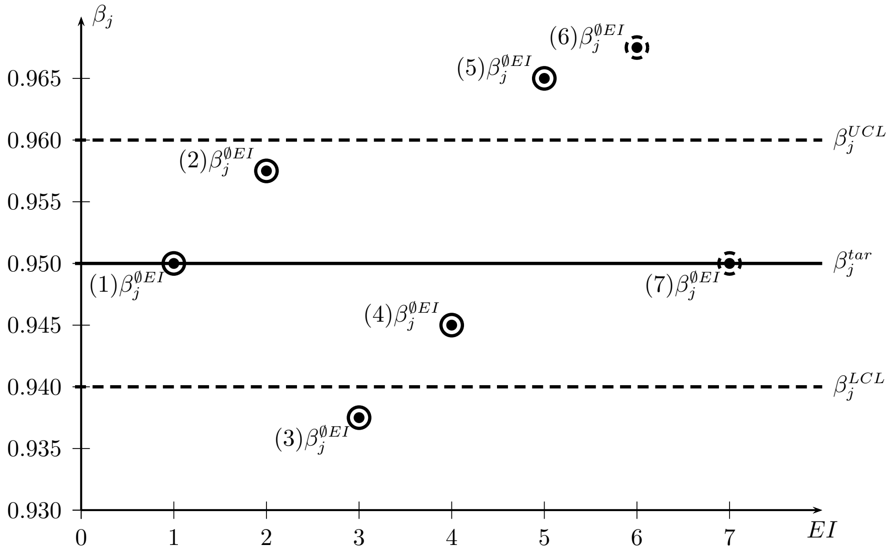


Figure 3 An illustrative example of $\beta_j^{\varnothing EI}$ resulting from rolling schedules

In the second re-plan - at the end of the second period - the re-plan procedure is repeated. However, assuming a much higher demand than expected in the second period, not all demand could be fulfilled directly from stock. This caused backorders of product j in the second period. Hence, the calculation of $\beta_j^{\varnothing EI}$ at the end of the second period, which is now based on the data of the previous periods $t = -7, .., 0, 1, 2$, results in a decrease of $(3)\beta_j^{\varnothing EI}$. According to the out of stock situation of product j in the second period, $\beta_j^{exp}(\beta_j^{exp} = 0) < \beta_j^{min}$. Thus, RI_j is set to 1, which results in a production of product j for the third period. This production decision is in line with the former production plan. On the one hand, the lot size in the third period also has to fulfill the backorders of the second period. On the other hand, it has to cover the expected demand of the next two periods ($t = 3$ and 4) as well as the required safety stocks of this production cycle. The next production is then scheduled for the fifth period.

Once again, $\beta_j^{\varnothing EI}$ is recalculated at the end of the third period. Assuming that the demand in the third period is higher than expected but lower then the inventory level, all demand is fulfilled directly from stock, which in turn increases (4)$\beta_j^{\varnothing EI}$. However, due to the high demand, $\beta_j^{exp} < \beta_j^{min}$ for the fourth period. In this case, it cannot be ensured that $\beta_j^{\varnothing EI} \geq \beta_j^{LCL}$ at the end of the fourth period. Therefore, RI_j is set to 1, which in turn leads to a pulled forward production in the fourth period. Meaning, that the formerly planned production cycle is aborted prematurely due to the LCM. Since of the $TBO = 2$, the next production is scheduled for the sixth period.

Again, we assume a higher actual demand than expected in the fourth period and that it is fulfilled directly from stock. Therefore, the recalculation of $\beta_j^{\varnothing EI}$ results in a further increase of the control parameter, see (5)$\beta_j^{\varnothing EI} = 0.965$ in Figure 3. However, the high demand leads to $\beta_j^{exp} < \beta_j^{min}$ for the fifth period. Thus, RI_j is set to 1 and a pulled forward production takes place in the fifth period. Since $\beta_j^{\varnothing EI} > \beta_j^{UCL}$, the UCM is activated. Thus, the safety stocks, which are included in the lot size of the fifth period, are now calculated on the basis of β_j^{min} instead of β_j^{tar}. Regarding the underlying $TBO = 2$ the next production is scheduled for the seventh period.

Referring to the current high $\beta_j^{\varnothing EI}$ at the beginning of the fifth period and the produced lot size, which will cover the expected demand of the fifth and the sixth period as well as the adjusted safety stocks, the probability of a premature termination of the production cycle indicated by RI_j at the end of the fifth period is negligible. Hence, the production plan is assumed to be stable. Due to the adjusted (and reduced) safety stocks of product j, we expect that $\beta_j^{\varnothing EI}$ will be repositioned into the control limits by the end of the current production cycle, see dashed circles in Figure 3. In addition, the reduced safety stocks decrease the inventory level, which in turn reduces the inventory holding costs.

In this example we demonstrated the functionality of the control limits as well as the corresponding control mechanisms. The UCM only has an impact on the planning process in a few re-plans.

Whereas, the LCM has always an impact on the re-planning process to ensure that $\beta_j^{\varnothing EI} \geq \beta_j^{LCL}$. Since the control limits are applied as intervention limits, it is possible that $\beta_j^{\varnothing EI}$ can reach values outside these limits. However, due to the control mechanisms one can expect that the parameter $\beta_j^{\varnothing EI}$ will be repositioned into the control limits within the next periods.

5. Computational study

We will now analyse the performance of *rolling schedules (RS) with the stabilized cycle* strategy in a computational study and compare it with *RS with fixed frozen horizons (FH)* strategies as well as with the *static uncertainty* strategy.

5.1. Test instances

We investigate test instances with six products, one capacitated production resource and an evaluation interval of one year, divided into 48 periods. We assume that the periods' demand are normally distributed and will be forecasted for the upcoming planning interval of twelve periods. The demand forecast as well as the coefficient of variation are assumed to be constant and equal

Table 1 Parameters of the data set

Number of products, $\bar{j}$	6
Number of periods within the evaluation interval, EI	48
Number of periods within the planning interval, $\bar{t}$	12
Given β service level, β_j^{tar}	0.95
Machine utilization	$(70\%, 85\%)$
Mean period demand, μ	Constant (1000)
Coefficient of variation, VC	Constant (0.2)
Time between order, TBO	$(2, 3, 5, 2/3/5)$

for all products and periods. Once, the demand forecast for a period is known, it will not be updated at a later time. A parameter overview of the dataset can be found in Table 1. Parameter sc_j represents the setup costs of a product j and is set to 1000$ for all products. We assume no setup times. The inventory holding costs per product are calculated according to equation (40). In equation (40), $E\{d_j\}$ stands for the expected demand of a product j and is set to 1000 units for every period and product for all test instances.

$$hc_j = (2 \cdot sc_j)/(E\{d_j\} \cdot TBO_j^2) \tag{40}$$

The production rate κ_j of a product j is calculated regarding the fixed machine capacity, the fixed average machine utilization and the cumulated expected average demand of all products (Seipl 2009, pp. 267). The initial inventories at the beginning of the first evaluation interval are uniformly distributed subject to the economic order quantity for every product (Seipl 2009, pp. 265). The last setup period of a product in the previous evaluation interval is calculated based on the initial inventory. The initial inventory of the products at the beginning of subsequent evaluation intervals (e.g., 2, 3,...) are set to the final inventory of the previous evaluation interval. A product's assumed TBO varies for different test instances (Table 1). By combining three different uniform options of the TBO (2, 3 and 5) as well as a mixture of the three (2/3/5) with two different machine utilizations, eight test instances have been generated.

The number of repetitions for each test instance is set to 61 evaluation intervals. This can be seen as a simulation of 61 consecutive years. When analysing test instances every evaluation interval is regarded as a sample subject to its performance indicators. In general every simulation environment has a transient phase in the beginning. To reduce the impact of the transient phase, we exclude the first simulated evaluation interval of every test instance from the analysis.

5.2. Strategies

In the following we study two settings of the *static uncertainty* strategy, three settings of the *RS with fixed FH* strategy and finally *RS with the stabilized cycle* strategy.

5.2.1. *Static uncertainty* strategy settings

We consider two different settings of the *static uncertainty* strategy as benchmarks for *RS with the stabilized cycle* strategy. In both strategy settings we use the *S-CLSP* of Tempelmeier (2011) to determine setup and lot size decisions.

In the first setting of the *static uncertainty* strategy, the planning interval of the $S\text{-}CLSP$ is set to the evaluation interval ($S\text{-}CLSP^{EI}$). Since the $S\text{-}CLSP^{EI}$ is used in a consecutive planning process over 61 evaluation intervals, it is insufficient to plan with product-specific target inventory levels of zero. Therefore, we set them to the initial inventory levels of the first evaluation interval.

In a second setting ($S\text{-}CLSP^{PI}$), a planning interval of twelve periods is used. This is equal to the length of the planning interval of the *RS with fixed FH* strategy settings. Again the target inventory levels are set to the initial inventory levels of the first evaluation interval. Once, the $S\text{-}CLSP^{PI}$ is solved, all decisions for the upcoming twelve periods are implemented regardless of the actual period demands. Hence, based on updated information, a next plan is generated every twelve periods for the next twelve periods.

However, when using the $S\text{-}CLSP$ in a *static uncertainty* strategy within a consecutive planning, it cannot be ensured that the actual inventory levels at the end of a planning interval enable a feasible solution for the production plan of the next planning interval. In case a test instance contains at least one capacity-infeasible production plan, it is excluded from the analysis.

5.2.2. *Rolling schedule with fixed frozen horizons* strategy settings

First, we study *RS with fixed FH* with period-based frozen horizons. In this setting only the first period's decisions of the planning interval are implemented, whereas decisions for later periods are subject to revision. According to the findings of Xie et al. (2003) it is beneficial to set the number of periods within the re-planning interval equal to the frozen horizon. Thus, the re-planning interval is set to one period within this strategy. For determining setup periods and lot sizes we use the $CLSP$ model formulation of Billington et al. (1983) extended by *static* safety stocks, *looking beyond the planning horizon* approach, and soft capacity constraints. The penalty costs pc_t are set to a relative high constant value for each period (41). This strategy setting is subsequently called $CLSP^{1.PE}_{SSS}$.

$$pc_t = \bar{t} \cdot \bar{j} \cdot sc_j + 1 \tag{41}$$

In a second experiment, we use the same setting as in the $CLSP^{1.PE}_{SSS}$ but now considering *TBO-SS* instead of *static* safety stocks ($CLSP^{1.PE}_{TBO\ SS}$). With $CLSP^{1.PE}_{TBO\ SS}$ we evaluate the impact of *TBO-SS* on the performance of rolling schedules.

The third experiment considers the same setting as in the $CLSP^{1.PE}_{TBO\ SS}$ with the exception of product-specific, order-based frozen horizons instead of period-based ones ($CLSP^{1.PC}_{TBO\ SS}$). The length of the product-specific frozen horizons is equal the length of the first production cycle of each product, if started in the first period of the planning interval. Since we consider a multi-item lot-sizing model with product-specific frozen horizons, a re-planning interval of one period is required. However, only decisions for products and periods beyond the frozen horizons are considered in the re-plans. Assuming an infinite production capacity, this strategy setting can be interpreted as a dynamic $(r_{j,t}, S_{j,t})$ inventory policy, in which $r_{j,t}$ is the optimal TBO_j for product j - produced in period t - and $S_{j,t}$ is the optimal product- and period-specific *order-up-to-level*, which is sufficient to fulfill the demand subject to β_j^{tar} and the TBO_j. A comparison between a single-item uncapacitated lot-sizing model and a dynamic (r_t, S_t) inventory policy can be found in Herpers (2009). However, our computational study focuses on the performance of a multi-item, capacitated, dynamic $(r_{j,t}, S_{j,t})$ inventory policy in rolling schedules, which - to the authors best knowledge - has not been studied before.

5.2.3. *Rolling schedules with the stabilized cycle* strategy

For *RS with the stabilized cycle* strategy the product-specific control limits (β_j^{UCL} and β_j^{LCL}) are set to $\pm 0.5\%$ of β_j^{tar}. As in the *RS with fixed FH* strategy settings the re-planning interval is set to one period. Note that no fixed frozen horizon has to be determined in advance if this strategy is used. Furthermore, the $CLSP$ - extended by *TBO-SS*, *looking beyond the planning horizon* approach, and soft capacity constraints - is used to determine setup periods and lot sizes in *RS with the stabilized cycle* strategy ($CLSP^{LC\&UC}_{TBO\ SS}$).

An overview of all studied strategy settings is shown in Table 2.

5.3. Computational results

The computational study has been performed on an *Intel(R) Core(TM) i7-4770* processor with a clock speed of *3.4 GHz* and *16.0 GB RAM* under *Windows 7 Professional*. We make use of the simulation environment of Seipl (2009), which is implemented in the programming language *Java*.

Table 2 Strategy settings used within the computational study

	Safety Stocks	Planning Interval	Frozen Horizon	Re-planning Interval	Strategy
$S\text{-}CLSP^{PI}$	TBO-dependent	12 Periods	12 Periods	12 Periods	*Static uncertainty*
$S\text{-}CLSP^{EI}$	TBO-dependent	48 Periods	48 Periods	48 Periods	*Static uncertainty*
$CLSP^{1.PE}_{SSS}$	Static (s.t. optimal TBO)	12 Periods	Period-based, first period	First period	*RS with fixed FH*
$CLSP^{1.PE}_{TBO\ SS}$	TBO-dependent	12 Periods	Period-based, first period	First period	*RS with fixed FH*
$CLSP^{1.PC}_{TBO\ SS}$	TBO-dependent	12 Periods	Order-based, first production cycle	First period	*RS with fixed FH*
$CLSP^{LC\&UC}_{TBO\ SS}$	TBO-dependent	12 Periods	Order-based with revisions, $\geq$ first period	First period	*RS with the stabilized cycle*

Moreover, the extended $CLSP$'s, which are used to determine setup periods and lot sizes within the *RS with fixed FH* strategy settings and *RS with the stabilized cycle* strategy, are implemented in *Xpress-IVE (Version 1.24.06)* and solved by the *Xpress Optimizer (Version 27.01.02)*. The maximal computational time per re-plan was set to 200 seconds. However, we observe that almost all re-plans yield an optimal solution. Thus, the mean optimality gap is negligible. For solving the $S\text{-}CLSP$'s within the *static uncertainty* strategy settings the CG ABC_β heuristic of Tempelmeier (2011) is used.

To ensure a fair comparison among the six strategy settings, we exclude all evaluation intervals from the analysis of a strategy setting in which additional capacity units are needed. However, none of the *RS with fixed FH* strategy settings require additional capacity units. The same also goes for *RS with the stabilized cycle* strategy. Unfortunately, it is impossible to generate 61 consecutive capacity feasible production plans for the $S\text{-}CLSP^{EI}$ for test instances with a machine utilization of 85%. Therefore, these test instances are excluded from the analysis of the $S\text{-}CLSP^{EI}$. The same applies to the $S\text{-}CLSP^{PI}$ for the test instance with a machine utilization of 85% and a TBO of 2.

The performance of the six strategy settings is analysed regarding two performance indicators. First, we consider the actual mean costs per evaluation interval, which contain inventory holding and setup costs. Second, we take the actual $\beta_j^{\varnothing EI}$ per product and evaluation interval into consideration. The actual mean costs per evaluation interval are shown in Table 3 and the actual average $\beta_j^{\varnothing EI}$ as well as the downside deviation (DD) of $\beta_j^{\varnothing EI}$ from β_j^{tar} in percentage points are presented in Table 4.

Table 3 Computational results of the actual mean costs

Strategy		*RS with fixed FH*			*RS with the stabilized cycle*	*Static uncertainty*	
TBO	Machine Utilization	$CLSP^{1.PE}_{SSS}$	$CLSP^{1.PE}_{TBO\ SS}$	$CLSP^{1.PC}_{TBO\ SS}$	$CLSP^{LC\&UC}_{TBO\ SS}$	$S\text{-}CLSP^{EI}$	$S\text{-}CLSP^{PI}$
Periods	∅	∅ [$]	∅△ [%]	∅△ [%]	∅△ [%]	∅△ [%]	∅△ [%]
2	70%	300.874	-0.32	-24.14	-20.74	14.86	-10.19
3	70%	206.077	-0.89	-20.98	-17.31	1.80	-10.09
5	70%	122.866	0.07	-15.43	-9.42	2.04	3.32
2 / 3 / 5	70%	207.631	-0.50	-20.54	-16.29	12.07	-4.74
2	85%	300.888	-0.16	-23.87	-19.00	-	-
3	85%	219.695	0.46	-25.68	-19.37	-	0.06
5	85%	131.438	0.36	-20.64	-9.95	-	22.12
2 / 3 / 5	85%	213.189	0.37	-21.40	-15.32	-	11.41
Average		212.832	-0.08	-21.58	-15.93	7.69	1.70

On average the *static uncertainty* strategy settings yield the highest mean costs per evaluation interval (Table 3). Within the *static uncertainty* strategy an increasing length of the planning interval results in increasing costs. The *RS with fixed FH* strategy settings with *period-based* frozen horizons ($CLSP^{1.PE}_{SSS}$,$CLSP^{1.PE}_{TBO\ SS}$) yield the highest mean costs among the *RS with fixed FH* strategy settings and *RS with the stabilized cycle* strategy. When comparing all six strategy settings the $CLSP^{1.PC}_{TBO\ SS}$ results in the least mean costs in every test instance. The same applies to the next best strategy - the $CLSP^{LC\&UC}_{TBO\ SS}$ - in comparison with the remaining four strategy settings. The reason for the cost increase of the $CLSP^{LC\&UC}_{TBO\ SS}$ is, that it aborts a formerly planned production cycle in situations in which $\beta_j^{\varnothing EI}$ is expected to fall below the lower control limit. Thus, the number of setups might increase and may result in increasing mean costs compared to the $CLSP^{1.PC}_{TBO\ SS}$. A further observation for the $CLSP^{LC\&UC}_{TBO\ SS}$ is that the number of violations of the lower control limit increases with higher machine utilizations.

As regards the compliance with the given service level, all studied strategy settings exceed β_j^{tar} on average over all products, evaluation intervals, and test instances. However, not every strategy

Table 4 Computational results of $\beta_j^{\varnothing EI}$

Strategy		*RS with fixed FH*						*RS with the stabilized cycle*		*Static uncertainty*			
TBO	Machine Utilization	$CLSP_{SSS}^{1.PE}$		$CLSP_{TBO\,SS}^{1.PE}$		$CLSP_{TBO\,SS}^{1.PC}$		$CLSP_{TBO\,SS}^{LC\&UC}$		$S\text{-}CLSP^{EI}$		$S\text{-}CLSP^{PI}$	
Periods	∅	μ [%]	DD [%]	μ [%]	DD [%]	μ [%]	DD [%]	μ [%]	DD [%]	μ [%]	DD [%]	μ [%]	DD [%]
2	70%	98.71	0.00	98.94	0.00	95.15	0.96	95.04	0.29	95.65	7.28	95.76	2.39
3	70%	99.09	0.00	99.31	0.00	95.12	1.06	95.19	0.27	95.63	6.42	95.96	1.76
5	70%	99.31	0.00	99.44	0.00	95.05	1.32	95.37	0.25	95.52	5.95	96.48	1.20
2 / 3 / 5	70%	99.17	0.00	99.27	0.00	95.02	1.32	95.23	0.23	95.51	7.09	96.03	2.03
2	85%	97.75	0.00	98.50	0.00	95.12	0.99	94.99	0.32	-	-	-	-
3	85%	98.56	0.00	98.89	0.00	95.10	1.13	95.15	0.28	-	-	96.05	2.21
5	85%	99.11	0.00	99.44	0.00	94.73	2.21	95.39	0.24	-	-	97.19	0.92
2 / 3 / 5	85%	98.78	0.00	99.06	0.00	95.14	1.16	95.13	0.29	-	-	96.42	1.80
Average		98.81	0.00	99.11	0.00	95.05	1.27	95.19	0.27	95.58	6.69	96.27	1.76

setting ensures $\beta_j^{\varnothing EI} \geq \beta_j^{tar}$ for each product and every evaluation interval. As expected *RS with the stabilized cycle* strategy ($CLSP_{TBO\,SS}^{LC\&UC}$) leads to a sharp decrease of the downside deviation of $\beta_j^{\varnothing EI}$ from the β_j^{tar} (∅0.27%) in all studied test instances compared with the $CLSP_{TBO\,SS}^{1.PC}$ (∅1.27%) and the *static uncertainty* strategy settings ($EI:$∅6.69%; $PI:$∅1.76%) (Table 4). As mentioned before, the production plans of the *static uncertainty* strategy settings are - once they have been planned - fixed for the planning interval. This results in large $\beta_j^{\varnothing EI}$ downside deviations for all test instances. Meaning, that the probability of a $\beta_j^{\varnothing EI}$ being lower than β_j^{tar} is quite high. The same applies to the $CLSP_{TBOSS}^{1.PC}$, where the first production cycle is frozen regardless of the actual demands. In contrast, the *RS with fixed FH* strategy settings with a period-based frozen horizon ($CLSP_{SSS}^{1.PE}$, $CLSP_{TBO\,SS}^{1.PE}$) react to the actual demand of the past period in every re-plan. This leads to an over-accomplishment of β_j^{tar} in every evaluation interval and no downside deviation of $\beta_j^{\varnothing EI}$ from β_j^{tar}.

Moreover, the results show that the *static uncertainty* strategy settings have difficulties to generate capacity and service level feasible production plans. The number of plans, which result in infeasible solutions regarding the β service level constraints of the $S\text{-}CLSP$ are shown in Table 6 in the Appendix A. Therefore, the *static uncertainty* strategy seems to be inappropriate for production planning in industrial practice. Instead, our computational results have shown that a good compromise between costs and downside deviation of the β_j service levels can be achieved by *RS with the stabilized cycle* strategy.

To study the impact of demand uncertainties and rolling schedules on costs, we compare the six strategy settings with a deterministic $CLSP$ (Appendix B) for the test instance with mixed $TBO's$ and a high machine utilization (Table 5). Here, the deterministic $CLSP$ is solved consecutively with a planning interval that equals the evaluation interval of 48 weeks. As deterministic demands actual demands that have been realized in the stochastic settings are used. Thus, the deterministic $CLSP$ provides a lower bound for the costs while exactly meeting given β service levels.

Table 5 Costs of uncertainty strategies in comparison with the deterministic $CLSP$

Strategy		deterministic			*RS with fixed FH*			*RS with the stabilized cycle*	*Static uncertainty*	
TBO	Machine Utilization	$CLSP^{deter}$			$CLSP^{1.PE}_{SSS}$	$CLSP^{1.PE}_{TBO\ SS}$	$CLSP^{1.PC}_{TBO\ SS}$	$CLSP^{LC\&UC}_{TBO\ SS}$	$S\text{-}CLSP^{EI}$	$S\text{-}CLSP^{PI}$
Periods	⌀	⌀ [$]	⌀ GAP [%]	max GAP [%]	⌀△ [%]	⌀△ [%]	⌀△ [%]	⌀△ [%]	⌀△ [%]	⌀△ [%]
2 / 3 / 5	85%	149.609	0,86	1,77	42,50	43,03	12,01	20,66	-	58,75

For a better understanding of the trade-off between actual costs per evaluation interval and the actual $\beta_j^{\varnothing EI}$ per product, we plot the performance indicators in an X/Y-diagram (Figure 4). It shows the results of all feasible strategy settings for the test instance with a machine utilization of 85% and mixed $TBO's$. For every evaluation interval the actual $\beta_j^{\varnothing EI}$ for every product together with the corresponding actual costs are represented by a single “dot”. To provide a better overview, the diagram contains two types of horizontal lines. The dashed lines visualise the chosen control limits for $\beta_j^{\varnothing EI}$, which are used for *RS with the stabilized cycle* strategy ($CLSP^{LC\&UC}_{TBO\ SS}$), whereas the solid lines visualise an (assumed) acceptable ± 0.01 tolerance range from β_j^{tar}.

A more detailed analysis of the results - shown in Figure 4 -, is presented in a standard graphical box-plot analysis performed by the statistical computing language *R (Version 3.1.2)*. The box-plot shows the resulting distribution of the analysed 60 evaluation intervals. There is one graphical box-plot for the $\beta_j^{\varnothing EI}$ analysis (Figure 5) and another for the costs analysis (Figure 6).

Figure 4 **Trade-off between costs and $\beta_j^{\varnothing EI}$ service level for mixed $TBO's$ and a machine utilization of 85%**

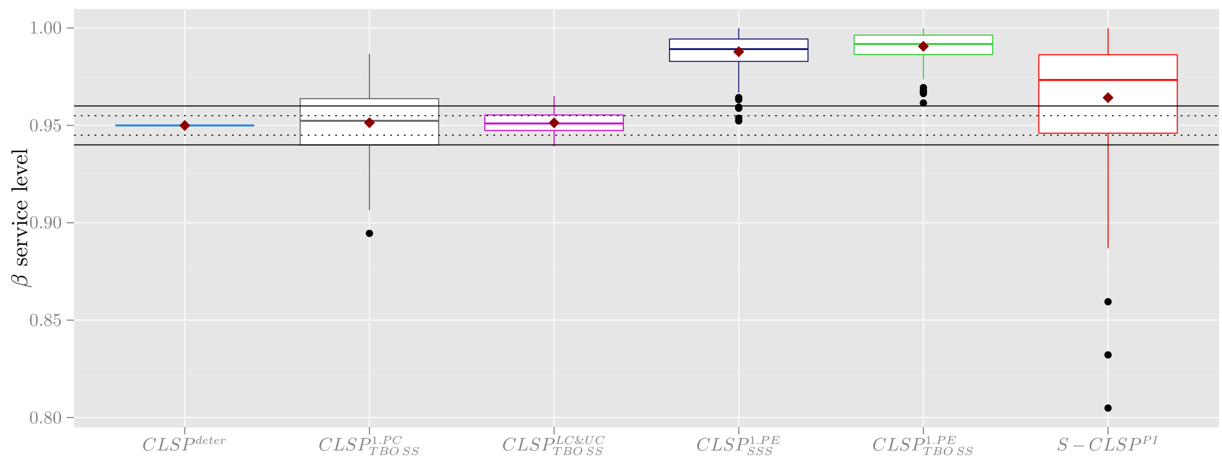


Figure 5 **$\beta_j^{\varnothing EI}$ service level box-plot analysis for mixed $TBO's$ and a machine utilization of 85%**

6. Conclusions and future research

In classical rolling schedules involving deterministic lot-sizing models with service level constraints about half of the production cycles are aborted prematurely because demand is higher than

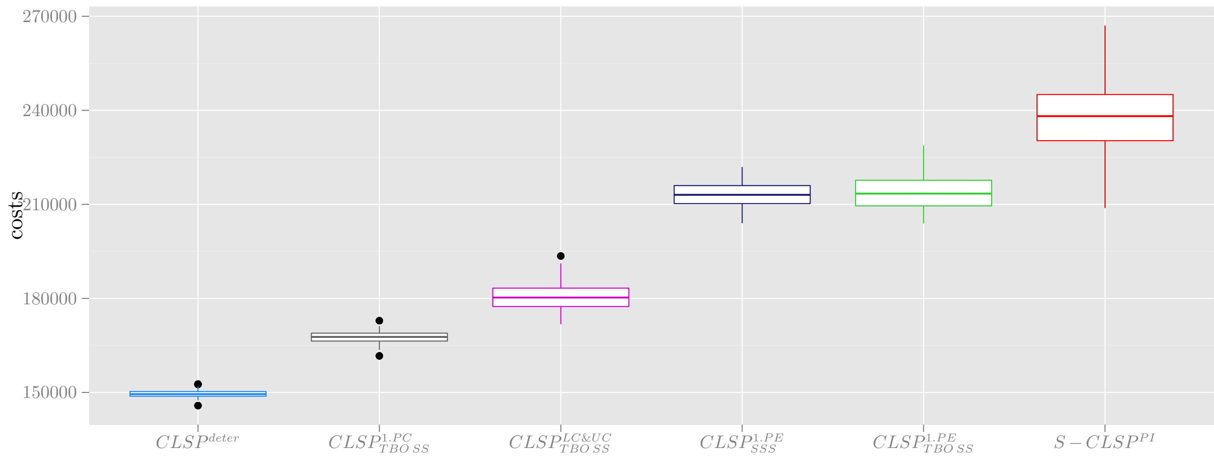


Figure 6 **Cost box-plot analysis for mixed $TBO's$ and a machine utilization of** 85%

expected. This results in unstable production plans and in higher setup and holding costs than anticipated. On the other hand actual service levels will be (much) higher than required.

A favorable improvement over classical rolling schedules is to employ (product-specific) order-based frozen horizons which are (re-)set to the length of a products production cycle whenever a corresponding setup is planned in the first period of the planning interval. These order-based frozen horizons are fixed even if service levels may be violated. This strategy yields the lowest costs in our computational study. This is in line with the findings of Sridharan and Berry (1990) and Zhao and Lee (1993) for uncapacitated lot-sizing problems. However, actual service levels vary largely around the given service level with a large portion of downside deviations. This might be counterproductive if service levels must be met in a given evaluation interval (e.g., due to periodic audits by customers).

It turned out that the *stabilized cycle* strategy proposed in this paper is a good compromise. Here, the current production cycle (i.e., the first setup in the planning interval) is kept as long as the service level stays within given upper and lower control limits. As our computational study demonstrates, the proposed control mechanisms are able to keep the downside deviations from given β service levels low while there is only a moderate increase in costs compared to rolling schedules with order-based frozen horizons. All other strategies tested have resulted in considerably worse solutions with respect to costs and/or the downside deviations from given service levels.

Other than most previous studies we have addressed capacitated production systems here. Due to large computational efforts when running rolling schedules over sixty years we have created only eight test instances with varying TBO and capacity utilization rates of 0.7 and 0.85 while considering a β service level of 0.95 with control limits of (0.95 ± 0.005) and an evaluation interval of 48 weeks. Although further tests might be desirable, the rationale of the *stabilized cycle* strategy as well as our computational tests indicate that this strategy should do well also in other planning environments. Though one should bear in mind that "some" slack capacity is needed for the *stabilized cycle* strategy to work economically in the backorder case.

The purpose of our paper has been to present general extensions for big bucket models to improve the performance of rolling schedules. We have demonstrated our proposals by means of the $CLSP$ modeled along the lines of Billington et al. (1983). It may be argued that there are tighter model formulations, like the shortest path introduced by Eppen and Martin (1987). However, these model formulations are less commonly used in practice than the *big M* formulation of Billington et al. (1983), especially in case more complex production planning problems have to be solved.

Future research should test the *stabilized cycle* strategy in different planning environments, like tighter capacity utilization rates, alternative demand forecasting procedures, seasonal demand, various capacitated lot-sizing models, and hierarchical production planning.

Acknowledgments

We are indebted to Horst Tempelmeier for providing the software to solve the S-$CLSP$ in our computational tests. Furthermore, we are most grateful that Christian Seipl made his simulation environment for rolling schedules available to us.

Appendix A: Number of service level infeasible plans of the *static uncertainty* strategy settings

Table 6 Number of service level infeasible plans of the static uncertainty strategy settings

TBO	Machine Utilization	$S\text{-}CLSP^{PI}$	$S\text{-}CLSP^{EI}$
Periods	∅	number of infeasible plans	
2	70%	0 of 240	0 of 60
3	70%	9 of 240	4 of 60
5	70%	23 of 240	4 of 60
2 / 3 / 5	70%	18 of 240	15 of 60
2	85%	capacity infeasible	capacity infeasible
3	85%	60 of 240	capacity infeasible
5	85%	96 of 240	capacity infeasible
2 / 3 / 5	85%	92 of 240	capacity infeasible

Appendix B: Deterministic $CLSP$ with β service level constraints

Sets

$J = \{j | j = 1, .., \bar{j}\}$ set of products

$T = \{t | t = 1, .., \bar{t}\}$ set of periods

Data

b_t production capacity in period t

β_j^{tar} given β service level of product j

$bo_{j,0}^{cum}$ backorders of product j at the beginning of a planning interval

$bonus_{j,t,t+\tau}$ (negative) bonus payment for the last production cycle of product j

$d_{j,t}$ demand of product j in period t

hc_j inventory holding costs per period t and product unit j

κ_j production coefficient of product j

$m_{j,t}^X$ Big M for lot size decisions

$m_{j,t}^{BO}$ Big M for backorder decisions

sc_j setup costs of product j

$\underline{t}$	the smallest period t for a product j for which $t+\tau^{\mu}_{j,t} \geq \bar{t}+1$ is valid
τ^{BO}_j	maximal number of periods product j may be backordered (here: $\tau^{BO}_j = \bar{t}$)
τ^{max}_j	maximal number of periods product j may be stored (here: $\tau^{max}_j = \bar{t}$)
$\tau^{\mu}_{j,t}$	static optimal TBO of product j in period t

Variables

$BO_{j,t}$	backorders of product j at the end of period t
$BO^{cum}_{j,t}$	existing backorders of product j at the end of period t cumulated over past periods
$I_{j,t}$	inventory of product j at the end of period t
$V_{j,t,t+\tau}$	1, if a setup for product j is scheduled in period t while the next setup is scheduled in period $t+\tau$, 0 otherwise
$W_{j,t}$	1, if $BO^{cum}_{j,t-1} - XB_{j,t} \leq 0$ (i.e., the backorder of j are met at the end of t), 0 otherwise
$X_{j,t}$	lot size of product j in period t
$XB_{j,t}$	portion of $X_{j,t}$ to satisfy the existing backorders cumulated over past periods
$XD_{j,t}$	portion of $X_{j,t}$ to satisfy the demand of period t and to build up inventory
$Y_{j,t}$	1, if a setup for product j takes place in period t, 0 otherwise
Z^{deter}	total inventory holding and setup costs including $bonus_{j,t,t+\tau}$

$$\min Z^{deter} = \sum_{j=1}^{\bar{j}} \sum_{t=1}^{\bar{t}} hc_j \cdot I_{j,t} + \sum_{j=1}^{\bar{j}} \sum_{t=1}^{\bar{t}} sc_j \cdot Y_{j,t} + \sum_{j=1}^{\bar{j}} \underbrace{\sum_{t=1}^{\bar{t}}}_{\text{if } t+\tau^{\mu}_{j,t} \geq \bar{t}+2} \sum_{\tau=\bar{t}-t+2}^{\tau^{\mu}_{j,t}} bonus_{j,t,t+\tau} \cdot V_{j,t,t+\tau} \tag{42}$$

subject to

$$X_{j,t} - m^X_{j,t} \cdot Y_{j,t} \leq 0 \quad \forall j \in J, t \in T \tag{43}$$

$$\sum_{j=1}^{\bar{j}} \kappa_j \cdot X_{j,t} \leq b_t \quad \forall t \in T \tag{44}$$

$$I_{j,t-1} + XD_{j,t} = (d_{j,t} - BO_{j,t}) + I_{j,t} \quad \forall j \in J, t \in T \tag{45}$$

$$BO_{j,t} \leq d_{j,t} \quad \forall j \in J, t \in T \tag{46}$$

$$BO^{cum}_{j,t-1} + BO_{j,t} - XB_{j,t} = BO^{cum}_{j,t} \quad \forall j \in J, t \in T \tag{47}$$

$$XD_{j,t} + XB_{j,t} = X_{j,t} \quad \forall j \in J, t \in T \tag{48}$$

$$XD_{j,t} \leq m^X_{j,t} \cdot W_{j,t} \quad \forall j \in J, t \in T \tag{49}$$

$$BO^{cum}_{j,t-1} - XB_{j,t} \leq m^{BO}_{j,t-1} \cdot (1 - W_{j,t}) \quad \forall j \in J, t \in T \tag{50}$$

$$W_{j,t} \leq Y_{j,t} \quad \forall j \in J, t \in T \tag{51}$$

$$I_{j,0} - BO^{cum}_{j,0} + \sum_{t=1}^{\bar{t}} X_{j,t} = \sum_{t=1}^{\bar{t}} d_{j,t} + I_{j,\bar{t}} \quad \forall j \in J \tag{52}$$

$$\sum_{t=1}^{\bar{t}} BO_{j,t} \leq (1 - \beta^{tar}_j) \cdot \sum_{t=1}^{\bar{t}} d_{j,t} \quad \forall j \in J \tag{53}$$

$$I_{j,\bar{t}} \geq \underbrace{\sum_{t=\underline{t}}^{\bar{t}}}_{\text{if } t+\tau^{\mu}_{j,t} \geq \bar{t}+2} \sum_{\tau=\bar{t}-t+2}^{\tau^{\mu}_{j,t}} \left(\sum_{s=\bar{t}+1}^{t+\tau-1} d_{j,s} \right) \cdot V_{j,t,t+\tau} \quad \forall j \in J \tag{54}$$

$$\sum_{t=\underline{t}}^{\bar{t}} \sum_{\tau=\bar{t}-t+1}^{\tau^{\mu}_{j,t}} V_{j,t,t+\tau} = 1 \quad \forall j \in J \tag{55}$$

$$\sum_{\tau=\bar{t}-t+1}^{\tau^{\mu}_{j,t}} V_{j,t,t+\tau} = Y_{j,t} \quad \forall j \in J, t = \underline{t}, .., \bar{t} \tag{56}$$

$$BO^{cum}_{j,\bar{t}} = 0 \quad \forall j \in J \tag{57}$$

$$BO^{cum}_{j,0} = bo^{cum}_{j,0} \quad \forall j \in J \tag{58}$$

$$Y_{j,t}, W_{j,t} \in \{0,1\} \quad \forall j \in J, t \in T \tag{59}$$

$$V_{j,t,t+\tau} \in \{0,1\} \quad \forall j \in J, t = \underline{t}, .., \bar{t}, \tau = 1, .., \tau^{\mu}_{j,t} \tag{60}$$

$$BO_{j,t}, BO^{cum}_{j,t}, I_{j,t}, X_{j,t}, XB_{j,t}, XD_{j,t} \geq 0 \quad \forall j \in J, t \in T \tag{61}$$

with

$$m^{X}_{j,t} = \min\{\frac{b_t}{\kappa_j}; \sum_{s=t}^{\min\{t+\tau^{max}_j-1, \bar{t}+\tau^{\mu}_{j,t}-1\}} d_{j,s}\} \tag{62}$$

$$m^{BO}_{j,t} = \min\{(1 - \beta^{tar}_j) \cdot \sum_{t=1}^{\bar{t}} d_{j,t}; \underbrace{bo^{cum}_{j,0}}_{\text{if } t-\tau^{BO}_j+1<1} + \sum_{s=\max\{1, t-\tau^{BO}_j+1\}}^{t} d_{j,s}\} \tag{63}$$

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