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## Forward-Backward Asymmetry of Dilepton Angular Distribution in the Decay $b \rightarrow s l^+ l^-$

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### Abstract

The angular distribution of  $l^+ l^-$  in the decay  $b \rightarrow s l^+ l^-$  is studied. We point out that a large forward-backward asymmetry in the angular distribution of the dileptons is expected from the short-distance contribution in the standard model for heavy top quark mass. The asymmetry is a measure of the contribution of the  $Z$  and  $W^+ W^-$  exchange diagrams, as well as, the top quark mass.

### 1. Introduction

Rare  $B$ -decays, in particular those involving the flavour changing neutral current (FCNC)  $b$ -quark transitions  $b \rightarrow s \gamma$  and  $b \rightarrow s l^+ l^-$ , provide important testing grounds for the standard model at the quantum (loop) level, since such transitions are forbidden in the Born approximation. The rates and distributions in these decays thus provide sensitivity to possible higher mass scales and/or new interactions. In the standard model the so-called short-distance contributions to these decays are dominated by the top quark whose mass is expected to lie in the range  $100 \text{ GeV} < m_t < 200 \text{ GeV}$ . Precise measurements of rare  $B$ -meson transitions will not only provide a good estimate of the top quark mass but also of the  $W^+ W^-$  and  $Z$ -exchange diagrams.

In this note, we concentrate on the rare  $B$ -decays,  $b \rightarrow s l^+ l^-$  ( $l = e, \mu$ ). These processes have been studied extensively; in particular, the decay rates and the differential invariant dilepton mass distributions and their dependence on the top quark mass have been investigated in earlier work.<sup>[1] [2] [3] [4] [5] [6] [7]</sup> The long distance contribution to the decays  $b \rightarrow s l^+ l^-$  is, however, still somewhat controversial. In particular, the relative sign between the short- and long-distance contributions, which reflects itself in the dilepton invariant mass distribution, is not yet a settled matter.<sup>[8] [9]</sup>

Here, we present a unified treatment of both the short-distance and long-distance QCD corrections in the effective Hamiltonian approach. In this method the heavy degrees of freedom (in the present context top quark and  $W^\pm$  bosons) are integrated out and the resulting effective Hamiltonian consists of operators involving only the light quanta with (renormalization group) improved Wilson coefficients. The same diagrammatic approach is then used to estimate the long-distance contribution from the intermediate  $J/\psi$ ,  $\psi'$  and the charmed hadron continuum. This framework, in our opinion, is free of ambiguities, as far as the relative sign of the two contributions is concerned. The main result of this paper is a detailed study of the angular distribution of  $l^+ l^-$  in the inclusive

decays  $b \rightarrow s l^+ l^-$ , obtaining a large forward-backward asymmetry in the angular distribution of the dileptons for heavy top quark mass  $\frac{m_b}{M_W} > 1$ . In the standard model, the decays in question occur through the  $\gamma$ ,  $Z$  and  $W^+ W^-$  intermediate states. The asymmetry is related to the circumstance that the right-chiral and left-chiral couplings of the leptonic current are different due to the  $Z$  and  $W^+ W^-$  exchanges. Therefore, the asymmetry is a direct measure of the contribution of these diagrams. Since, as shown below, the aforementioned asymmetry is driven by the top quark, it is a genuine short-distance effect and its magnitude is a good measure of the top quark mass.

## 2. Invariant dilepton mass distribution and forward-backward asymmetry in $b \rightarrow s l^+ l^-$

In what follows we shall ignore the  $B$ -hadron wave function effects and consider the decay of a  $b$ -quark and set  $m_b = m_B$ , so that the kinematics and the resulting dilepton spectra of the decay  $b \rightarrow s l^+ l^-$  mimic the  $B$ -hadron decays  $B_{d,u} \rightarrow X_s + l^+ l^-$ . We start by defining the kinematic variables needed to study the forward-backward asymmetry. The decay distributions of interest are characterized by two Mandelstam variables  $s$  and  $u$ :

$$\begin{aligned} s &= (s^+ + s^-)^2 & u &= (p - s^-)^2 - (p - s^+)^2 \\ (4m_l^2 \leq s \leq (m_b - m_s)^2, & & -u(s) \leq u \leq u(s)) & \\ u(s) &= \sqrt{\{s - (m_b + m_s)^2\} \{s - (m_b - m_s)^2\}} \end{aligned} \quad (1)$$

where  $s^+$ ,  $s^-$  and  $p$  are the four-momenta of  $l^+$ ,  $l^-$  and the  $b$ -quark ( $B$ -meson), respectively. The variable  $u$  is related to the angle between the momentum of the  $B$ -meson (or the outgoing  $s$ -quark) and that of  $l^+$  in the center of mass frame of the dileptons  $l^+ l^-$ :

$$z \equiv \cos \theta = \frac{u}{u(s)} \quad (2)$$

The forward-backward asymmetry is obtained by integrating the double differ-

ential branching ratio  $\left(\frac{d^2 BR}{ds dz}\right)$  with respect to the angular variable ( $z$ ),

$$A(\hat{s}) \equiv \frac{\int_0^1 dz \frac{d^2 BR}{ds dz} - \int_{-1}^0 dz \frac{d^2 BR}{ds dz}}{\int_0^1 dz \frac{d^2 BR}{ds dz} + \int_{-1}^0 dz \frac{d^2 BR}{ds dz}}, \quad (3)$$

where  $\hat{s} = \frac{s}{m_b^2}$ . In the region where  $A(\hat{s})$  is positive, the number of  $l^+$  scattered in the forward hemisphere is more than the one in the backward hemisphere in the center of mass frame of  $l^+ l^-$ .

We now record the formulae for the differential branching ratio, angular distribution and asymmetry for the decays  $b \rightarrow s l^+ l^-$ . Following ref. (4), we work in the framework of the (one-loop) QCD corrected effective weak Hamiltonian, obtained by integrating out the top quark and the  $W^\pm$  bosons. This leads to the following invariant amplitude for  $b \rightarrow s l^+ l^-$ ,

$$\begin{aligned} M &= 2\sqrt{2}G_F \frac{1}{\sin^2 \theta_W} \frac{\alpha}{4\pi} (-V_{cb}^* V_{cs}) \{A \bar{s}_L \gamma_\mu b_L \bar{l}_R \gamma^\mu l_R + B \bar{s}_L \gamma_\mu b_L \bar{l}_R \gamma^\mu l_R \\ &\quad + C \bar{s}_i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_s L + m_b R) \bar{l} \gamma^\mu l\}, \end{aligned} \quad (4)$$

where  $L = (1 - \gamma_5)/2$ ,  $R = (1 + \gamma_5)/2$ ,  $q^\mu = s^{+\mu} + s^{-\mu}$ ,  $V_{ij}$  are the Cabibbo-Kobayashi-Maskawa matrix elements, and  $\sin^2 \theta_W$  is the weak angle in the standard model with  $\sin^2 \theta_W \simeq 0.23$ . The form factors in Eq. (4) are given by,

$$\begin{aligned} A &= B + \bar{C}^{Boz}(x_t) + \bar{C}^Z(x_t) \\ B &= -\sin^2 \theta_W [F_1(s)(x_t) + 2\bar{C}^Z(x_t) - \frac{4}{9} \{\ln(x_t) + 1\}] \\ &\quad + \{3C_1(m_b) + C_2(m_b)\} \{g(\hat{m}_c, \hat{s}) + \frac{3}{\alpha^2} \kappa \sum_{V_i = \Psi, \psi} \frac{\pi \Gamma[V_i \rightarrow l^+ l^-] M_{V_i}}{M_{V_i}^2 - q^2 - iM_{V_i} \Gamma_{V_i}}\} \\ &\quad + \frac{4\pi}{\alpha_s(M_W)} \left\{ -\frac{4}{33} (1 - \eta^{-\frac{11}{3}}) + \frac{8}{87} (1 - \eta^{-\frac{23}{3}}) \right\} C_2(M_W) \\ C &= [\eta^{-\frac{11}{3}} \{F_2(x_t) - \frac{116}{135} (\eta^{\frac{10}{3}} - 1) C_2(M_W) - \frac{58}{189} (\eta^{\frac{23}{3}} - 1) C_2(M_W)\}] \sin^2 \theta_W \\ x_t &= \left(\frac{m_t}{M_W}\right)^2 \end{aligned}$$

where  $C_1$  and  $C_2$  are the QCD corrected Wilson coefficients:  $C_1(m_b) = -\frac{1}{2}(\eta^{-\frac{8}{3}} -$

$\eta^{\frac{12}{13}}$ ,  $C_2(m_b) = -\frac{1}{2}(\eta^{-\frac{9}{13}} + \eta^{\frac{12}{13}})$ , with  $\eta = \frac{\alpha_s(m_b)}{\alpha_s(M_W)}$  and  $C_2(M_W) = -1$ . Note that the expression in the braces multiplying the factor  $3C_1(m_b) + C_2(m_b)$  represents the long distance contribution to the decay amplitude for  $b \rightarrow s l^+ l^-$ . The pole contributions from the  $J/\psi$  and  $\psi'$  with the Breit-Wigner form are explicitly indicated and  $g(z, \hat{s})$  represents the contribution from the  $c\bar{c}$  continuum obtained from the electromagnetic penguin diagrams, following Ref. [4].

$$\begin{aligned}
g(z, \hat{s}) = & -\frac{4}{9} \ln z^2 - \frac{8}{27} - \frac{16z^2}{9\hat{s}} \\
& + \frac{2}{9} \sqrt{1 - \frac{4z^2}{\hat{s}}} \left( 2 + \frac{4z^2}{\hat{s}} \right) \left\{ \ln \frac{|1 + \sqrt{1 - \frac{4z^2}{\hat{s}}}|}{|1 - \sqrt{1 - \frac{4z^2}{\hat{s}}}|} + i\pi \right\} \quad (\hat{s} \geq 4z^2) \\
& - \left[ \frac{4}{9} \ln z^2 - \frac{8}{27} - \frac{16z^2}{9\hat{s}} + \frac{4}{9} \sqrt{\frac{4z^2}{\hat{s}}} + \frac{4z^2}{9\hat{s}} \right] \arctan \frac{1}{\sqrt{\frac{4z^2}{\hat{s}} - 1}} \quad (\hat{s} \leq 4z^2)
\end{aligned} \tag{6}$$

It is well known<sup>[10]</sup> that the Wilson coefficient sum  $3C_1(m_b) + C_2(m_b)$  depends very sensitively on the QCD scale parameter  $\Lambda_{QCD}$ , as well as the renormalization point  $\mu$ . For instance, taking  $\Lambda_{QCD} = 400$  MeV and the scale  $\mu = m_b$  one has  $3C_1(m_b) + C_2(m_b) = -0.17$ , while for  $\Lambda_{QCD} = 100$  MeV one obtains  $3C_1(m_b) + C_2(m_b) = -0.41$ , and for  $\Lambda_{QCD} = 100$  MeV and using the renormalization point  $\mu = 2m_b$  one finds  $3C_1(2m_b) + C_2(2m_b) = -0.58$ . Although the QCD corrected inclusive nonleptonic decay rate is stable against changes in  $\Lambda_{QCD}$ , the semi-inclusive channels  $B \rightarrow J/\psi X_s$  and  $B \rightarrow \psi' X_s$  have rather large rate uncertainties due to cancellations in the relevant combination of the Wilson coefficients. Since the leading order renormalization of the particular combination is very significant, in our opinion, higher order QCD corrections need to be calculated to estimate the semi-inclusive decay channels in question quantitatively.

On the other hand, in the present note we are mainly concerned with the

short distance part, in particular the dilepton asymmetry. To circumvent the (yet unresolved) problem concerning the semi-inclusive rate we shall use the data on these decays. Using the branching ratios from the Particle Data Group:<sup>[13]</sup>  $BR(B \rightarrow J/\psi X \rightarrow l^+ l^- X) = BR(B \rightarrow J/\psi X) BR(J/\psi \rightarrow l^+ l^-) \sim 0.01 \times 0.07 = 7 \times 10^{-4}$  fixes the constant  $\kappa$  introduced in the resonance part of the amplitude for  $b \rightarrow s l^+ l^-$ . Numerically, a value  $\kappa(3C_1(m_b) + C_2(m_b)) = -1$  reproduces the data well.

The relative phase between the  $c\bar{c}$  continuum and the pole terms in the present work has been determined with the help of the dispersion analysis as in Ref. [3], though the actual contributions presented here are quite different. Other functions in Eq. (5) can be extracted from Ref. [13],<sup>[13]</sup>

$$\begin{aligned}
\overline{C}^{Box}(x) + \overline{C}^Z(x) &= \frac{1}{4}x + \frac{3}{4}\frac{x}{1-x} + \frac{3}{4}\left(\frac{x}{1-x}\right)^2 \ln x \\
\overline{C}^Z(x) &= \frac{1}{4}x + \frac{3}{8}\frac{x}{1-x} + \frac{3}{8}\frac{2x^2-x}{(1-x)^2} \ln x \\
F_1^{(*)}(x) &= \frac{63x - 151x^2 + 82x^3}{36(1-x)^3} + \frac{63x - 138x^2 + 59x^3 + 10x^4}{36(1-x)^4} \ln x \\
F_2(x) &= \frac{7x - 5x^2 - 8x^3}{12(1-x)^3} + \frac{2x^2 - 3x^3}{2(1-x)^4} \ln x
\end{aligned} \tag{7}$$

The double differential distribution (normalized to the branching ratio) can be easily obtained from Eq. (5):

$$\begin{aligned}
\frac{dBR}{ds dz} &= \frac{BR(B \rightarrow l\nu + X)}{|V_{ub}|^2 + |V_{cb}|^2 P(\hat{m}_c^2)} \frac{3\alpha^2}{32\pi^2} |V_c|^2 \hat{u}(\hat{s}) \left( \frac{1}{\sin^2 \theta_W} \right)^2 \\
&+ (|A|^2 + |B|^2) \{ (1 - \hat{m}_s^2)^2 - (\hat{s}^2 + \hat{u}(\hat{s})^2 z^2) \} \\
&+ |C|^2 \left\{ \frac{1 + \hat{m}_s^2}{2} (\hat{s}^2 - \hat{u}(\hat{s})^2 z^2 - 2(1 + \hat{m}_s^2)\hat{s} + (1 - \hat{m}_s^2)^2) \right. \\
&- (1 + \hat{m}_s^4 + 6\hat{m}_s^2)\hat{s} + (1 + \hat{m}_s^2)(1 - \hat{m}_s^2)^2 \} \\
&+ 4Re\{(A^* + B^*)C\} \{ (\hat{m}_s^2 + 1)\hat{s} - (1 - \hat{m}_s^2)^2 \} \\
&+ 2(|A|^2 - |B|^2)\hat{s}\hat{z}(\hat{s}z - 4Re\{(A^* - B^*)C\}) \{ (1 + \hat{m}_s^2)\hat{u}(\hat{s})z \}
\end{aligned} \tag{8}$$

where  $\hat{u}(\hat{s}) = \sqrt{(\hat{s} - (1 + \hat{m}_s)^2)(\hat{s} - (1 - \hat{m}_s)^2)}$ ,  $V_c = V_{cs}^* V_{cb}$  and  $P(\hat{m}_c^2)$  is a

phase space factor, which we take to be  $P(\hat{m}_c^2) = 0.55$ . The explicit expressions for the dilepton invariant mass distribution and the asymmetry defined in Eq. (3) are:

$$\begin{aligned}
\frac{dBR}{d\hat{s}} &= \int_{-1}^1 \frac{d^2 BR}{d\hat{s}dz} = \frac{BR(B \rightarrow l\bar{\nu} + X)}{|V_{ub}|^2 + |V_{cb}|^2 P(\hat{m}_c^2)} \frac{\alpha^2}{8\pi^2} |V_c|^2 \hat{u}(\hat{s}) \left( \frac{1}{\sin^2 \theta_W} \right)^2 \\
&+ |C|^2 \frac{2}{3} \{ -(1 + \hat{m}_s^2)^2 - (1 + 14\hat{m}_s^2 + \hat{m}_s^4)\hat{s} + 2(1 + \hat{m}_s^2)(1 - \hat{m}_s^2)^2 \} \\
&+ 6Re\{(A^* + B^*)C\} \{ (1 + \hat{m}_s^2)\hat{s} - (1 - \hat{m}_s^2)^2 \}, \\
&\int_0^1 \frac{d^2 BR}{d\hat{s}dz} - \int_{-1}^0 \frac{d^2 BR}{d\hat{s}dz} = \frac{BR(B \rightarrow l\bar{\nu} + X)}{|V_{ub}|^2 + |V_{cb}|^2 P(\hat{m}_c^2)} \frac{3\alpha^2}{32\pi^2} |V_c|^2 \hat{u}(\hat{s})^2 \left( \frac{1}{\sin^2 \theta_W} \right)^2 \\
&[2(|A|^2 - |B|^2)\hat{s} - 4Re\{(A^* - B^*)C\}(1 + \hat{m}_s^2)]. \tag{9}
\end{aligned}$$

In the limit  $m_s = 0$ , the asymmetry assumes a very simple expression,

$$\begin{aligned}
A(\hat{s}, m_s = 0) &= \frac{3}{2} \frac{(|A|^2 - |B|^2)\hat{s} - 2Re\{(A^* - B^*)C\}}{(|A|^2 + |B|^2)(1 + 2\hat{s}) + |C|^2 \frac{2}{3}(1 + 2\hat{s}) - 6Re\{(A^* + B^*)C\}} \\
&= \frac{3}{2} \frac{(\overline{C}^{Boz} + \overline{C}^Z) \{ (\overline{C}^{Boz} + \overline{C}^Z) \{ (\overline{C}^{Boz} + \overline{C}^Z) \hat{s} - 2C \} + 2ReB \} \hat{s} - 2C \}}{2|B|^2(2\hat{s} + 1) + \{ \overline{C}^Z(2\hat{s} + 1) - 6C \} (\overline{C} + 2ReB) + |C|^2 \frac{2}{3}(1 + 2\hat{s})}, \tag{10}
\end{aligned}$$

where  $\overline{C} = \overline{C}^{Boz} + \overline{C}^Z$ . Eq. (10) shows that the asymmetry is proportional to  $\overline{C}^{Boz} + \overline{C}^Z$ . The asymmetry in the dilepton angular distribution can be qualitatively understood as follows. As shown above, the decays  $b \rightarrow s l^+ l^-$  occur through  $\gamma$ ,  $Z$  and  $W^+ W^-$  exchange diagrams. For low  $m_t$  ( $\frac{m_t}{M_W} < 1$ ) the photon contribution dominates and the vector-like interactions to the leptonic current remain substantial, consequently the asymmetry is small. However, for  $\frac{m_t}{M_W} > 1$ , the contribution from the  $Z$ -exchange diagrams becomes important and the coefficient of the left-handed leptonic current grows as  $m_t^2$ , leading to a large asymmetry. (A part of the coefficient for the vector current also behaves in the

same way because  $Z$  also couples to the vector current.) We also note that the asymmetry depends on the invariant dilepton mass. At and near the  $J/\psi$  and  $\psi'$  peaks, the asymmetry is small since the resonances couple to the vector leptonic currents.

#### 4. Numerical results and discussion

We have used the following parameters in our numerical calculations:

$$\begin{aligned}
m_b &= m_B = 5.28 \text{ GeV} \\
m_c &= 1.50 \text{ GeV} \\
m_s &= 0.50 \text{ GeV} \\
\Lambda \overline{MS} &= 0.1 \text{ GeV} \\
\eta &= \alpha_s(m_b)/\alpha_s(m_W) = 1.66 \\
V_{cs} &= 0.97, \quad V_{cb} = 0.045 \\
BR(B \rightarrow l\bar{\nu} + X) &= 0.12 \tag{11}
\end{aligned}$$

Our results are summarized in Figs. (1)–(3). In Fig. (1), we show the invariant dilepton mass distribution,  $dBR/dm_{ll}^2$ , corresponding to Eq. (9), for three assumed values of the top quark mass,  $m_t = 100, 150, 200 \text{ GeV}$ . Away from the resonance regions,  $m_{ll}^2 = m_{J/\psi, \psi'}^2$ , the dilepton mass distribution is sensitive to the top quark mass, as already noted in literature. We remark that we have a constructive (positive) interference, supporting the sign in Refs. [3],[8],[9], and in disagreement with Ref. [6]. The double differential distribution  $d^2 BR/d\hat{s}dz|_{\hat{s}=0.3}$  is shown in Fig. (2a) for  $m_t = 100, 150, 200 \text{ GeV}$ , whereas Fig. (2b) shows the same double differential distribution for a fixed value of  $m_t = 150 \text{ GeV}$  and three different values of the variable  $\hat{s}$ , as indicated. Finally, in Fig. 3, we show the asymmetry for the decays  $b \rightarrow s l^+ l^-$ , defined in Eq. (3), in three different invariant dilepton mass ranges for the assumed values of the top quark mass,  $m_t = 100, 150, 200 \text{ GeV}$ . The branching ratios for the processes  $b \rightarrow s l^+ l^-$  are given in Table (1) with the indicated cuts on the dilepton mass.

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The present (published) best limits on the inclusive FCNC  $b$ -decays involving dileptons have been obtained at the CERN SPPS:  $BR(B_{d,u} \rightarrow X_s + \mu^+ \mu^-) < 5.0 \times 10^{-5}$  (90% C.L.),<sup>[14]</sup> (see also earlier references in the Particle Data Group [12]) to be compared with the SM estimates:  $BR(B_{d,u} \rightarrow X_s + \mu^+ \mu^-) = 7.0 \times 10^{-6}$  (for  $m_t = 150 \text{ GeV}$ ), and  $BR(B_{d,u} \rightarrow X_s + e^+ e^-) = 1.2 \times 10^{-5}$  (for  $m_t = 150 \text{ GeV}$ ).<sup>[15]</sup> We mention here that the present best limit on the FCNC radiative  $B$ -decays  $B \rightarrow X_s + \gamma$  due to the CLEO collaboration is:  $BR(B \rightarrow X_s + \gamma) < 8.4 \times 10^{-4}$  (90% C.L.),<sup>[16]</sup> which is a factor  $\sim 2$  away from the expectations in the standard model [15].

The forthcoming run of the Fermilab Tevatron collider ( $p\bar{p}$ ,  $\sqrt{s} = 1.8 \text{ TeV}$ ) is expected to increase the present experimental sensitivity in the dilepton channel by one to two orders of magnitude. Likewise, the planned threshold  $e^+e^- B$ -factories and the proton-proton colliders, LHC and SSC, are expected to increase the sensitivity in this channel to a level of  $10^{-8}$  or better.

We are hopeful that the FCNC  $b$ -quark decays would be measured in one or more of the present and future  $B$ -facilities. In that event, the dilepton invariant mass distribution and the forward-backward asymmetry of the dileptons in the decays  $b \rightarrow s l^+ l^-$  studied here would be good measures of  $m_t$ . Conversely, knowing  $m_t$  directly - a possibility at the Tevatron collider if the present bounds on the top quark mass  $m_t = 130 \pm 30 \text{ GeV}$  from the LEP data are correct<sup>[17]</sup> - the experimental observation of the rare  $B$ -decays  $b \rightarrow s l^+ l^-$  and  $b \rightarrow s \gamma$  would provide precision tests of the standard model in the crucial and as yet untested FCNC sector of  $B$ -decays.

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$m_t$ (GeV)	(i)	(ii)	(iii)	(iv)	(v)
100	$1.8 \times 10^{-6}$	$6.8 \times 10^{-4}$	$4.3 \times 10^{-7}$	$3.3 \times 10^{-5}$	$1.8 \times 10^{-7}$
150	$2.7 \times 10^{-6}$	$6.8 \times 10^{-4}$	$6.6 \times 10^{-7}$	$3.3 \times 10^{-5}$	$4.1 \times 10^{-7}$
200	$4.3 \times 10^{-6}$	$6.8 \times 10^{-4}$	$1.1 \times 10^{-6}$	$3.3 \times 10^{-5}$	$8.5 \times 10^{-7}$

Table 1: Branching ratio for the decay  $B \rightarrow \ell^+ \ell^- X_s$  with  $\ell = e, \mu$  in different regions of the dilepton invariant mass. (i):  $1.0 \text{ GeV}^2 \leq s \leq (M_{J/\psi} - \delta)^2$ , (ii):  $(M_{J/\psi} - \delta)^2 \leq s \leq (M_{J/\psi} + \delta)^2$ , (iii):  $(M_{J/\psi} + \delta)^2 \leq s \leq (M_{\psi'} - \delta)^2$ , (iv):  $(M_{\psi'} - \delta)^2 \leq s \leq (M_{\psi'} + \delta)^2$ , (v):  $(M_{\psi'} + \delta)^2 \leq s \leq (M_b - M_s)^2$ , where we assume an energy resolution of  $\delta = 20 \text{ MeV}$  at the  $J/\psi$  and  $\psi'$  resonances.

### Figure Captions

**Figure (1):** The differential branching ratio  $dBR/d\hat{s}$  as a function of the scaled invariant dilepton mass  $\hat{s} = s/m_b^2$  in the decay  $b \rightarrow s \ell^+ \ell^-$ . Assumed top quark mass values  $m_t = 100, 150, 200 \text{ GeV}$  are indicated on the curves.

**Figure (2a):** The angular distribution  $d^2BR/dz d\hat{s}$  in the decay  $b \rightarrow s \ell^+ \ell^-$ , for a fixed value of the scaled dilepton invariant mass  $\hat{s} = 0.3$ . Assumed top quark mass values  $m_t = 100, 150, 200 \text{ GeV}$  are indicated on the curves.

**Figure (2b):** The angular distribution  $d^2BR/dz d\hat{s}$  in the decay  $b \rightarrow s \ell^+ \ell^-$  for a fixed value of the top quark mass,  $m_t = 150 \text{ GeV}$ , and the indicated values of the scaled invariant dilepton mass,  $\hat{s} = 0.3, 0.4, 0.6$ .

**Figure (3):** Forward-backward asymmetry of the dileptons in the decays  $b \rightarrow s \ell^+ \ell^-$ ,  $A(\hat{s})$ , as a function of the scaled invariant dilepton mass,  $\hat{s}$ , for the indicated values of the top quark mass.



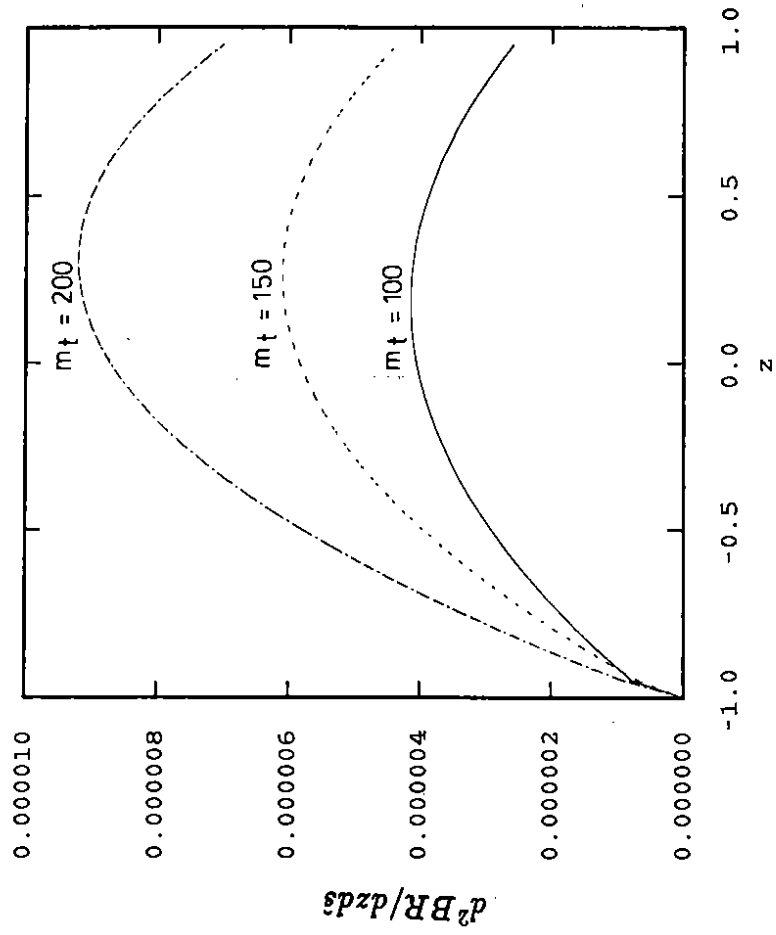


Figure (2a)

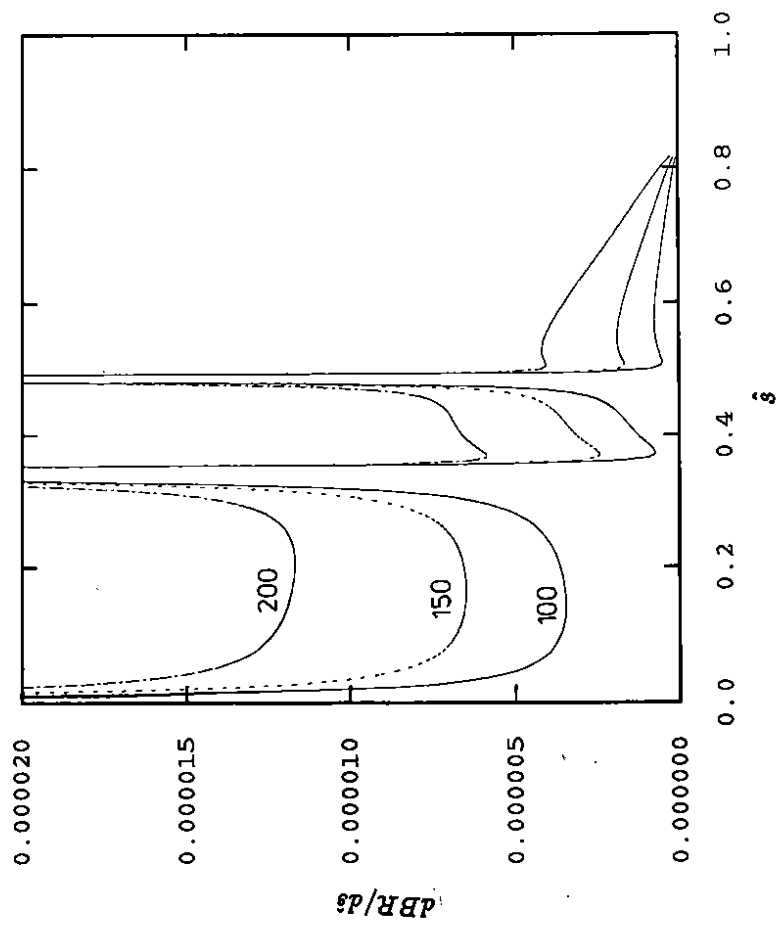


Figure (1)

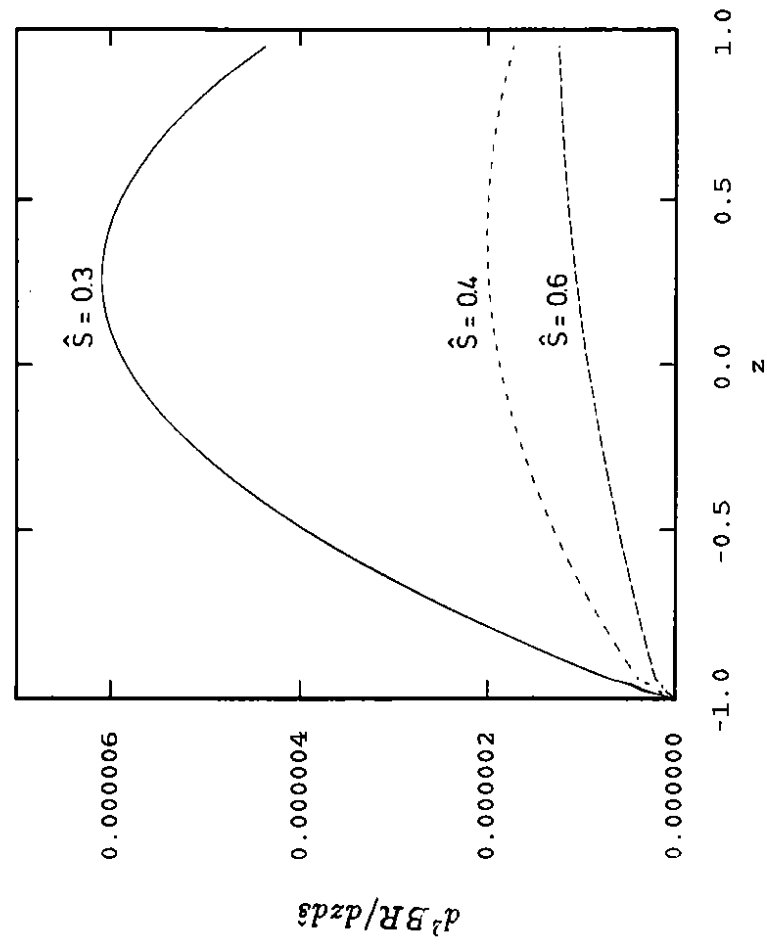


Figure (2b)

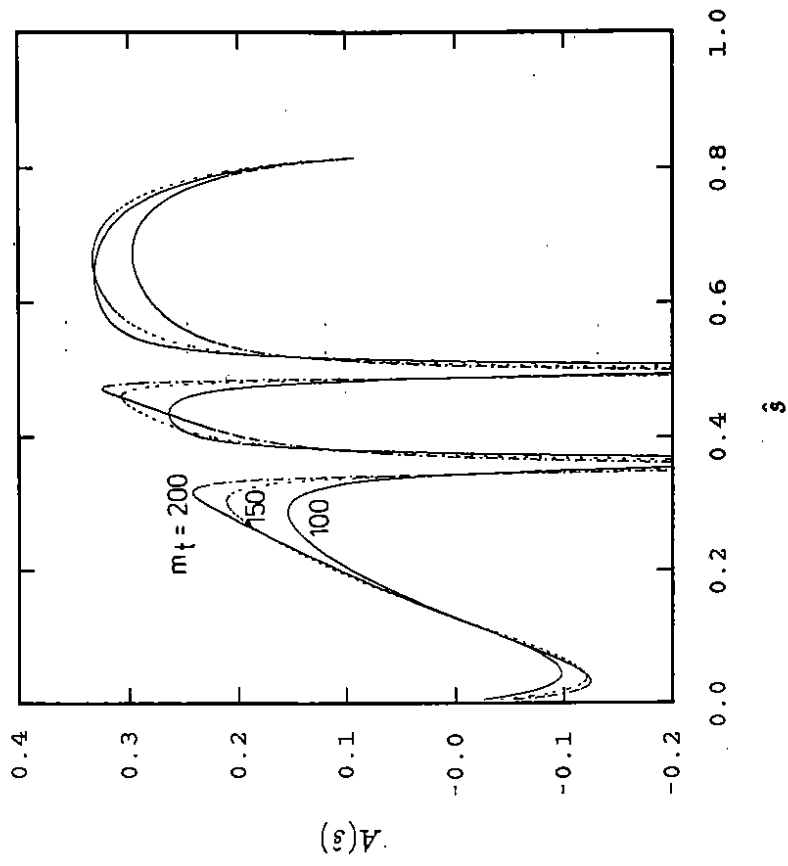


Figure (3)