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Measuring the Weak Isospin of b Quarks

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Abstract

From measurements of the Z-decay width to b quarks $\Gamma(Z \to b\bar{b})$ and the forward-backward asymmetry $A_{FB}(b)$ of b quarks at LEP, PETRA, PEP and TRISTAN, the weak isospin components of the left- and right-handed b quarks can unambiguously be determined: $I_3^L(b) = -1/2$ and $I_3^R(b) = 0$. From the non-zero value of $I_3^L(b)$ the top quark can be inferred as isospin partner to the b quark.

For quite some time there has been considerable interest in measurements of the weak isospin quantum numbers of b quarks. If the iso-multiplet structure of the first two families is reiterated again in the third family, the left-handed b quark is the lower component of an isodoublet, demanding the existence of top quarks as the upper iso-partner to the b quarks. The right-handed particles are both iso-singlets.

Several arguments have been advanced in the past which experimentally rule out the assignment $I_3^L(b) = 0$ to the left-handed b quarks. They are based on the assumption of the familiar $SU(2)\times U(1)$ gauge structure of the electroweak interactions. If both right- and left-handed components of the b quarks are assumed to be isosinglets, several consequences can be derived which are in conflict with experimental observations. (i) Assigning zero-isospin quantum numbers to left-handed b quarks breaks the GIM mechanism which demands the left-handed quarks to belong to the same isomultiplet in all three families. If GIM is broken the mixing between the quarks induces flavor-changing neutral currents which give rise to large branching ratios of B decays to charged lepton pairs[1]: $BR(B \to l^+l^-X)/BR(B \to l\nu_l X) > 0.12$. This ratio is four orders of magnitude bigger than the bound recently set by UA1 [2] : $BR(B \to \mu^+\mu^-X)/BR(B \to \mu\nu_\mu X) < 5.0 \cdot 10^{-5}/(0.110 \pm 0.009)$. (ii) The partial Z-decay width to b quarks were reduced by more than an order of magnitude if b quarks were isosinglets [3]. (iii) The forward-backward asymmetry of b quarks in e^+e^- annilation is determined by the Z-axial charge of b quarks which is given by the difference between left- and right-handed isospin quantum numbers. Were the isospin of b_L zero and equal to the isospin of b_R, the forward-backward asymmetry would vanish across the entire e^+e^- energy range. This however is in clear conflict with non-zero values of the forward-backward asymmtry as observed at PETRA, PEP, TRISTAN and LEP [for a summary see Ref. [4]].

Additional arguments for top quarks follow from the observation of rapid $B - \bar{B}$ oscillations [5] and the theoretical requirement of the electroweak gauge theory to be anomaly-free, demanding the sum over all electric charges in the third family to be zero.

In this note we exploit recent LEP data on the width $\Gamma(Z \to b\bar{b})$ and the forward-backward asymmetry $A_{FB}(b)$ of b quarks from LEP and PETRA, PEP, TRISTAN, to determine accurately the isospin components $I_3^L(b)$ and $I_3^R(b)$ of the left- and right-handed b quarks . [Earlier analyses based on various subsets of data were discussed in Refs. [6].]

The decay width and the forward-backward asymmetry are given by the Z-vectorial and axial charges in the [improved] Born approximation [see e.g. [7]] as

$$\Gamma(Z \to b\bar{b}) = \frac{G_F M_Z^3}{2\pi\sqrt{2}}[v_b^2 + a_b^2] \tag{1}$$

$$A_{FB}(b) = \frac{3}{4}\,\frac{2v_e a_e}{v_e^2 + a_e^2}\,\frac{2v_b a_b}{v_b^2 + a_b^2} \tag{2}$$

The Born approximation, as defined in this form, is not only sufficient for our analysis aiming at half-integer quantum numbers, but we are forced to this approximation if we refrain from major extensions of the Higgs sector to keep the model renormalizable for generalized isospin assignments. Expressing the Z-charges by the isospin component of the b quark [briefly denoted I_3^L and I_3^R from now on],

$$v_b = (I_3^L + I_3^R) - 2e_b \sin^2\overline{\theta}_W \quad (3)$$

$$a_b = I_3^L - I_3^R \quad (4)$$

[and for the leptons $v_e = -1/2 + 2\sin^2\overline{\theta}_W$, $a_e = -1/2$], the width defines a circle in the (I_3^L, I_3^R) plane while the forward-backward asymmetry defines a pair of straight lines,

$$(I_3^L + \frac{1}{3}\sin^2\overline{\theta}_W)^2 + (I_3^R + \frac{1}{3}\sin^2\overline{\theta}_W)^2 = R^2 \quad (5)$$

$$|I_3^R + \frac{1}{3}\sin^2\overline{\theta}_W| = \gamma\, |I_3^L + \frac{1}{3}\sin^2\overline{\theta}_W| \quad (6)$$

The radius (squared) is given by

$$R^2 = \frac{\pi\sqrt{2}\,\Gamma(Z \to b\overline{b})}{G_F M_Z^3} \quad (7)$$

and the slope parameter (squared)

$$\gamma^2 = \left[1 - \frac{4A_{FB}(b))}{3}\frac{v_e^2 + a_e^2}{2v_e a_e}\right] \Big/ \left[1 + \frac{4A_{FB}(b)}{3}\frac{v_e^2 + a_e^2}{2v_e a_e}\right] \quad (8)$$

The center of the circle as well as the crossing point of the lines are displaced from the origin of the (I_3^L, I_3^R) plane down to the third quadrant by a small shift $-\frac{1}{3}\sin^2\overline{\theta}_W$ in both variables. The physical solution for the isospin components are those values where the circle and the straight lines both cross one and the same point with half-integer coordinates. The width alone allows for two solutions where L and R are interchanged; this ambiguity is resolved by the forward-backward asymmetry which is asymmetric in L and R and selects finally one single point.

It is interesting to note the high sensitivity of the width and of the forward-backward asymmetry to the pairs of possible half-integer isospin values. This is demonstrated in Table 1 for the lowest values of the quantum numbers. The analysis appears quite robust.

Corroborating evidence can be derived from the forward-backward asymmetries of b quarks measured at PETRA, PEP and TRISTAN. While the FB asymmetry on the Z is due to the coherent superposition of vectorial and axial Z charges, the asymmetry at the low PETRA energies is a result of the interference between vectorial γ exchange and axial Z exchange. At TRISTAN all amplitudes contribute with similar strength and the asymmetry is of the general form

$$A_{FB}(b) = \frac{3}{4}\frac{-2e_e e_b a_e a_b Re\chi + 4v_e v_b a_e a_b|\chi|^2}{e_e^2 e_b^2 - 2e_e e_b v_e v_b Re\chi + (v_e^2 + a_e^2)(v_b^2 + a_b^2)|\chi|^2} \quad (9)$$

with the electric charges $e_e = -1$, $e_b = -1/3$ and

$$\chi = -\frac{1}{4\sin^2\overline{\theta}_W \cos^2\overline{\theta}_W}\,\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}$$

$\Gamma_{b\overline{b}}$ (MeV)		I_3^L = −1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	+1
I_3^R	+1	3386	2048	1705	2357	4003
	$+\frac{1}{2}$	2048	710	367	1019	2665
	0	1705	367	24	676	2322
	$-\frac{1}{2}$	2357	1019	676	1327	2974
	−1	4003	2665	2322	2974	4621

A_{FB}^b		I_3^L = −1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	+1
I_3^R	+1	0.000	-0.068	-0.102	-0.045	0.016
	$+\frac{1}{2}$	0.068	0.000	-0.097	0.031	0.076
	0	0.102	0.097	0.000	0.100	0.103
	$-\frac{1}{2}$	0.045	-0.031	-0.100	0.000	0.057
	−1	-0.016	-0.076	-0.103	-0.057	0.000

Table 1: Values of the partial width $\Gamma(Z \to b\overline{b})$ and the forward-backward asymmetry $A_{FB}(b)$ on the Z for the lowest possible isospin quantum numbers of the left- and right-handed b quarks. The electroweak mixing angle is chosen as $\sin^2\overline{\theta}_W = 0.2327$.

For a given experimental value of $A_{FB}(b)$, eq. (9) corresponds to a conic section in the (I_3^L, I_3^R) plane, i.e. an ellipse, hyberbola or a pair of straight lines. At low PETRA energies, the expression is reduced to a simple straight line

$$I_3^R \approx I_3^L + \frac{A_{FB}(b)}{9Re\chi} \quad (10)$$

We now confront this discussion with data from e^+e^- storage rings. First we want to explore the constraints obtained from LEP. To determine $I_3^L(b)$ and $I_3^R(b)$ from the measured values for $\Gamma(Z \to b\overline{b})$ and $A_{FB}(b)$ we base our analysis on recent LEP averages, summarized in Table 2. The values are given for the quark final states, including the correction for B-$\overline{B}$ oscillations χ_B in the transition from hadron to quark final states [see Ref. [7]]. To compare these values directly to the formulae (1) and (2), we have to apply QED and QCD corrections to the measured values:

$$\Gamma(Z \to b\overline{b}) \;=\; \Gamma(Z \to b\overline{b})^{LEP}/r_{QCD} \;=\; 345 \pm 18\ MeV$$

$$A_{FB}(b) \;=\; \frac{A_{FB}(b)^{LEP}}{(1 - \alpha_s/\pi)} + \Delta_{QED} \;=\; 0.135 \pm 0.024$$

with r_{QCD}=1.045, $\alpha_s(M_Z^2)$=0.12 and Δ_{QED}=0.004 to account for a shift in $A_{FB}(b)$ due to photonic corrections. Table 4 column 2 displays the result of a fit to the LEP data. In this fit the LEP averages for a^2_{lept} and v^2_{lept} have been included as constraints. The value for $\sin^2\overline{\theta}_W(b)$ has been approximated by $\sin^2\overline{\theta}_W(lept) = 0.2327^{+0.0027}_{-0.0018}$, derived from the ratio of v^2_{lept}/a^2_{lept} assuming the Standard Model isospin assignments for leptons. The nonvanishing χ^2 for this fit is a consequence of the fact, that for fixed a_e and v_e the condition $2a_b v_b/(a_b^2 + v_b^2) < 1$ imposes an upper bound on $A_{FB}(b)$, which is exceeded slightly by the average value of the data. The result of this fit can also be expressed in terms of the radius R and the slope parameter γ of equation (7),(8):

$$R = 0.416^{+0.015}_{-0.018} \qquad \gamma = 0.00 \pm 0.20$$

The corresponding 1-σ bounds in the (I_3^L, I_3^R)-plane are shown in Figure 1. There are two cross-over regions for the bounds from $\Gamma(Z \to b\overline{b})$ and $A_{FB}(b)$, but only one contains a grid point of half integer isospin values I_3^L and I_3^R, whereas the other one, corresponding to solution II in Table 4, is far off any such grid point.

The ambiguity can however also be solved purely experimentally by including the measurements of $A_{FB}(b)$ from PETRA, PEP and TRISTAN, which are summarized in Table 3. In Figure 1 we show as an example the 1-σ bounds from data of $A_{FB}(b)$ at 35 GeV. As explained above, the contribution of the various terms to the forward-backward asymmetry in equation (10) is energy dependent. Therefore the data for $A_{FB}(b)$ from PETRA, PEP and TRISTAN alone are sufficient in principle to derive I_3^L and I_3^R simultaneously. The result of a fit to $A_{FB}(b)$ from these storage rings is given in Table 4 column 3. The 90% confidence region extends along the band for $A_{FB}(b)$ at 35 GeV and includes only 2 pairs of half-integer isospin values.

Combining the measurements from LEP with those at lower energies, only one solution for the weak isospins of the left- and right-handed b quarks remains finally. The result of a fit to the combined data set is given in Table 4 column 4, the 90 % confidence region in the (I_3^L, I_3^R)-plane is indicated by the small hatched area in Figure 1. The only solution compatible with data is the isospin assignment of the Standard Model:

$$[I_3^L(b)]_{exp} = -0.490^{+0.015}_{-0.012} \quad \Rightarrow \quad I_3^L(b) = -1/2$$
$$[I_3^R(b)]_{exp} = -0.028 \pm 0.056 \quad \Rightarrow \quad I_3^R(b) = 0$$

This analysis of LEP data in conjunction with $A_{FB}(b)$ measurements from PETRA, PEP and TRISTAN corroborates in a most transparent way the iso-multiplet structure of the Standard Model for the b quark in the third family, underlining the existence of the top quark as isopartner of the b quark.

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observable	reference	result
$A_{FB}(b)$	[19]	0.126 ± 0.022
$\Gamma(Z \to b\overline{b})$(MeV)	[20]	361 ± 19
χ_B	[21]	0.144 ± 0.020
M_Z(GeV)	[19]	91.175 ± 0.021
a_e^2	[19]	0.2492 ± 0.0012
v_e^2	[19]	0.0012 ± 0.0003

Table 2: LEP results used in the analysis. The value for $A_{FB}(b)$ is corrected for $B\overline{B}$ mixing.

$\sqrt{s}(GeV)$	reference	$A_{FB}(b)$
29	[8],[9],[10],[11]	-0.055 ± 0.086
35	[12],[13],[14],[15]	-0.228 ± 0.053
44	[13],[12]	-0.489 ± 0.156
55	[16],[17],[18]	-0.844 ± 0.185

Table 3: PETRA / PEP / TRISTAN results for $A_{FB}(b)$. The values quoted include the correction for $B\overline{B}$ mixing based on the LEP average $\chi_B = 0.144 \pm 0.020$.

	LEP only solution I	LEP only solution II	PETRA + PEP + TRISTAN	LEP + PETRA + PEP + TRISTAN
I_3^L	$-0.493^{+0.011}_{-0.013}$	$0.339^{+0.013}_{-0.011}$	-0.29 ± 0.16	$-0.490^{+0.015}_{-0.012}$
I_3^R	-0.077 ± 0.088	-0.077 ± 0.088	$0.15^{+0.17}_{-0.14}$	-0.028 ± 0.056
a^2_{lept}	0.2492 ± 0.0012	0.2492 ± 0.0012	0.2492 ± 0.0012	0.2492 ± 0.0012
v^2_{lept}	0.0014 ± 0.0003	0.0014 ± 0.0003	0.0012 ± 0.0003	0.0014 ± 0.0003
χ^2/NDOF	$1.4/(4-4)$	$1.4/(4-4)$	$2.6/(6-4)$	$6.0/(8-4)$

Table 4: Fit results for the weak isospin of the b quark. In all fits the LEP results for a^2_{lept} and v^2_{lept} have been included as constraint.

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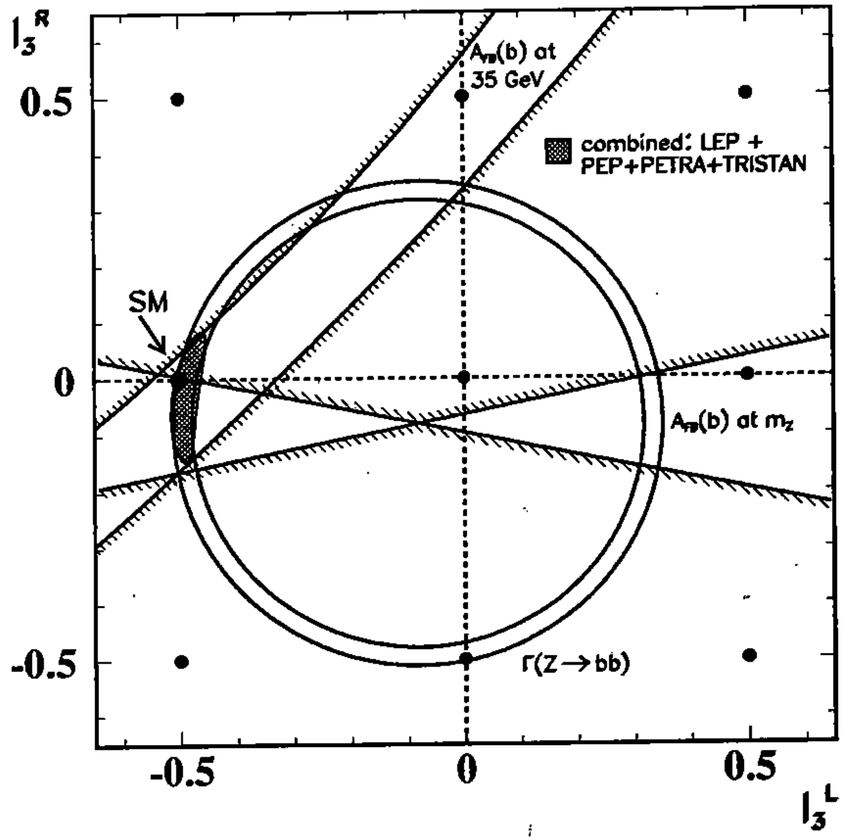

Figure 1: Experimental constraints for the isospin of the b-quark in the (I_3^L, I_3^R)-plane. The circle and the pair of straight lines, centered in $e_b \sin^2\bar{\theta}_W = -0.08$, represent 1-$\sigma$ bounds derived from the LEP averages for $\Gamma(Z \to b\bar{b})$ and A_{FB}(b). The effective electroweak mixing angle is chosen as $\sin^2\bar{\theta}_W(b) \approx \sin^2\bar{\theta}_W(\text{lept}) = 0.2327$ as obtained from the ratio $v_{\text{lept}}^2/a_{\text{lept}}^2$ measured at LEP. The grid points mark the half-integer values of I_3^L and I_3^R allowed by the isospin algebra. Only one of the two cross-over regions contains a grid point, which coincides with the Standard Model isospin assignment for the b-quarks, $I_3^L(b) = -1/2$ and $I_3^R(b) = 0$. Experimentally the $b\bar{b}$ asymmetry at PETRA, PEP and TRISTAN energies can separate between the two solutions mathematically allowed by LEP data. As an example we show the 1-σ bounds derived from the measured FB asymmetry at a c.m. energy of 35 GeV. Combining LEP, PETRA, PEP and TRISTAN data results in the 90% confidence region in the (I_3^L, I_3^R)-plane indicated by the hatched area.