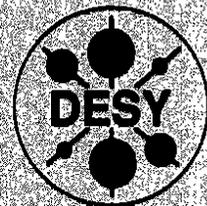


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Third Threshold in the Weak Interactions?

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Third threshold in the weak interactions ?

on the occasion of M. Veltman's 60th birthday

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1 Introduction

As the experiments at LEP indicate the Standard Model of the weak interactions is in agreement with all data. What has not been tested so far is the existence of the vector boson self couplings. While the three vector boson coupling will be studied at LEP200, the four vector boson coupling can only be studied at future accelerators like the LHC, SSC or the NLC. Within the Standard Model the vector-boson self couplings are completely determined by the gauge structure of the theory and deviations of its values are only possible through radiative corrections, which tend to be small. An exception to this situation can in principle arise if the Higgs particle is very heavy, because then strong couplings are present. Since the Higgs particle has not been found so far and since problems like the generation of a large cosmological constant are associated with the Higgs sector it motivates us to consider the Standard Model without the Higgs particle. This model is then non-renormalizable and divergences arise. An alternative way is to study the situation where the Higgs mass becomes infinite, which at least at the tree level is equivalent to having no Higgs at all. This limit was first studied by Veltman [1]. He found that radiative corrections indeed blow up but only logarithmically. As a consequence the vector boson couplings stay small unless one makes the Higgs mass unreasonably large. This observation is now known as Veltman's screening theorem. In order to generate large self couplings of the vector bosons one therefore has to find a way to avoid this theorem. In section 2 we discuss in some detail how Veltman's theorem arises and we try to put limits on the generality of the theorem. A careful study shows that the theorem can be circumvented if extra strong interactions are present in the Higgs sector. This conclusion was also reached in a previous paper [2], where an explicit model was constructed. More precisely in that paper it was shown that strong interactions among the vector bosons can be generated if there is a hierarchy of strong interactions in the Higgs sector. This feature is likely to be generally valid and we therefore call it Hill's theorem. The model is discussed in section 3. In section 4 we then study the dynamics of Hill's model in more detail and we find that resonances exist in the four vector boson scattering following an argument presented in [3].

2 Veltman's screening theorem

It is known that the weak interactions are mediated by massive vector bosons. In the Standard Model the mass of the vector bosons arises through the mechanism of spontaneous symmetry breaking. A doublet of bosonic fields is introduced, which receives a vacuum expectation value, because the minimum of the potential is not located at zero. As a consequence the vector bosons receive a mass, but there is also a particle left, the so called Higgs boson. As there is no experimental evidence of the Higgs boson so far it is natural to study the Standard Model without the Higgs boson. At the tree level this is equivalent to studying the Standard Model with the Higgs boson mass becoming very large.

We study the possibility of the existence of non-standard four vector boson couplings induced by strongly interacting Higgs particles. We show that such interactions can exist if there is a hierarchy of strong interactions and masses in the Higgs sector, thereby circumventing Veltman's screening theorem. We show that resonances can be formed in this case.

Abstract

At the one-loop level one can take two approaches. In the first approach one takes the Standard Model and calculates the Higgs mass dependence of the radiative corrections to vector boson interactions. In the limit of large Higgs mass the Higgs sector becomes strongly interacting and therefore the decoupling theorem [4] is not valid. As a consequence corrections growing with the Higgs mass are present. It was found in [1] that these corrections grow only logarithmically with the Higgs mass, which make them very hard to see experimentally. The Higgs effects are screened. A precise study of the way these infinities arise was done in a non-gauged model in [5].

In the second approach one looks at the Standard Model without the Higgs boson altogether. This model is nonrenormalizable and therefore infinities arise. The possible infinities were classified in [6] for the SU(2) theory and in [7] for the full SU(2) \times U(1) model. Since the theory is nonrenormalizable a precise definition of the infinities is needed in this case. Dimensional regularization [8] was used and it was found that infinities arise in the form of poles in $n - 4$, where n is the dimension of space-time.

A priori it is not clear that these approaches should give the same result. In order to see how this comes about we describe here both the Standard Model and the model without the Higgs boson in a gauge-invariant way. The Standard Model is a gauged linear σ -model

$$\mathcal{L} = -\frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - \frac{\lambda}{8}(\Phi^\dagger\Phi - f^2)^2 \quad (2.1)$$

$$\Phi = (\sigma + i\vec{\tau}\cdot\vec{\phi}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.2)$$

At the tree level, the Higgs particle can be removed from (2.1) by taking the limit $\lambda \rightarrow \infty$ or, equivalently, $m_H^2 \rightarrow \infty$. The Standard Model then turns into a gauged nonlinear σ -model

$$\mathcal{L} = \frac{f^2}{4}T_\tau(D_\mu U)^\dagger(D^\mu U) \quad (2.3)$$

$$U = \sqrt{1 - \vec{\pi}^2} + i\vec{\tau}\cdot\vec{\pi} \quad (2.4)$$

$$\vec{\pi} = \frac{\vec{\phi}}{f}$$

which is equivalent to massive Yang-Mills theory. It can be seen on the formal level that the Standard Model reduces to (2.3) in the limit $\lambda \rightarrow \infty$ by noticing that the potential acts like a constraint in this case. Explicit calculations show that at the one-loop level the effects of a heavy Higgs particle are given by terms involving higher order derivative terms of U . In the unitary gauge these terms correspond to extra interactions beyond the ordinary gauge couplings. If we limit ourselves to the SU(2) model, which we do here and in the following for simplicity, the extra effects are summarized by the following effective Lagrangian

$$\mathcal{L}_{eff} = \alpha_1 T_\tau(V_\mu V^\mu)T_\tau(V_\nu V^\nu) + \alpha_2 T_\tau(V_\mu V^\nu)T_\tau(V_\nu V^\mu) + g\alpha_3 T_\tau(F_{\mu\nu}[V^\mu, V^\nu]) \quad (2.5)$$

where

$$V_\mu = (D_\mu U)U^\dagger \quad (2.6)$$

and

$$F_{\mu\nu} = (\partial_\mu - \frac{ig}{2}\vec{W}_\mu\cdot\vec{\tau})\frac{\vec{W}_\nu\cdot\vec{\tau}}{2i} - (\mu \leftrightarrow \nu) \quad (2.7)$$

Explicit calculation in the linear model gives

$$\alpha_1 = \frac{1}{384\pi^2} \ln(m_H^2/M_W^2) + \mathcal{O}(1), \quad (2.8)$$

$$\alpha_2 = \frac{1}{192\pi^2} \ln(m_H^2/M_W^2) + \mathcal{O}(1),$$

$$\alpha_3 = -\frac{1}{384\pi^2} \ln(m_H^2/M_W^2) + \mathcal{O}(1).$$

In the nonlinear model one gets the same coefficients with the following replacement, for the moment not worrying about the $\mathcal{O}(1)$ terms,

$$\text{linear model: } \ln(m_H^2/M_W^2) \longleftrightarrow \text{nonlinear model: } -\frac{2}{n-4} \quad (2.9)$$

The possibility of calculating the divergences directly in the nonlinear model seems to indicate that the coefficients are universal. We wish to study whether this is indeed the case. While in the calculation of α_2 and α_3 only irreducible graphs contribute it is not too surprising that the relation (2.9) is valid. For the coefficient α_1 also reducible graphs involving Higgs exchange (fig.1) appear and a delicate cancellation between the graphs and the renormalization counterterm for the Higgs mass is present in this case. Therefore it is natural to consider extensions involving extra strong interactions in the Higgs sector, which may change the coefficient α_1 strongly. This will be done in the next chapter. Since we will later be considering the possibility of resonances in the longitudinal vector boson scattering, we remark here that only the coefficients α_1 and α_2 contribute in this case. In the end the possibility of resonance formation in the isospin $I = 1$ channel will be determined by the following combination of parameters

$$\beta = 128\pi^2(\alpha_2 - 2\alpha_1).$$

Note that β is independent of $\ln(m_H^2)$.

3 Hill's theorem; avoiding the screening theorem

In this chapter we study the possibility of generating strong interactions in the gauge sector via radiative corrections in the Higgs sector. As argued in the previous sector

to get large effects one has to find a way to change the Higgs propagator in order to avoid the cancellations of the Standard Model. A simple way to do this is to add a strongly interacting singlet field to the Higgs sector [2]. We call this field the X field. The corresponding Lagrangian is

$$\mathcal{L} = -\frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) - \frac{1}{2}(\partial_\mu X)^2 - \frac{\lambda_1}{8}(\Phi^\dagger \Phi - f_1^2)^2 - \frac{\lambda_2}{8}(2f_2 X - \Phi^\dagger \Phi)^2 + \mathcal{L}_{gauge} \quad (3.1)$$

Even though no X self couplings are present this model is renormalizable, because the X field is a singlet under the gauge group. Therefore no divergent X^4 interactions can be induced via gauge boson loops. Explicit power counting of divergences bears out this argument [2].

Within this model we are interested in the limit

$$\lambda_2 \gg \lambda_1 > 0, \quad (3.2)$$

because here there is a hierarchy of coupling strengths and one can expect effects to be present even at lower energies. In this limit the mass eigen values in the boson sector become

$$m_+^2 \approx \lambda_2(f_1^2 + f_2^2), \quad (3.3)$$

$$m_-^2 \approx \frac{\lambda_1 f_1^2 f_2^2}{f_1^2 + f_2^2}. \quad (3.4)$$

We note that the mass of the Higgs and the mass of the X are each some combination of m_+ and m_- . In the absence of the X particle, which corresponds to $\lambda_2 = 0$, we have $m_+ = 0$ and $m_- = m_H$.

It is clear from these formulae that there is in this limit also a hierarchy of mass scales in the higgs boson sector $m_+ \gg m_- > m_W$, so that one can speak of a third threshold in the weak interactions. For future reference we define here

$$\alpha = \frac{f_1^2}{f_1^2 + f_2^2}. \quad (3.5)$$

In this limit the one loop corrections to the α_i 's, due to the contribution of the X particle, become

$$\alpha_1 = \frac{1}{384\pi^2} \{ \ln(m_-^2/m_W^2) - \frac{1}{2}\alpha(6+\alpha) \ln(m_+^2/m_-^2) + 3\delta_H \} + \mathcal{O}(1), \quad (3.6)$$

$$\alpha_2 = \frac{1}{192\pi^2} \{ \ln(m_-^2/m_W^2) + \alpha^2 \ln(m_+^2/m_-^2) \} + \mathcal{O}(1), \quad (3.7)$$

$$\alpha_3 = -\frac{1}{384\pi^2} \{ \ln(m_-^2/m_W^2) + \alpha \ln(m_+^2/m_-^2) \} + \mathcal{O}(1). \quad (3.8)$$

where we defined

$$\delta_H = \frac{\lambda_2}{\lambda_1} \left\{ -\frac{9}{2}\alpha^2 \left(2 - \frac{\pi}{\sqrt{3}} \right) - \frac{1}{2}(4 - 2\alpha + \alpha^2) \ln(1 - \alpha) \right. \\ \left. + \alpha^2 \left(1 - \sqrt{\frac{3+\alpha}{1-\alpha}} \operatorname{arctg} \sqrt{\frac{1-\alpha}{3+\alpha}} \right) + \alpha^2 \ln \alpha - \frac{1}{6}(1-\alpha)(3+17\alpha) \right. \\ \left. + 4\pi \sqrt{\frac{\lambda_2}{\lambda_1}} \alpha^{3/2} (1-\alpha)^{3/2} \right\} + \mathcal{O}(\lambda_1). \quad (3.9)$$

We see therefore that deviations from the Standard Model are present. For the coefficients α_2 and α_3 these deviations are only logarithmic, but for α_1 they grow with the ratio of the coupling constants λ_2/λ_1 and therefore arbitrarily large effects can be expected. A further simplification appears when one takes $\alpha \rightarrow 0$, i.e. $f_2 \gg f_1$. In that case the whole effect of the extra interactions beyond the Standard Model can be summarized by the parameter

$$\beta = 128\pi^2(\alpha_2 - 2\alpha_1) = \lambda_2/\lambda_1. \quad (3.10)$$

Therefore a large β can appear if there is a hierarchy of coupling strengths, thereby violating the screening theorem. A similar effect appears in a related model [9] where a slightly different scalar is introduced, the so called U particle. Therefore we claim that the following statement holds: "The screening theorem can be avoided if a hierarchy of strong interactions exists in the Higgs sector". We call this Hill's theorem.

In the next chapter we will study the consequence of Hill's theorem on the formation of resonances for longitudinally polarized vector boson scattering.

4 Resonances in vector boson scattering

Let us consider the amplitude for longitudinally polarized vector boson scattering in the large Higgs mass limit, where the interacting vector bosons have an energy much larger than their mass. We thus consider the energy region

$$m_W \ll \sqrt{s} \ll m_{\text{Higgs}}.$$

The tree amplitude grows like the energy squared and the unitarity limit is reached around 1 TeV. Thus if no Higgs has been found below 1 TeV, new physics will have to show up in the TeV region. This new physics could manifest itself for example through the occurrence of resonances in the different isospin channels for $W_L W_L$ scattering. In ref.[3] it was shown that the occurrence of a resonance in the isospin I=1 channel is sensitive to the details of the Higgs sector. As an example of such model dependence we consider in the following the effect that the X-particle may have.

In the energy region below 1 TeV we assume that physics is described by some effective Lagrangian, for example the Standard Model in the large Higgs mass limit.

Furthermore we assume that in this energy region the one loop correction, being of the order of 10 %, is a reasonable approximation. Using partial wave analysis, it is then possible to describe vector boson dynamics in the TeV region.

In general the amplitude for $W_L W_L$ scattering, including the one loop correction, may be written as

$$A(W_L^a W_L^b \rightarrow W_L^c W_L^d) = \frac{s}{f_1^2} - \frac{1}{96\pi^2 f_1^4} \cdot \{3s^2(\ln s - \beta_1) + t(t-u)(\ln t - \beta_2) + u(u-t)(\ln u - \beta_2)\}. \quad (4.1)$$

Here all model dependence is contained in the parameters β_1 and β_2 . Expressed in terms of α_1 and α_2 we have

$$\begin{aligned} \beta_1 &= 64\pi^2(4\alpha_1 + \alpha_2), \\ \beta_2 &= 192\pi^2\alpha_2. \end{aligned} \quad (4.2)$$

Different models give different values for β_1 and β_2 . The question is now how the occurrence of a resonance in the different isospin channels depends on these two parameters. Using partial wave analysis, it was derived in refs. [3,10] that in the isospin I=1 channel a resonance is located at

$$s = \frac{32\pi^2 v^2}{\beta - \frac{1}{9}}, \quad (4.3)$$

where \sqrt{s} is the center of mass energy and

$$\begin{aligned} \beta &= \beta_2 - \beta_1 \\ &= 128\pi^2(\alpha_2 - 2\alpha_1). \end{aligned} \quad (4.4)$$

For example, when $\beta = 5$, a resonance occurs around 2 TeV. For $\beta < 0$, no resonance appears at any energy. Similarly, the occurrence of a resonance in the I=0 channel for a particular value of \sqrt{s} depends on another linear combination of β_1 and β_2 .

Let us summarize what values we have for β according to various models.

(1) Standard Model in the large Higgs mass limit, linear σ model;

$$\begin{aligned} \beta_1 &= \ln(m_H^2) + \frac{4}{3} + 9 \left(\frac{\pi}{\sqrt{3}} - 2 \right), \\ \beta_2 &= \ln(m_H^2) - \frac{2}{3}, \\ \beta &= -2 - 9 \left(\frac{\pi}{\sqrt{3}} - 2 \right) = -0.32. \end{aligned} \quad (4.5)$$

We see that here no resonance can be formed in the I=1 channel at any energy.

(2) Standard Model in the large Higgs mass limit, non-linear σ model;

$$\beta_1 = \Delta + \frac{11}{6},$$

$$\begin{aligned} \beta_2 &= \Delta + \frac{13}{6}, \\ \beta &= \frac{1}{3}. \end{aligned} \quad (4.6)$$

In this case we find that, according to eq.(4.3) a resonance in the I=1 channel will occur around 9 TeV.

(3) Contribution of the U-particle [9]; to the Standard Model add a piece :

$$\mathcal{L}(U) = -\frac{1}{2}(\partial_\mu U)^2 - \frac{1}{2}m_U^2 U^2 - gg_U r m_W U^2 H - \frac{1}{4}g^2 g_U r^2 U^2 (H^2 + \phi^2) \quad (4.7)$$

with $r = m_H^2/4m_W^2$ and g_U is the parameter associated with the U-particle. Thus the U is coupled to the Higgs with a strength proportional to m_H^2 . In the limit $m_U = m_H \rightarrow \infty$, we obtain

$$\begin{aligned} \beta_1 &= g_U^2 \left(\frac{\pi}{\sqrt{3}} - 2 \right), \\ \beta_2 &= 0, \\ \beta &= -g_U^2 \left(\frac{\pi}{\sqrt{3}} - 2 \right). \end{aligned} \quad (4.8)$$

The contribution to β is always positive, and depending on the value of g_U , a resonance can occur at any value of s .

(4) Contribution of the X particle. Eq.(4.2) gives β_1 and β_2 in terms of α_1 and α_2 , which in the limit $\lambda_2 \gg \lambda_1 \gg 0$ are given by eqs. (3.6) and (3.7). According to eq.(4.4) we find

$$\beta = \alpha(2 + 3\alpha) \ln(m_+^2/m_-^2) - 2\delta_H. \quad (4.9)$$

In the limit $f_2 \gg f_1$ we have

$$\beta = \frac{\lambda_2}{\lambda_1}. \quad (4.10)$$

Just like in the case of the U-particle, β is completely arbitrary.

We can now make the following remarks. First of all the reason that the isospin I=1 channel is interesting is now obvious; β is independent of $\ln(m^2)$ and thus sensitive to the model considered. This is not the case for example for the I=0 channel, where one finds that the location of a resonance depends on $\ln(m_H^2)$. Next, from eqs.(4.5) and (4.6), if we consider the Standard Model in the large Higgs mass limit, we find that the result depends on which way the large Higgs mass limit is taken and therefore β , although a calculable number, is an arbitrary parameter. Indeed, through the U or the X particle, the value for β can be altered and can in principle be any positive or negative number. In order for a resonance to occur in the I=1 channel, β must be > 0 . Hill's Theorem states that a large and positive value for β is only possible if there are extra strong interactions present in the Higgs sector. For example for the case of the U particle, eq.(4.8) was derived given that $m_U = m_H \rightarrow \infty$, resulting in a positive contribution to

β . On the other hand, we find that $\beta < 0$, when the mass m_U is well below a given choice of m_H . For the case of the X particle, we have plotted in fig. 2 β as a function of λ_2/λ_1 for various values of α . We notice that for example for $\alpha \approx 1$, which corresponds to $f_2/f_1 < 1$ β is always negative when $\lambda_2 > \lambda_1$. In this case no resonance occurs in the I=1 channel.

In fig. 3 we have plotted the I=1 amplitude as a function of \sqrt{s} for various values of β . We observe that the resonance is located at lower energies for higher values of β . We can therefore put an upper limit on β . Up to ~ 100 -200 GeV no bound state of two vector bosons has been observed. Therefore $\beta_{\max} \sim 500$. This can be roughly translated into a ratio $m_+/m_- = 50$. Therefore if the second threshold is at 1 TeV the new physics corresponding to the strong interactions between singlet and doublet fields would appear below 50 TeV.

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Figure Captions

- Figure 1: Various Higgs reducible graphs contributing to the four vector boson vertex.
- Figure 2: β as a function of λ_2/λ_1 for various values of α .
- Figure 3: Absolute value of the I=1 partial wave amplitude as a function of \sqrt{s} for various values of α .

Fig. 2

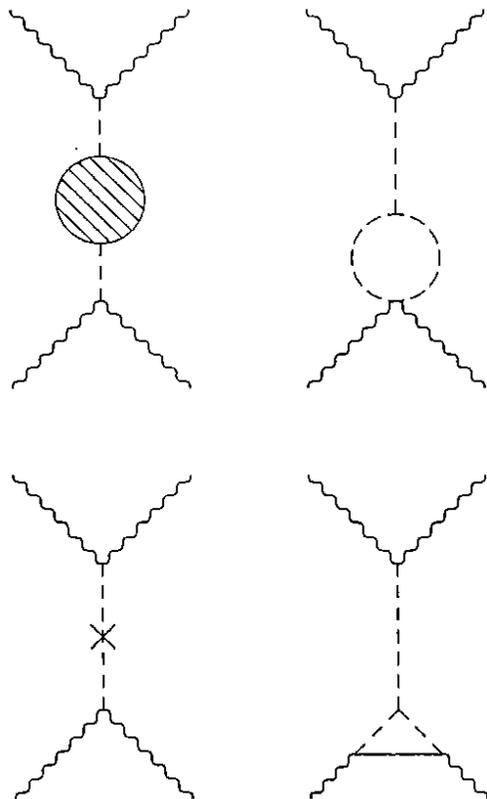
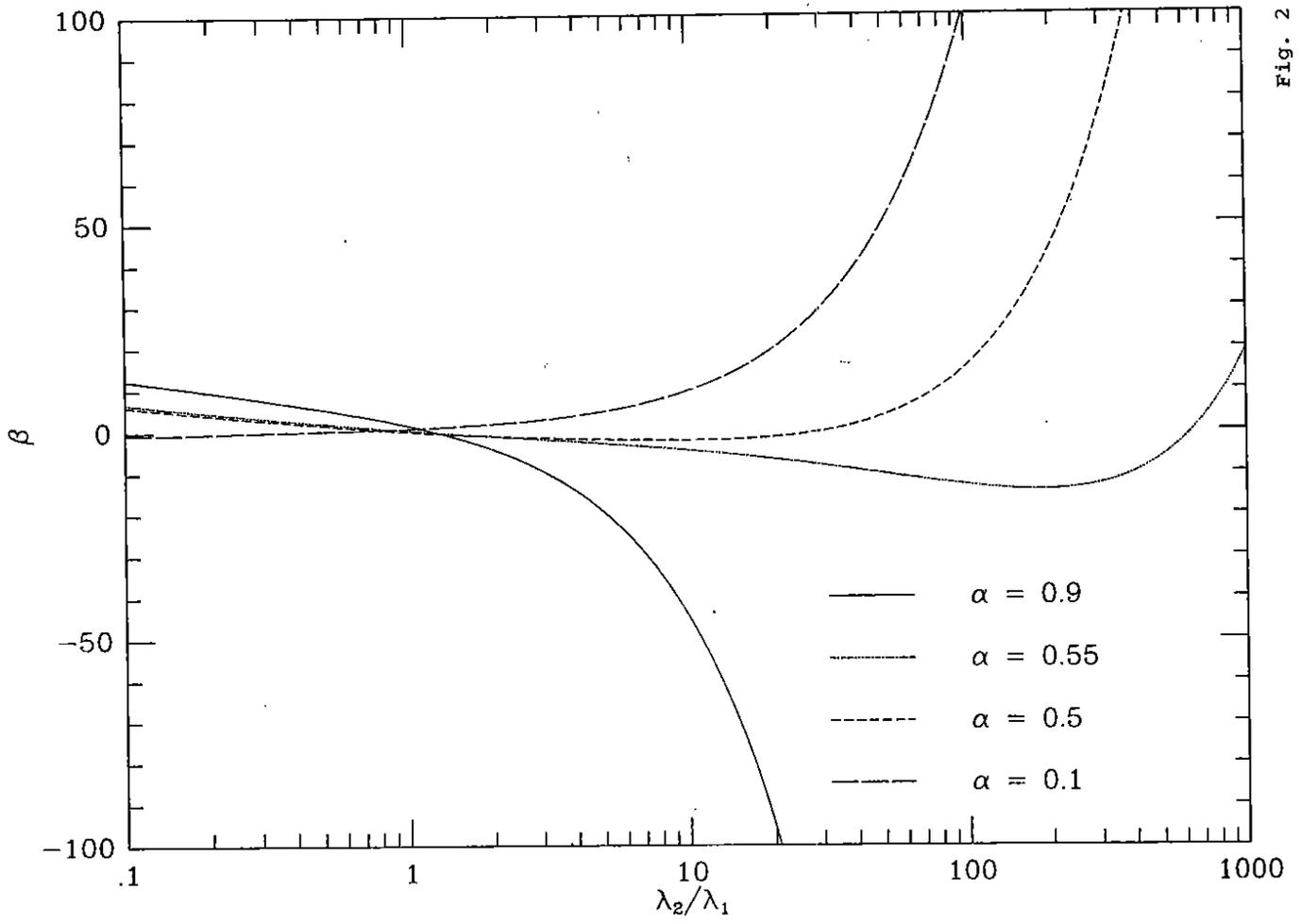


Fig. 1

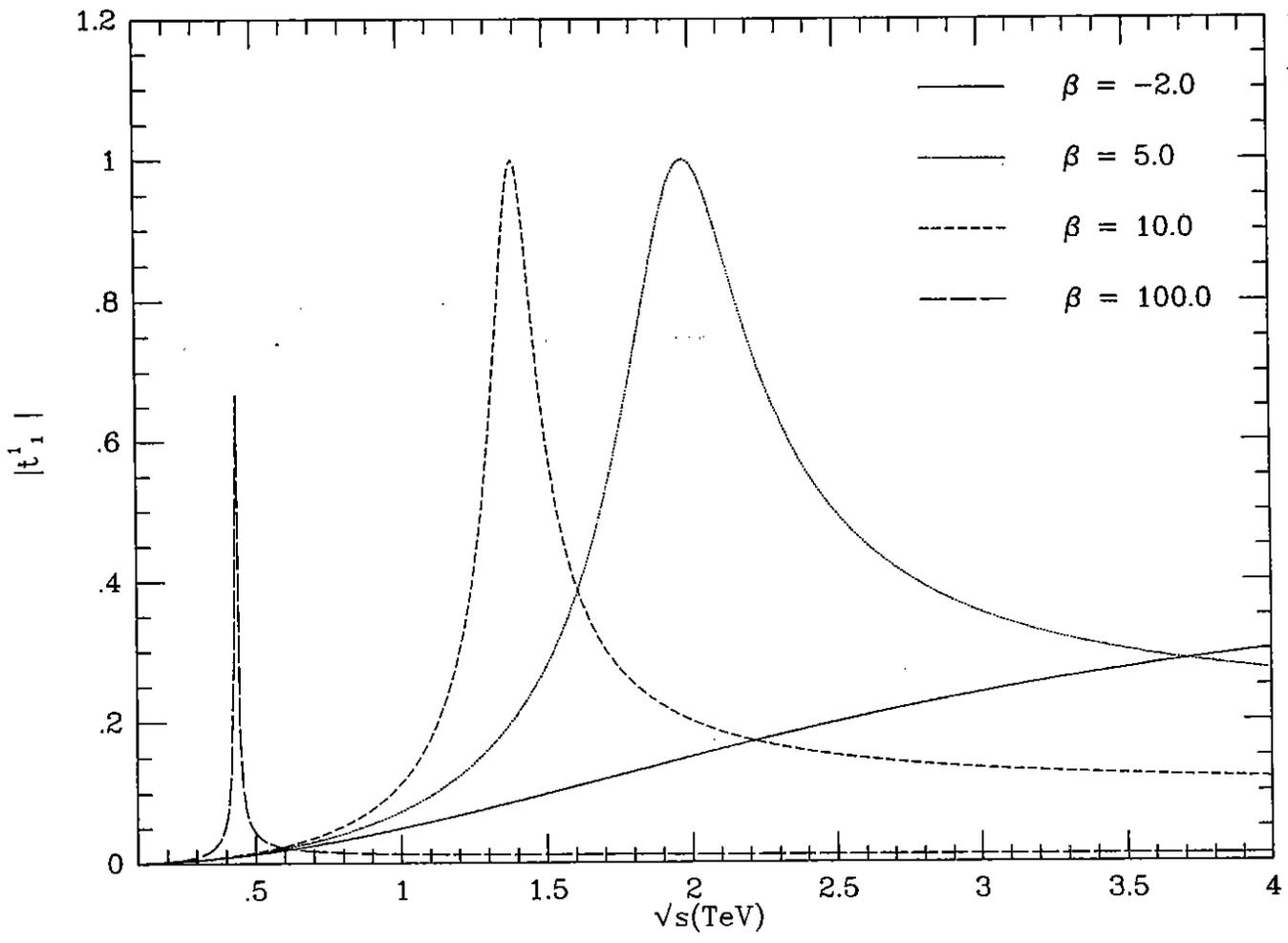


Fig. 3