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KMS STATES FOR DIRAC QUANTUM FIELD IN RINDLER SPACETIME

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Abstract: One considers the theory of the quantized Dirac field in Rindler spacetime. Working in the framework of Haag's local definiteness principle, one computes the KMS states, using the scaling limit procedure. The result is quite surprising that the so-called Hawking temperature from the scalar case is unacceptable.

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The massless spin -1/2 field is quantized in the Rindler wedge. The Rindler wedge is the region $x > |t|$ of the Minkowski spacetime and is covered by coordinates (τ, ρ, y, z) which are related to Minkowski coordinates by

$$x = \rho \cosh \tau \;, \quad t = \rho \sinh \tau \tag{1}$$

In these coordinates the Minkowski line element is

$$ds^2 = -\rho^2 d\tau^2 + d\rho^2 + dy^2 + dz^2 \tag{2}$$

from which it is seen that the manifold is static, admitting a Killing vector field $\partial/\partial\tau$ and the paths $\rho = $ const., (y,z)=const. are worldlines of constant proper acceleration ρ^{-1}.
One works here in the Majorana representation in which four component spinor ψ as well as the Dirac matrices, which satisfy the anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu} I \tag{3}$$

are real. The $\gamma^\mu (\mu = \tau, \rho, y, z)$ are related to the Minkowski space Dirac matrices $\gamma^a (a = 0,1,2,3)$ by a vierbein field

$$\gamma^\mu = L^\mu{}_a \gamma^a \tag{4}$$

Covariant differentiation of spinors is defined so that the γ^μ are covariantly constant and then

$$\psi_{;\mu} = \psi_{,\mu} - B_\mu \psi \;, \tag{5}$$

with

$$B_\mu = \frac{1}{4} L^{\nu}{}_{a;\mu} L_{\nu b} \gamma^a \gamma^b \, , \tag{6}$$

where

$$L^{\nu}{}_{a;\mu} = L^{\nu}{}_{a,\mu} + \Gamma^{\nu}{}_{\mu\lambda} L^{\lambda}{}_{a} \tag{7}$$

with $\Gamma^{\nu}{}_{\mu\lambda}$ the Christoffel symbols.
With the choice of vierbein

$$L^{\mu}{}_{a} = \mathrm{diag}\left(\rho^{-1}, 1, 1, 1\right) \tag{8}$$

the only nonvanishing component of the spinor connection is

$$B_\tau = \frac{1}{2}\gamma^0\gamma^1 \tag{9}$$

The adjoint spinor is defined to be

$$\bar{\Psi} = \Psi^{\sim}\gamma^0 \, , \qquad \Psi^{\sim} \equiv \Psi^{+} \tag{10}$$

We will solve the massless Dirac equation, following [1] ,

$$\gamma^\mu \Psi_{;\mu} = 0 \tag{11}$$

by means of the ansatz

$$\Psi = i\gamma^\nu u_{;\nu} \tag{12}$$

so that (11) becomes, taken into account (8):

$$-\frac{1}{\rho^2}\frac{\partial^2 u}{\partial \tau^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{1}{\rho^2}\gamma^0\gamma^1\frac{\partial u}{\partial \tau} - \frac{1}{4\rho^2}u + \frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{13}$$

If we look for particular solutions of the normal modes form

$$u = u(\rho|\omega, \vec{k}^{\perp})\, e^{-i\omega\tau + i\vec{k}^{\perp}\cdot\vec{x}^{\perp}} \tag{14}$$

then (13) becomes:

$$\left\{\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \left(\omega^2 - i\omega\gamma^0\gamma^1 - \frac{1}{4} - k^2\rho^2\right)\right\} u(\rho|\omega, \vec{k}^{\perp}) = 0 \tag{15}$$

This is a modified Bessel equation of order

$$\nu \equiv i\left(\omega^2 - i\omega\gamma^0\gamma^1 - \frac{1}{4}\right)^{1/2} = \begin{cases} \pm\left(i\omega + \frac{1}{2}\gamma^0\gamma^1\right) \\ \pm\left(\frac{1}{2} + i\omega\gamma^0\gamma^1\right) \end{cases} \tag{16}$$

with four independent solutions, but with the help of the identity

$$f(\gamma^0\gamma^1) = \frac{1}{2}\left(1+\gamma^0\gamma^1\right) f(1) + \frac{1}{2}\left(1-\gamma^0\gamma^1\right) f(-1) \tag{17}$$

it can be easy shown that only two of these four solutions are linearly independent as functions of ρ.
Requiring the normal modes for Ψ to be bounded as $\rho\to\infty$ then we choose for u

$$u = e^{-i\omega\tau + i\vec{k}^{\perp}\cdot\vec{x}^{\perp}} K_{i\omega + \frac{1}{2}\gamma^0\gamma^1}(k\rho)\,\chi \, , \tag{18}$$

where χ is a constant arbitrary spinor.

Now, substituting (18) into (12) and making use of the recurence relation

$$\left[\gamma^1 \frac{\partial}{\partial\varrho} - \frac{1}{\varrho}\gamma^0\left(i\omega + \tfrac{1}{2}\gamma^0\gamma^1\right)\right] K_{i\omega+\frac{1}{2}\gamma^0\gamma^1}(k\varrho) = -k K_{i\omega+\frac{1}{2}\gamma^0\gamma^1}(k\varrho)\gamma^1 \quad (19)$$

it is found that the elementary solutions of (11) admit the form

$$\Psi(x|\omega,\vec{k}^{\perp},\lambda) = N(\vec{k}^{\perp}) e^{-i\omega\tau + i\vec{k}^{\perp}\cdot\vec{x}^{\perp}} K_{i\omega+\frac{1}{2}\gamma^0\gamma^1}(k\varrho) P(\vec{k}^{\perp})\chi(\vec{k}^{\perp},\lambda), \quad (20)$$

where

$$P(\vec{k}^{\perp}) = \tfrac{1}{2}\left(1 + i k^{-1}\gamma^1 \vec{k}^{\perp}\cdot\vec{\gamma}\right) \quad (21)$$

Since $P(\vec{k}^{\perp})$ satisfies

$$P + P^{\sim} = 1, \quad PP^{\sim} = 0, \quad P^2 = P, \quad (22)$$

it follows that P is a projection matrix which projects out a 2-dimensional subspace of the spin space, and we can choose the $\chi(\vec{k}^{\perp},\lambda)$, $(\lambda = \pm 1)$ to form an orthonormal basis of the unit eigenvalue eigenspace of $P(\vec{k}^{\perp})$. Therefore

$$P(\vec{k}^{\perp})\chi(\vec{k}^{\perp},\lambda) = \chi(\vec{k}^{\perp},\lambda),$$

$$\chi^{+}(\vec{k}^{\perp},\lambda)\chi(\vec{k}^{\perp},\lambda') = \delta_{\lambda\lambda'}, \quad (23)$$

$$\sum_{\lambda=\pm1}\chi(\vec{k}^{\perp},\lambda)\chi^{+}(\vec{k}^{\perp},\lambda) = P(\vec{k}^{\perp}).$$

We observe that $P^{+}(\vec{k}^{\perp}) = P(\vec{k}^{\perp})$ and then

$$P^{\sim}(\vec{k})\chi^{*}(\vec{k}^{\perp},\lambda) = \chi^{*}(\vec{k}^{\perp},\lambda) \quad (24)$$

i.e. the complex-conjugate spinors form the corresponding basis for the unit eigenvalue eigenspace of $P^{\sim}(\vec{k}^{\perp})$.

Finally, the normalization factor $N(\vec{k}^{\perp})$ in (20) is determined from the canonically conserved inner product.

$$\int_{\Sigma} d\Sigma_{\mu}\,\overline{\Psi}(x|\omega_1,\vec{k}_1^{\perp},\lambda_1)\gamma^{\mu}\Psi(x|\omega_2,\vec{k}_2^{\perp},\lambda_2) = \quad (25)$$

$$= -\frac{\pi^2}{2}\left(\cosh\omega_1\pi\right)^{-1}\delta_{\lambda_1\lambda_2}\delta(\omega_1-\omega_2)\delta^2(\vec{k}_1^{\perp}-\vec{k}_2^{\perp}).$$

When Σ is taken to be a $\tau =$ const. hypersurface, (25) becomes:

$$\int_0^{\infty}\varrho\, d\varrho\int d^2\vec{x}^{\perp}\,\overline{\Psi}(x|\omega_1,\vec{k}_1^{\perp},\lambda_1)\gamma^{\tau}\Psi(x|\omega_2,\vec{k}_2^{\perp},\lambda_2) =$$

$$= -\frac{\pi^2}{2}\left(\cosh\omega_1\pi\right)^{-1}\delta_{\lambda_1\lambda_2}\delta(\omega_1-\omega_2)\delta^2(\vec{k}_1^{\perp}-\vec{k}_2^{\perp}).$$

Since $\gamma^{\tau} = \frac{1}{\varrho}\gamma^0$ and $\overline{\Psi} = \Psi^{+}\gamma^0$, $(\gamma^0)^2 = -1$, then we obtain

$$-\int_0^{\infty} d\varrho\int d^2\vec{x}^{\perp}\,\Psi^{+}(x|\omega_1,\vec{k}_1^{\perp},\lambda_1)\Psi(x|\omega_2,\vec{k}_2^{\perp},\lambda_2) =$$

$$= -\frac{\pi^2}{2}\left(\cosh\omega_1\pi\right)^{-1}\delta_{\lambda_1\lambda_2}\delta(\omega_1-\omega_2)\delta^2(\vec{k}_1^{\perp}-\vec{k}_2^{\perp}).$$

Using further (20) and (23) and the completeness relation [1]

$$\int_0^{\infty} du\, K_{i\omega_1-\frac{1}{2}\gamma^0\gamma^1}(u)\, K_{i\omega_2+\frac{1}{2}\gamma^0\gamma^1}(u) = \frac{\pi^2}{2}\left\{\frac{\delta(\omega_1-\omega_2)}{\cosh\omega_1\pi} - \frac{i\gamma^0\gamma^1}{\sinh\omega_1\pi - \sinh\omega_2\pi}\right\}$$

with

$$\chi^{+}(\vec{k}_1^{\perp},\lambda_1)\gamma^0\gamma^1\chi(\vec{k}_1^{\perp},\lambda_2)=0$$

it follows that we can take $N(\vec{k}^{\perp})$ real with value

$$N(\vec{k}^{\perp})=\frac{\sqrt{k}}{2\pi}. \tag{26}$$

The elementary solutions of (11) admit the form

$$\Psi(x|\omega,\vec{k}^{\perp},\lambda)=\frac{\sqrt{k}}{2\pi}e^{-i\omega\tau+i\vec{k}^{\perp}\cdot\vec{x}^{\perp}}K_{i\omega+\frac{1}{2}\gamma^0\gamma^1}(k\rho)\chi(\vec{k}^{\perp},\lambda), \tag{27}$$

and the quantized spinor field operator may be written as

$$\Psi(x)=\sum_{\lambda}\int_0^{\infty}d\omega\int d^2\vec{k}^{\perp}\{a(\omega,\vec{k}^{\perp},\lambda)\Psi(x|\omega,\vec{k}^{\perp},\lambda)+ \\ +a^{*}(\omega,\vec{k}^{\perp},\lambda)\Psi^{*}(x|\omega,\vec{k}^{\perp},\lambda)\} \tag{28}$$

with the canonical anticommutation relations

$$\{a(\omega_1,\vec{k}_1^{\perp},\lambda_1),\ a^{*}(\omega_2,\vec{k}_2^{\perp},\lambda_2)\}=\delta_{\lambda_1\lambda_2}\delta(\omega_1-\omega_2)\delta^2(\vec{k}_1^{\perp}-\vec{k}_2^{\perp}) \tag{29}$$

imposed.

Now one follows the algebraic approach, having at the basis the principle of local definiteness [2]. The expectation value of $\Psi(x_1)\Psi^{+}(x_2)$ in a state can be written with (28) and (13)

$$\langle\Psi(\tau_1,\rho_1,\vec{x}_1^{\perp})\Psi^{+}(\tau_2,\rho_2,\vec{x}_2^{\perp})\rangle=$$

$$=\sum_{\lambda_1\lambda_2}\int_{-\infty}^{+\infty}d\omega_1 d\omega_2\int d^2\vec{k}_1^{\perp}d^2\vec{k}_2^{\perp}\,\Xi(\omega_1,\vec{k}_1^{\perp},\lambda_1|\omega_2,\vec{k}_2^{\perp},\lambda_2)\times \\ \times\frac{\sqrt{k_1k_2}}{(2\pi)^2}e^{i(\vec{k}_1^{\perp}\cdot\vec{x}_1^{\perp}-\vec{k}_2^{\perp}\cdot\vec{x}_2^{\perp}-\omega_1\tau_1+\omega_2\tau_2)}\times \\ \times K_{i\omega_1+\frac{1}{2}\gamma^0\gamma^1}(k_1\rho_1)\chi(\vec{k}_1^{\perp},\lambda_1)\chi^{+}(\vec{k}_2^{\perp},\lambda_2)K_{i\omega_2-\frac{1}{2}\gamma^0\gamma^1}(k_2\rho_2), \tag{30}$$

where, as in [2],

$$\Xi=\begin{cases}\langle a(\omega_1,\vec{k}_1^{\perp},\lambda_1)a^{*}(\omega_2,\vec{k}_2^{\perp},\lambda_2)\rangle\equiv\Xi^{++}, & \omega_1,\omega_2>0,\\ \langle a^{*}(|\omega_1|,-\vec{k}_1^{\perp},\lambda_1)a(|\omega_2|,-\vec{k}_2^{\perp},\lambda_2)\rangle\equiv\Xi^{--}, & \omega_1,\omega_2<0,\\ \langle a(\omega_1,\vec{k}_1^{\perp},\lambda_1)a(|\omega_2|,-\vec{k}_2^{\perp},\lambda_2)\rangle\equiv\Xi^{+-}, & \omega_1>0,\omega_2<0,\\ \langle a^{*}(|\omega_1|,-\vec{k}_1^{\perp},\lambda_1)a^{*}(\omega_2,\vec{k}_2^{\perp},\lambda_2)\rangle\equiv\Xi^{-+}, & \omega_1<0,\omega_2>0.\end{cases} \tag{31}$$

It is clear that CAR (29) impose the restriction

$$\Xi^{++}(\omega_1,\vec{k}_1^{\perp},\lambda_1|\omega_2,\vec{k}_2^{\perp},\lambda_2)+\Xi^{--}(-\omega_1,-\vec{k}_1^{\perp},\lambda_1|-\omega_2,-\vec{k}_2^{\perp},\lambda_2)= \\ =\delta_{\lambda_1\lambda_2}\delta(\omega_1-\omega_2)\delta^2(\vec{k}_1^{\perp}-\vec{k}_2^{\perp}). \tag{32}$$

We shall consider the additional restrictions on the state arising from the conditions in the tangent space [2], only for the case of "equilibrium states" with respect to the "time translation" associated with the timelike Killing vector field

$$\alpha_{\tau'}\Psi(\tau,\rho,\vec{x}^{\perp})=\Psi(\tau+\tau',\rho,\vec{x}^{\perp}). \tag{33}$$

The KMS-condition [3]

$$\int \langle B\alpha_\tau(A)\rangle_\beta e^{-i\omega\tau} d\tau = \left\{\int \langle \alpha_\tau(A)B\rangle_\beta e^{-i\omega\tau} d\tau\right\} e^{\beta\omega} \quad (34)$$

allows us now to express the expectation value of a product by that of an anticommutator. Indeed, from (34) one obtains:

$$\int \langle \alpha_\tau(A)B\rangle_\beta e^{-i\omega\tau} d\tau = \frac{1}{1+e^{\beta\omega}} \int \langle\{\alpha_\tau(A), B\}\rangle_\beta e^{-i\omega\tau} d\tau$$

Integrating further over ω from $-\infty$ to $+\infty$ and on the right-hand side changing then ω in $-\omega$, one obtains finally

$$\langle AB\rangle_\beta = \frac{1}{2\pi} \int \frac{e^{\beta\omega}}{e^{\beta\omega}+1} \langle\{\alpha_\tau(A), B\}\rangle_\beta e^{i\omega\tau} d\tau d\omega. \quad (35)$$

With this, the left-hand side of (30) becomes:

$$\langle \Psi(\tau_1,\vec{x}_1)\Psi^+(\tau_2,\vec{x}_2)\rangle_\beta = \frac{1}{2\pi}\int \frac{e^{\beta\omega}}{e^{\beta\omega}+1} \langle\{\Psi(\tau_1+\tau,\vec{x}_1), \Psi^+(\tau_2,\vec{x}_2)\}\rangle_\beta e^{i\omega\tau} d\tau d\omega$$

$$= \frac{1}{2\pi i} \int \frac{e^{\beta\omega}}{e^{\beta\omega}+1} S(\tau_1+\tau,\vec{x}_1|\tau_2,\vec{x}_2)\gamma^0 e^{i\omega\tau} d\tau d\omega, \quad (36)$$

where $\vec{x}_i = (\rho_i, \vec{x}_i^{\perp})$ $(i=1,2)$ and

$$\frac{1}{i} S(x_1|x_2)\gamma^0 = \{\Psi(x_1), \Psi^+(x_2)\}.$$

Now, in the Rindler case one obtains:

$$\frac{1}{i}\int S(\tau_1+\tau,\vec{x}_1|\tau_2,\vec{x}_2)\gamma^0 e^{i\omega\tau} d\tau = \frac{1}{2\pi} e^{-i\omega(\tau_1-\tau_2)} \times$$

$$\times \int k\, d^2\vec{k}^{\perp} K_{i\omega+\frac{1}{2}\gamma^0\gamma^1}(k\rho_1) P(\vec{k}^{\perp}) K_{i\omega-\frac{1}{2}\gamma^0\gamma^1}(k\rho_2) e^{i\vec{k}^{\perp}\cdot(\vec{x}_1^{\perp}-\vec{x}_2^{\perp})} . \quad (37)$$

with $P(\vec{k}^{\perp})$ from (21).

As in [2] one denotes

$$\tau_2 - \tau_1 = s z^0, \quad \rho_2 - \rho_1 = s z^1, \quad \vec{x}_2^{\perp} - \vec{x}_1^{\perp} = s\vec{z}^{\perp},$$

and evaluates the singularity as $s \to 0$. Therefore

$$\langle \Psi(\tau_1,\vec{x}_1)\Psi^+(\tau_2,\vec{x}_2)\rangle_\beta = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} d\omega \frac{e^{\beta\omega}}{e^{\beta\omega}+1} e^{i\omega s z^0} \times$$

$$\times \int k\, d^2\vec{k}^{\perp} K_{i\omega+\frac{1}{2}\gamma^0\gamma^1}(k\rho_1) P(\vec{k}^{\perp}) K_{i\omega-\frac{1}{2}\gamma^0\gamma^1}(k\rho_2) e^{-is(\vec{k}^{\perp}\cdot\vec{z}^{\perp})} . \quad (38)$$

We see that in these integrals over ω and $\vec{k}^{\perp}$, finite ranges of ω and $\vec{k}^{\perp}$ give no singularity in s, only the asymptotic part for large ω and large $\vec{k}^{\perp}$ is relevant in the limit.
In that region, however, the Fermi factor

$$e^{\beta\omega} / (e^{\beta\omega} + 1)$$

becomes independent of β , namely 1 for $\omega \to +\infty$ and 0 for $\omega \to -\infty$, for positive values of β . Therefore, for any (positive) value of β the singular part of (38) will be precisely the vacuum expectation value of the Dirac massless field, which is valid for all positive temperatures in the inner points of the Rindler wedge.
On displacement of the contour for the ω -integration according to

$$\omega \longrightarrow \omega' = \omega - \frac{1}{2} i\gamma^0\gamma^1 \quad (39)$$

the equation (38) becomes:

$$\langle \Psi(\tau_1,\vec{x}_1)\Psi^+(\tau_2,\vec{x}_2)\rangle_\beta = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} \frac{e^{\frac{\beta}{2}i\gamma^0\gamma^1}\cdot e^{\beta\omega'}}{e^{\frac{\beta}{2}i\gamma^0\gamma^1}\cdot e^{\beta\omega'}+1}\, e^{is\omega' z^0} \cdot e^{-\frac{sz^0}{2}\gamma^0\gamma^1} d\omega' \times$$

$$\times \int k\, d\vec{k}^{2\perp} K_{i\omega'}(k\rho_1) P(\vec{k}^\perp) K_{-i\omega'+\gamma^0\gamma^1}(k\rho_2) e^{-is(\vec{k}^\perp\cdot\vec{z}^\perp)} . \quad (40)$$

Now, for small argument [2]

$$K_{i\omega'}(k\rho_1) \longrightarrow \pi\delta(\omega')$$

and (40) becomes in the limit $\rho_1 \to 0$:

$$\langle \Psi(\tau_1,0,\vec{x}_1^\perp)\Psi^+(\tau_2,\rho_2,\vec{x}_2^\perp)\rangle_\beta = \frac{1}{8\pi}\frac{e^{\frac{\beta}{2}i\gamma^0\gamma^1}}{e^{\frac{\beta}{2}i\gamma^0\gamma^1}+1} e^{-\frac{sz^0}{2}\gamma^0\gamma^1} \times$$

$$\times \int k\, d^2\vec{k}^\perp \left(1 + ik^{-1}\gamma^1\vec{k}^\perp\cdot\vec{\gamma}\right) K_{\gamma^0\gamma^1}(ksz^1) e^{-is\vec{k}^\perp\cdot\vec{z}^\perp}$$

$$\xrightarrow[s\to 0]{} \frac{1}{8\pi} e^{\frac{\beta}{2}i\gamma^0\gamma^1}\left(e^{\frac{\beta}{2}i\gamma^0\gamma^1}+1\right)^{-1}\int k\, d^2\vec{k}^\perp\left(1+ik^{-1}\gamma^1\vec{k}^\perp\cdot\vec{\gamma}\right)K_{\gamma^0\gamma^1}(ksz^1)e^{-is\vec{k}^\perp\cdot\vec{z}^\perp} . \quad (41)$$

Using standard formels in the theory of Bessel functions [4] , one can compute the integral in (41) and find:

$$\langle \Psi(\tau_1,0,\vec{x}_1^\perp)\Psi^+(\tau_2,\rho_2,\vec{x}_2^\perp)\rangle_\beta \xrightarrow[s\to 0]{} \frac{1}{4s^3} e^{\frac{\beta}{2}i\gamma^0\gamma^1}\left(e^{\frac{\beta}{2}i\gamma^0\gamma^1}+1\right)^{-1} \times$$

$$\times \left\{ \frac{2z^1}{\left[(z^1)^2+|\vec{z}|^2\right]^2} + \gamma^1\gamma^2 \frac{3\pi|\vec{z}^\perp|}{4(z^1)^4}\, {}_2F_1\left(\frac{5}{2},\frac{3}{2};2;-\frac{|\vec{z}^\perp|^2}{(z^1)^2}\right)\right\}. \quad (42)$$

One observes also that

$$\left(e^{\frac{\beta}{2}i\gamma^0\gamma^1}+1\right)^{-1} = \left(1+e^{-\frac{\beta}{2}i\gamma^0\gamma^1}\right)\Big/\left[2\left(1+\cos\tfrac{\beta}{2}\right)\right]. \quad (43)$$

Then, finally, for $\vec{z}^\perp = 0$ one obtains:

$$\langle \Psi(\tau_1,0,\vec{x}_1^\perp)\Psi^+(\tau_2,\rho_2,\vec{x}_1^\perp)\rangle_\beta \xrightarrow[s\to 0]{} \frac{1}{4(s\rho_2)^3}\left[1 + i\,\frac{\sin\frac{\beta}{2}}{1+\cos\frac{\beta}{2}}\gamma^0\gamma^1\right]. \quad (44)$$

One observe easy that β cannot admit the value 2π , obtained in the case of the massless scalar field in the Rindler wedge [2] .

From (44) one can say that in order to have independence of the representation used for the γ-Dirac matrices, one obtains:

$$\beta = 4n\pi \quad , \quad n \in \{1,2,\dots\} . \quad (45)$$

In other words, the temperatures of the KMS states are:

$$T_n = 1/4n\pi \quad , \quad n\in\{1,2,\dots\} .$$

One can conclude, if there are no mistakes in the above computations and admiting however, the validity of local definiteness principle as a basic criterium to restrict the class of physical states of the full theory, that the scaling limit procedure used in the derivation of the "Hawking temperature" for the massless free field is not appropiate.

For the moment, it is not quite clear, if the result from this paper may be also in contradiction with the result of Bisognano and Wichmann [5] invoked by Sewell [6] in his rigorous derivation of Hawking temperature.

In a forthcomming paper, one proposes to study the stability of the local definiteness principle on the lines of [7] , for the free Dirac field on a Robertson-Walker spacetime using the concept of an adiabatic vacuum state.

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