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RARE DECAYS OF B-MESONS

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Introduction

This article is dedicated to the memory of Andrei D. Sakharov, a great scientist and human rights activist. Sakharov was blessed with the rare gift of prophetic prediction in matters concerning both science and society. His paper in 1967 on the baryon asymmetry of the universe [1] relating it to the baryon instability, CP-violation, and thermodynamic non-equilibrium, was a very long shot. In view of subsequent theoretical developments in grand unified theories of elementary particle physics and cosmology, where the Sakharov conditions can be accommodated, this paper represents indeed a very fine example of scientific genius and prophecy. His political judgement, exemplified by his visionary essay *Progress, Coexistence, and Intellectual Freedom*, written in 1968, was equally stunning [2]. Among other topics Sakharov was also very much interested in physics of the heavy quarks. In this paper we review theoretical predictions about an interesting aspect of heavy quark physics, namely rare phenomena in the decays of B-mesons involving flavour changing neutral current (FCNC) processes.

Rare B-decays, in particular, those involving the b-quark transitions $b \rightarrow (s,d) + \gamma$ and $b \rightarrow (s,d) + \ell\bar{\ell}$ $(\ell = e, \mu, \tau, \nu)$, provide important testing grounds for the standard model at the quantum (loop) level, since such transitions are forbidden in the Born approximation [3,4,5,6,7,8]. The rates and distributions in these decays provide sensitivity to possible new interactions at a much higher mass scale than can be probed directly at present energies, thus providing a window on the farther landscape in particle physics. In the standard model this landscape is dominated by the top quark whose mass is expected to lie in the range 100 $GeV < m_t < 200$ GeV. Precise measurements of rare B-meson transitions will not only provide a good estimate of the top quark mass but also of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements V_{td}, V_{ts}, V_{tb}, which in turn would provide very stringent tests of the unitarity of the CKM matrix.

We shall concentrate here on the decay processes $b \rightarrow s + X$ (with $X = \ell^+\ell^-, \nu\bar{\nu}, \gamma$) and the ones involving a d-quark, $b \rightarrow d + X$ (with $X = \ell^+\ell^-, \nu\bar{\nu}, \gamma$). The two classes of the FCNC B-decays, which can be experimentally distinguished by the strangeness quantum number $S=-1$ (for $b \to s + X$) and $S=0$ (for $b \to d + X$), can be called the CKM-allowed and -suppressed transitions, respectively. This distinction is based on the unitarity constraints in the standard model and the realization that their branching ratios are proportional to $|V_{ts}/V_{bc}|^2 \simeq 1$ (for $b \to s + X$) and $|V_{td}/V_{bc}|^2 = O(sin^2\,\theta_C)$ (for $b \to d + X$), as can be most easily seen in the Wolfenstein parametrization of the CKM matrix [9]. It can be shown in the standard model that the relative rates satisfy (upto small $SU(3)$-breaking effects denoted by δ_i):

$$\Gamma(b \to d+\gamma)/\Gamma(b \to s+\gamma) = \frac{|V_{td}|^2}{|V_{ts}|^2}(1+\delta_1) \quad (1)$$

$$\Gamma(b \to d+\ell\bar{\ell})/\Gamma(b \to s+\ell\bar{\ell}) = \frac{|V_{td}|^2}{|V_{ts}|^2}(1+\delta_2) \quad (2)$$

Similar relations also apply to the ratios of the exclusive rare decays, for example, $\Gamma(B \to \rho+\gamma)/\Gamma(B \to K^*+\gamma)$ and $\Gamma(B \to \rho + \ell\bar{\ell}\ (\ell=e,\mu,\tau,\nu))/\Gamma\ (B \to K^* + \ell\bar{\ell}\ (\ell=e,\mu,\tau,\nu))$. They are reminiscent of the relation for the $B^0-\bar{B}^0$ mass mixing ratio, x_d/x_s, with $x_{d,s} = (\Delta M_{d,s})/(\Gamma_{d,s})$, which is predicted to be: $x_d/x_s = |V_{td}|^2/|V_{ts}|^2(1+\delta)$, with δ again an $SU(3)$-breaking parameter [10]. It is important to check all these predictions of the standard model though theoretical estimates of x_s, pitching it around $x_s = O(10)$, would make its measurements very formidable. Rare B-decays provide an alternative though also challenging means to constrain the CKM matrix elements, V_{td}, V_{ts}, and V_{tb}.

We shall use here the effective Hamiltonian formalism for weak decays based on integrating out the heavy degrees of freedom [3,4,5,6]; the resulting Hamiltonian can then be written in terms of operator product involving light quark fields. In the present context, the heavy degrees are the W-boson and the top quark. The Wilson coefficients in the operator product expansion have been evaluated at the leading logarithm level by taking into account two-loop contributions to the anomalous dimension matrix. The resulting (QCD-improved) coefficients have been used to calculate the rare decay rates and distributions. These contributions represent the so-called short distance piece. The long distance contributions, due to the intermediate states consisting of light hadrons, are expected to be very significant for the process $b \to (s,d) + \ell\bar{\ell}\ (\ell=e,\mu)$ but they are generally estimated to be small for the radiative B-decays $b \to (s,d)+\gamma$, as well as, for those involving a neutrino pair in the decays $b \to (s,d) + \nu\bar{\nu}$ [11,12,13].

While inclusive radiative rare B-decay rates are probably reasonably estimated from the quark decays $b \to (s,d)+\gamma$, after including the QCD corrections in the Wilson coefficients, the gluon bremsstrahlung contributions are mandatory to get a non-trivial photon energy spectrum [7]. For the rare B-decays involving lepton pairs, $b \to (s,d) + \ell\bar{\ell}\ (\ell=e,\mu,\tau,\nu)$, the need for including the gluon bremsstrahlung corrections is probably less urgent though certainly desirable. The final spectra in the decays must also include the hadron wave-function effects, which are in general model dependent. However, this dependence can be very much reduced by constraining the wave function from existing data like, for example, using the inclusive lepton energy spectra in the CC semileptonic decays $B \to (X_c, X_u) + \ell\nu_\ell$. The resulting E_γ-spectrum

in this way has been computed in ref. [7]. A comparison of this with the corresponding spectrum from the dominant background process $B \to X_c + \gamma$, estimated using the quark decay $b \to c+d+\bar{u}+\gamma$ [14], shows that a good separation of signal and background events can be made for energetic photons with energy in excess of $\sim 2\ GeV$ in the B-hadron rest frame. We stress on the measurements of the inclusive photon energy spectrum in rare B-decays to get enhanced experimental sensitivity on such decays, and also because of the relative model independence of the spectrum.

To predict the exclusive rare B-decay rates one has to evaluate the hadronic matrix elements of the operators in the effective theory. To that end various approaches have been developed, mostly based on hadronic wave-function models using the basic quark decay diagrams [7,12,13]. In this context we note that significant progress has been made recently in developing an effective field theory governing the heavy hadron decays, by using the static limit for the heavy quarks in QCD [15,16]. The static limit provides the first term of a systematic $1/m_Q$ expansion, where m_Q is the quark mass. In a recent paper this method was applied to study the exclusive rare decays $B \to K^*\gamma$ and $B \to (K,K^*)\ell\bar{\ell}\ (\ell=e,\mu,\nu)$ [8], which we use here. Estimates of the branching ratios for $B^0_{s,d} \to \gamma\gamma$ and the purely leptonic decay rates for the neutral B-hadrons, $B^0_{s,d} \to \ell^+\ell^-$ with $(\ell=e,\mu,\tau)$, first calculated in [17], are also included for completeness.

This paper is organized as follows. First, we discuss the decay rates for $b \to (s,d)+\gamma$ in the lowest order (1 loop) and including the QCD corrections in the effective Hamiltonian method. The photon energy spectrum in the inclusive decays $B \to X_s + \gamma$ is evaluated in this approach and the dominant background from the CC decays $B \to X_c + \gamma$ is presented. Next, we discuss the calculations for the inclusive decays $b \to s + \ell\bar{\ell}\ (\ell=e,\mu,\nu)$, including the QCD corrections. Finally, we summarize rate estimates for the exclusive rare decays of the B-meson, $B \to K^*\gamma$, and $B \to (K,K^*)\ell\bar{\ell}$ $(\ell=e,\mu,\nu)$, as well as, $B^0_{s,d} \to \gamma\gamma$ and $B^0_{s,d} \to \ell^+\ell^-$ with $(\ell=e,\mu,\tau)$. The various inclusive and exclusive rates are given in table 1 and compared with the presently available experimental bounds.

Radiative Rare B-Decays

We first discuss the lowest order (1-loop) calculations for the FCNC inclusive radiative decays $B \to (X_d, X_s)+\gamma$. These decays are modelled after the b-quark decays. The matrix element for the process $b \to s+\gamma$ in the lowest order can be written as:

$$\mathcal{M}(b \to s\ +\gamma) = \frac{G_F}{\sqrt{2}}\frac{e}{2\pi^2}\sum_i V_{ib}V_{is}^{*}F_2^i(x_i)\,q^\mu\epsilon^\nu\bar{s}\sigma_{\mu\nu}(m_bR\ +\ m_sL)b \quad (3)$$

where G_F is the Fermi coupling constant, $L=(1-\gamma_5)/2$, $R=(1+\gamma_5)/2$, $x_i = m_i^2/m_W^2$, q_μ and ϵ_μ are, respectively, the photon four-momenta and polarization vectors, and the sum is over the charge $+2/3$ quarks, u, c, and t. The Inami-Lim function $F_2^i(x_i)$ derived from the penguin diagrams is given by[19]

$$F_2(x) = \frac{x}{24(x-1)^4}\left[6x(3x-2)\log x-(x-1)(8x^2+5x-7)\right] \quad (4)$$

In writing the expression for $F_2^i(x_i)$ above, we have left out a constant piece (independent of x_i), since its contribution in (3) sums to zero due to unitarity of the CKM matrix. For small x_i one has $F_2^i(x_i) \sim x_i$; hence the contribution of the u and c quarks in (3) can be neglected. Retaining only the top quark contribution in the amplitude one has:

$$\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \lambda_{ts} F_2^t(x_t) q^\mu \epsilon^\nu \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b \tag{5}$$

with $\lambda_{ts} = V_{tb} V_{ts}^*$. The amplitude for the CKM-suppressed FCNC radiative transition $b \to d + \gamma$ is obtained by replacing the s-quark variables by the d-quark ones. The CKM factor in this case is $\lambda_{td} \equiv V_{tb} V_{td}^*$. The functional dependence of the other FCNC processes $b \to (s,d) + \ell\bar{\ell}$ ($\ell = e, \mu, \tau, \nu$) on x_i is different, due to the inclusion of the box diagrams in addition to the electroweak penguins, but the behaviour of the resulting functions for small x_i is similar to the one in $F_2(x_i)$ and hence also in these processes the contribution of the intermediate u- and c-states can be neglected. The widths $\Gamma(b \to s + \gamma)$ and $\Gamma(b \to d + \gamma)$ in the lowest order are (dropping the superscript on F_2^t):

$$\Gamma(b \to s + \gamma) = |\lambda_{ts}|^2 \frac{G_F^2 m_b^5 \alpha}{32\pi^4} |F_2(x_t)|^2 \tag{6}$$

$$\Gamma(b \to d + \gamma) = |\lambda_{td}|^2 \frac{G_F^2 m_b^5 \alpha}{32\pi^4} |F_2(x_t)|^2 \tag{7}$$

From the expression for $F_2(x_t)$ it is easy to see that for low values of x_t ($x_t \ll 1$) one has a quartic suppression in m_t of the decay rates $\Gamma(b \to s+\gamma)$ and $\Gamma(b \to d+\gamma)$; this goes over to a constant behaviour for $x_t \gg 1$. So, while intrinsically there is a large m_t-dependence in the radiative rates for both the $b \to s+\gamma$ and $b \to d+\gamma$ transitions, this dependence in the m_t-range of present phenomenological interest is rather modest.

The rates for rare B-decays may be expressed in terms of the branching ratio for the inclusive CC semileptonic B-decays $B \to (X_{c,u}) \ell \nu_\ell$, which have been well measured. This gives:

$$BR(b \to s + \gamma) = 6 \frac{\alpha}{\pi} \frac{|\lambda_{ts}|^2}{|V_{bc}|^2} \frac{|F_2(x_t)|^2}{f(m_c/m_b)} (0.12) \tag{8}$$

$$\frac{BR(b \to d + \gamma)}{BR(b \to s + \gamma)} = \frac{|V_{td}|^2}{|V_{ts}|^2} \tag{9}$$

where the function $f(m_c/m_b)$ includes the phase space suppression in the CC semileptonic decay $b \to c + \ell\nu_\ell$ (numerically it is estimated to be $f(m_c/m_b) \simeq 0.44$), we have dropped the small contribution $\propto V_{bu}^2$ in the CC semileptonic decay width. The number in paranthesis corresponds to the B-hadron semileptonic branching ratio [23]. The second equality follows from (6) and (7).

The QCD corrections to the decay widths being discussed have been calculated in a number of papers. We first discuss the (by now well known) perturbative QCD corrections in the two-body decay $b \to s+\gamma$ and then take up the gluon bremsstrahlung contributions from the decay $b \to s + \gamma + g$ and the resulting photon energy spectrum.

To that end, one defines an appropriate operator basis. This consists of four fermion operators, involving the light quark fields and the covariant derivatives of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ groups, as well as the magnetic moment operators involving the photon and gluon fields. However, since the initial and final particles (quarks, photons and gluons) are on the mass-shell, one could use the equations of motion to get rid of the covariant derivatives and reduce the number of independent operators. To leading order in the small (weak)-mixing angles, a complete set of operators relevant for the processes $B \to X_s + \gamma$ is contained in the effective Hamiltonian [3,4,5,6]:

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} \lambda_t \sum_{j=1}^{8} C_j(\mu) O_j(\mu) \tag{10}$$

with $C_j(\mu)$ = Wilson coefficients at the scale μ, the various operators are defined as:

$$\begin{aligned}
O_1 &= (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}) \\
O_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta}) \\
O_3 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\left[(\bar{u}_{L\beta}\gamma_\mu u_{L\beta}) + \cdots + (\bar{b}_{L\beta}\gamma_\mu b_{L\beta})\right] \\
O_4 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})\left[(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma_\mu b_{L\alpha})\right] \\
O_5 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\left[(\bar{u}_{R\beta}\gamma_\mu u_{R\beta}) + \cdots + (\bar{b}_{R\beta}\gamma_\mu b_{R\beta})\right] \\
O_6 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})\left[(\bar{u}_{R\beta}\gamma_\mu u_{R\alpha}) + \cdots + (\bar{b}_{R\beta}\gamma_\mu b_{R\alpha})\right] \\
O_7 &= (e/16\pi^2)\, \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) b_\alpha F_{\mu\nu} \\
O_8 &= (g_S/16\pi^2)\, \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) T^A_{\alpha\beta} b_\beta G^A_{\mu\nu}
\end{aligned} \tag{11}$$

and g_S denotes the QCD coupling constant. Pertubative QCD corrections, contained in the Wilson coefficients $C_j(\mu)$, have been evaluated to leading logarithmic accuracy. It is known that the operators $O_3, ..., O_6$ only get contributions from operator mixing; as these coefficients are small, their effect is usually neglected. The coefficients $C_1(\mu), C_2(\mu), C_7(\mu)$ and $C_8(\mu)$, obtained by integrating out the top quark and the W-boson simultaneously, are given in refs. [4,5]. At $\mu = m_b$, which is the relevant scale for the b quark decay, these coefficients read as follows:

$$\begin{aligned}
C_1(m_b) &= \frac{1}{2}\left[\eta^{-6/23} - \eta^{12/23}\right] C_2(m_W) \\
C_2(m_b) &= \frac{1}{2}\left[\eta^{-6/23} + \eta^{12/23}\right] C_2(m_W) \\
C_7(m_b) &= \eta^{-16/23}\left\{C_7(m_W) - \frac{58}{135}\left[\eta^{10/23} - 1\right] C_2(m_W) - \frac{29}{189}\left[\eta^{28/23} - 1\right] C_2(m_W)\right\} \\
C_8(m_b) &= \eta^{-14/23}\left\{C_8(m_W) - \frac{11}{144}\left[\eta^{8/23} - 1\right] C_2(m_W) + \frac{35}{234}\left[\eta^{26/23} - 1\right] C_2(m_W)\right\}
\end{aligned} \tag{12}$$

with $\eta = \frac{\alpha_S(m_b)}{\alpha_S(m_W)}$. At the scale $\mu = m_W$, where the matching conditions from the lowest order (1 loop) results are imposed [19], we have (again to leading logarithmic accuracy):

$$C_j(m_W) = 0 \quad , \quad j = 1, 3, 4, 5, 6$$

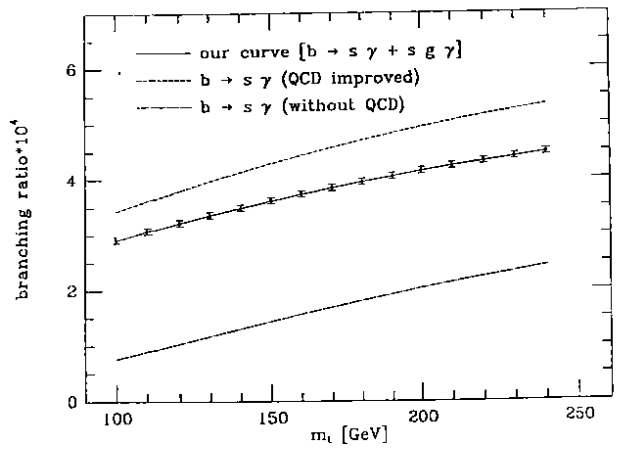

Figure 1: Inclusive branching ratio for the decays $B \to \gamma + X_s$ as a function of the top quark mass m_t. The charm quark mass dependence for $m_c = 1.5 \pm 0.2\ GeV$ is indicated by error bars. Also shown are the lowest order and QCD improved results for the two body decays $b \to s + \gamma$ (from ref [7]).

$$
\begin{aligned}
C_2(m_W) &= 1 \\
C_7(m_W) &= F_2(x) \\
C_8(m_W) &= -\frac{x}{8(x-1)^4}\left[6x\log x + (x-1)(x^2-5x-2)\right] \qquad (13)
\end{aligned}
$$

with $x = m_t^2/m_W^2$. For the two body decays $b \to s + \gamma$ (likewise for $b \to d + \gamma$), only the magnetic moment operator O_7 contributes and hence the QCD corrected rate can be represented by similar formulæ as given above for the lowest order, with the function $F_2(x_t)$ replaced by the QCD corrected function $c_7(m_b)$. The branching ratios for the decay $b \to s + \gamma$ (1 loop) and $b \to s + \gamma$ (QCD corrected) are shown in fig. 1 as functions of m_t. The net result of the QCD corrections for the two body radiative b-quark decays is very significant, increasing the rate by ~ 6 for $m_t = 100\ GeV$ to ~ 3 for $m_t = 200\ GeV$, assuming $\Lambda_{QCD} = 200\ MeV$.

It should be remarked here that there are two approximations that have gone into the derivation of the QCD corrections in (12). The first one is due to setting $\mu = m_W \approx m_t$ in the evaluation of QCD effects. This amounts to neglecting the running of the QCD coupling $\alpha_S(Q^2)$ between m_t amd m_W. Since the running is anyway small in the top quark mass range of current interest $89\ GeV < m_t < 200\ GeV$, the residual correction from this approximation is expected to be small (within 10%). These expectations have been confirmed in a recent calculation [20], in which an effective 5-quark Hamiltonian for the process $b \to s + \gamma$ has been obtained by integrating out first a heavy top quark with $m_t^2 \gg m_W^2$ and then integrating out the W-boson. The two-step integration procedure results in increasing the radiative decay width Γ (

$b \to s + \gamma$), which could be as large as 14% for $m_t = 250\ GeV$ and $\Lambda_{\overline{MS}} = 300\ MeV$. The second approximation is due to neglecting the effect of the mixing of the operators $O_3 ... O_6$ with the rest, which modifies the expression given above for $C_7(m_b)$. This approximation (i.e. truncating the operator basis) has also been recently investigated and the resulting correction to the decay rate Γ ($b \to s + \gamma$) is found to be $O(10\%)$ [21], reducing the estimates presented above. Since the two mentioned corrections are both $O(10\%)$, and they have opposite signs, the radiative rare b-decay rates presented in fig. 1 are not significantly altered.

Photon Energy Spectrum in FCNC Rare B-Decays

In this section we discuss the inclusive photon energy spectrum from the rare B-decays, $B \to \gamma + X_s$. Extension to the case $B \to \gamma + X_d$ due to the radiative quark decays $b \to d + \gamma$ and $b \to d + \gamma + g$ is straightforward, though the resulting spectrum has still to be computed. There are three inputs needed for a satisfactory calculation of the inclusive photon energy spectrum in a QCD-based framework:

- The $O(\alpha\alpha_S)$ perturbative QCD contribution, including the real (bremsstrahlung) and virtual corrections.
- An improved treatment of the photon energy spectrum near the end-point $x_\gamma \to 1$, where $x_\gamma = E_\gamma / E_\gamma^{max}$.
- Incorporation of the B-hadron wave-function effects.

The photon energy spectrum in perturbative QCD has been computed in ref. [7]. The relevant expressions are too long to be included in this paper and we restrict ourselves here to a discussion of the salient features. We recall that the dominant contribution to the rate and spectra from the decays $b \to s + \gamma$ and $b \to s + \gamma + g$ involve the operators O_1, O_2, O_7, and O_8. The infra-red and ultra-violet singularities in the spectrum and decay rate have been regulated using the dimensional regularization method, and the results correspond to using the so-called $\overline{MS}$ scheme [22].

Concerning the second point above, we note that the photon energy spectrum in $b \to s + \gamma + g$ has a nonintegrable infrared singularity for $x_\gamma \to 1$. Adding the process $b \to s + \gamma$ with its virtual corrections, the divergences cancel in a distribution sense and one gets a finite expression for $\frac{d\Gamma}{dx_\gamma}$ but the end-point spectrum feels the left-over effects of the infrared singularity. The leading behaviour of the photon spectrum near $x_\gamma \to 1$ can be traced back to the quark splitting $s \to s + g$, having the typical Altarelli-Parisi behaviour, which causes $\frac{d\Gamma}{dx_\gamma}$ to rise very steeply near the end-point, $x_\gamma \simeq 1$. This is all too familiar a situation in the calculation of energy spectra of gauge bosons in fixed order perturbation theory and the usual remedy of this (perturbative) malaise is to resum the leading (infrared) logarithms, thereby obtaining a better description of the spectra near the end-point.

We now discuss the B-meson wave function effects on the photon energy spectrum. These effects can at best be estimated in a model. However, one could fix the model parameters from independent data such as the lepton energy spectra in the CC semileptonic decays of the B-hadrons. A simple model, used in ref. [24,25] to implement the D- and B-meson bound state effects on the inclusive lepton energy spectra, was harnessed in ref. [7] to calculate the wave function effects on the photon energy and hadron mass spectra in the decays $B \to X_s + \gamma$. In this model the b-quark is given a non-zero momentum having a Gaussian distribution:

$$\phi(p) = \frac{4}{\sqrt{\pi}{p_F}^3}\exp\left(\frac{-p^2}{{p_F}^2}\right) \quad ; \quad p = |\vec{p}| \tag{14}$$

The energy-momentum constraint is imposed in the form:

$$W^2 = {M_B}^2 + {m_q}^2 - 2M_B\sqrt{p^2 + {m_q}^2} \tag{15}$$

where M_B is the B-meson mass, W, the effective mass of the b-quark, and m_q, the mass of the spectator quark in the B-meson, $B = b\bar{q}$ (for technical details see [7]).

There are two important parameters that influence the decay rates and shapes of the spectra, namely m_t and p_F, out of which p_F has been experimentally constrained to be $0.21\ GeV \leq p_F \leq 0.39\ GeV$, using a recent analysis of the CLEO data [27](the numbers from the ARGUS analysis are similar [26]). The other parameters used in the numerical estimates are: $m_W = 80.2$ GeV, $G_F = 1.16637 \cdot 10^{-5}$ GeV^{-2}, $\alpha = 1/137.036$, $\alpha_S = 0.23$ (corresponding to $N_f = 5$ and $\Lambda = 0.15$ GeV), $|V_{ub}| \approx 0.0075$ and $|V_{cb}| = |V_{ts}| \approx 0.045$.

The inclusive photon energy spectrum in the process $B \to X_s + \gamma$, for $p_F = 0.30\ GeV$ and $m_t = 100,\ 140,\ 200\ GeV$, is shown in fig. 2. We note that the shape of the spectrum is not very sensitive to m_t (though the rate shows more sensitivity); this can be attributed to the wave function effects which don't depend on m_t and dominate for large x_γ, overpowering the m_t-dependence of the perturbative QCD contribution. We also show here an estimate of the photon energy spectrum from the background processes $B \to X_c + \gamma$ computed in ref. [14] using the quark decay, $b \to c + d + \bar{u} + \gamma$. A clear separation between the background ($B \to X_c + \gamma$) and signal ($B \to X_s + \gamma$) is obtained for energetic photons having energy in excess of the kinematically allowed maximum for the indicated background process, $E_\gamma = (m_B^2 - m_{D^*}^2)/2m_B \simeq 2.0\ GeV$.

The same calculation also yields the invariant mass distribution of the hadrons recoiling against the photon in the inclusive process $B \to X_s + \gamma$. This distribution (which can be seen in ref. [7]) is fairly broad (with the full width at half maximum $\simeq 1\ GeV$), which one could interpret as a statement that the decays $B \to X_s + \gamma$ are expected to be dominated by multibody decays with an overall strangeness quantum number $S = -1$. Thus, one should anticipate a substantial penalty due to the small branching ratios in any specific channel. In particular, models giving large branching ratio for the decay $B \to K^* + \gamma$ are not tenable.

The decay width for $B \to X_s + \gamma$ is not effected by the exponentiation of the end-point spectrum near $x_\gamma \to 1$; likewise the width is also not sensitive to the B-meson wave-function effects (for details see [7]). The numerical results for the $O(\alpha\alpha_S)$

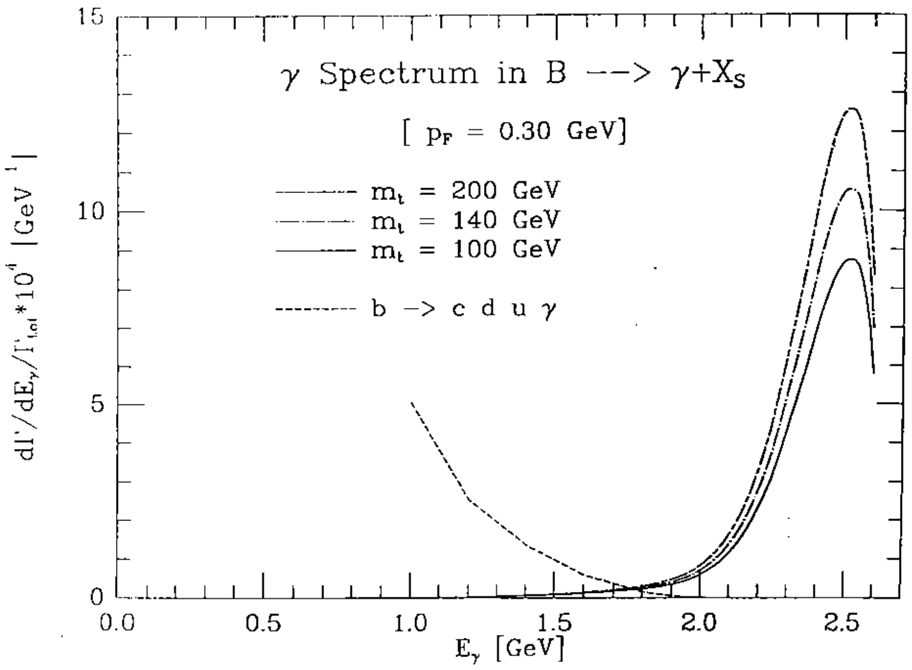

Figure 2: Inclusive photon energy spectrum for the process $B \to X_s + \gamma$ using perturbative QCD and the B-meson wave function model described in the text, with the parameter p_F set to 0.3 GeV and three representative values of the top quark mass, as indicated. The dashed curve corresponds to the photon energy spectrum from the background process $B \to X_c + \gamma$.

corrected branching ratio $BR(B \to \gamma + X_s)$ as a function of the top quark mass is shown in fig. 1. The dependence of the branching ratio on the charm quark mass in the range 1.3 GeV $\leq m_c \leq$ 1.7 GeV is indicated by the error bars. Comparison of this result with the QCD improved results for the two-body decay $b \to s+\gamma$ in fig. 1 shows that the additional corrections lower the previously discussed estimates of the inclusive branching ratio $BR(B \to \gamma + X_s)$ by about 15 % across the top quark mass range. To quote a number from the complete calculation: $BR(B \to \gamma + X_s) = 3.5 \times 10^{-4}$ for $m_t = 140$ GeV, and it rises to about 4.5×10^{-4} for $m_t = 250\ GeV$. Taking into account all the uncertainties, a firm prediction for the FCNC inclusive radiative B-decay in the standard model is:

$$BR(B \to X_s + \gamma) = (3-5) \times 10^{-4} \tag{16}$$

Finally, one could integrate the hadron mass spectrum in the range $m_K + m_\pi \leq m_{X_s} \leq 1.0\ GeV$ to get an upper limit on the branching ratio for $B \to K^* + \gamma$. The upper limit so obtained, however, may fringe on the actual branching ratio, as suggested by the recoil hadron mass spectrum in the semileptonic D-decays, which in the stated range is completely saturated by the K^* -resonance. This yields [7]: BR $(B \to K^* + \gamma) \simeq (3-6) \times 10^{-5}$. (Other estimates of this branching ratio are similar.) This is to be contrasted with the present experimental bound $BR(B \to K^* + \gamma) < 2.8 \times 10^{-4}$(@ 90% CL) [23], which is a factor $\sim (5-10)$ away from theoretical expectations.

FCNC B-Decays involving Dileptons

The effective Hamiltonian for the rare decays $b \to (s,d) + \ell\bar{\ell}$ $(\ell = e, \mu, \tau, \nu)$ is derived as before by integrating out the top quark and the W-bosons. In the approximation of keeping only dimension-6 operators, the appropriate basis for the FCNC B-decays involving dileptons consists of twelve operators and the effective Hamiltonian may be written as [5]:

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{j=1}^{12} C_j(\mu) O_j(\mu) \tag{17}$$

Detailed considerations, however, show that the coefficients of some of the operators and their mixing with the remaining ones are small and the basis may be truncated. The operators of interest, in addition to O_1, O_2, and O_7 already listed in the previous section, are given by

$$O_8 = \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu L b)(\bar{e}\gamma_\mu e) \tag{18}$$

$$O_9 = \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu L b)(\bar{e}\gamma_\mu \gamma_5 e) \tag{19}$$

and the Wilson coefficients are [3,4]

$$C_8(m_b) = C_8(M_W) + \frac{4\pi}{\alpha_s(M_W)}\left[\frac{4}{33}\left(1-\eta^{-11/23}\right) - \frac{8}{87}\left(1-\eta^{-29/23}\right)\right] \tag{20}$$

$$C_9(m_b) = C_9(M_W) \tag{21}$$

Here attention is drawn to the point that O_8 in the effective Hamiltonian here and the corresponding Hamiltonian for $b \to s + \gamma$ are different operators, hence also the Wilson coefficients C_8 in the two cases are different. Again, $\mu = m_b$ has been assumed to have the large logarithms of the type $\ln(M_W^2/m_b^2)$ included in the Wilson coefficients and not in the matrix elements of the operators. Since the operators O_j are used to calculate the matrix elements for the transition $b \to s$+(virtual) photon, we have to take a one loop matrix element of the operators O_1 and O_2. Note that the coefficient $C_9(m_b)$, which enters in the decays $B^0_{d,s} \to \ell^+\ell^-$ does not get renomalized due to QCD corrections. The decay rate for $b \to (s,d)\nu\bar{\nu}$, likewise, remains unchanged from its one-loop value, as shown below.

The coefficients $C_8(m_W)$ and $C_9(m_W)$ appearing above can be written in terms of three functions B, C and D, following ref. [5].

$$C_8(M_W) = \frac{1}{\sin^2\theta_W}B(x) + \frac{-1+4\sin\theta_W}{\sin^2\theta_W}C(x) + D(x) + \frac{4}{9} \tag{22}$$

$$C_9(M_W) = \frac{-1}{\sin^2\theta_W}B(x) + \frac{1}{\sin^2\theta_W}C(x) \tag{23}$$

with

$$B(x) = \frac{1}{4}\left[\frac{x}{x-1} - \frac{x}{(x-1)^2}\ln x\right] \tag{24}$$

$$C(x) = \frac{x}{4}\left[\frac{-x/2+3}{x-1} - \frac{3x/2+1}{(x-1)^2}\ln x\right] \tag{25}$$

$$D(x) = \left[\frac{19x^3/36 - 25x^2/36}{(x-1)^3} + \frac{x^4/6 - 5x^3/3 + 3x^2 - 16x/9 + 4/9}{(x-1)^4}\ln x\right] \tag{26}$$

The matrix elements for the processes of interest can now be written:

$$\mathcal{M}(b \to s + \ell^+\ell^-) = 2\sqrt{2}G_F \frac{\alpha}{sin^2\,\theta_W}\lambda_{ts}\frac{-1}{4\pi}\overline{\mathcal{M}} \tag{27}$$

where we have again dropped the small terms due to the intermadiate u-quark, and have used the CKM unitarity constraint to relate $V_{cb}V_{cs}^*$ to λ_{ts}. The reduced matrix element $\overline{\mathcal{M}}$ can be shown to be:

$$\overline{\mathcal{M}} = C_A\, \bar{s}_L\gamma_\mu b_L\, \bar{\ell}_L\gamma^\mu\ell_L + C_B\, \bar{s}_L\gamma_\mu b_L\, \bar{\ell}_R\gamma^\mu\ell_R + 2\,sin^2\theta_W C_7(\mu)\bar{s}i\sigma_{\mu\nu}q^\nu/q^2(m_s L + m_b R)b\,\bar{\ell}\gamma_\mu\ell \tag{28}$$

The functions C_A, C_B are given by:

$$C_A = sin^2\theta_W(-C_8(m_b) + C_9(m_b) - (3C_1(m_b) + C_2(m_b))g(m_c/m_b, q^2)) \tag{29}$$

$$C_B = sin^2\theta_W(-C_8(m_b) - C_9(m_b) - (3C_1(m_b) + C_2(m_b))g(m_c/m_b, q^2)) \tag{30}$$

and the function $g(m_c/m_b, q^2)$ arises from the one-loop matrix element of the four-quark operators [5]:

$$g(m_c/m_b, y) = -\left\{\frac{4}{9}\ln\left(\frac{m_c^2}{m_b^2}\right) - \frac{8}{27} - \frac{16}{9}\frac{m_c^2}{y} + \frac{2}{9}\sqrt{\left|1 - \frac{4m_c^2}{y}\right|}\left(2 + \frac{4m_c^2}{y}\right)G(y)\right\} \tag{31}$$

with

$$G(y) = \ln \left| \frac{1+\sqrt{1-\frac{4m_c^2}{y}}}{1-\sqrt{1-\frac{4m_c^2}{y}}} \right| + i\pi \text{ for } y > 4m_c^2 \tag{32}$$

$$G(y) = 2\arctan \left(\frac{1}{\sqrt{1-\frac{4m_c^2}{y}}} \right) \text{ for } y < 4m_c^2 \tag{33}$$

Note that $g(m_c/m_b, q^2)$ is renormalization scheme dependent; this dependence will be cancelled by that of the Wilson coefficient C_8. However, the present calculations have not been carried through to the desired accuracy and hence they leave an irritating (regularization) scheme dependence in the rates. For the top quark mass range of interest, this dependence is estimated to be $O(20\%)$ [5].

The invariant dilepton mass distribution in the inclusive decays can now be calculated easily with the help of matrix elements, given above. The calculations so far, however, take into account only the short distance contribution. In addition to this, there are long distance contributions from the intermediate $c\bar{c}$ state, $b \to sc\bar{c} \to s\ell\bar{\ell}$, since the $c\bar{c}$ state would contribute through the $J/\psi, \psi'$, and charmed hadrons above the $D\bar{D}$ threshold. The observed rates, $BR(B \to J/\psi + X) \sim 1\%$ and $BR(B \to \psi' + X) \sim 0.1\%$ [23] suggest that the long distance contribution to $b \to s + \ell\bar{\ell}$ ($\ell = e, \mu$) is as large as 10^{-3}, to be compared with the corresponding short distance contribution which are estimated to be $O(10^{-5})$. It has, therefore, been anticipated in a number of papers that the interference between the dispersive part of the hadronic contribution and the short distance amplitude would cause a large effect in the dilepton mass spectrum[11, 12,13]. Unfortunately, the situation about the relative sign of the two amplitudes has been controversial, though recent analyses of this problem presented in [18] support constructive (positive) interference between the resonant(l.d.) and the non-resonant (s.d.) contributions. The dilepton distribution, away from the resonances, is sensitive to the top quark mass as well as to non-standard interactions, such as due to additional Higgs doublets [5]. The branching ratios for the decays $B \to X_s + \ell\bar{\ell}(\ell = e,\ \mu)$ for the short distance contribution are listed in table 1. For the inclusive decays, one estimates $BR(B_{d,u} \to X_s + e^+e^-) = (1-2) \times 10^{-5}$ and $BR(B_{d,u} \to X_s + \mu^+\mu^-) = (6-8) \times 10^{-6}$, for the top quark mass range presently being entertained. These estimates are to be contrasted with a recent upper limit for the (averaged) B-meson branching ratio from the UA1 collaboration: $BR(B \to X + \mu^+\mu^-) = (5) \times 10^{-5}$ [28], indicating that the present experimental sensitivity is an order of magnitude away in this channel. The positive aspect of searches in the dilepton channel is that these are also accessible in the hadron colliders.

Exclusive Rare B-Decays

In this section we summarize the exclusive branching ratios for the rare B-decays, calculated recently in the limit of heavy b- and s-quarks. We also give results

for the purely leptonic decys $B_s^0 \to \ell^+\ell^-$ and the two-photon decay $B_s^0 \to \gamma\ \gamma$. The bilinear operators involving heavy quarks, can be denoted generically as: $\bar{h}'_{v'}\Gamma h_v$, where Γ denotes some combination of the Dirac gamma matrices and v and v' are the velocities of the heavy quarks of type h and h'. It is known that, neglecting $1/m_Q$-corrections, all the matrix elements of this type of operators are described in terms of a single universal function ξ, the so called Isgur-Wise function [16], which depends only on the product of the two velocities $v \cdot v'$. The matrix element of the operators are:

$$< H'^{(*)}(v',\epsilon')|\bar{h}'_{v'}\Gamma h_v|H^{(*)}(v,\epsilon) >= \xi(v\cdot v')\ \mathrm{Tr}\ \left\{ \bar{H}'^{(*)}(v',\epsilon')\Gamma H^{(*)}(v,\epsilon) \right\} \tag{34}$$

In the heavy quark limit, the rare B decays may be related to the semileptonic decays of the D- and B-mesons, since all these decays are described by the same function. This determines the Isgur-Wise function upto a parametrization. We shall henceforth assume a monopole form:

$$\xi(v\cdot v') = \frac{w_0^2}{w_0^2 - 2 + 2v\cdot v'} \tag{35}$$

and the best fit of the semileptonic D-decay data is obtained for $w_0 \approx 1.80$ [8]. We now review the exclusive rare B-decay rates.

Decay Rate for $B \to K^*\gamma$: In this case, only the operator O_7 from the list given for $b \to s + \gamma$ contributes.

$$< K^*\gamma|H_{eff}|B >= -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* C_7(m_b) < K^*\gamma|O_7(m_b)|B > \tag{36}$$

The hadronic matrix element needed for (36) is now evaluated in terms of the Isgur-Wise function. This then leads to the following decay rate:

$$\Gamma(B \to K^*\gamma) = \frac{1}{16\pi^4} G_F^2 \alpha |\xi(v\cdot v')|^2 |V_{tb}|^2 |V_{ts}|^2 |C_7(m_B)|^2 \times (k\cdot v)^3 m_B^2 \frac{m_K}{m_B}\left(1+\frac{m_B}{m_K}\right)^2 \tag{37}$$

where $v\cdot v' = \frac{1}{2m_Bm_K}(m_B^2 + m_K^2) \approx 3.04$. Defining the exclusive to inclusive decay rate ratio :

$$R(B \to K^*\gamma) \equiv \frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)} \tag{38}$$

one has:

$$R(B \to K^*\gamma) = \frac{m_K}{4m_B}|\xi(v\cdot v')|^2 \left(1+\frac{m_B}{m_K}\right)^2 \tag{39}$$

where, for simplicity we have used the simple analytic form for the QCD corrected rate for $b \to s + \gamma$, since the residual corrections are small. This gives $BR(B \to K^*\gamma) = 7.5 \times 10^{-5}$ and $R(B \to K^*\gamma) = 18\%$, for $m_t = 150\ GeV$ [8]. These numbers are in reasonable agreement with the estimates, described earlier, $BR(B \to K^*\gamma) = (4-6) \times 10^{-5}$ and $R(B \to K^*\gamma) = O(15)\%$ [7].

Decay Rates for $B \to K\ell^+\ell^-$, $B \to K^*\ell^+\ell^-$: In these decays all the five operators given above in the context of the inclusive decays $b \to s + \ell\bar{\ell}$ ($\ell = e, \mu, \tau, \nu$) are relevant. The contribution of O_1 and O_2 is via a one loop diagram which renders the invariant mass spectrum of the lepton pair nontrivial. The hadronic matrix elements are obtained as before from the Isgur-Wise function and are given in ref. [8]. The branching ratios for the exclusive decays $B \to K^* + \ell\bar{\ell}$ ($\ell = e, \mu$) and $B \to K + \ell\bar{\ell}$ ($\ell = e, \mu$) are given in table 1 for $m_t = 150\ GeV$. Note that the differential rate for $B \to K^* + e^+e^-$, as well as, $B \to X + e^+e^-$ becomes singular for $m_{ll} \to 0$ and thus the total rate depends logarithmically on the kinematic lower bound, namely $4m_\ell^2/m_B^2$. This is the reason why the two decay rates involving e^+e^- and $\mu^+\mu^-$ in these cases differ from each other. The rate for $B \to K + e^+e^-$ does not become singular in this limit, and thus the difference between the rates for $B \to K + e^+e^-$ and $B \to K + \mu^+\mu^-$ is negligible.

Decay Rates for $B \to K\nu\bar{\nu}$ and $B \to K^*\nu\bar{\nu}$: These processes are experimentally very difficult to measure, except probably in a high luminosity LEP environment, involving the decay $Z^0 \to b\bar{b}$, where such events have a nice tag: large missing energy and momentum in a b-quark jet with no charged lepton. From the theoretical point of view, on the other hand, these decays are much simpler to deal with, since the operator basis given for $b \to s + \ell\bar{\ell}$ ($\ell = e, \mu, \tau, \nu$) reduces to only one operator, which is the difference of O_8 and O_9. Thus, a left handed current for the neutrinos appears which couples pointlike to the left handed current of the $b \to s$ transition.

The Wilson coefficient of this operator is D_9 [29]

$$D_9(M_W) = \frac{1}{\sin^2\theta_w}\left[\frac{1}{8}x + \frac{3}{8}\frac{x(x-2)}{(x-1)^2}\ln x + \frac{3}{8}\frac{x}{x-1}\right] \tag{40}$$

D_9, like C_9, is not affected by QCD scaling corrections. This gives

$$H_{eff} = -\frac{8G_F}{\sqrt{2}}V_{tb}V_{ts}^* D_9(M_W)\left(\frac{\alpha}{4\pi}\right)(\bar{s}\gamma_\mu Lb)(\bar{\nu}\gamma_\mu L\nu) \tag{41}$$

The hadronic matrix elements have been given in ref. [8], where the differential distribution in the missing invariant mass and the decay rates are presented. The branching ratios for the inclusive decay $B \to X_s + \nu\bar{\nu}$ and the exclusive decays $B \to K + \nu\bar{\nu}$ and $B \to K^* + \nu\bar{\nu}$ are given in table 1, for $m_t = 150\ GeV$.

Decay Rates for $B_s^0 \to \ell^+\ell^-$ and $B_s^0 \to \gamma\gamma$: These decays were discussed some time ago in ref. [17] in the lowest (1 loop) order. With the help of the effective Hamiltonian formalism developed since then, it is easy to show that the decay rates don't get renormalized by QCD effects, and one has:

$$\Gamma(B_s^0 \to \tau^+\tau^-) = \frac{G_F^2}{2\pi}\left(\frac{\alpha}{4\pi}\right)^2 f_B^2 m_B m_\tau^2\sqrt{1 - \frac{4m_\tau^2}{m_B^2}}|V_{tb}V_{ts}^*|^2|C_9(m_W)|^2 \tag{42}$$

Here f_B is the B-meson pseudoscalar coupling constant for which we assume $f_B = 200\ MeV$, and numerically $C_9(m_W) = 3.73$ for $m_t = 150\ GeV$. The branching ratio for

Decay Modes	BR	Experimental Upper Limits (90 % C.L.)
$B_{d,u} \to X_s\gamma$	4.2×10^{-4}	–
$B_{d,u} \to K^*\gamma$	7.0×10^{-5}	2.8×10^{-4}[23]
$B_{d,u} \to X_s e^+e^-$	1.2×10^{-5}	–
$B_{d,u} \to X_s\mu^+\mu^-$	6.7×10^{-6}	5.0×10^{-5}[28]
$B_{d,u} \to K e^+e^-$	4.4×10^{-7}	5.0×10^{-5}[23]
$B_{d,u} \to K\mu^+\mu^-$	4.4×10^{-7}	1.5×10^{-4}[23]
$B_{d,u} \to K^*e^+e^-$	3.7×10^{-6}	–
$B_{d,u} \to K^*\mu^+\mu^-$	2.3×10^{-6}	2.3×10^{-5}[28]
$B_{d,u} \to X_s\nu\bar{\nu}$	6.6×10^{-5}	–
$B_{d,u} \to K\nu\bar{\nu}$	5.2×10^{-6}	–
$B_{d,u} \to K^*\nu\bar{\nu}$	2.0×10^{-5}	–
$B_s \to \gamma\gamma$	2.0×10^{-8}	–
$B_s \to \tau^+\tau^-$	1.8×10^{-7}	–
$B_s \to \mu^+\mu^-$	8.3×10^{-10}	–
$B_s \to e^+e^-$	2.0×10^{-14}	–

Table 1: Estimates of the branching fractions for FCNC B-decays in the standard model for m_t = 150 GeV and f_B = 200 MeV. Experimental upper limits are also listed.

$B_s^0 \to \ell^+\ell^-$ can be written as

$$BR(B_s^0 \to \tau^+\tau^-) = 1.8 \times 10^{-7}\left(\frac{f_B(GeV)}{0.2}\right)^2\left(\frac{|C_9(m_W)|}{3.73}\right)^2 \tag{43}$$

The rates for the decays $B_s^0 \to \mu^+\mu^-$ and $B_s^0 \to e^+e^-$ are suppressed compared to the corresponding rate for $\tau^+\tau^-$ by $(m_\mu/m_\tau)^2$ and $(m_e/m_\tau)^2$, respectively, and they are indeed very small, as can be seen in table 1. Again, one can express the decay rates $\Gamma(B_d^0 \to \ell^+\ell^-)$ in terms of the rates for $\Gamma(B_s^0 \to \ell^+\ell^-)$ using the relation in the CKM model: $\frac{\Gamma(B_d^0 \to \ell^+\ell^-)}{\Gamma(B_s^0 \to \ell^+\ell^-)} = \frac{|V_{td}|^2}{|V_{ts}|^2}(1+\delta)$, with δ an $SU(3)$-breaking parameter. Since the present unitarity bound on the CKM ratio is $\frac{|V_{td}|^2}{|V_{ts}|^2} \leq 0.16$, one expects the branching ratios for $B_d^0 \to \ell^+\ell^-$ to be suppressed by an order of magnitude compared to the corresponding decays $B_s^0 \to \ell^+\ell^-$, given in table 1.

Finally, the branching ratio for the process $B_s^0 \to \gamma\gamma$ is:

$$BR(B_s^0 \to \gamma\gamma) = 1.5 \times 10^{-8}\left(\frac{f_B(GeV)}{0.2}\right)^2\left(\frac{|I^+(m_t)|}{0.43}\right)^2 \tag{44}$$

where the function $I^+(m_t)$ is given in ref. [17]. The presently available bounds from experimental searches are also listed in table 1.

Concluding Remarks

We have presented estimates for a number of inclusive and exclusive rates and distributions involving FCNC B-decays in the standard model. None of these decays have been measured so far. The rare B-decays discussed here have branching ratios which are spread over ten orders of magnitude! Some of them could cryptically be called *The rarest of rarities!* like $B_{d,s} \to e^+e^-$, which may be considered as defining the yonder in the field. They will remain numbers of academic interest if FCNC processes are described by the standard model alone. However, it is not hard to imagine models where the helicity suppressions are not operative. Some other decays, however, could aptly be called *Rare decays around the corner*, like $B \to X_s + \gamma$ and $B \to K^* + \gamma$, having branching ratios probably less than an order of magnitude away from the present experimental limits. We have mostly concentrated in this report on this category. These decays test the standard model at the quantum level; they may also turn out to be places where new physics may cast its shadow in the near future. It is, however, clear from the foregoing that experimental sensitivity to these processes, particularly the more interesting CKM-suppressed $b \to d+X$ transitions, can only be achieved in high luminosity B-physics facilities, such as threshold B-factories, high-luminosity upgrade of LEP-100, and hadron colliders such as the Tevatron, LHC and SSC, with dedicated detectors for B-physics studies. We hope that the results summarized in this report will motivate the experimental physics community and the machine builders in providing the appropriate environments, where at least some of the issues discussed here could be meaningfully addressed.

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References

[1] A. D. Sakharov, Pis 'ma ZhETF 5 (1967) 32.

[2] A. D. Sakharov, Progress, Coexistance and Intellectual Freedom (New York: W.W. Norton) (1968).

[3] M. Shifman, A.I. Vainshtain and V. Zakharov, Phys. Rev. D18 (1978) 2583; B. Campbell and P. J. O'Donnell, Phys. Rev. D25 (1982) 1989; P. J. O'Donnell, Phys. Lett. B175 (1986) 369; S. Bertolini, F. Borzumati and A. Masiero, Phys. Rev. Lett. 59 (1987) 180; N. Deshpande et al., Phys. Rev. Lett. 59 (1987) 183; N. Deshpande and J. Trampetic, Phys. Rev. Lett. 60 (1988) 2583; R. Grigjanis et al., Phys. Lett. B213 (1988) 355; W. Jaus and D. Wyler, Phys. Rev. D41 (1990) 3405; S. Bertolini et al., MPI München Report MPI-PAE/PTh43/90 (1990).

[4] B. Grinstein, M. J. Savage and M. B. Wise, Nucl. Phys. B319 (1989) 271.

[5] B. Grinstein, R. Springer and M. B. Wise, Nucl. Phys. B339 (1990) 269.

[6] G. Cella et al., Phys. Lett. B248 (1990) 181.

[7] A. Ali and C. Greub, Zeit. f. Phys. C49 (1991) 431; Phys. Lett. B259 (1991) 182.

[8] A. Ali and T. Mannel, DESY Report 91-014 (1991) (to appear in Phys. Lett. B).

[9] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[10] A. Ali and Z.Z. Aydin Nucl. Phys. B148 (1979)165.

[11] C.S. Lim, T. Morozumi and A.I. Sanda, Phys. Lett. B218 (1989) 343.

[12] C. Dominguez, N. Paver and Riazuddin, Phys. Lett. B214 (1988) 459.

[13] N. Deshpande, Oregon Inst. Theor. Phys. Report OITS 413 (1989).

[14] G. Eilam et al., Univ. of Western Ontario Report (1990).

[15] See, for example, J.D. Bjorken, SLAC Report SLAC-PUB-5389 (1990).

[16] N. Isgur and M. Wise, Phys. Lett. B232 (1989) 113; Phys. Lett. B237 (1990) 527.

[17] B.A. Campbell and P.J.O'Donnell, Phys. Rev. D25 (1982)25.

[18] A. Ali, T. Mannel, and T. Morozumi, DESY Report (to be published); P.J.O'Donnell and H.K.K. Tung, Phys. Rev. D43 (1991) R2067; N. Paver and Riazuddin, ICTP Trieste Report(1991).

[19] T. Inami and C.S. Lim, Progr. Theor. Phys. 65 (1981) 297.

[20] P. Cho and B. Grinstein, Harvard Univ. Report, HUTP-91/A020(1991).

[21] M. Misiak, Warsaw Univ. Report, IFT-9/91(1991).

[22] W.A. Bardeen et al., Phys. Rev. D18 (1978) 3998.

[23] Particle Data Group, Phys. Lett. B 239 (1990).

[24] A. Ali and E. Pietarinen, Nucl. Phys. B154 (1979) 519.

[25] G. Altarelli et al., Nucl. Phys. B208 (1982) 365.

[26] H. Albrecht et al. (ARGUS), Phys. Lett. B234 (1990) 409; DESY Report, DESY 90-088 (1990).

[27] R. Fulton et al. (CLEO), Phys. Rev. Lett. 64 (1990) 16.

[28] C. Albajar et al. (UA1), Phys. Lett. B262 (1991) 163.

[29] W. Hou, R. Willey and A. Soni, Phys. Rev. Lett. 58 (1987) 1608.