DEUTSCHES ELEKTRONEN – SYNCHROTRON

DESY 91-119
MZ-TH/91-35
October 1991

Heavy Baryons in the Heavy Quark Effective Theory

J. G. Körner
Inst. für Physik, Johannes Gutenberg-Universität, Mainz
and
Deutsches Elektronen-Synchrotron DESY, Hamburg

G. Thompson
Inst. für Physik, Johannes Gutenberg-Universität, Mainz

ISSN 0418-9833

NOTKESTRASSE 85 · D-2000 HAMBURG 52

DESY
Bibliothek
Notkestrasse 85
D–2000 Hamburg 52
Germany

DESY 91-119
MZ-TH/91-35
October 1991

ISSN 0418-9833

HEAVY BARYONS IN THE HEAVY QUARK EFFECTIVE THEORY[a]

Jürgen G. Körner[b,c] and G. Thompson[d]
Institut für Physik, Johannes Gutenberg-Universität,
Staudingerweg 7, Postfach 3980, D-6500 Mainz, Germany

Abstract: We give a mini-review of recent results on current-induced transitions between heavy baryons (and between heavy and light baryons) in the light of the new spin and flavour symmetries of the Heavy Quark Effective Theory (HQET). We discuss the structure of the 1/m corrections to the heavy mass limit and outline a diagrammatic proof that there are no O(1/m) corrections to the Voloshin-Shifman normalization condition at zero recoil.

a) Invited talk given by J.G. Körner at "4th International Symposium at Heavy Flavour Physics", 25.-29. June, 1991, Orsay (France)
b) also at Deutsches Elektronen-Synchroton, DESY Hamburg
c) Supported in part by the BMFT, FRG under contract 06MZ760
d) Supported by the BMFT, FRG under contract 06MZ760

The properties of QCD in the infinite quark mass limit are currently the subject of intense study[1-6]. In the infinite quark mass limit, the heavy quark sector of QCD becomes independent of quark masses and the effective Lagrangian of the heavy quark effective theory (HQET) exhibits new spin and flavour symmetries[1-6]. Semi-leptonic (s.l.) transitions between heavy baryons (and between heavy and light baryons) are quite important in this context[7-15]. Their study provides information on heavy quark systems complementary to that drawn from the study of s.l. heavy meson transitions which were studied first.

It is quite intriguing that many of the ideas of the HQET date back as far as 1937, then of course in the context of QED[16]. In the Block-Nordsieck approach to soft photon radiation it was the electron that was "infinitely" heavy (on the scale of the soft photons) so it could be treated as a classical source of radiation. In fact the Block-Nordsieck model was already formulated in terms of an effective theory with the electron degrees of freedom removed from the field theory (see also Ref.17). What used to be called the eikonal approximation is now referred to as on-mass-shell propagation of heavy quarks with no velocity loss ("velocity superselection rule"). The quantum mechanical Foldy-Wouthuysen transformation has turned into the field theoretical 1/m expansion. Also, the new spin-flavour symmetries are easily appreciated in the context of soft photon radiation from the e, μ and τ: the soft photon spectrum emitted from a e, μ and a τ looks the same regardless of the leptons' spin orientation as long as they move with the same velocity. It is only the velocity dependent Lorentz boost that matters.

To begin with we discuss the field theoretic equivalent of the Foldy-Wouthuysen transformation. We shall employ the path integral language to obtain on all orders 1/m expansion of the fermionic action. The power of the path integral approach[18] may be judged by the fact that the all orders proof of Ref.18 takes up only ≅ 20% of the page size of the O(1/m) proof in Ref.19 where a Hamiltonian approach was used. The heavy mass expansion may be obtained from QCD on performing a Foldy-Wouthuysen transformation on the heavy quark field Ψ_Q. The transformation (change of variable) is

$$\Psi_Q = \exp\left[i(\not{D} - \not{v}\, v\cdot D)/2m_o\right]\Psi'_Q$$
$$\bar{\Psi}_Q = \bar{\Psi}'_Q \exp\left[-i(\overleftarrow{\not{D}} - \not{v}\, v\cdot\overleftarrow{D})/2m_o\right] \qquad (1)$$

where $v^2 = 1$ is the heavy quark velocity and $\int fDg = -\int f\overleftarrow{D}g$. The heavy quark action

$$S = \int \bar{\Psi}_Q(i\not{D} - m_o)\Psi_Q \qquad (2)$$

becomes

$$S = \int \bar{\Psi}'_Q\,(i\not{v}\, v\cdot D - m_o)\,\Psi'_Q + \sum_{i=1}^{\infty} \bar{\Psi}'_Q\, O_i\, \Psi'_Q (2m_o)^{-i} \qquad (3)$$

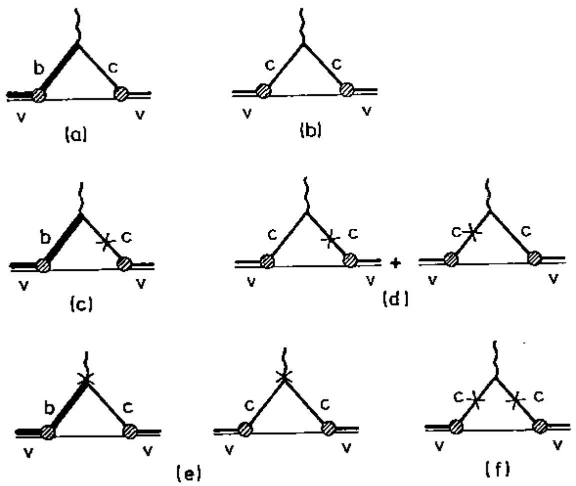

Fig.1: O(1) ((a) and (b)), $O(1/m_c)$ ((c),(d) and (e)) and $O(1/m_c^2)$ (f) contributions to mesonic $b \to c$ and $c \to c$ transitions at zero recoil (v=v') in the $v \cdot A = 0$ gauge.

giving a form of the action which makes explicit the mass perturbations. The operators O_i are easily determined from the transformation, for example

$$O_1 = -\frac{1}{2} D^2 + \frac{1}{2} (v \cdot D)^2 + \frac{1}{4} g \sigma_{\mu\nu} F^{\mu\nu} \quad . \tag{4}$$

The others are obtained on using the Baker-Hausdorff formula[18]. We are working within the path integral framework, so one thing that must be checked is whether the path integral measure is invariant under the change of variable. Using dimensional regularization this is shown to be the case in Ref.18, however, other regularization schemes may require the addition of local counterterms.

In other formulations of the effective theory, where the operators O_i are determined on dimensional grounds, the relative co-efficients of the operators that make up the O_i are determined by "matching conditions". These matching conditions arise on a comparision of the effective theory with the true theory. In our approach no matching is required, it is built in, as we derive the operators directly from the QCD Lagrangian. Various issues, such as the gauge dependence and renormalizability of the effective theory compared to the complete theory are addressed in Ref.18.

Voloshin and Shifman had shown that there is a zero recoil normalization condition for bottom meson to charm meson transitions to leading order of $1/m$[20]. The relevant flavour changing transition is shown in Fig.1a and corresponds to the matrix element $\langle(c\bar{q})|\bar{\Psi}_c J_\mu \Psi_b|(b\bar{q})\rangle$. At zero recoil we have $v_b = v_c = v$. So the heavy c and b quark effective actions have the same form at O(1),

with the same velocity. If we pick the gauge $v \cdot A = 0$ for the gluons we see that the heavy quarks decouple from the gluons to this order. This is the reason that there are no gluon lines in Fig.1a. The charm or bottom charge normalization diagram Fig.1b corresponds to $\langle(c\bar{q})|\int d^3x \bar{\Psi}_c(x)\, \gamma_0 \Psi_c(x)|(c\bar{q})\rangle = 1$ at zero recoil. As only the time component of the vector current contributes to $b \to c$ transitions at zero recoil, the $b \to c$ transition Fig.1a is also normalized at zero recoil.

The normalization survives $O(1/m_c)$ corrections[18,21,23,24]. There are two sources for $O(1/m_c)$ corrections. The first is represented in Fig.1e and comes from the transformation of the current (for $v = (1,\vec{0})$)

$$\bar{\Psi}_b J_\mu \Psi_c = \bar{\Psi}'_b J_\mu \Psi'_c + \bar{\Psi}'_b J_\mu (D_i \gamma_i / 2m_c)\, \Psi'_c + O(1/m_c^2) \quad . \tag{5}$$

The vector contribution of (5) vanishes as the $(b\bar{q})$ and $(c\bar{q})$ wave functions have the projectors $(1+\gamma_0)$ (coming from the propagator $(1+\not{v})/(v^2-1)$ with the pole removed by going on-shell and using LSZ), and $(1+\gamma_0)\gamma_0\gamma_i(1+\gamma_0)$ is zero. Notice that for the charm charge we have $\int \bar{\Psi}_c(x)\gamma_0\Psi_c(x) = \int \bar{\Psi}'_c(x)\gamma_0\Psi'_c(x)$, so there are no corrections either. The second source of $1/m_c$-corrections comes from the fact that there can be an insertion of O_1 in the charm propagator. For $(c\bar{q}) \to (c\bar{q})$ this is shown in Fig.1d and can be seen to be twice the $(b\bar{q}) \to (c\bar{q})$ contribution Fig.1c. But as this $O(1/m_c)$ insertion must vanish for $(c\bar{q}) \to (c\bar{q})$ from charge normalization it must also vanish for $(b\bar{q}) \to (c\bar{q})$. At $O((1/m_c)^2)$ the diagonal and nondiagonal charm propagator insertions are no longer proportional to one another as the sample diagram 1f shows.

As a simple example of how the O(1) normalization condition survives at $O(1/m_c)$ in the case of baryons we consider the $\Lambda_b \to \Lambda_c$ transition[22,25,26]. At lowest order one has $\langle\Lambda_c|J_\mu|\Lambda_b\rangle = \bar{u}\gamma_\mu(1-\gamma_5)uF$[7,9,12], where F is normalized to one at zero recoil. The general matrix element reads

$$\langle\Lambda_c|J_\mu|\Lambda_b) = \bar{u}[\gamma_\mu(F_1^V + F_1^A\gamma_5) + v_\mu(F_2^V + F_2^A\gamma_5) + v'_\mu(F_3^V + F_3^A\gamma_5)]u \quad . \tag{6}$$

At zero recoil v=v', $\bar{u}\gamma_\mu u = \bar{u}uv_\mu$ and $\bar{u}\gamma_5 u = 0$, so we will only be able to determine corrections to the pseudo-threshold s-wave amplitude combinations $F_1^V + F_2^V + F_3^V$ and F_1^A. First we establish that there are no current corrections. The corrections would have the form $\bar{u}\gamma_\mu(1-\gamma_5)\gamma_i u$ which would be contracted with v_i from the wave function. But v_i is zero at no recoil. As in the mesonic case the vector current propagator insertions in $\Lambda_b \to \Lambda_c$ and $\Lambda_c \to \Lambda_c$ are proportional to one another and thus vanish. Thus one finds $F_1^V + F_2^V + F_3^V = F + O(1/m_c^2)$ at zero recoil. A similar exercise for the axial vector piece F_1^A shows that $F_1^A = F + O(1/m_c^2)$ at zero recoil. The cases $\Sigma_b \to \Sigma_c$ and $\Sigma_b \to \Sigma_c^*$ require a bit more spinology. They are discussed in Ref.26.

Various equivalent approaches have been used to derive the HQET baryonic form factor structure away from zero recoil. These are the i) algebraic approach using spin commutation relations[9)], ii) group theoretic approach using tensor techniques[10,11)], iii) Bethe-Salpeter approach[12)], and, iv) the helicity matching approach[7,8,27)].

Technically the group theoretic approach of Refs.10,11 is the simplest. The spin wave functions of the Λ-type and Σ-type $J^P = \frac{1}{2}^+$ ground state baryons are represented by u and by $(\gamma_\alpha+v_\alpha)\gamma_5 u$, respectively, and the ground state $J^P = \frac{3}{2}^+$ baryon is represented by its Rarita-Schwinger spinor u_α. The HQET form factor structure can then immediately be written down by considering independent contractions of Lorentz indices. One has

$$\langle\Lambda_c|J_\mu|\Lambda_b\rangle = \bar{u}F_\Lambda\ \gamma_\mu(1-\gamma_5)u \tag{7a}$$

$$\begin{aligned}\langle\Sigma_c,\Sigma_c{}^*|j_\mu|\Sigma_b\rangle = (\bar{u}\gamma_5(v'^{\alpha}+\gamma^\alpha) + \bar{u}^\alpha)(-4F_1\ g_{\alpha\beta} \\ + F_2\ v_\alpha v'_\beta)\ \gamma_\mu(1-\gamma_5)(v^\beta+\gamma^\beta)\ \gamma_5 u\end{aligned} \tag{7b}$$

Note that one may not use γ-matrices for the contraction as they would bring in spin interactions on the heavy quark legs which are absent in the static approximation. One thus has three universal form factors with the normalization conditions $F_\Lambda = F_1 = 1$ at zero recoil. Ref.15 contains a derivation of the form factor structure (7) using Bethe-Salpeter amplitudes for the heavy baryon bound state systems. The form factors are thereby related to wave function overlap integrals which are computable for any given model of the bound state wave functions. Further assumptions on the spin structure of the bound states reduces the number of independent form factors in (7) from three to two and three to one[12].

The heavy to light form factor structure may be obtained from (7) by allowing for spin interactions of the light active quark[11,12,15)]. This amounts to the replacement $F_\Lambda \to F'_\Lambda + \not{v}F''_\Lambda$, $F_1 \to F'_1 + \not{v}F''_1$ and $F_2 \to F'_2 + \not{v}F''_2$. Now there is no normalization condition for the form factors. Also the $\Sigma_b \to \Sigma_u$ and $\Sigma_b \to \Sigma_u^*$ form factors are not related. Phenomenological consequences of the heavy to heavy (including $1/m_c$ corrections) and the heavy to light baryonic form factor structure are presently being worked out[28].

References

1. E. Eichten, Nucl.Phys. B (Proc.Suppl.) 4(1988)70; G.P. Lepage and B. Thacker, Nucl.Phys. B (Proc.Suppl.) 4(1988)119; E. Eichten and B. Hill, Phys.Lett. 234B(1990)511; B. Grinstein, Nucl.Phys. B339(1990)253
2. N. Isgur and M.B. Wise, Phys.Lett. 237B(1990)527
3. H. Georgi, Phys.Lett. 240B(1990)447
4. J.D. Bjorken, invited talk given at "Les Recontre de la Valle d'Aoste La Thuile", Editions Frontieres, Gif-sur-Yvette (1990)583
5. F. Hussain, J.G. Körner, K. Schilcher, G. Thompson and Y.L. Wu, Phys.Lett. 249B(1990)295

6. A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl.Phys. B343(1990)1
7. F. Hussain, J.G. Körner and R. Migneron, Phys.Lett. B248(1990)406
8. F. Hussain, J.G. Körner and G. Thompson, Ann.Phys. 206(1991)334
9. N. Isgur and M.B. Wise, Nucl.Phys. B348(1991)276
10. H. Georgi, Nucl.Phys. B348(1991)293
11. T. Mannel, W. Roberts and Z. Ryzak, Nucl.Phys. B355(1991)38
12. F. Hussain, J.G. Körner, M. Krämer and G. Thompson, Z.Phys. C51(1991)321
13. F. Hussain and J.G. Körner, Z. Phys. C51(1991)607
14. J.G. Körner and H.W. Siebert, Ann.Rev.Nucl.Part.Sci. 41(1991)511
15. F. Hussain, D. Liu, M. Krämer, J.G. Körner and S. Tawfiq, Nucl.Phys. B (in press)
16. F. Bloch and A. Nordsieck, Phys.Rev. 52(1937)54
17. N. Bogoliubov and D.V. Shirkov, "Introduction to the Theory of Quantized Fields", Interscience Publishers Inc. (New York, 1959)
18. J.G. Körner and G. Thompson, Phys.Lett. 264b(1991)185
19. F. Feinberg, Phys.Rev. D17(1978)2659
20. M.B. Voloshin and M.A. Shifman, Yad.Fis. 45(1987)463 (Sov.J.Nucl.Phys. 45(1987)292)
21. M.E. Luke, Phys.Lett. 252B(1990)447
22. C.G. Boyd and D.E. Brahm, Phys.Lett. 254B(1991)468
23. C.G. Boyd and D.E. Brahm, Phys.Lett. 257B(1991)393
24. R.F. Lebed and M. Suzuki, Phys.Rev. D44(1991)829
25. H. Georgi, B. Grinstein and M.B. Wise, Phys.Lett. 252B(1990)456
26. S. Balk, J.G. Körner and G. Thompson, to be published
27. J.G. Körner, Nucl.Phys. B21(1991)366 (Proc.Suppl.)
28. J.G. Körner and M. Krämer, Mainz preprint, MZ-TH/91-05 and in preparation