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Self-Insurance, Self-Protection, and Increased Risk Aversion: An Intertemporal Reinvestigation

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Zusammenfassung / Abstract

This paper studies the effect of increased risk aversion on self-insurance and self-protection in a two-period framework. Here risk management incentives and consumption smoothing incentives are traded off, and the monotonic relationship between self-insurance and risk aversion may no longer hold as more risk-averse agents cannot always afford spending more on self-insurance. A very similar relationship holds for self-protection making self-insurance and self-protection much more alike in a two-period model. We also extend the model to a joint analysis of self-insurance/self-protection and saving decisions.

Keywords: self-insurance; self-protection; risk aversion; time structure.

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Abstract

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1 Introduction

To mitigate risks individuals may either invest in reducing the size of a potential loss (self-insurance or loss reduction) or in reducing the probability of a hazardous event (self-protection or loss prevention).[1] Dionne and Eeckhoudt (1985) derive the well-known and intuitive comparative static result that more risk-averse agents invest more in self-insurance but not necessarily more in self-protection.[2]

The timing of prevention decisions has mostly been abstracted from in the economic literature so far. However, in many instances it makes sense to think of self-insurance or self-protection as expenditures incurred today to mitigate future risks. Indeed, as the term "prevention" literally suggests, it is to some extent sustainable in many situations. Eeckhoudt and Gollier (2005) demonstrate in a static model that a more prudent agent invests less in self-protection. In a simple two-period framework, however, Menegatti (2009) shows that a more prudent agent invests *more* in self-protection! Therefore, it is justified to ask whether the impact of risk-aversion is the same in a dynamic setting compared to a static one. This paper extends the prominent results of Dionne and Eeckhoudt (1985) and studies the impact of increased risk-aversion on self-insurance and self-protection decisions when such decisions are more naturally defined as investments in future states of the world.

2 The Models

2.1 Self-Insurance

Consider two points in time, t_1 and t_2, which we call today and tomorrow. A representative individual owns initial wealth w_i in t_i. She faces the risk of losing l with probability $p \in (0, 1)$ in the second period. She can invest today in self-insurance y, which reduces the size of the loss to $l(y)$ ($l' < 0, l'' > 0$) but comes at a cost of $c(y)$ ($c' > 0, c'' > 0$). Utility U over both periods is separable and future utility is discounted by δ ($0 < \delta \leq 1$). The individual is risk-averse and she maximizes expected utility of final wealth($U' > 0, U'' < 0$):

$$\max_y U(w_1 - c(y)) + \delta\Big(pU(w_2 - l(y)) + (1-p)U(w_2)\Big).$$

The first-order condition (foc) is

$$-c'(y)U'(w_1 - c(y)) - \delta p l'(y)U'(w_2 - l(y)) = 0.$$

The individual balances marginal cost from investing in self-insurance and marginal benefits of incurring a smaller loss.[3]

Let us now consider an agent with utility V exhibiting greater risk-aversion than U; due to Pratt (1964), V can be represented as a concave transformation of U, i.e., $V = k(U)$ with $k' > 0, k'' < 0$. It follows that[4]

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[1] See Ehrlich and Becker (1972).

[2] Brys and Schlesinger (1990) rationalize this intuition by showing that investing in (actuarially fair) self-insurance can be interpreted as mean-preserving contraction, whereas the investment in (actuarially fair) self-protection is neither a mean-preserving contraction nor a mean-preserving spread: it rather consists of both at different levels of the final wealth distribution which explains the ambiguous effect of increasing risk aversion on preventive effort.

[3] The second-order condition is satisfied, i.e., $-c''(y)U'(w_1-c(y))+c'(y)^2U''(w_1-c(y))-\delta p l''(y)U'(w_2-l(y)) + \delta p l'(y)^2U''(w_2-l(y)) < 0$.

[4] All proofs can be found in the appendix.

Proposition 1. *V invests more in self-insurance than U if and only if U's period-1 final wealth exceeds period-2 final wealth in the loss state, i.e., $w_1 - c(y_U) > w_2 - l(y_U)$.*

As intuition suggests, increasing risk-aversion leads to more spendings on self-insurance if and only if the loss state is the worst state. In this situation, the risk management component of self-insurance dominates. However, if current consumption is too low, more risk-averse agents will *reduce* self-insurance to increase consumption today. Then the consumption smoothing component dominates. Therefore, in a two-period model, increased risk-aversion can be associated with lower self-insurance expenditures.

2.2 Self-Protection

Consider now the possibility of self-protection by allowing the individual to invest x today in reducing the probability of a hazardous event tomorrow (without changing the loss size). The loss probability is given by $p(x)$ ($p' < 0, p'' > 0$) and self-protection comes at a cost of $c(x)$ ($c' > 0, c'' > 0$). The individual's objective is

$$\max_x U(w_1 - c(x)) + \delta\Big(p(x)U(w_2 - l) + (1 - p(x))U(w_2)\Big),$$

with foc

$$-c'(x)U'(w_1 - c(x)) + \delta p'(x)(U(w_2 - l) - U(w_2)) = 0.$$

Again, the individual balances marginal cost from investing in self-protection against marginal benefit of enjoying a lower loss probability.[5]

Consider again a more risk-averse agent with utility $V = k(U)$:

Proposition 2. *V invests more (less) in self-protection than U if and only if k' evaluated at U's utility of wealth today is below (above) 1.*

Similar to the static model studied by Dionne and Eeckhoudt (1985), the effect of increased risk-aversion on self-protection is ambiguous and crucially depends on k'. This condition can be interpreted in the same manner: If current consumption is too low, increased risk-aversion lowers self-protection to save on consumption today.[6] Then the consumption smoothing incentive dominates. If consumption today is sufficiently high, more risk-aversion increases self-protection. Then the risk management incentive dominates. As a consequence, self-insurance and self-protection are more alike in a dynamic model compared to a static one.

3 The Models with Saving

We now extend the basic two-period self-insurance model by introducing a mechanism (i.e. a bank account) that allows the individual to transfer wealth between periods. Let s be the individual's savings in period 1 and $r \geq 0$ the interest rate. The individuals's objective then changes to

$$\max_{y,s} U(w_1 - c(y) - s) + \delta\Big(pU(w_2 - l(y) + (1 + r)s) + (1 - p)U(w_2 + (1 + r)s)\Big),$$

---

[5]The second-order condition is satisfied, i.e., $-c''(x)U'(w_1 - c(x)) + c'(x)^2 U''(w_1 - c(x)) + \delta p''(x)(U(w_2 - l) - U(w_2)) < 0$.

[6]Looking into the proof, the intermediate value theorem establishes the existence of a $\xi \in (w_2 - l, w_2)$ for which $k'(U(\xi)) = 1$. This is the crucial threshold for consumption today.

with associated focs

$$-c'(y)U'(w_1 - c(y) - s) - \delta p l'(y)U'(w_2 - l(y) + (1+r)s) = 0,$$
$$-U'(w_1 - c(y) - s) + \delta(1+r)\Big(pU'(w_2 - l(y) + (1+r)s) + (1-p)U'(w_2 + (1+r)s)\Big) = 0.$$

Self-insurance is used to equate marginal cost from spending in self-insurance with marginal benefits from enjoying a lower loss; saving is used to smooth consumption (to equate (expected) marginal utility levels across different points in time). We extend the result of Menegatti and Rebessi (2011) to the case of two-period self-insurance by noting that

$$\frac{dy}{ds} = -\frac{EU_{ys}}{EU_{yy}} = -\frac{c'(y)U''(w_1 - c(y) - s) - \delta(1+r)p l'(y)U''(w_2 - l(y) + (1+r)s)}{EU_{yy}} < 0.$$

As a result, saving and self-insurance are substitutes in an intertemporal setting!

Incorporating saving into our two-period self-protection model is straightforward. However, investigating the role of increased risk-aversion is not without difficulty, as the amount of optimal savings is also influenced by properties of the third derivative of utility (Kimball (1990)). As a specific example, let us look at quadratic utility. Let $U(w) := w - \alpha w^2$ with $\alpha > 0$, but sufficiently small for risk aversion. Then, it follows that

Proposition 3.   *a) If $w_1 - c(y) - s > w_2 - l(y) + s(1+r)$ and $1/(1+r) < \delta$, increased risk-aversion will lead to higher spendings on self-insurance and reduced savings. If $w_1 - c(y) - s < w_2 - l(y) + s(1+r)$ and $1/(1+r) > \delta$, increased risk aversion will lower expenditures on self-insurance and increase savings.*

   b) If $w_1 - c(x) - s > w_2 - 0.5l + s(1+r)$ and $1/(1+r) < \delta$, increased risk-aversion will lead to higher spendings on self-protection and reduced savings. If $w_1 - c(x) - s < w_2 - 0.5l + s(1+r)$ and $1/(1+r) > \delta$, increased risk aversion will lower expenditures on self-protection and increase savings.

In both cases, the first condition states that absolute spendings on prevention are currently modest and hence that current consumption is high enough to afford increasing preventive investments. The second condition is about the attractiveness of saving: if future utility is discounted stronger than by the risk-free rate, saving is relatively unattractive. Combined, spendings on prevention rise and savings fall with risk aversion. For the second parts of the proposition the argument is reversed. We see again that self-insurance and self-protection are very much alike here and that affordability from today's perspective is decisive for the question whether self-protection is increasing or decreasing in risk aversion.

4   Conclusion

Although in simple static models the relationship between risk aversion and self-insurance is monotonic, this may not hold in a dynamic context due to consumption smoothing incentives: more risk-averse individuals may want to lower self-insurance expenditures in favor of current consumption. Importantly, self-insurance and self-protection are very much alike in a two-period sense since for both techniques an endogenous threshold on current consumption is decisive on whether higher risk aversion lowers or raises risk management expenditures. This argument holds even if consumption can be smoothed through saving.

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Appendix

Proof of Proposition 1: We evaluate the foc of V at U's optimal level of self-insurance y_U:

$$-c'(y_U)k'(U(w_1 - c(y)))U'(w_1 - c(y)) - \delta p l'(y_U)k'(U(w_2 - l(y)))U'(w_2 - l(y))$$

which is positive, yielding a higher level of self-insurance for agent V, if and only if $k'(U(w_1 - c(y))) < k'(U(w_2 - l(y)))$. This is equivalent to having $w_1 - c(y_U) > w_2 - l(y_U)$. $\square$

Proof of Proposition 2: By an affine transformation, we can pick V such that $V(w_2 - l) = U(w_2 - l)$ and $V(w_2) = U(w_2)$ without changing preferences. Now we evaluate the foc of agent V at the optimal level of self-protection, x_U, of agent U to obtain

$$-c'(x_U)k'(U(w_1 - c(x_U)))U'(w_1 - c(x_U)) + \delta p'(x_U)\Big(U(w_2 - l) - U(w_2)\Big).$$

By substituting in the foc of U this becomes

$$-c'(x_U)U'(w_1 - c(x_U))\Big(k'(U(w_1 - c(x_U))) - 1\Big),$$

which is positive if and only if $k'(U(w_1 - c(x_U)))$ is below 1. $\square$

Proof of Proposition 3: The two-dimensional version of the implicit function theorem yields

$$\begin{pmatrix} \frac{dy}{d\alpha} \\ \frac{ds}{d\alpha} \end{pmatrix} = -\frac{1}{D}\begin{pmatrix} EU_{ss} & -EU_{ys} \\ -EU_{ys} & EU_{yy} \end{pmatrix} \cdot \begin{pmatrix} EU_{y\alpha} \\ EU_{s\alpha} \end{pmatrix},$$

where D denotes the determinant of the Hessian of EU. All the signs can be unambiguously determined except the signs of the entries of the last vector. Now

$$
\begin{aligned}
EU_{y\alpha} &= 2\Big(c'(y)(w_1 - c(y) - s) + \delta p l'(y)(w_2 - l(y) + s(1 + r))\Big) \\
&= \frac{1}{\alpha}(c'(y) + \delta p l'(y)),
\end{aligned}
$$

by substituting in the foc with respect to y. Exploiting the foc again this is positive if and only if

$$
U'(w_1 - c(y) - s) < U'(w_2 - l(y) + s(1 + r)),
$$

leading to the condition that compares consumption levels. Furthermore,

$$
\begin{aligned}
EU_{s\alpha} &= 2\Big((w_1 - c(y) - s) - \delta(1 + r)(p(w_2 - l(y) + s(1 + r)) + (1 - p)(w_2 + s(1 + r)))\Big) \\
&= \frac{1}{\alpha}(1 - \delta(1 + r))
\end{aligned}
$$

by substituting in the foc for saving. This is negative if and only if the second condition holds. The proposition is obtained by resolving the matrix operation and using the sign conditions.

For self-protection, we obtain

$$
\begin{aligned}
EU_{x\alpha} &= 2c'(w_1 - c(x) - s) + \delta p'((w_2 + s(1 + r))^2 - (w_2 - l + s(1 + r))^2) \\
&= \frac{1}{\alpha}\Big(c' + \delta p' l\Big)
\end{aligned}
$$

by substituting in the foc. Exploiting the foc again this is positive if and only if

$$
U'(w_1 - c(x) - s) < \frac{U(w_2 + s(1 + r)) - U(w_2 - l - s(1 + r))}{l},
$$

which yields the consumption condition after some algebra. The procedure with respect to saving is identical to the above.

$\square$

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