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Branching Ratios and CP Asymmetries in the Decay $B \to VV$

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Abstract

We carry out a systematic study of branching ratios, angular correlations, and CP asymmetries in the decay of neutral and charged B mesons to final states consisting of two vector mesons. The renormalization group improved effective Hamiltonian is evaluated in the vacuum insertion (factorization) approximation. OZI suppressed and annihilation terms are neglected. Current matrix elements are evaluated using the wave functions of Bauer, Stech and Wirbel. Branching ratios and angular correlations among subsequent decays of the vector mesons are calculated for 34 channels and a comparison is made with the data. As a first approximation, the calculational scheme provides a useful framework with which to organize the data. Interesting direct CP asymmetries are particularly evident in $K^*\omega$ and $K^*\rho$ final states, where branching ratios are moderate. They are excellent probes of penguin term influence on decay amplitudes. Even larger direct asymmetries are present in $\omega\rho$ and $\rho\rho$ final states where, however, branching ratios are low and results are very model dependent. We show how B^0 - $\bar{B}^0$ mixing phases are influenced by phases in the direct amplitudes. The effect is particularly strong for $\bar{K}^{*0}\bar{D}^{*0}$ final states.

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1 Introduction

The nonleptonic weak decays of B mesons are very interesting for several reasons. First, CP violation in the B-meson system will eventually give us information about the CP violating phase in the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix.[1] Second, nonleptonic weak decays will give additional clues for determining the absolute values of the quark mixing parameters, in particular the fundamental ratio $|V_{ub}/V_{cb}|$, although it is expected that more solid information on this ratio will come from semileptonic B decays. Last, the dynamics of the nonleptonic weak decays in the framework of the standard model is not yet well understood. One of the problems in calculating the transition amplitudes for nonleptonic weak decays is that one needs to evaluate the hadronic matrix elements of certain four–quark operators which can be done in QCD only with non–perturbative methods. It is clear that in order to gain information on the $|V_{ub}/V_{cb}|$ ratio or on the CP violating phase from nonleptonic B decays further progress is needed in computing the relevant hadronic matrix elements for these decays.

The usual route to calculating these hadronic matrix elements for B decays is to start from the effective, QCD corrected, Hamiltonian for the $\Delta b = 1$ nonleptonic decays in the six–quark model (i.e. including the t quark).[2],[3] This gives the weak Hamiltonian in terms of four–quark operators. For computing the hadronic matrix elements of these four–quark operators the factorization approximation is used.[4] Then the hadronic matrix elements are given in terms of current matrix elements (matrix elements of two–quark operators) as they appear also in semileptonic decays. These current matrix elements are much easier to calculate and many models have been proposed for them.

In this paper we intend to apply this framework for calculating nonleptonic $\bar{B}^o$ and B^- decays into two vector mesons. Actually this has been done in the past by various authors [5] in particular by Bauer, Stech and Wirbel (BSW)[6]. Concerning current matrix elements we shall use their results, which they obtained from relativistic oscillator wave functions at infinite momentum. It is the purpose of this paper to calculate systematically all nonleptonic decays of $\bar{B}^o$ and B^- mesons into two vector particles V_1 and V_2 independent of whether they have large or small branching ratios. However, we exclude from our consideration channels which arise purely from penguin diagrams. Since in the near future due to the efforts of the ARGUS and CLEO collaboration B decays with branching ratios as small as 10^{-4} seem experimentally feasible we find it timely to get an overview about all two-body vector decays of B's which are possible on the basis of the complete effective weak Hamiltonian for $\Delta b = 1$ nonleptonic decays in the context of CKM mixing matrix.

In addition to rates we shall calculate the full angular distribution which can be obtained by measuring the combined angular correlations of the decay products of V_1 and V_2. These coefficients of the angular correlations serve as further tests of the combined short distance weak effective Hamiltonian factorization approach together with the BSW current matrix elements. In addition we intend to look at asymmetries which occur in the decay of the B into two vector mesons which in the absence of unitary phases are signals of CP violation in the b sector. These CP-odd asymmetries can originate only through interference of at least two amplitudes contributing to the same process with different phases coming either from genuine CP violating effects in the CKM matrix or from unitary phases from hadron dynamics. To generate the CP violating effects, in the absence of unitarity phases, we need the full effective weak Hamiltonian including QCD renormalization effects when flavor symmetry breaking (FSB) is considered. For the b sector the FSB effects enter via the penguin diagrams as proposed by Shifman, Vainshtein and Zakharov [7] for the s sector, and via box diagrams. This full $\Delta b = 1$ effective weak Hamiltonian has been calculated by Ponce [3]following the work of Gilman and Wise [2] for the strange sector, where FSB effects enter only via the penguin diagrams.

Concerning final state interactions and strong phases, our philosophy in this work is to present our results for the most part as if they were not there. If they can be calculated, their effects can be included as sketchedin Section 3. Attempts at this difficult calculation have been made along two approaches: (1) absorptive parts of penguin diagrams at the quark level [8] [9] and (2) K- matrix formalism [10], [11] at the hadron level. We intend to return to this matter in a future publication.

We shall try several approximate versions of the Ponce Hamiltonian in order to get an overview on the dependence of QCD corrections and/or penguin terms. In addition we shall invoke also the $N_c \to \infty$ limit when evaluating the hadronic matrix element to have a realistic model for some of the suppressed B decays [6].

In Section 2 we briefly recapitulate the weak effective $\Delta b = 1$ Hamiltonian and discuss the influence of the penguins and/or QCD corrections in the different sectors. The resulting effective Hamiltonian is then applied to the calculation of two–vector meson decays of $\bar{B}^0$ and B^- mesons. Here we use the factorization or vacuum saturation approximation (VSA). (In the text the final formulas are given only for one decay channel of $\bar{B}^0$ and B^-. The results of all the many other channels are put into Appendix B.) Section 3 contains an outline of the calculation of helicity amplitudes in terms of the invariant amplitudes of current matrix elements as used in the BSW model. The angular correlations are also written down in this section. Their derivation is relegated to Appendix A. The matrix elements in the VSA are written down in Section 4. We report our results in Section 5 and discuss their relevance for future experiments and refinement of the theory beyond the approximations for the calculation of weak hadronic matrix elements of B mesons used in this effort. Section 6 is a brief summary and overview with some concluding remarks.

2 The $\Delta b = 1$ Effective Hamiltonian

In this section we begin with a short description of the different pieces of the $\Delta b = 1$ weak Hamiltonian. In the absence of strong interaction effects and in the limit $m_W \to \infty$ the nonleptonic Hamiltonian has the usual current × current form

$$H_{wk} = \frac{G}{\sqrt{2}} \left(J_\mu J^{\mu +} + h.c. \right) \tag{2.1}$$

where in the CKM model the current J_μ is :

$$J_\mu = \bar{u}\gamma_\mu (1-\gamma_5)\tilde{d} + \bar{c}\gamma_\mu (1-\gamma_5)\tilde{s} + \bar{t}\gamma_\mu (1-\gamma_5)\tilde{b}$$

with color indices summed over. Here $\tilde{d}$, $\tilde{s}$, $\tilde{b}$, are the transformed eigenstates of the weak interactions which are related to the strong interaction eigenstates d, s and b by the unitary CKM matrix V [1]. G is the Fermi coupling constant. The part of (2.1) responsible for $\Delta b = 1$ transitions has six different pieces which, following Ponce [3], are

$$\Delta s = 0,\ \Delta b = -\Delta c = 1 :$$

$$H_1 = \frac{G}{\sqrt{2}} V_{ud}^* V_{cb} \left(\bar{d}u\right) (\bar{c}b) \tag{2.2a}$$

$$\Delta c = 0,\ \Delta b = \Delta s = 1 :$$

$$H_2 = \frac{G}{\sqrt{2}} \left\{ V_{us}^* V_{ub} (\bar{s}u)\ (\bar{u}b) + V_{cs}^* V_{cb} (\bar{s}c)\ (\bar{c}b) + V_{ts}^* V_{tb}\ (\bar{s}t)\ (\bar{t}b) \right\}. \tag{2.2b}$$

$$\Delta c = \Delta s = 0,\ \Delta b = 1 :$$

$$H_3 = \frac{G}{\sqrt{2}} \left\{ V_{ud}^* V_{ub} (\bar{d}u)(\bar{u}b) + V_{cd}^* V_{cb} \left(\bar{d}c\right) (\bar{c}b) + V_{td}^* V_{tb} \left(\bar{d}t\right) (\bar{t}b) \right\} \tag{2.2c}$$

$$\Delta b = \Delta s = -\Delta c = 1 :$$

$$H_4 = \frac{G}{\sqrt{2}} V_{us}^* V_{cb} (\bar{s}u)\ (\bar{c}b) \tag{2.2d}$$

$$\Delta b = \Delta c = \Delta s = 1$$

$$H_5 = \frac{G}{\sqrt{2}} V_{ub}^* V_{cs} (\bar{s}c)\ (\bar{u}b) \tag{2.2e}$$

$$\Delta s = 0,\ \Delta b = \Delta c = 1$$

$$H_6 = \frac{G}{\sqrt{2}} V_{cd}^* V_{ub} (\bar{d}c)\ (\bar{u}b) \tag{2.2f}$$

where V_{ij} is the ij element of the matrix V. $(\bar{a}b)(\bar{c}d)$ refers to the color singlet structure $\bar{a}_i\gamma_\mu(1-\gamma_5)b_i\bar{c}_j\gamma^\mu(1-\gamma_5)d_j$ (sum over i and j understood).

When strong interactions are present the $H_1, H_2, \ldots, H_6$ are modified. At very high energies, i.e. for energies above m_W, the strong interaction effects are negligible and the effective weak Hamiltonian for the $\Delta b = 1$ sector is described by (2.2a–f). For energies below m_W the strong- interaction renormalization effects will change the weak Hamiltonian. The operators in (2.2) get different coefficients and in addition new operators are produced with coefficients which are obtained from solutions of the renormalization- group equations (RGE), i.e.

$$H^{eff}_{wk} = \sum_i a_i O_i \tag{2.3}$$

where at short distances only a finite number of new operators, i.e. only the lowest–dimension operators O_i are relevant. The coefficients a_i are functions of m_W, of the subtraction point μ and for the cases of FSB they are functions of the heavy quark masses m_t, m_b and m_c. In this respect the various pieces of H^{eff}_{wk} look quite different after renormalization. Only for H_2 and H_3 do the FSB effects enter via the box and penguin diagrams and new operators O_i appear. For all the other cases, H_1, H_4, H_5, and H_6, there are no penguin contributions. Then the effect of the QCD corrections is as follows. (We consider H_1 as an example. The pieces H_4, H_5, and H_6 are treated in an analogous manner.) In the RGE procedure the operators that renormalize multiplicatively are

$$O_{1\pm} = (\bar{d}u)(\bar{c}b) \pm (\bar{d}b)(\bar{c}u) \tag{2.4}$$

Taking the subtraction point $\mu = m_c$ the weak interaction Hamiltonian H_1 in (2.2a) is changed through QCD corrections into the following form:

$$H_1^{eff} = \frac{G}{2\sqrt{2}} V^*_{ud} V_{cb} \{c_- O_{1-} + c_+ O_{1+}\} \tag{2.5}$$

where for $\alpha_s = 1$, $m_W = 100$ GeV, $m_t = 30$ GeV, $m_b = 5$ GeV, $m_c = \mu = 2$ GeV the coefficients $c_\pm$ take the following values: $c_+ = 0.774$, $c_- = 1.669$ [3]. For $c_+ = c_- = 1$ we recover the uncorrected H_1 in (2.2a). For H_4, H_5 and H_6 the analysis is exactly the same and follows by changing $O_{1\pm} \to O_{4\pm}, O_{5\pm}, O_{6\pm}$ with the same coefficients $c_\pm$ as given above. Since Ponce's calculation the limits on the top mass have steadily increased. Thus the coefficients would change especially since now it is believed that $m_t > m_W$. We have examined the changes which would occur if Ponce's calculation were to be repeated with much larger m_t. It was found that the changes are not significant in particular in view of uncertainties due to the scale of μ in connection with Λ_{QCD}, i.e. α_s. Therefore we decided in this survey that we would continue to use Ponce's coefficients which in any case will be modified for phenomenological reasons in various models given below.

In the case (2.2b) and (2.2c) penguin diagrams enter and the analysis becomes rather complicated. Let us consider (H_2), for example. For energies above m_W the effective weak Hamiltonian is given by (2.2b) which, using the relation $V^*_{us} V_{ub} + V^*_{cs} V_{cb} + V^*_{ts} V_{tb} = 0$ following from unitarity of the CKM matrix V, is written as

$$\begin{aligned} H_2 &= -\frac{G}{\sqrt{2}}\{V^*_{cs}V_{cb}((\bar{s}u)(\bar{u}b)-(\bar{s}c)(\bar{c}b)) \\ &\quad + V^*_{ts}V_{tb}((\bar{s}u)(\bar{u}b)-(\bar{s}t)(\bar{t}b))\} \\ &= -\frac{G}{2\sqrt{2}}\left\{V^*_{cs}V_{cb}\left(O^c_{2+}+O^c_{2-}\right)+V^*_{ts}V_{tb}\left(O^t_{2+}+O^t_{2-}\right)\right\} \end{aligned} \tag{2.6}$$

where

$$O^q_{2\pm} = \left[(\bar{s}u)(\bar{u}b) \pm (\bar{s}b)(\bar{u}u)\right] - \left[(\bar{s}q)(\bar{q}b) \pm (\bar{s}b)(\bar{q}q)\right] \tag{2.7}$$

For B decays we are interested in H_2 down to energies $\kappa = m_c$. In this case the penguin contributions introduce new operators so that H^{eff}_2 takes the following form [3]:

$$H^{eff}_2 = -\frac{G}{2\sqrt{2}}\left\{V^*_{cs}V_{cb}\left(c_+O^c_{2+} + c_-O^c_{2-}\right) + V^*_{ts}V_{tb}\sum_{i=1}^{6} c_iO_i\right\} \tag{2.8}$$

with

$$\begin{aligned} O_1 &= (\bar{s}b)_L(\bar{u}u)_L & O_4 &= (\bar{s}_ib_j)_L \textstyle\sum_q(\bar{q}_jq_i)_L \\ O_2 &= (\bar{s}_ib_j)_L(\bar{u}_ju_i)_L & O_5 &= (\bar{s}\lambda^ab)_L \textstyle\sum_q(\bar{q}\lambda^aq)_R \\ O_3 &= (\bar{s}b)_L \textstyle\sum_q(\bar{q}q)_L & O_6 &= (\bar{s}b)_L \textstyle\sum_q(\bar{q}q)_R \end{aligned} \tag{2.9}$$

The coefficients $c_\pm$, c_1, c_2, c_3, c_4, c_5 and c_6 have been calculated by Ponce [3]:

$$\begin{aligned} c_- &= 0.774, & c &= 1.669, & c_1 &= -0.443, & c_2 &= 2.316, \\ c_3 &= 0.037, & c_4 &= -0.084, & c_5 &= -0.052, & c_6 &= -0.010 \end{aligned} \tag{2.10}$$

We notice that $c_\pm$ are unchanged as compared to $c_\pm$ in H_1. Down to $\kappa = m_c$ the operators $O^c_\pm$ still renormalize multiplicatively as in H_1. The subscripts L and R refer to the Dirac operators $L = \gamma_\mu(1-\gamma_5)$ and $R = \gamma_\mu(1+\gamma_5)$, respectively.

The $\Delta c = \Delta s = 0$, $\Delta b = 1$ sector is treated analogously. The Hamiltonian H_3 in (2.2c) is written as

$$H_3 = -\frac{G}{2\sqrt{2}}\left\{V^*_{cd}V_{cb}\left(O^c_{3-}+O^c_{3+}\right) + V^*_{td}V_{tb}\left(O^t_{3+}+O^t_{3-}\right)\right\} \tag{2.11}$$

where

$$O_3^{q\pm} = \left[(\bar{d}u)\,(\bar{u}b) \pm (\bar{d}b)(\bar{u}u)\right] - \left[(\bar{d}q)\,(\bar{q}b) \pm (\bar{d}b)(\bar{q}q)\right] \tag{2.12}$$

Penguin diagrams are present again. For H_3^{eff} we have the analogous formula as (2.8) with the replacement of the s quark, in all places where it occurs in the operators and the CKM matrix elements, by the d quark.

In the following we shall consider all $\bar{B}^0$ and $\bar{B}^-$ decays into two vector mesons which can proceed through $H_1, H_2, \ldots, H_6$. We shall limit ourselves to those channels which have contributions from W emission, i.e. $O_\pm$, and possibly penguin contributions, i.e. through operators O_3, O_4, O_5, O_6 . We shall not consider channels that can occur via penguins only, since they have extremely small branching ratios. As we shall see, some of the decays which are calculated also have rather small branching ratios too. We include them in order to find out the origin of the reduction of these branching ratios. Of course in some of the cases, as for example, the decays induced by H_6, this is obvious because of very small CKM mixing elements. We group the various decays into those without penguin contributions, i.e. those induced by H_1, H_4, H_5 and H_6 and the decays with penguin terms which are calculated from H_2 and H_3. The list of transitions with mixed contributions internal, W emission and penguins is given in table 1 ordered according whether they proceed through H_2 or H_3, respectively. The transitions without penguins ordered with respect to H_1, H_4, H_5 and H_6 can be read off from table 5.

A large fraction of these decays have been considered in the past [5,6] mostly to estimate decay rates. Recently Chau et al. have completed a calculation of B decay branching ratio from a similar calculational point of view for charmless b-decays [12]. The interference of internal W emission and penguins was calculated also in some detail by Valencia [13] for the special channel $B^- \to \omega K^{*-}$ using the Ponce Hamiltonian H_2 with the aim to obtain predictions for angular correlations and CP violation signals. As it will be apparent later, this work is an extension of Valencia's work in several respects.

In the next two sections we calculate the matrix elements for the decays listed in table 2 and table 5.

3 Matrix Elements and Decay Correlations.

To calculate decay rates and angular correlations we need the matrix element

$$< V_1(\lambda_1)\, V_2(\lambda_2)\, |H_{wk}^{eff}|\bar{B}^0 >$$

and similarly for the B^- decay. λ_1 and λ_2 are the helicities of the final state vector particles V_1 and V_2, with four momenta p_1 and p_2, respectively. In the $\bar{B}^o$ rest system ($\vec{p}_1 - \vec{p}_2 = 0$) we have $\lambda_1 = \lambda_2 \equiv \lambda$. We use the notation

$$H_\lambda = < V_1(\lambda)\, V_2(\lambda)\, |H_{wk}^{eff}|\bar{B}^0 \tag{3.1}$$

for the helicity matrix element, $\lambda = 0, \pm 1$. The rate Γ of the decay $\bar{B}^o \to V_1 V_2$ is given in terms of the H_λ by

$$\Gamma = \frac{1}{8\pi m^2} p \left(|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2\right) \tag{3.2}$$

where $p = |\vec{p}_1|$ is the momentum of V_1 in the $\bar{B}^0$ rest system. We have three independent helicity amplitudes H_0, H_{+1}, and H_{-1}. They can be expressed by three invariant amplitudes a, b, c, which are easier to calculate. They are defined by the following decomposition

$$H_\lambda = \epsilon_{1\mu}(\lambda)^* \, \epsilon_{2\nu}(\lambda)^* \left[a g^{\mu\nu} - \frac{b}{m_1 m_2} p^\mu p^\nu + \frac{ic}{m_1 m_2} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_\beta\right] \tag{3.3}$$

where $p = p_1 + p_2$ is the $\bar{B}^o$ four momentum. The masses of V_1 and V_2 are m_1 and m_2 , respectively. m is the mass of the decaying $\bar{B}^o$. a, b, c are defined as in ref. 8. The relations among $H_{\pm 1}$, H_0 and a, b, c are:

$$H_{\pm 1} = a \pm \sqrt{x^2 - 1}\; c$$

$$H_0 = -ax - b\left(x^2 - 1\right) \tag{3.4}$$

where

$$x = \frac{p_1 p_2}{m_1 m_2} = \frac{m^2 - m_1^2 - m_2^2}{2 m_1 m_2} \tag{3.5}$$

so that

$$p^2 = \frac{m_1^2 m_2^2}{m^2}\left(x^2 - 1\right) \tag{3.6}$$

We also use the reduced width

$$\hat{\Gamma} = |H_{+1}|^2 + |H_{-1}|^2 + |H_0|^2 \quad (3.7)$$

The helicity amplitudes $\bar{H}_\lambda$ for the decay of $B^0 \to \bar{V}_1\bar{V}_2$, where $\bar{V}_1$ and $\bar{V}_2$ are the antiparticles of V_1 and V_2 respectively have the same decomposition as (3.3) with $a \to \bar{a}, b \to \bar{b}$ and $c \to -\bar{c}$.

The coefficients a, b and c describe the $s-, d-$ and $p-$ wave contribution of the two final vector particles. They have phases δ from strong interactions, for example, coming from final state interaction of the two vector particles V_1 and V_2 and weak phases ϕ originating from the CP violating phase in the CKM matrix. a, b and c may consist of several interfering amplitudes of different isospin i. Then the phase structure of a, b and c is:

$$a = \sum_i |a_i| e^{i\delta_i^0 + i\phi_i^0}$$

$$b = \sum_i |b_i| e^{i\delta_i^2 + i\phi_i^2}$$

$$c = \sum_i |c_i| e^{i\delta_i^1 + i\phi_i^1} \quad (3.8)$$

The upper indexes in δ_i^0 and ϕ_i^0 etc. denote the angular momenta in a, b and c. For the particle decay $B^{0} \to \bar{V}_1\bar{V}_2$, the amplitudes $\bar{a}, \bar{b}$ and $\bar{c}$ we have the same phase structure as in (3.8) with the weak phases changing sign $\phi_i^{0,1,2} \to -\phi_i^{0,1,2}$. This means the $\bar{a}_i$, $\bar{b}_i$ and $\bar{c}_i$ have the same strong interaction phase and the opposite weak phase as a_i, b_i and c_i. For the case that there are no strong interaction phases, which we shall assume in the models considered later, we have

$$\bar{a} = a^*,\ \bar{b} = b^* \ and\ \bar{c} = c^*$$

Since there is a sign change in front of $\bar{c}$ in $\bar{H}_\lambda$ we have for the case of vanishing strong phases $\delta_i^{0,1,2}$:

$$\bar{H}_{\pm 1} = H^*_{\mp 1} \qquad \bar{H}_0 = H_0^* \quad (3.9)$$

Next we present the formulas for the angular decay distributions. The exact form of these angular distributions depends on the spins of the decay products of the decaying vector mesons V_1 and V_2. We considered five different cases, depending whether one of the vector mesons decays into two pseudoscalar mesons, into e^+e^-, into one pseudoscalar meson plus a photon or into three pseudoscalar mesons. These four specific cases, which cover also all other cases in table 2 and table 5 are:

$$\begin{array}{llll}
(i) & B \to K^*\psi, & K^* \to K\pi, & \psi \to e^+e^- \\
(ii) & B \to K^*\rho, & K^* \to K\pi, & \rho \to \pi\pi \\
(iii) & B \to D^*D_s^*, & D^* \to D\pi, & D_s^* \to D\gamma \\
(iv) & B \to K^*\omega, & K^* \to K\pi, & \omega \to \pi^+\pi^-\pi^0 \\
(v) & B \to \omega\psi, & \omega \to \pi^+\pi^-\pi^0, & \psi \to e^+e^- \\
(vi) & B \to \omega\omega, & \omega \to \pi^+\pi^-\pi^0 &
\end{array} \quad (3.10)$$

The derivation of the angular distributions is straightforward and is sketched in Appendix A for the case (i). Here we give only the results. For $B \to K^*\psi \to (K\pi)(e^+e^-)$ the differential decay distribution looks as follows:

$$\frac{d^3\Gamma}{dcos\theta_1 dcos\theta_2 d\phi} = \frac{p}{16\pi^2 m^2}\frac{9}{8} \times$$
$$\Big\{\frac{1}{4}sin^2\theta_1\left(1+cos^2\theta_2\right)\left(|H_{+1}|^2+|H_{-1}|^2\right) + cos^2\theta_1 sin^2\theta_2 |H_0|^2$$
$$-\frac{1}{2}sin^2\theta_1 sin^2\theta_2[cos2\phi\ Re\left(H_{+1}H^*_{-1}\right) - sin2\phi\ Im\left(H_{+1}H^*_{-1}\right)]$$
$$-\frac{1}{4}sin2\theta_1 sin2\theta_2[cos\phi\ Re(H_{+1}H_0^* + H_{-1}H_0^*)$$
$$-sin\phi\ Im(H_{+1}H_0^* - H_{-1}H_0^*)]\Big\} \quad (3.11)$$

In (3.11) θ_1 is the polar angle of the K momentum in the rest system of the K^* meson with respect to the helicity axis, i.e. the momentum $\vec{p}_1$. Similarly θ_2 and ϕ are the polar and azimuthal angle of the positron e^+ in the ψ rest system with respect to the helicity axis of the e^+, i.e. ϕ is the angle between the planes of the two decays $K^* \to K\pi$ and $\psi \to e^+e^-$ (or $\mu^+\mu^-$). The integration over angles θ_1, θ_2 and ϕ yields the integrated width (3.2). The ratios

$$\frac{\Gamma_T}{\Gamma} = \frac{|H_{+1}|^2 + |H_{-1}|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} \tag{3.12}$$

and

$$\frac{\Gamma_L}{\Gamma} = \frac{|H_0|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} \tag{3.13}$$

measure the amount of transversely (longitudinally) polarized K^* (*or* ψ). We have $\Gamma = \Gamma_L + \Gamma_T$. Later we shall give numerical results for Γ_T/Γ and similarly for the coefficients of the $cos2\phi$, $sin2\phi$, $cos\phi$ and $sin\phi$ terms in (3.11) which are defined as:

$$\alpha_2 = \frac{Re\left(H_{+1}H^*_{-1}\right)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}$$

$$\beta_2 = \frac{Im\left(H_{+1}H^*_{-1}\right)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}$$

$$\alpha_1 = \frac{Re\left(H_{+1}H^*_0 + H_{-1}H^*_0\right)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}$$

$$\beta_1 = \frac{Im\left(H_{+1}H^*_0 - H_{-1}H^*_0\right)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} \tag{3.14}$$

For case (ii) $B \to K^*\rho \to (K\pi)(\pi\pi)$ the decay angular distribution has the following form:

$$\frac{d^3\Gamma}{dcos\theta_1 dcos\theta_2 d\phi} = \frac{p}{16\pi^2 m^2}\frac{9}{4}$$
$$\left\{\frac{1}{4}sin^2\theta_1\, sin^2\theta_2\left(|H_{+1}|^2 + |H_{-1}|^2\right) + cos^2\theta_1\, cos^2\theta_2|H_0|^2\right.$$
$$+\frac{1}{2}sin^2\theta_1 sin^2\theta_2\left[cos2\phi\; Re\left(H_{+1}H^*_{-1}\right) - sin2\phi\; Im\left(H_{-1}H^*_{-1}\right)\right.$$
$$\left.+\frac{1}{4}\sin 2\theta_1\; sin2\theta_2[cos\phi\; Re\left(H_{+1}H^*_0 + H_{-1}H^*_0\right) - sin\phi\; Im\left(H_{+1}H^*_0 - H_{-1}H^*_0\right)]\right\} \tag{3.15}$$

Clearly (3.15) can be used also for all other decays where V_1 and V_2 decay into two pseudoscalar mesons. The decay distribution of $B \to D^* D^*_s$ (iii) with $D^* \to D\pi(\theta_1)$ and $D^*_s \to D_s\gamma(\theta_2, \phi)$ is the same as (3.11), i.e. the distribution for $B \to K^*\psi$ with $K^* \to K\pi(\theta_1)$ and $\psi \to e^+e^-(\theta_2,\phi)$. It is clear that the formula (3.11) can also be used for $B \to D^*D^*$ where one of the D^* decays into $D\pi$ and the other D^* decays into $D\gamma$. The angular distribution of $B \to K^*\omega$ (ii) is the same as the distribution for case (iv), independent of whether one defines the direction (θ_2, ϕ) by the momentum of one of the outgoing pions, for example, the momentum of the π^+, or by the normal of the decay plane formed by the momenta of π^+, π^- and π^0 in the ω rest system. The distribution for case (v) is identical to the distribution of case (i) and that of (vi) is the same as the angular distribution for case (ii).

In general the dominant terms in the angular correlations are Γ_T/Γ, Γ_L/Γ, α_1 and α_2. The terms β_1 and β_2 are small since they are nonvanishing only if the helicity amplitudes H_{+1}, H_{-1} and H_0 or the invariant amplitudes a, b and c, respectively have different phases. These phases can originate from strong or weak interactions (see (3.8)). In the models considered later we shall have no strong interaction phases. In this case the coefficients β_1 and β_2 are nonvanishing only through the CP violating phase of the CKM matrix under the condition that they contribute differently to a, b and c or H_{+1}, H_{-1} and H_0 respectively. As we shall see later this will happen only for a very small list of $\bar{B}^o$ and $\bar{B}^-$ decays. Under the simplifying assumption of no strong interaction phases, the asymmetry coefficients β_1 and β_2 will give us the amount of CP–violation to be expected due to our present knowledge of CKM coefficients and current matrix elements.

A different signature for CP violation is obtained when one considers neutral B mesons only. Then it is possible to generate interference via mixing by looking at final states that can occur from B^0 and $\bar{B}^0$ decays. For this we consider the time evolution of B^0 and $\bar{B}^0$ mesons. This is generally written in the following form [14]:

$$|B^0(t)> = g_+(t)|B^0> + \frac{q}{p}\, g_-(t)|\bar{B}^0>$$

$$|\bar{B}^0(t)> = \frac{p}{q}g_-(t)|B^0> + g_+(t)|\bar{B}^0> \tag{3.16}$$

where $|B^0>\left(|\bar{B}^0>\right)$ are the $B^0\left(\bar{B}^0\right)$ states at $t = 0$, $q/p = \frac{1-\epsilon}{1+\epsilon}$, and

$$g_\pm(t) = \frac{1}{2}e^{-\Gamma t/2}e^{im_1 t}\left(1 \pm e^{-\Delta\Gamma t/2}e^{i\Delta m t}\right) \tag{3.17}$$

with $\Gamma_i, m_i, i = 1,2$ the width and mass of two neutral mass eigenstates B_i and $\Delta\Gamma = \Gamma_2 - \Gamma_1$, *and* $\Delta m = m_2 - m_1$. For the $B^0 - \bar{B}^0$ system we can neglect $\Delta\Gamma$ and obtain for the time dependence of the decay rate of B^0 and $\bar{B}^0 (\Gamma = \Gamma_1 \simeq \Gamma_2)$ into a final state f and its CP conjugate $\bar{f}$:

$$\Gamma\left(B^0(t)\to f\right) \propto \frac{1}{2}\, e^{-\Gamma t} \times$$
$$\left\{|A(f)|^2 + |\frac{q}{p}\bar{A}(f)|^2 + \left(|A(f)|^2 - |\frac{q}{p}\bar{A}(f)|^2\right) cos\Delta mt\right.$$
$$\left. +2\, Im\left(\frac{q}{p}\bar{A}(f)\, A^*(f)\right) sin\Delta mt\right\} \tag{3.18}$$

$$\Gamma\left(\bar{B}^0(t)\to \bar{f}\right) \propto \frac{1}{2}\, e^{-\Gamma t}$$
$$\left\{|\bar{A}\left(\bar{f}\right)^2| + |\frac{p}{q}A\left(\bar{f}\right)|^2 + \left(|\bar{A}\left(\bar{f}\right)|^2 - |\frac{p}{q}\, A\left(\bar{f}\right)|^2\right) cos\Delta mt\right.$$
$$\left. +2\, Im\left(\frac{p}{q}\, A\left(\bar{f}\right)\bar{A}^*\left(\bar{f}\right)\right) sin\Delta mt\right\} \tag{3.19}$$

with $A(f) = A(B^0\to f)$ and $\bar{A}(f) = A\left(\bar{B}^0 \to f\right)$. For $\Delta\Gamma \simeq 0$ the factor $\frac{q}{p}$ is just a phase: $\frac{p}{q} = \left(\frac{q}{p}\right)^*$ and we have $Im\left(\frac{p}{q}A\left(\bar{f}\right)\bar{A}^*\left(\bar{f}\right)\right) = -Im\left(\frac{q}{p}\bar{A}\left(\bar{f}\right)A^*\left(\bar{f}\right)\right)$. According to (3.9) the helicity amplitudes $\bar{H}_\lambda$ for the decay $B^0 \to \bar{V}_1\bar{V}_2$ are connected to H_λ, the decay amplitudes of $\bar{B}^0 \to V_1V_2$, so that in the case of common final states for $\bar{B}^0$ and B^0 decays we get for the quantity $\Delta\hat{\Gamma}$ which determines the difference of $\Gamma(B^0(t)\to f) - \Gamma\left(\bar{B}^0(t)\to f\right)$:

$$\frac{\Delta\hat{\Gamma}}{\hat{\Gamma}} = \frac{Im\left(\frac{q}{p}[H_0^2 + 2H_{+1}H_{-1}]\right)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} \tag{3.20}$$

This quantity depends on the mixing phase $\frac{q}{p}$ and the weak phase of $H_0^2 + 2H_{+1}H_{-1}$. In some of the considered decays H_0, H_{+1} and H_{-1} have equal weak phases. Then $\Delta\hat{\Gamma}/\hat{\Gamma}$ is maximal if $H_{+1} = H_{-1}$ [15] [16].

In the next section we shall calculate the helicity amplitudes H_λ with the help of the vacuum saturation approximation, which we referred to as factorization in the introduction.

4 Matrix Elements in the Vacuum Saturation Approximation.

To calculate the helicity amplitudes H_λ we use the vacuum saturation approximation (VSA). It is clear that we cannot expect this approximation to give accurate predictions for the

helicity amplitudes in all cases which we consider. According to more recent investigation the factorization approximation seems most accurate for decays in which at least one vector mesons contains a heavy quark, as for example in the final states $D^*\rho$ etc. [17]. In all other cases we must expect important corrections. For these cases we shall regard the VSA as giving us order of magnitude estimates for the H_λ and also for the resulting decay rates and angular correlation coefficients. Because the weak interaction hamiltonian has in general the form (2.8), within the VSA the decay amplitudes will be linear combination of the following factors

$$\begin{aligned} F_1 &= < V_1|j_\mu|B >< V_2|\bar{j}^\mu|0 > \\ F_2 &= < V_2|j_\mu|B >< V_1|\bar{j}^\mu|0 > \\ F_3 &= < V_1V_2|j_\mu|0 >< 0|\bar{j}^\mu|0 > \\ F_4 &= < V_1V_2|S|0 >< 0|P|B > \end{aligned} \tag{4.1}$$

The amplitude F_4 with scalar and pseudoscalar amplitudes occurs when one Fierz arranges O_5 or O_6 in (2.9). In our final evaluation we shall neglect the contributions of F_3 and F_4 since the momentum dependence of the form factors suppresses these terms with the typical factor $m_P{}^2/(m_B^2 - m_P^2)$ where $m_P << m_B$ is the form factor mass in F_3 and F_4. It is clear that this argument becomes invalid for such cases where F_1 and F_2 are suppressed for other reasons. Nevertheless to get an overview about the various decays we shall concentrate on the contributions of F_1 and F_2 and neglect the weak annihilation terms F_3 and F_4. The currents j_μ in F_1 and F_2 are always left-handed currents.

In the following we shall write the matrix elements for two specific channels which are induced by H_2^{eff} as given in (2.8) and by H_3^{eff} which follows from (2.11) and (2.12), respectively. The first example is $\bar{B}^0 \to \bar{K}^{*0}\psi$. The matrix element for this decay in VSA is:

$$\begin{aligned} &< \bar{K}^{*0}\psi\, |H_2^{eff}|\, \bar{B}^0 > = -\ G_f\sqrt{2} \\ &\{< \bar{K}^{*0}\, |(\bar{s}b)_\mu|\, \bar{B}^0 >< \bar{\psi}\, |(\bar{c}c)^\mu|\, 0 > [-a_-\ A_c + (a_3 + c_6)\, A_t] \\ &+ < \psi\, |(\bar{d}b)_\mu|\, \bar{B}^0 >< \bar{K}^{*0}\, |(\bar{s}d)^\mu\, |0 > a_4A_t \\ &+ < \bar{K}^{*0}\psi\, |(\bar{s}d)_\mu\, |0 >< 0|(\bar{d}b)^\mu|\bar{B}^0 > a_4A_t \\ &+ < \bar{K}^{*0}\psi\, |(\bar{s}d)\, |0 >< 0|\left(\bar{d}\gamma_5 b\right)|\bar{B}^0 > a_5A_t\} \end{aligned} \tag{4.2}$$

In (4.1) we have introduced the following combinations of the QCD coefficients $c_+, c_-, c_1, \ldots, c_6$:

$$\begin{aligned} a_\pm &= \tfrac{2}{3}c_+ \pm \tfrac{1}{3}c_- \\ a_1 &= \tfrac{1}{6}c_1 + \tfrac{1}{2}c_2 \\ a_2 &= \tfrac{1}{2}c_1 + \tfrac{1}{6}c_2 \\ a_3 &= \tfrac{1}{2}c_3 + \tfrac{1}{6}c_4 \\ a_4 &= \tfrac{1}{6}c_3 + \tfrac{1}{2}c_4 \\ a_5 &= \tfrac{16}{9}c_5 + \tfrac{1}{3}c_6 \end{aligned} \tag{4.3}$$

Some of these definitions do not occur in (4.1) but are needed for the other channels which are collected in appendix B. A_c and A_t stand for the following combination of CKM elements:

$$A_c = V_{cs}^* V_{cb}, \qquad A_t = V_{ts}^* V_{tb} \tag{4.4}$$

In (4.2) the first term on the right–hand side is the term with the structure of F_1 . The second term has the structure F_2 and the third and the fourth term have the structure of F_3 and F_4, respectively. The current matrix element of the second term is OZI forbidden and will be neglected. Since in the numerical evaluation we shall neglect also the terms with the structure F_3 and F_4 , i.e. the annihilation terms, we are left here only with the first term with the structure F_1. Therefore all helicity matrix elements will have the same phase for this case so that the CP asymmetries β_1 and β_2 vanish. Of course, if we would include the OZI forbidden matrix element and/or the annihilation terms, such CP asymmetries, although very small, could be generated. In this work we shall concentrate on the dominant terms and will leave further improvements to later work.

Concerning QCD coefficients we observe that the penguin effects are small in this case since $(a_3 + c_6)$ is small compared to a_- (see (2.10)). Thus the branching ratio is determined essentially by the factor $a_- A_c$. The QCD coefficient a_- is non-dominant, i.e. for the coefficients (2.10) we have $|a_-| << a_+$. How the rates predicted with the coefficients (2.10) compare to experimental data will be considered in the next section.

Next we give the analogous formula for the decay $\bar{B}^o \to D^{*+}D^{*-}$, which is generated by the effective interaction H_3^{eff}. The result is:

$$< D^{*+}D^{*-}|H_3^{eff}|\bar{B}^0 > = \; - G/\sqrt{2}$$

$$\Big\{< D^{*+}|(\bar{c}b)_\mu|\bar{B}^0 >< D^{*-}|(\bar{d}c)^\mu|0 > [-a_+\hat{A}_c + a_4\hat{A}_t]$$

$$+ < D^{*+}D^{*-}|(\bar{u}u - \bar{c}c)_\mu|0 >< 0|(\bar{d}b)^\mu|\bar{B}^0 > a_-\hat{A}_c$$

$$+ < D^{*+}D^{*-}|(\bar{u}u)_\mu|0 >< 0|(\bar{d}b)^\mu|\bar{B}^0 > a_2\hat{A}_t$$

$$+ < D^{*+}D^{*-}|\sum_q(\bar{q}q)_\mu|0 >< 0|(\bar{d}b)^\mu|\bar{B}^0 > a_3\hat{A}_t$$

$$+ < D^{*+}D^{*-}|\sum_q(\bar{q}\ q)_\mu^R|0 >< 0|(\bar{d}b)^\mu|\bar{B}^0 > c_6\hat{A}_t$$

$$+ < D^{*+}D^{*-}|\left(\bar{d}d\right)|0 >< 0|\left(\bar{d}\gamma_5 b\right)\ \bar{B}^0 > a_5\hat{A}_t\Big\} \tag{4.5}$$

with

$$\hat{A}_c = V_{cd}^* V_{cb}, \quad A_t = V_{td}^* V_{tb} \tag{4.6}$$

Compared to A_c and A_t in (4.4) the coefficients $\hat{A}_c$ and $\hat{A}_t$ are Cabibbo suppressed. Except for the first term on the right–hand side of (4.5) all other contributions are annihilation terms which will be neglected in the numerical evaluation. The QCD coefficient a_+ is large. The penguin terms proportional to a_4 are small compared to the dominant a_+ term. With annihilation terms neglected the CP asymmetries β_1 and β_2 (see eq.(3.14)) vanish.

The matrix elements of all other decays generated by H_2^{eff} and H_3^{eff}, which also have penguin contributions and which are listed in table 5 are written down in appendix B.

The structure of the decay matrix elements generated by $H_1^{eff}, H_4^{eff}, H_5^{eff}$ and H_6^{eff} is much simpler since penguin contributions are absent. As an example we write here the matrix element for the decay $B^- \to \rho^- D^{*0}$ which occurs via H_1^{eff}. It has the following form

$$< \rho^- D^{*0}|H_1^{eff}|B^- >= \frac{G}{\sqrt{2}} V_{ud}^* V_{cb}$$

$$\Big[< \rho^-|(\bar{d}b)_\mu|B^- >< D^{*0}|(\bar{c}u)^\mu|0 > a_-$$

$$+ < D^{*0}|(\bar{c}b)_\mu|B^- >< \rho^-|(\bar{d}u)^\mu|0 > a_+\Big] \tag{4.7}$$

The branching ratio of this decay is large since the matrix element is proportional to $V_{ud}^* V_{cb} \simeq V_{cb}$ and to a_+. In this matrix element only one combination of CKM elements is present so that no interference of different CKM elements can occur. Therefore β_1 and β_2 vanish identically because the different helicity matrix elements are all real in our approach. The matrix elements of all other decay channels listed in table 2 are given in appendix B.

In order to calculate the matrix elements (4.2), (4.5) and (4.7) (and all the others listed in appendix B) we need the current matrix elements between a vector particle and the vacuum (see also (4.1)) which has the form:

$$< V_2(\lambda)\,|\bar{j}_\mu|0> \;=\; \epsilon_2(\lambda)^*_\mu\, m_2 f_{v_2} \tag{4.8}$$

and the current matrix element between the initial B and a vector particle $V_1(\lambda)$ in the final state. This is parametrized by three invariant form factors $A_1(q^2), A_2(q^2)$ and $V(q^2)$ following the work of BSW [6]:

$$\begin{aligned} &< V_1(\lambda)\,|j_\mu|\bar{B}^0 \;> = \\ &\epsilon_1(\lambda)^{*\nu}\left\{(m+m_1)\,A_1(q^2)g_{\mu\nu} \;-\; \frac{2A_2(q^2)}{m+m_1}p_\mu p_\nu \;+\; \frac{2iV(q^2)}{m+m_1}\epsilon_{\mu\nu\rho\sigma}p_1^\rho p^\sigma\right\} \end{aligned} \tag{4.9}$$

where $q = p - p_1 = p_2$. Thus the form factors A_1, A_2 and V are needed for $q^2 = m_2^2$. The first two terms in (4.9) describe the contribution of the axial current and the last term proportional to $V(q^2)$ is the contribution of the vector part of j_μ. To obtain the structure F_1 in (4.1) we must multiply (4.8) and (4.9) with the result:

$$\begin{aligned} F_1 &= f_{v_2} m_2 \epsilon_1(\lambda)^{*\mu}\,\epsilon_2(\lambda)^{*\nu} \\ &\left[(m+m_1)\,A_1(m_2^2)g_{\mu\nu} - \frac{2A_2(m_2^2)}{m+m_1}p_\mu p_\nu \;-\; \frac{2iV(m_2^2)}{(m+m_1)}\epsilon_{\mu\nu\rho\sigma}p_1^\rho p^\sigma\right] \end{aligned} \tag{4.10}$$

The contribution to F_2 in (4.1) is calculated analogously. From (4.10) we read off the invariant amplitudes a, b and c defined in (3.3):

$$\begin{aligned} a &= m_2 f_{v_2}(m+m_1)\cdot A_1(m_2^2) \\ b &= -2m_1 m_2^2 f_{v_2} A_2(m_2^2)\,/\,(m+m_1) \\ c &= -2m_1 m_2^2 f_{v_2} V(m_2^2)\,/\,(m+m_1) \end{aligned} \tag{4.11}$$

For a, b and c coming from F_2 we must replace $m_1 \to m_2$ and $f_{v_2} \to f_{v_1}$. For the evaluation of a, b and c and from them $H_{\pm1}$ and H_0 according to (3.4) and (3.5) we need f_{v_2} and $A_1(m_2^2), A_2(m_2^2)$ and $V(m_2^2)$. These four constants for various choices of vector mesons V_1 and V_2 have been calculated by BSW [6]. They calculated A_1, A_2 and V at zero momentum transfer $q^2 = 0$ using relativistic oscillator wave functions. The q^2 dependence is approximated with a singe pole ansatz according to

$$\begin{aligned} A_i(q^2) &= A_i(0)\,\frac{m_P{}^2}{m_P{}^2 - q^2} \\ V(q^2) &= V(0)\,\frac{m_P^2}{m_P^2 - q^2} \end{aligned} \tag{4.12}$$

We take the same pole masses m_P as BSW [6]. They tabulated $A_1(0)$, $A_2(0)$, $V(0)$ for $B \to D^*, K^*, \rho, \omega$ and f_{v_2} for $\rho^-, K^{*0}, \omega, \phi, \psi, D^{*0}$ and D_s^{*+} which we use in our calculations. We shall neglect strong final state interaction and annihilation terms as already remarked earlier. Concerning final state interaction we shall add some remarks when we compare our results to experimental data in the next section.

5 Results

In this section we present results for the following quantities of interest for existing and future experiments: (i) the branching fractions of all decay channels, (ii) the quantity $\Delta\Gamma/\Gamma$ which characterizes CP violation via mixing of B^0 and $\bar{B}^0$, (iii) the ratio Γ_T/Γ which determines the polar angular distribution and (iv) the coefficients of the azimuthal angular distribution.

5.1 Models for QCD Coefficients

Actually we present results for three distinct models, not all of which turn out to be realistic if we confront them with existing data for branching fractions. These three models involve different assumptions concerning the short distance coefficients $c_\pm, c_1, c_2, c_3, c_4, c_5$ and c_6 as defined in Section 2. The first model has the coefficients $c_\pm, c_1, ..., c_6$ given by first order QCD ($(O(\alpha_s))$. In this case $c_+ = c_- = 1, c_1 = 0, c_2 = 2$ and $c_3 : c_4 : c_5 : c_6 = -2 : 6 : 3 : 0$ with c_5 given by

$$c_5 = -\frac{\alpha_s(\mu^2)}{12\pi}\log\frac{m_t^2}{\mu^2}$$

We choose $\mu = m_c = 1.5\,GeV$, $m_t = 150\,GeV$ with $\Lambda = 0.3\,GeV$ and α_s given by the lowest order formula. It is unclear what is the best choice for μ. We have taken the mass of the c-quark as scale in α_s and note that the coefficients do not depend very much on the scale which enters only logarithmically. With these numbers we get ($N_f = 4$) $\alpha_s = 0.47$, $c_5 =$

-0.114, $c_3 = 0.076$ and $c_4 = -0.228$.

The second model incorporates the QCD coefficients as given by Ponce [3] which contain all higher-order QCD corrections in leading-logarithm approximation. It is well known that this model has problems accounting for the decays with branching ratios which are proportional to a_-^2 (see the definition of a_- in (4.3)) [18,19]. a_- has a rather small value $|a_-| = 0.04$ for the QCD corrected short distance coefficients. This strongly suppresses such decays as $\bar{B}^0 \to \omega D^{*0}, \bar{B}^0 \to \rho^0 D^{*0}, \bar{B}^0 \to \bar{K}^{*0} D^{*0}$ and similar decays induced by H_5^{eff} and H_6^{eff}. We see from table 3 that $Br(\bar{B}^0 \to \rho^0 D^{*0}) = 5.43 \times 10^{-6}$ in this case, whereas with $O(\alpha_s)$ coefficients where $a_- = 1/3$ we get $Br(\bar{B}^0 \to \rho^0 D^{*0}) = 3.71 \times 10^{-4}$ (see table 2). This branching fraction is still too small to be measured in existing experiments and we have no data for comparison. Such a comparison is possible if we consider the decay $\bar{B}^0 \to K^{*0}\psi$ which has a measured branching ratio equal to $(0.11 \pm 0.05 \pm 0.03)\%$ [19] and $(0.11 \pm 0.05 \pm 0.02)\%$ [18], respectively. In the second model the branching ratio is ratio is 6.35×10^{-5} which is about a factor 20 too small.

There is a well known analogous effect in nonleptonic D decays. [6]. Therefore several authors advocated the following modification of the short-distance QCD coefficients [6,20]: only terms which are dominant in the $1/N_c$ expansion are taken into account. For example in the coefficient a_+ (see (4.3)) which is $a_+ = \frac{2}{3}c_+ + \frac{1}{3}c_- = \frac{1}{2}(c_+ + c_-) + \frac{1}{2N_c}(c_+ - c_-)$ the term proportional to the color factor $\frac{1}{N_c}$ arises from the color mismatch in color singlets after Fierz transformation [6,20,21]. These terms, however, appear together with color octet contributions resulting from the Fierz transformation which do not contribute in the VSA. This procedure which is consistent in the vacuum saturation approximation may give different results for the octet contribution in a more complete evaluation [22]. Empirically it has been found in connection with nonleptonic D decays that the approximation where the $1/N_c$ terms are neglected gives a satisfactory description of all measured branching ratios [6,20]. Therefore we consider this leading $1/N_c$ approximation as the third possibility to evaluate nonleptonic B decays. These three versions are labelled in the tables as $O(\alpha_s)$ QCD coefficients with Fierz terms (tables 2,5), QCD coefficients with Fierz terms (tables 3, 6) and QCD coefficients without Fierz terms (tables 4, 7, 8). The QCD coefficients for these three models and their relation to the coefficients used in the BSW model are exhibited in table 1. The branching ratio of $\bar{B}^0 \to K^{*0}\psi$ in the QCD model without Fierz terms comes out as 0.716% (see table 7), now larger than the experimental value but in much betterr agreement than the QCD model without Fierz terms.

Before we discuss the results we must specify further input. The CKM elements are calculated with the Wolfenstein representation [23] using the form in which the imaginary part of the unitarity relation is satisfied to order λ^5 and the real part to order λ^3. The CKM matrix element are calculated from the following input: (i) $\rho > 0$ with $\lambda = 0.220, A = 0.971, \rho = 0.350$ and $\eta = 0.357$ and in a second run with (ii) $\rho < 0$ with $\lambda = 0.220, A = 0.971, \rho = -0.460$ and $\eta = 0.196$. These values are taken from a recent analysis of Lusignoli et al. [24]. The CKM elements are calculated with the Wolfenstein representation [23] using the form in which the imaginary part of the unitarity relation is satisfied to order λ^5 and the real part to order λ^3. (In order to fulfill the unitarity relationship in the real part to order λ^4 we augmented V_{ts} with an additional real term proportional to λ^4.) In this representation $V_{ud}, V_{us}, V_{cd}, V_{ts}$ and

V_{tb} are real. For the choices (i) and (ii) $V_{cb} = 0.047$ and $V_{ub} = 0.0051$ which can be compared to the values obtained from semileptonic B decays [25,26]. The current matrix elements are calculated from the BSW model [6] as a starting point of phenomenology. These consist of matrix elements of currents between the vacuum and one of the vector mesons and of matrix elements of the weak currents between the initial B meson and the other vector meson. We consider the BSW input as a useful first approximation to get an overview of the strength of the various decays.

In the following we shall comment on the results for the three models. We start with the decays with no penguin contributions which are induced by H_1^{eff}, H_4^{eff}, H_5^{eff} and H_6^{eff}. The results are presented in table 2 ($O(\alpha_s)$ with Fierz terms), table 3 (QCD coefficients with Fierz terms) and table 4 (QCD Coefficients without Fierz terms). These transitions also have no annihilation terms and their matrix elements are very simple in the VSA. Let us look first at the decays induced by H_1^{eff} which are the most promising ones in the no-penguin group. In sections 5.2 - 5.6 we discuss the results for the case (i) with positive ρ. In Section 5.7 we discuss the changes that result when the negative ρ solution is used.

5.2 Sector $\Delta s = 0$, $\Delta b = -\Delta c = 1$ (H_1)

The branching ratios differ model to model, in particular for $\bar{B}^0 \to \omega D^{*0}$ and $\bar{B}^0 \to \rho^0 D^{*0}$ since they are proportional to $|a_-|^2$ which is much smaller in the model of QCD coefficients with Fierz terms. The branching ratios of the other two decays: $\bar{B}^0 \to \rho^- D^{*+}$ and $\bar{B}^- \to \rho^- D^{*0}$ are in reasonable agreement with each other and with experimental data: $Br(\bar{B}^0 \to \rho^- D^{*+}) = (0.7 \pm 0.3 \pm 0.3)\%$ [18], $(1.9 \pm 0.9 \pm 1.3)\%$ [19], $Br(B^- \to \rho^- D^{*0}) = (1.0 \pm 0.6 \pm 0.4)\%$ [18]. These two branching ratios depend mostly on $|a_+|^2$ and therefore are less dependent on the choice of the QCD coefficients. We remark that the decay rate for $B^- \to \rho^- D^{*0}$ depends on a_+ and a_- and allows a determination of the ratio a_-/a_+. If we average the CLEO and ARGUS results we get $a_-/a_+ = (-0.2 + 1.3(-0.5))$. Using the experimental information on the ratio of the branching ratios for the decays $\bar{B}^0 \to K^{*0}\psi$ and $\bar{B}^0 \to \rho^- D^{*+}$ we find

$$0.097 \leq |a_2/a_1| \leq 0.275.$$

The angular correlations are rather insensitive to the choice of the QCD coefficients. In all these models Γ_T/Γ, the $\cos\phi$ and the $\cos 2\phi$ coefficients, α_1 and α_2, are very similar. We have $\Gamma_T/\Gamma < 0.3$ which means that the longitudinal transition matrix element H_0 dominates. Therefore the $\cos\phi$ coefficient (α_1) is much larger than the $\cos 2\phi$ correlation term (α_2). The α_1 coefficient is negative and large. The dominance of the longitudinal transition matrix elements originates from the BSW current matrix elements and is independent of the short-distance coefficients. $\Delta\Gamma/\Gamma$ is also large negative. The helicity matrix elements H_0, H_{+1} and H_{-1} are dominantly real, so that $\Delta\Gamma$ is essentially given by the imaginary part of $q/p = V_{td}^2/|V_{td}|^2$. For ρ and η as obtained in solution (i) (ρ positive) we have $Im\, V_{td}^2/|V_{td}|^2 = -0.844$. We see that $\Delta\Gamma/\Gamma$ is somewhat reduced compared to this value. This has its origin in the fact that in these transitions $H_{+1} \neq H_{-1}$, in fact $H_{+1}/H_{-1} \simeq 0.1$ [29]. For $H_{+1} = H_{-1}$ the CP asymmetry would be maximal. We see, however, that the dilution through terms proportional to $(H_{+1} - H_{-1})$ [16] is moderate (see eq(3.20)).

Since the amplitudes for $\bar{B}^0 \to \omega D^{*0}$ and $\bar{B}^0 \to \rho^0 D^{*0}$ involve only one CKM combination, namely $V_{ud}V_{cb}^*$, $\Delta\Gamma/\Gamma$ is independent of final state strong interaction phases. This is also true for the transition $\bar{B}^0 \to \rho^0 D^{*0}$ which has different isospin amplitudes ($I = 1/2$ and $I = 3/2$). The CP-conjugated mode has the same final state phase, but has the complex conjugate factor $V_{ud}V_{cb}^*$, which in Wolfenstein's choice of phases is real. Therefore in $\Delta\Gamma$ the strong interaction phase drops out [27] and the direct $\Delta b = 1$ weak phase is real so that $\Delta\Gamma/\Gamma$ is determined by the phase of $V_{td}^2/|V_{td}|^2$.

There is an additional contribution to the CP eigenstate of $\rho^0 + D^{*0}$ and $\omega + D^{*0}$ coming from transitions by H_6^{eff}. These transitions $\bar{B}^0 \to \omega\bar{D}^{*0}$ and $\bar{B}^0 \to \rho^0\bar{D}^{*0}$ may have different strong interaction phases. They interfere with the transitions induced by H_1^{eff} and would invalidate the statement about $\Delta\Gamma/\Gamma$ being independent of final state phases. However, the transitions induced by H_6^{eff} are highly suppressed by the CKM - combination $V_{cd}^* V_{ub}$, which leads to a reduction of the branching ratios by the factor 5×10^{-4}, so that their contribution is negligible compared to the unsuppressed modes induced by H_1^{eff}. The discussion of the results for the second and third group, induced by H_4^{eff} and H_5^{eff}, respectively, is very similar.

5.3 Sector $\Delta b = \Delta s = -\Delta c = 1\,(H_4)$

The transitions $\bar{B}^0 \to K^{*-}D^{*+}$ etc. are CKM suppressed as compared to the transitions $\bar{B}^0 \to \rho^- D^{*+}$ etc. by the factor $V_{us}/V_{ud} \simeq \lambda$ which explains the reduction of the branching ratios. The rate for $\bar{B}^0 \to \bar{K}^{*0}D^{*0}$ is reduced further since the matrix element is proportional to a_- (see (B.23)), whereas the other two decay channels in this group depend on a_+. In the more realistic model with no Fierz terms the decay rate for $B^- \to K^{*-}D^{*0}$ is reduced compared to the rate of $\bar{B}^0 \to K^{*-}D^{*+}$ since the former involves destructive interference of terms proportional to a_+ and a_- (see (B.24)). Except for $\bar{B}^0 \to \bar{K}^{*0}D^{*0}$ the decays of this group should be observable in the near future.

Concerning angular correlations the decays induced by H_4^{eff} show the same pattern as the decays induced by H_1^{eff}: Γ_T/Γ is small, i.e. H_0 dominates over $H_{\pm 1}$. Therefore the coefficient, α_1 also negative, is large compared to α_2. Since the longitudinal transition matrix element dominates over the transverse ones the CP-asymmetry $\Delta\Gamma/\Gamma$ is reduced very little as compared to the maximally possible value given by the imaginary part of $V_{td}^2/|V_{td}|^2$, similar to the transitions induced by H_1^{eff}. Finally we remark that in all four groups the asymmetries proportional to $\sin\phi$ $[\beta_1]$ and $\sin 2\phi$ $[\beta_2]$ vanish. This is obvious since all three helicity amplitudes have the same phase so that no imaginary parts are generated in our models. This could be different in models with explicit strong interaction phases, as for example generated by rescattering into different channels.

5.4 Sectors $\Delta b = \Delta c = \Delta s = 1\,(H_5)$ and $\Delta s = 0, \Delta b = \Delta c = 1(H_6)$

The transitions induced by H_5^{eff} are reduced further since their matrix elements are proportional to $V_{ub}^* V_{cs}$. Compared to the transitions from H_4^{eff} this amounts to a reduction of the rate by the factor $\rho^2 + \eta^2 \simeq 0.25$. Both transitions in this group are proportional to a_- which leads to further reduction.

Concerning Γ_T/Γ, and the $\cos\phi$ and $\cos 2\phi$ correlation terms, the results are equal to the results for $\bar{B}^0 \to \bar{K}^{*0}D^{*0}$ in the H_4^{eff}-group. However $\Delta\Gamma/\Gamma$ is fairly large and has a different sign than in the $H_1^{eff}-$, $H_4^{eff}-$ and $H_6^{eff}-$ groups. This comes from the fact that $V_{ub}^* V_{cs} \simeq A\lambda^3(\rho - \eta)$, i. e. the CKM matrix element has now an imaginary part of comparable magnitude to the real part. Therefore we have an interesting interference of the phase of $q/p = V_{td}^2/|V_{td}|^2$ with the phase of the transition matrix elements generated by the CKM angles. This phenomenon is independent of the QCD coefficients since only a_- enters in the transition matrix element.

Actually the analysis of this interference pattern is more complicated since both transitions, the transition induced by H_4^{eff} and the transition induced by H_5^{eff} contribute to the transition going to the CP eigenstate combination contained in $\bar{K}^{*0} + D^{*0}$ and $\bar{K}^{*0} + \bar{D}^{*0}$. The analysis is complitely equivalent to the analysis of the final state $\bar{K}^0 + D^0$, $\bar{K}^0 + \bar{D}^0$ as discussed in detail by Gronau and London [28]. We shall not repeat it here.

The decays in the sector H_6 are very strongly Cabibbo suppressed. Their influence on $B^0 - \bar{B}^0$ mixing was discussed already in Section 5.2.

Now we come to the discussion of the results for transitions induced by H_2^{eff} and H_3^{eff}, which are influenced by penguin effects.

5.5 Sector $\Delta c = 0$, $\Delta b = \Delta s = 1\,(H_2)$

The results for the three models, again for ρ positive, are found in table 5, 6 and 7. Let us first look at the transitions in the group H_2^{eff}. In this group interesting azimuthal asymmetries proportional to $\sin\phi$ and $\sin 2\phi$ are generated. In our three models these asymmetries are totally due to the CP violating phase in the CKM elements. Four of the transitions, namely those with the final states $K^*\psi$ and $D^*D_s^*$, have vanishing $\sin\phi$ and $\sin 2\phi$ terms. This is due to our approximation of neglecting OZI forbidden and annihilation matrix elements, so that no interfering contributions with different CKM phases and different helicity structure are present (see (4.2), (B.5), (B.6), (B.7) and (B.8)). Concerning improving this approximation the most promising cases are the transitions $B^- \to K^{*-}\psi$ and $B^- \to D^{*0}D_s^{*-}$ where the neglected terms have at least large QCD coefficients (see(B.6) and (B.8))

As can be seen from the tables the transitions to these final states. $K^*\psi$ and $D^*D_s^*$, have the largest branching ratios in the H_2^{eff} group. The transitions to $D^*D^*_s$ are proportional to a_+ and have the largest branching ratio and are the same for $\bar{B}^0$ and B^- decay. Because this rate is proportional to a_- it is approximately equal in all the QCD models. It is largest in the QCD model without Fierz terms where $Br(\bar{B}^0 \to D^{*+}D_s^{*-}) = Br(B^- \to D^{*0}D_s^{*-}) = 2.40\%$ in agreement with the experimental branching ratios of the ARGUS collaboration: $Br(B^0 \to D^{*-}D_s^{*+}) = (2.6 \pm 1.3)\%$ and $Br(B^- \to D^{*0}D_s^{*+}) = (3.1 \pm 1.6)\%$ [30]. However, the branching ratios obtained for the other two models are also consistent with the data.

The transitions to the final state $K^*\psi$ are proportional to a_-. As it was already mentioned this QCD coefficient is very small in the QCD model with Fierz terms. Therefore in this model the branching fraction for $B \to K^*\psi$ is too small at least by a factor of 10. In the other two models it is approximately consistent with the experimental data as was already mentioned above. In the more detailed results not presented here we observe that one of the transverse helicity matrix elements, namely H_{+1} is small, whereas the other one is comparable to the longitudinal matrix element. Therefore in both transitions $B \to D^*D_s^*$ and $B \to K^*\psi$ we obtain $\Gamma_T/\Gamma \simeq 0.5$. The $\cos\phi$-term is larger than the $\cos 2\phi$-term only by a factor of 3 for $B \to D^*D_s^*$ and a factor of 5 for $B \to K^*\psi$, so that the $\cos 2\phi$-term has some effect on the azimuthal distribution. These results are independent of the QCD model. $\Delta\Gamma/\Gamma$ for $\bar{B}^0 \to \bar{K}^{*0}\psi$ is large, comparable to the numbers we encountered in the non-penguin group. Therefore the dilution factor, approximately 0.8, is again unimportant for detecting CP violation via $B^0 - \bar{B}^0$ mixing, so that the decay channel $B \to K^*\psi$ is as useful for CP studies as the decay $B \to K\psi$. Actually this dilution factor may be even larger than 0.8 as we shall see below.

The polar angle distribution of the e^+ in $B \to K^*\psi$ has been measured recently by the ARGUS collaboration. If this is written in the form

$$d\Gamma/d\cos\theta_2 \propto (1 + \alpha \cos^2\theta_2)$$

the preliminary analysis of the ARGUS data yields $\alpha = -1.17 \pm 0.17$ [30]. α is directly related to Γ_T/Γ by:

$$\alpha = \frac{3\Gamma_T - 2\Gamma}{2\Gamma - \Gamma_T}$$

Taking $\Gamma_T/\Gamma = 0.429$ from table 5 (or 6 or 7) we predict $\alpha = -0.454$ which is larger than the experimental number. Whereas experimentally the tranverse helicity contribution Γ_T vanishes, it is 43% in the models. If this experimental result persists this would be a serious problem for all of these models. Our prediction for Γ_T/Γ depends heavily on the BSW current matrix elements and not on the QCD coefficients.

The decays $B \to K^*\omega$ and $B \to K^*\rho$ are most interesting from the point of view of detecting direct CP violation through azimuthal asymmetries. We see from table 7 that the $\sin\phi$-term may be as large as 5.5×10^{-3} (for $B^- \to K^{*-}\omega$). The branching ratios come out also reasonably large, of the order of 10^{-4}. Unfortunately the asymmetry is largest when the branching ratio is the smallest. The reduction of branching ratios of the four decays $\bar{B}^0$, $B^- \to K^*\rho, K^*\omega$ as compared to the other decays in the H_2^{eff} group has its origin, as was remarked already in Appendix B (after (B.2)), in a cancellation of two contributions with different QCD factors. Let us take for example the transition $\bar{B}^0 \to \bar{K}^{*0}\omega$ where the matrix element is written down in (B.1). When penguin and box diagram effects are neglected we have $a_- = a_2$, as for example in the BSW model [6], so that in (B.1) the dominant term is proportional to $a_-(A_c + A_t) = -a_- A_u = -a_- V_{us}^* V_{ub} \sim \lambda^4$ due to the unitarity of the CKM matrix. The penguin and box diagram contributions have the effect that a_- and a_2 differ. In the QCD model without Fierz terms ($N_c = \infty$) we have according to table 1, $a_- - a_2 = -0.23$. This way the $V_{us}^* V_{ub}$ suppression is partly lifted. In addition there are other small terms proportional to $(a_3 + c_6) A_t$ which bring in unsuppressed CKM matrix elements.

In the three other cases the situation is completely analogous. The amount of suppression can easily seen by comparing the branching ratios of the four decays in table 5 ($O(\alpha_s)QCD$) with those in table 7 (QCD without Fierz terms). The difference is up to one order of magnitude. Therefore these four decays are ideal cases to study the interference of penguin effects with the usually stronger direct decay contributions. In all four decays the longitudinal matrix element is dominant. Therefore Γ_T/Γ and the $\cos 2\phi$ terms are small. The $\cos\phi$ terms are appreciable and independent of the QCD models. $\Delta\Gamma/\Gamma$ is also large and the dilution factor is approximately 1, so that these decays are also well suited to study CP violation with $B^0 - \bar{B}^0$ mixing.

There exist interesting experimental limits for the branching ratios: $\mathrm{Br}(B^0 \to K^{*0}\rho^0) < 4.6 \times 10^{-4}$[31], 6.7×10^{-4} [32] compared to our value 1.4×10^{-4} in table 7. This value is higher than that obtained by other authors [12]. The reason is that we have $a_- \neq a_2$ as discussed above. There is also an upper limit for $\mathrm{Br}(B^+ \to K^{*+}\omega) < 1.3 \times 10^{-4}$ [31] to be compared with $\mathrm{Br}(B^- \to K^{*-})\omega) = 6.1 \times 10^{-5}$ in table 7.

5.6 Sector $\Delta s = \Delta c = 0,\ \Delta b = 1\,(H_3)$

As the last point we discuss the results for the transitions induced by H_3^{eff}. These transitions are also influenced by penguin effects. As one can see in table 7, for instance, the branching ratios vary tremendously in this group, by almost seven orders of magnitude. The dominant decays are $B \to D^*D^*$. Compared to $B \to D^*D_s^*$ the branching ratio is reduced by the Cabibbo supresion factor λ^2. Since we neglect annihilation terms this decay depends only on one amplitude so that no $\sin\phi$ and $\sin 2\phi$ terms can occur. The angular correlation terms: Γ_T/Γ, α_1, α_2 and also $\Delta\Gamma/\Gamma$ are very similar to those in the $B \to D^*D_s^*$ transitions. The next important group concerning branching ratios are the transitions $B \to \omega\psi$ and $B \to \rho\psi$. The branching ratios are up to 0.03% (see table 7). Due to the neglect of OZI forbidden matrix elements there is only one amplitude left (see (B.13), (B.14) and (B.17) in Appendix B), so that no $\sin\phi$ and $\sin 2\phi$ terms can be generated. The other angular correlation coefficients look similar to those for the decay $B \to D^*D^*$.

The decays $B \to \omega\rho$, $\rho\rho$, and $\omega\omega$ have very small branching ratios, in particular the decay $B^o \to \omega\rho^o$. The matrix elements of all these decays are proportional to $\hat{A}_c + \hat{A}_t = -\hat{A}_u = -V_{ud}^* V_{ub} \sim \lambda^3$ when penguin effects are neglected. This leads already to strong suppression of these decays. Now let us look at each of these six channels separately. The rate for the decay $\bar{B}^0 \to \omega\rho^0$ is further reduced since the dominant contribution proportional to $(a_-\hat{A}_c + a_2\hat{A}_t)$ (see (B.9)) cancels out for $m_\omega = m_\rho$ due to an opposite sign of the first two terms and to the fact that in the BSW model f_v and the current matrix elements are almost equal. Therefore in the limit $m_\omega = m_\rho$ this transition is almost dominated by penguin terms. This makes it understandable that the predictions for all quantities of this decay, in particular the branching ratio, depend very strongly on the QCD model (compare table 5, 6, and 7). The $\sin\phi$-term also varies quite strongly with the QCD model. In the QCD model without Fierz terms it is almost as large as 0.1. Unfortunately the rate is extremely small in this case due to additional cancellations. It is clear that the exact numbers can not be trusted in this channel since neglected annihilation terms and inaccuracies in the VSA will have large effects on these results.

The decays $\bar{B}^0 \to \rho\rho$ and $\bar{B} \to \omega\omega$ have their natural rate and are not reduced by additional cancellations. The reduction due to the factor $V_{ud}^* V_{ub}$ is partially lifted by the penguin contributions. That the branching ratio of these decays in table 5 and 6 for two of the three QCD models come out almost the same is accidental. In table 5 the coefficient $a_- = a_2$ (see table 1) but $a_- = 1/3$. In table 5 $(a_- - a_2)$ is almost equal to $(a_- - a_2)$ for table 7, but in the QCD model with Fierz terms the contribution of the terms proportional to $a_- V_{ud}^* V_{ub}$ are very small.

The decays $\bar{B}^0 \to \rho^+\rho^-$, $\bar{B}^0 \to \omega\rho^0$, and $B^- \to \rho^0\rho^-$ have larger branching ratios since their matrix elements have contributions proportional to a_+. Otherwise no additional reduction occurs except for $a_+ = a_1$ when the matrix elelments are proportional to $a_+ V_{ud}^* V_{ub}$. There is no $sin\phi$- and $sin2\phi$-term for the decay $\bar{B}^o \to \rho^+\rho^-$ because we neglect annihilation terms. For the decays $B^- \to \omega\rho^-$ and $B^- \to \rho^o\rho^-$ there would be no CP-odd terms β_1 and β_2 either, if $m_\omega = m_\rho$. For $B^- \to \rho^o\rho^-$ the mass splitting is even smaller and and therefore β_1 and β_2 are reduced even further. Of course for these very rare decays further improvements of the decay model are needed before the results can be trusted.

For some of the decays in this sector we found upper limits measured by the ARGUS collaboration . They are: $Br(\bar{B}^0 \to \rho^0\rho^0) < 2.8 \times 10^{-4}$[33] to be compared with the value 8.2×10^{-6} in table 7, $Br(\bar{B}^+ \to \rho^+\rho^0) < 1.0 \times 10^{-3}$ which is to be compared with the value 1.1×10^{-5}. The limits are still more than two orders of magnitude above the theoretical predictions.

5.7 Results for Negative ρ

We have also calculated results for the other choice of the CKM matrix elements as reported by Lusignoli et al.[24] and given earlier in this section (ρ negative.) In most cases the results change only with respect to $\Delta\Gamma/\Gamma$, with all other quantities remaining unchanged. The change in $\Delta\Gamma/\Gamma$ is caused by the change in

$$Im\frac{q}{p} = \frac{-2\eta(1-\rho)}{(1-\rho)^2+\eta^2}$$

which equals -0.264 as compared to -0.844 for the positive ρ solution considered above. This automatically changes $\Delta\Gamma/\Gamma$ by a factor of 0.31 when there are no important phases in the helicity matrix elements. This is not the case for $\bar{B}^o \to \bar{K}^{*o}\bar{D}^{*o}$. Here $\Delta\Gamma/\Gamma$ changes from 0.428 ($\rho > 0$) (see tables 2,3,4) to -0.701. Similarly $\Delta\Gamma/\Gamma$ changes for the transitions in the H_6^{eff} group. There $\Delta\Gamma/\Gamma = -0.432$ ($\rho > 0$) (see tables 2,3,4) is changed to 0.398 ($\rho > 0$) but otherwise the angular correlations remain completely unchanged and the branching ratios are quite similar. Thus in the transitions induced by the H_5^{eff} and the H_6^{eff} there are siginificant phases in the helicity matrix elements which influence $\Delta\Gamma/\Gamma$.

For the transitions influenced by penguins the situation is quite similar except that now also the $sin\phi$ $[\beta_1]$ and $sin2\phi$ $[\beta_2]$ terms are present and are quite different, as expected. We show the complete results for QCD without Fierz terms in table 8 ($\rho < 0$) from which the changes can be seen by comparison with table 7 ($\rho > 0$).

6 Summary and Concluding Remarks

In this study of all two-body vector-vector decay channels of charged and neutral B mesons we have found a remarkable span of branching ratios (from 10^{-9} to 10^{-2}) even though we have not calculated rates solely dominated by penguins (e.g. $B \to K^*\phi$). We have found direct CP asymmetries arising from interference of penguins with W emission diagrams with different weak phases in several channels, notably in $B \to K^*\omega, K^*\rho$ where branching ratios are moderate and angular distribution measurements may soon be feasible. All angular distributions are dominated by $\cos\phi$ over $\cos 2\phi$ and $\sin\phi$ over $\sin 2\phi$ becauseof the helicity structure of the BSW form factors. For this reason also, the polarization is dominantly longitudinal with many rates exclusively longitudinal. In comparing these results with the data on branching ratios, limits and polarization we found that they could for the most part be accomodated within the models but that the model of QCD coefficients without Fierz terms clearly was preferred to the QCD coefficients with Fierz terms model. However, the lack of transverse polarization found by ARGUS in $B \to K^*\psi$ can not be accomodated within any of these models.

The next steps in this program are clear and fall into two classes. First, the RGE should be run for a large top mass. The coefficients will not grossly change from Ponce's calculation but there may be intersesting differences. A related issue concerns the absorptive parts of penguin diagrams. The second class of improvement is connected with the hadronization procedure by which the weak effective Hamiltonian is evaluated in hadron states. We have (1) used factorization (2) discarded annihilation terms, and (3) used the BSW current form factors and decay matrix elements. Factorization has survived some experimental test [34] but further theoretical analysis is needed to define its domain of validity and calculate higher order corrections. Heavy quark theory can help in this regard in certain channels [35],[36]. We are currently applying these methods. Annihilation terms are sometimes accompained by large QCD coefficients and have no further OZI suppression. Methods must be found to estimate these amplitudes. Finally, BSW wave functions and decay constants must be validated by other theoretical methods, or checked experimentally, where it is known already that some are not correct. In this initial study we have not put in these refinements for the sake of simplicity. A more careful phenomenological analysis of key rates will appear in a future publication.

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Appendix A: Angular Distributions

In this appendix we give a short derivation of the angular distribution (3.11) for the decay $B \to K^*\psi \to (K\pi)(e^-e^-)$. For its derivation we use the usual density matrix formalism [37].

We denote the momentum of the K^* meson by $\vec{p_1}$, its helicity by λ_1 and the momentum and helicity of the ψ particle by $\vec{p_2}$ and λ_2 respectively. In the B rest system $\vec{p_1} + \vec{p_2} = 0$ and since B has spin zero we have $\lambda_1 = \lambda_2$. Thus the decay matrix element for the decay $B \to K^*\psi$ depends only on λ_2 and the c.m.momentum $|\vec{p}| = |\vec{p_1}| = |\vec{p_2}|$. Let $h_{\lambda_2,\lambda_{2'}}$ be the density matrix elements for the decay $B \to K^*\psi$

$$h_{\lambda_2,\lambda_{2'}} = H_{\lambda_2} H_{\lambda_{2'}} \tag{A.1}$$

where H_{λ_2} is the decay matrix element as introduced in Section 3. The angular distribution is called $W(\theta_1, \theta_2, \phi)$, where θ_1 is the polar angle of the K momentum in the rest system of the K^* with respect to the helicity axis, which is the direction of the momentum $\vec{p_1}$ and θ_2 and ϕ are the polar and azimuthal angle of the positron momentum $\vec{p_a}$ in the ψ rest system with respect to the helicity axis of the e^-. Then this angular distribution is calculated from:

$$W(\theta_1, \theta_2, \phi) = \sum_{\lambda_2, \lambda_{2'}} h_{\lambda_2\lambda_{2'}} A_{\lambda_2\lambda_{2'}}(\theta_1) B_{\lambda_2\lambda_{2'}}(\theta_2, \phi) \tag{A.2}$$

In (A.2) the matrix $A_{\lambda_2\lambda_{2'}}(\theta_1)$ stands for the density matrix for the decay $K^* \to K\pi$

$$A_{\lambda_2\lambda_{2'}}(\theta_2) = \frac{R(\theta_2)_{\lambda_2\lambda_{2'}}}{|T(p_k)|^2} \tag{A.3}$$

and $R(\theta_2)$ is the angular distribution for the decay $K^* \to K\pi$ which is

$$R(\theta_2)_{\lambda_2\lambda_{2'}} = \frac{3}{4\pi} |T(p_k)|^2 D^{1*}_{\lambda_2 0}(0, \theta_1, 0) D^1_{\lambda_{2'} 0}(0, \theta_1 0) \tag{A.4}$$

where $D^j_{mm'}(\alpha, \beta, \gamma) = e^{-i(m\alpha + m'\gamma)} d^j_{mm'}(\beta)$ are the usual rotation matrices and $T(p_K)$ is the reduced matrix element for the decay $K^* \to K\pi$ which depends only on the c.m. momentum $p_K = |\vec{p_K}| = |\vec{p_\pi}|$. Similarly the matrix $A_{\lambda_2\lambda_{2'}}(\theta_2, \phi)$ is the decay distibution for $\psi \to e^+e^-$ which is given by

$$B_{\lambda_2\lambda_{2'}}(\theta_2, \phi) = \frac{\sum_{\lambda_a, \lambda_b} R(\theta_2, \phi; \lambda_a, \lambda_b)_{\lambda_2\lambda_{2'}}}{\sum_{\lambda_a, \lambda_b} |T(p_a, \lambda_a, \lambda_b)|^2} \tag{A.5}$$

$\lambda_a, \vec{p_a}(\lambda_b, \vec{p_b})$ denote the helicity and momentum of $e^+(e^-)$ and $p_a = |\vec{p_a}| = |\vec{p_b}|$ is the c.m. momentum of the leptons. The matrix $R(\theta_2, \phi; \lambda_a, \lambda_b)_{\lambda_2\lambda_{2'}}$ is

$$R(\theta_2, \phi; \lambda_a, \lambda_b)_{\lambda_2\lambda_{2'}} = \frac{3}{4\pi} T(p_a; \lambda_a, \lambda_b) D^{1*}_{\lambda_2, \lambda_a - \lambda_b}(\phi, \theta_2, -\phi) D^1_{\lambda_{2'}, \lambda_a - \lambda_b}(\phi, \theta_2, -\phi) \tag{A.6}$$

$T(p_a; \lambda_a, \lambda_b)$ $\left(\lambda_a, \lambda_b = \pm\frac{1}{2}\right)$ is the reduced matrix element for the decay $\psi \to e^+e^-$. Only two of the four helicity matrix elements are independent since the decay $\psi \to e^+e^-$ conserves parity. We have

$$T(p_a; -\lambda_a, -\lambda_b) = T(p_a; \lambda_a, \lambda_b) \tag{A.7}$$

In addition in the limit of vanishing electron mass we have $T(p_a; \lambda_a, \lambda_a) = 0$ so that $B_{\lambda_2\lambda_{2'}}(\theta_2, \phi)$ becomes independent of the ψ decay matrix element.

The evaluation of the angular functions $A_{\lambda_2,\lambda_{2'}}(\theta_1)$ and $B_{\lambda_2,\lambda_{2'}}(\theta_2, \phi)$ yields:

$$\begin{aligned}
A_{1,1}(\theta_1) &= A_{-1,-1}(\theta_1) = -A_{1,-1}(\theta_1) = -A_{-1,1}(\theta_1) = \tfrac{3}{8\pi} sin^2\theta_1 \\
A_{0,0}(\theta_1) &= \tfrac{3}{8\pi} cos^2\theta_1 \\
A_{1,0}(\theta_1) &= A_{0,-1}(\theta_1) = -A_{-1,0}(\theta_1) = -A_{0,-1}(\theta_1) = -\tfrac{3}{4\sqrt{2}\pi} sin\theta_1 cos\theta_1
\end{aligned} \tag{A.8}$$

$$\begin{aligned}
B_{1,1}(\theta_2, \phi) &= B_{-1,-1}(\theta_2, \phi) = \tfrac{3}{16\pi}(1 + cos^2\theta_2) \\
B_{0,0}(\theta_2, \phi) &= \tfrac{3}{8\pi} sin^2\theta_2 \\
B_{1,-1}(\theta_2, \phi) &= B^*_{-1,1}(\theta_2, \phi) = \tfrac{3}{16\pi} sin^2\theta_2 e^{2i\phi} \\
B_{1,0}(\theta_2, \phi) &= B^*_{0,1}(\theta_2, \phi) = -B^*_{-1,0}(\theta_2, \phi) = -B_{0,-1}(\theta_2, \phi) = \\
&\tfrac{3}{8\pi\sqrt{2}} sin\theta_2 cos\theta_2 e^{2i\phi}
\end{aligned} \tag{A.9}$$

Substituting (A.8) and (A.9) into (A.2) gives

$$\begin{aligned}
W(\theta_1; \theta_2, \phi) = \frac{9}{8(2\pi)^2} \Big[& \tfrac{1}{4} sin^2\theta_1 (1 + cos^2\theta_2) [h_{1,1} + h_{-1,-1}] \\
& + cos^2\theta_1\ sin^2\theta_2 h_{0,0} - \tfrac{1}{2} sin^2\theta_1\ sin^2\theta_2 [cos2\phi\ Re h_{1,-1} - sin2\phi Im h_{1,-1}] \\
& - \tfrac{1}{4} sin\theta_1\ cos\theta_1\ sin\theta_2 cos\theta_2 [cos\phi\ Re\,(h_{1,0} + h_{-1,0}) \\
& - sin\phi\ Im\,(h_{1,0} - h_{-1,0})] \Big]
\end{aligned} \tag{A.10}$$

Renormalizing (A.10) so that the integration of $W(\theta_1; \theta_2, \phi)$ over θ_1, θ_2 and ϕ gives back the total width Γ in (3.2) brings us to the result (3.11). The derivation of (3.15) is performed analogously.

In the derivation given above all two–particle states are defined as in the Jacob–Wick convention [37] where the two-particle c.m. state at momentum direction $\vec{p}$ is obtained from the state where $\vec{p}$ is along the z direction by the rotation $R = (\phi, \theta, -\phi)$. In addition we defined the phase of the two–particle state as in Jacob and Wick [37]. This has consequences for the definition of the polarization vector of particle 2 in (3.3).

Appendix B: Decay Amplitudes

In this appendix we collect the formulas for the matrix elements of all the other decays of $\bar{B}^o$ and B^- induced by $H_1, H_2, \ldots, H_5$ which are listed in table 1 and table 4 and which have not been given in section 4. Although we shall neglect in this work all OZI forbidden terms and all annihilation contributions we shall give the complete formulas so that the reader can see in which cases there are additional contributions which can modify our predictions and which case this does not occur. Of course the former will happen mostly for H_2 and H_3 where penguin terms are present.

We start with the transitions induced by H_2^{eff} which are $\bar{B}^0 \to \bar{K}^{*0}\omega, \bar{K}^{*0}\rho^0$, $D^{*+}D_s^{*-}$ and $B^- \to K^{*-}\omega$, $K^{*-}\rho^0, K^{*-}\psi$ and $D^{*0}D_s^{*-}$. The matrix element for $\bar{B}^0 \to \bar{K}^{*0}\omega$ is:

$$
\begin{aligned}
&< \bar{K}^{*0}\omega|H_2^{eff}|\bar{B}^0 >= -G/\sqrt{2} \\
&\quad \Big\{< \bar{K}^{*0}|(\bar{s}b)_\mu|\bar{B}^0 >< \omega|(\bar{u}u)^\mu|0 > [a_-A_c + a_2A_t] \\
&\quad + < \bar{K}^{*0}|(\bar{s}b)_\mu|\bar{B}^0 >< \omega|\left(\bar{u}u + \bar{d}d\right)^\mu|0 > (a_3 + c_6)A_t \\
&\quad + < \omega|\left(\bar{d}b\right)_\mu|\bar{B}^0 >< \bar{K}^{*0}|(\bar{s}d)^\mu|0 > a_4A_t \\
&\quad + < \bar{K}^{*0}\omega|(\bar{s}b)_\mu|O >< 0|\left(\bar{d}b\right)^\mu|\bar{B}^0 > a_4A_t \\
&\quad + < \bar{K}^{*0}\omega|(\bar{s}d)|0 >< 0|\left(\bar{d}\gamma_5 b\right)|\bar{B}^0 > a_5A_t\Big\}
\end{aligned}
\tag{B.1}
$$

and for $\bar{B}^0 \to \bar{K}^{*0}\rho^0$ which looks similar:

$$
\begin{aligned}
&< \bar{K}^{*0}\rho^0|H_2^{eff}|\bar{B}^0 >= -G/\sqrt{2} \\
&\quad \Big\{< \bar{K}^{*0}|(\bar{s}b)_\mu|\bar{B}^0 >< \rho^0|(\bar{u}u)^\mu|0 > [a_-A_c + a_2A_t] \\
&\quad + < \rho^0|\left(\bar{d}b\right)^\mu|\bar{B}^0 >< \bar{K}^{*0}|(\bar{s}d)^\mu|0 > a_4A_t \\
&\quad + < \bar{K}^{*0}\rho^0|\left(\bar{d}s\right)_\mu|0 >< 0|\left(\bar{d}b\right)_\mu|\bar{B}^0 > a_4A_t
\end{aligned}
$$

$$
\quad + < \bar{K}^{*0}\rho^0|\left(\bar{d}s\right)|0 >< 0|\left(\bar{d}\gamma_5 b\right)|\bar{B}^0 > a_5A_t\Big\}
\tag{B.2}
$$

We remark that in both cases we have two interfering contributions even when annihilation terms are neglected. These terms, first and second term in (B.1) and (B.2), can lead to helicity amplitudes with different weak phases which can produce non–vanishing β_1 and β_2 terms. Without higher order QCD corrections we have $a_- = a_2$ $(c_+ = c_- = 1, c_1 = 0, c_2 = 2)$ so that the first term in (B.1) and (B.2) is proportional to $A_c + A_t = -A_u = -V_{us}^*V_{ub}$ due to the unitarity of the CKM matrix. This means that in zeroth order of QCD these two transitions are strongly suppressed since $V_{us}^*V_{ub} \sim \lambda^4$ (see sect. 5.1)). With penguins included this suppression is lifted and larger branching fractions are possible. The main point is that there is no OZI suppression in the second term in (B.1) and (B.2), respectively.
For comparison we write now the matrix elements for the corresponding B^- decays: $B^- \to K^{*-}\omega, B^- \to K^{*-}\rho^0$ and $\bar{B}^0 \to K^{*-}\rho^+$.

$$
\begin{aligned}
&< K^{*-}\omega|H_2^{eff}|B^- >= -G/\sqrt{2} \\
&\quad \Big\{< K^{*-}|(\bar{s}b)_\mu|B^- >< \omega|(\bar{u}u)^\mu|0 > [a_-A_c + a_2A_t] \\
&\quad + < K^{*-}|(\bar{s}b)_\mu|B^- >< \omega|\left(\bar{u}u + \bar{d}d\right)^\mu|0 > (a_3 + c_6)A_t \\
&\quad + < \omega|(\bar{u}b)_\mu B^- >< K^{*-}|(\bar{s}u)^\mu|0 > [a_+A_c + (a_1 + a_4)A_t] \\
&\quad + < K^{*-}\omega|(\bar{s}u)_\mu|0 >< 0|(\bar{u}b)^\mu|B^- > \left[a_+A_c + (a_1 + a_4)A_t\right] \\
&\quad + < K^{*-}\omega|(\bar{s}u)|0 >< 0|(\bar{u}\gamma_5 b)|B^- > a_5A_t\Big\}
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
&< K^{*-}\rho^0|H_2^{eff}|B^- >= -G/\sqrt{2} \\
&\quad \Big\{< K^{*-}|(\bar{s}b)_\mu|B^- >< \rho^0|(\bar{u}u)^\mu|0 > \left[a_-A_c + a_2A_t\right] \\
&\quad + < \rho^0|(\bar{u}b)_\mu|B^- >< K^{*-}|(\bar{s}u)^\mu|0 > \left[a_+A_c + (a_1 + a_4)A_t\right] \\
&\quad + < K^{*-}\rho^0|(\bar{s}u)_\mu|0 >< 0|(\bar{u}b)^\mu|B^- > \left[a_+A_c + (a_1 + a_4)A_t\right] \\
&\quad + < K^{*-}\rho^0|(\bar{s}u)|0 >< 0|(\bar{u}\gamma_5 b)|B^- > a_5A_t\Big\}
\end{aligned}
\tag{B.4}
$$

$$< K^{*-}\rho^{+}|H_2^{eff}|\bar{B}^0> = -G/\sqrt{2}$$

$$\Big\{<\rho^{+}|(\bar{u}b)_\mu|\bar{B}^0><K^{*-}|(\bar{s}u)^\mu|0>\left[a_-A_c+(a_1+a_4)A_t\right]$$

$$+<K^{*-}\rho^{+}|\left(\bar{d}s\right)_\mu|0><0|\left(\bar{d}b\right)^\mu|\bar{B}^0>a_4A_t$$

$$+<K^{*-}\rho^{+}|\left(\bar{d}s\right)|0><0|\left(\bar{d}\gamma_5 b\right)|\bar{B}^0>a_5A_t\Big\}$$

(B.5)

Next we write the matrix element for $B^- \to K^{*-}\psi$

$$< K^{*-}\psi|H_2^{eff}|B^-> = -G/\sqrt{2}$$

$$\Big\{<K^{*-}|(\bar{s}b)_\mu|B^-><\psi|(\bar{c}c)^\mu|0>\left[-a_-A_c+(a_4-c_6)A_t\right]$$

$$+<\psi|(\bar{u}b)_\mu|B^-><K^{*-}|(\bar{s}u)^\mu|0>\left[a_+A_c+(a_1-a_4)A_t\right]$$

$$+<K^{*-}\psi|(\bar{s}u)_\mu|0><0|(\bar{u}b)^\mu|B^->\left[a_+A_c+(a_1+a_4)A_t\right]$$

$$+<K^{*-}\psi|(\bar{s}u)|0><0|(\bar{u}\gamma_5 b)|B^->a_5A_t\Big\}$$

(B.6)

The second term is **OZI** forbidden and in addition suppressed in zeroth order QCD because of $A_c + A_t = -A_u$. The latter also applies to the third term, an annihilation contribution. The most dominant decays in this group are $\bar{B}^0 \to D^{*+}D_s^{*-}$ and $B^- \to D^{*0}D_s^{*-}$ which are calculated from the following formulas:

$$< D^{*+}D_s^{*-}|H_2^{eff}|\bar{B}^0> = -G/\sqrt{2}$$

$$\Big\{<D^{*+}|(\bar{c}b)_\mu|\bar{B}^0><D_s^{*-}|(\bar{s}c)^\mu|0>\left[-a_+A_c-a_4A_t\right]$$

$$+<D^{*+}D_s^{*-}|\left(\bar{d}s\right)_\mu|0><0|\left(\bar{b}d\right)^\mu|\bar{B}^0>a_4A_t$$

$$+<D^{*+}D_s^{*-}|(\bar{s}d)|0><0|\left(d\gamma_5 b\right)|\bar{B}^0>a_5A_t\Big\}$$

(B.7)

$$< D^{*0}D_s^{*-}|H_2^{eff}|B^-> = -G/\sqrt{2}$$

$$\Big\{<D^{*0}|(\bar{c}b)_\mu|B^-><D_s^{*-}|(\bar{s}c)^\mu|0>\left[-a_-A_c+a_4A_t\right]$$

$$+<D^{*0}D_s^{*-}|(\bar{s}u)_\mu|0><0|(\bar{u}b)^\mu|B^->\left[a_+A_c+(a_1+a_4)A_t\right]$$

$$+<D^{*0}D_s^{*-}|(\bar{s}u)|0><0|(\bar{u}\gamma_5 b)|B^->a_5A_t\Big\}$$

(B.8)

These two decays are influenced very little by penguin contributions. In (B.8) the annihilation term is additionally suppressed in zeroth order QCD since it is proportional to $A_c + A_t = -A_u$. β_1 and β_2 vanish when annihilation terms are neglected.

Next we write the matrix element for the decays induced by H_3^{eff}. Instead of A_c and A_t the CKM elements are $\hat{A}_c$ and $\hat{A}_t$ defined in (4.6). Compared to A_c and A_t they are Cabibbo suppressed by the factor $V_{cd}/V_{cs} \simeq -\lambda$. We group together $\bar{B}^0 \to \omega\rho^0$, $\rho^0\rho^0$, $\omega\omega$, $\rho^+\rho^-$, $\omega\psi$, $\rho^0\psi$. The matrix element for these six decays are:

$$< \omega\rho^0|H_3^{eff}|\bar{B}^0> = -G/\sqrt{2}$$

$$\Big\{<\omega|\left(\bar{d}b\right)_\mu|\bar{B}^0><\rho^0|(\bar{u}u)^\mu|0>[a_-\hat{A}_c+a_2\hat{A}_t]$$

$$+<\rho^0|\left(\bar{d}b\right)_\mu|\bar{B}^0><\omega|(\bar{u}u)^\mu|0>[a_-\hat{A}_c+(a_2+a_3+c_6)\hat{A}_t]$$

$$+<\rho^0|\left(\bar{d}b\right)_\mu|\bar{B}^0><\omega|\left(\bar{d}d\right)^\mu|0>(a_3+a_4+c_6)\hat{A}_t$$

$$+<\omega\rho^0|(\bar{u}u)_\mu|0><0|\left(\bar{d}b\right)^\mu|\bar{B}^0>[a_-\hat{A}_c+a_2\hat{A}_t]$$

$$<\omega\rho^0|\left(\bar{d}d\right)|0><0|\left(\bar{d}\gamma_5 b\right)|\bar{B}^0>a_5\hat{A}_t\Big\}$$

(B.9)

$$< \rho_1^0\rho_2^0|H_3^{eff}|\bar{B}^0> = -G/\sqrt{2}$$

$$\Big\{<\rho_1^0|\left(\bar{d}b\right)_\mu|\bar{B}^0><\rho_2^0|(\bar{u}u)^\mu|0>[a_-\hat{A}_c+a_2\hat{A}_t]$$

$$+<\rho_2^0|\left(\bar{d}b\right)_\mu|\bar{B}^0><\rho_1^0|(\bar{u}u)^\mu|0>[a_-\hat{A}_c+a_2\hat{A}_t]$$

$$+<\rho_1^0\rho_2^0|(\bar{u}u)_\mu|0><0|\left(\bar{d}b\right)^\mu|\bar{B}^0>[a_-\hat{A}_c+a_2\hat{A}_t]$$

$$+<\rho_1^0\rho_2^0|\left(\sum_q \bar{q}q\right)_\mu|0><0|\left(\bar{d}b\right)^\mu|\bar{B}^0>a_3\hat{A}_t$$

$$+<\rho_1^0\rho_2^0|\left(\sum_q \bar{q}q\right)_\mu^R|0><0|\left(\bar{d}b\right)^\mu|\bar{B}^0>c_6\hat{A}_t$$

$$+ < \rho_1^0 \rho_2^0 | \left(\bar{d}d\right) |0 >< 0| \left(\bar{d}\gamma_5 b\right) |\bar{B}^0 > a_5 \hat{A}_t \Big\} \tag{B.10}$$

$$< \omega_1\omega_2|H_3^{eff}|\bar{B}^0 >= -G/\sqrt{2}$$
$$\Big\{ < \omega_1| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \omega_2| (\bar{u}u)^\mu |0 > [a_- \hat{A}_c + a_2 \hat{A}_t]$$
$$+ < \omega_2| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \omega_1| (\bar{u}u)^\mu |0 > [a_- \hat{A}_c + (a_2 + a_3 + c_6) \hat{A}_t]$$
$$+ < \omega_2| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \omega_1| \left(\bar{d}d\right)^\mu |0 > (a_3 + a_4 + c_6) \hat{A}_t$$
$$+ < \omega_1\omega_2| (\bar{u}u)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > [a_- \hat{A}_c + a_2 \hat{A}_t]$$
$$+ < \omega_1\omega_2| \left(\sum_q \bar{q}q\right)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > a_3 \hat{A}_t$$
$$+ < \omega_1\omega_2| \left(\sum_q \bar{q}q\right)_\mu^R |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > c_6 \hat{A}_t$$
$$+ < \omega_1\omega_2| \left(\bar{d}d\right) |0 >< 0| \left(\bar{d}\gamma_5 b\right) |\bar{B}^0 > a_5 \hat{A}_t \Big\} \tag{B.11}$$

$$< \rho^+\rho^- |H_3^{eff}|\bar{B}^0 >= -G/\sqrt{2}$$
$$\Big\{ < \rho^+ | (\bar{u}b)_\mu |\bar{B}^0 >< \rho^- | \left(\bar{d}u\right)^\mu |0 > [a_+ \hat{A}_c + (a_3 + a_4) \hat{A}_t]$$
$$+ < \rho^+ \rho^- | (\bar{u}u)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > [a_- \hat{A}_c + a_2 \hat{A}_t]$$
$$+ < \rho^+ \rho^- | \left(\sum_q \bar{q}q\right)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > a_3 \hat{A}_t$$
$$+ < \rho^+ \rho^- | \left(\sum_q \bar{q}q\right)_\mu^R |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > c_6 \hat{A}_t$$
$$+ < \rho^+ \rho^- | \left(\bar{d}d\right) |0 >< 0| \left(\bar{d}\gamma_5 b\right) |\bar{B}^0 > a_5 \hat{A}_t \Big\} \tag{B.12}$$

The four decay matrix elements in (B.9-12) have the property that in zeroth order QCD they are proportional to $\hat{A}_c + \hat{A}_t = -\hat{A}_u = V_{ud}^* V_{ub} \simeq \lambda^3$. Therefore all these decays have small branching ratios. Next we give the matrix elements for the decays $\bar{B}^0 \to \omega\psi, \rho^0\psi$.

$$< \omega\psi|H_3^{eff}|\bar{B}^0 >= -G/\sqrt{2}$$
$$\Big\{ < \omega| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \psi| (\bar{c}c)^\mu |0 > [-a_- \hat{A}_c + (a_3 + c_6) \hat{A}_t]$$
$$+ < \psi| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \omega| (\bar{u}u)^\mu |0 > [a_- \hat{A}_c + (a_2 + a_3 + c_6) \hat{A}_t]$$
$$- < \psi| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \omega| \left(\bar{d}d\right)^\mu |0 > (a_3 + a_4 + c_6) \hat{A}_t$$
$$- < \omega\psi| (\bar{u}u)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > [a_- \hat{A}_c + a_1 \hat{A}_t]$$
$$- < \omega\psi| (\bar{c}c)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > a_- \hat{A}_c$$
$$+ < \omega\psi| \sum_q (\bar{q}q)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > a_3 \hat{A}_t$$
$$+ < \omega\psi| \sum_q (\bar{q}q)_\mu^R |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > c_6 \hat{A}_t$$
$$< \omega\psi| \left(\bar{d}d\right) |0 >< 0| \left(\bar{d}\gamma_5 b\right) |\bar{B}^0 > a_5 \hat{A}_t \Big\} \tag{B.13}$$

$$< \rho^0\psi|H_3^{eff}|\bar{B}^0 >= -G/\sqrt{2}$$
$$\Big\{ < \rho^0| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \psi| (\bar{c}c)^\mu |0 > [-a_- \hat{A}_c - (a_3 + c_6) \hat{A}_t]$$
$$+ < \psi| \left(\bar{d}b\right)_\mu |\bar{B}^0 >< \rho^0| (\bar{u}u)^\mu |0 > [a_- \hat{A}_c + a_2 \hat{A}_t]$$
$$+ < \rho^0\psi| (\bar{u}u)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > [a_- \hat{A}_c + a_2 \hat{A}_t]$$
$$- < \rho^0\psi| (\bar{c}c)_\mu |0 >< 0| \left(\bar{d}b\right)^\mu |\bar{B}^0 > a_- \hat{A}_c$$

$$+ < \rho^0\psi|\left(\bar{d}d\right)|0 >< |\left(\bar{d}\gamma_5 b\right)|\bar{B}^0 > a_5\hat{A}_t\Big\}$$

(B.14)

In both matrix elements the second non-annihilation diagram is OZI forbidden, so that the dominant part consists of just one amplitude.

Since the matrix element for $\bar{B}^0 \to D^{*+}D^{*-}$ was already given in (4.5) we are left with four matrix elements for B^- decays, namely for $B^- \to \omega\rho^-, \rho^0\rho^-, \rho^-\psi, D^{*0}D^{*-}$, which will be written now.

$$< \omega\rho^-|H_3^{eff}|B^- >= -G/\sqrt{2}$$

$$\Big\{< \omega|(\bar{u}b)_\mu|B^- >< \rho^-|\left(\bar{d}b\right)^\mu|0 > [a_+\hat{A}_c + (a_1 + a_4)\hat{A}_t]$$

$$+ < \rho^-|\left(\bar{d}b\right)_\mu|B^- >< \omega|(\bar{u}u)^\mu|0 > [a_-\hat{A}_c + (a_2 + a_3 + c_6)\hat{A}_t]$$

$$+ < \rho^-|\left(\bar{d}b\right)_\mu|B^- >< \omega|\left(\bar{d}d\right)^\mu|0 > (a_3 + a_4 + c_6)\hat{A}_t$$

$$+ < \omega\rho^-|\left(\bar{d}u\right)_\mu|0 >< 0|(\bar{u}b)^\mu|B^- > [a_+A_c + (a_1 + a_4)\hat{A}_t]$$

$$< \omega\rho^-|\left(\bar{d}u\right)|0 >< 0|(\bar{u}\gamma_5 b)|B^- > a_5\hat{A}_t\Big\}$$

(B.15)

$$< \rho^0\rho^-|H_3^{eff}|B^- >= -G/\sqrt{2}$$

$$\Big\{< \rho^0|(\bar{u}b)_\mu|B^- >< \rho^-|\left(\bar{d}u\right)^\mu|0 > [a_+\hat{A}_c + (a_1 + a_4)\hat{A}_t]$$

$$+ < \rho^-|\left(\bar{d}b\right)_\mu B^- >< \rho^0|(\bar{u}u)^\mu|0 > [a_-\hat{A}_c + a_2\hat{A}_t]$$

$$+ < \rho^0\rho^-|\left(\bar{d}u\right)_\mu|0 >< 0|(\bar{u}b)^\mu|B^- > [a_+\hat{A}_c + (a_1 + a_4)\hat{A}_t]$$

$$+ < \rho^0\rho^-|\left(\bar{d}u\right)|0 >< 0|(\bar{u}\gamma_5 b)|B^- > a_5\hat{A}_t\Big\}$$

(B.16)

In this matrix element we have interference only through the mass difference of ρ^0 and ρ^-, otherwise the helicity matrix elements generated from the first and second line in (B.15) differ

only by a common factor.

$$< \rho^-\psi|H_3^{eff}|B^- >= -G/\sqrt{2}$$

$$\Big\{< \rho^-|\left(\bar{d}b\right)_\mu|B^- >< \psi|(\bar{c}c)^\mu|0 > [-a_-\hat{A}_c + (a_3 + c_6)\hat{A}_t]$$

$$+ < \psi|(\bar{u}b)_\mu|B^- >< \rho^-|\left(\bar{d}u\right)^\mu|0 > [a_+\hat{A}_c + (a_1 + a_4)\hat{A}_t]$$

$$+ < \rho^-\psi|(du)_\mu|0 >< 0|(\bar{u}b)^\mu|B^- > [a_+\hat{A}_c + (a_2 + a_4)\hat{A}_t]$$

$$+ < \rho^-\psi|\left(\bar{d}u\right)|0 >< 0|(\bar{u}\gamma_5 b)|B^- > a_5\hat{A}_t\Big\}$$

(B.17)

The second term of the right-hand side is OZI forbidden.

$$< D^{*0}D^{*-}|H_3^{eff}|B^- >= -G/\sqrt{2}$$

$$\Big\{< D^{*0}|(\bar{c}b)_\mu|B^- >< D^{*-}|\left(\bar{d}c\right)^\mu|0 > [-a_+\hat{A}_c + a_4\hat{A}_t]$$

$$+ < D^{*0}D^{*-}|\left(\bar{d}u\right)_\mu|0 >< 0|(\bar{u}b)^\mu|B^- > [a_+\hat{A}_c + (a_1 + a_4)\hat{A}_t]$$

$$+ < D^{*0}D^{*-}|\left(\bar{d}u\right)|0 >< 0|(\bar{u}\gamma_5 b)|B^- > a_5\hat{A}_t\Big\}$$

(B.18)

This matrix element is dominated by the term proportional to $a_+\hat{A}_c$ so that penguin term are unimportant here.

In the following page we present the formulas for decays without penguin contributions induced by H_1^{eff}, H_4^{eff}, H_5^{eff} and H_6^{eff}. These matrix elements have far fewer terms. We start with the matrix elements for $\bar{B}^0 \to \omega D^{*0}, \rho^0 D^{*0}$ and for $B^- \to \rho^- D^{*0}$ (see table 2 for the list of all H_1^{eff} induced transitions)

$$< \omega D^{*0}|H_1^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}}V_{ud}^{*}V_{cb}$$
$$< \omega|\left(\bar{d}b\right)_\mu|\bar{B}^0 >< D^{*0}|(\bar{u}c)^\mu|0 > a_-$$

(B.19)

$$< \rho^0 D^{*0}|H_1^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}} V_{ud}^* V_{cb}$$
$$< \rho^0 D^{*0}|(\bar{d}b)_\mu|\bar{B}^0 >< D^{*0}|(\bar{u}c)^\mu|0 > a_- \tag{B.20}$$

$$< \rho^- D^{*+}|H_1^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}} V_{ud}^* V_{cb}$$
$$< D^{*+}|(\bar{c}b)_\mu \bar{B}^0 >< \rho^-|(\bar{u}d)^\mu|0 > a_- \tag{B.21}$$

The matrix elements for the transitions $\bar{B}^0 \to K^{*-}D^{*+}, \bar{K}^{*0}D^{*0}$ and $B^- \to K^{*-}D^{*0}$ induced by H_4^{eff} are:

$$< K^{*-} D^{*+}|H_4^{eff}|\bar{B}^0 > = \frac{G}{\sqrt{2}} V_{us}^* V_{cb} a_+$$
$$< D^{*+}|(\bar{c}b)_\mu \bar{B}^0 >< K^{*-}|(\bar{s}u)^\mu|0 > \tag{B.22}$$

$$< \bar{K}^{*0} D^{*0}|H_4^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}} V_{us}^* V_{cb} a_-$$
$$< \bar{K}^{*0}|(sb)_\mu|\bar{B}^0 >< D^{*0}|(\bar{c}u)^\mu|0 > \tag{B.23}$$

$$< \bar{K}^{*-} D^{*0}|H_4^{eff}|B^- >= \frac{G}{\sqrt{2}} V_{us}^* V_{cb}$$
$$\{< K^{*-}|(\bar{s}b)_\mu|B^- >< D^{*0}|(\bar{c}u)^\mu|0 > a_-$$
$$+ < D^{*0}|(\bar{c}b)_\mu|B^- >< K^{*-}|(\bar{s}u)^\mu|0 > a_+\} \tag{B.24}$$

The decays induced by H_5^{eff}, namely $\bar{B}^{*0} \to \bar{K}^{*0}\bar{D}^{*0}$ and $B^- \to K^{*-}\bar{D}^{*0}$ have, as pure $b \to u$ transitions, small branching ratios. The matrix elements are

$$< \bar{K}^{*0}\bar{D}^{*0}|H_5^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}} V_{ub}^* V_{cs} a_- < \bar{K}^{*0}|(\bar{s}b)_\mu|\bar{B}^0 >< \bar{D}^{*0}|(\bar{u}c)^\mu|0 > \tag{B.25}$$

$$< K^{*-}\bar{D}^{*0}|H_5^{eff}|B^- >= \frac{G}{\sqrt{2}} V_{ub}^* V_{cs} a_- < K^{*-}|(\bar{s}b)_\mu|B^- >< D^{*0}|(\bar{u}c)^\mu|0 > \tag{B.26}$$

The transitions originating from H_6^{eff} are suppressed even further since they have additional Cabibbo suppression as compared to the H_5^{eff} transitions. The matrix elements for $\bar{B}^0 \to \rho^+ D^{*-}, \rho^0\bar{D}^{*0}\omega\bar{D}^{*0}$ and $B^- \to \rho^-\bar{D}^{*0}, \rho^0 D^{*-}, \omega D^{*-}$ are:

$$< \rho^+ D^{*-}|H_6^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}} V_{cd}^* V_{ub} a_+$$
$$< \rho^+|(\bar{u}b)_\mu|\bar{B}^0 >< D^{*-}|\left(\bar{d}c\right)^\mu|0 > \tag{B.27}$$

$$< \rho^0 \bar{D}^{*0}|H_6^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}} V_{cd}^* V_{ub} a_-$$
$$< \rho^0|\left(\bar{d}b\right)_\mu|\bar{B}^0 >< \bar{D}^{*0}|(\bar{u}c)^\mu|0 > \tag{B.28}$$

$$< \omega\bar{D}^{*0}|H_6^{eff}|\bar{B}^0 >= \frac{G}{\sqrt{2}} V_{cd}^* V_{ub} a_-$$
$$< \omega|\left(\bar{d}b\right)_\mu|\bar{B}^0 >< D^{*0}|(\bar{u}c)^\mu|0 > \tag{B.29}$$

$$< \rho^- \bar{D}^{*0}|H_6^{eff}|B^- >= \frac{G}{\sqrt{2}} V_{cd}^* V_{ub} a_-$$
$$< \rho^-|\left(\bar{d}b\right)_\mu|B^- >< \bar{D}^*0|(\bar{u}c)^\mu|0 > \tag{B.30}$$

$$< \rho^0 D^{*-}|H_6^{eff}|B^- >= \frac{G}{\sqrt{2}} V_{cd}^* V_{ub} a_+ < \rho^0|(\bar{u}b)_\mu|B^- >< D^{*-}|\left(\bar{d}c\right)^\mu|0 > \tag{B.31}$$

$$< \omega D^{*-}|H_6^{eff}|B^- >= \frac{G}{\sqrt{2}} V_{cd}^* V_{ub} a_+ < \omega|(\bar{u}b)_\mu|B^- >< D^{*-}|\left(\bar{d}c\right)^\mu|0 > \tag{B.32}$$

References

[1] M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49 (1973) 652

[2] F. Gilman, M. Wise, Phys. Rev. D20 (1979) 2392

[3] W. Ponce, Phys. Rev. D23 (1981) 1134

[4] D. Fakirov, B. Stech, Nucl. Phys. B133 (1978) 315, N. Cabibbo, L. Maiani, Phys. Lett. 73 B (1978) 418 and earlier references given there.

[5] A. Ali et al., Z. Phys. C1 (1979) 269, J. G. Körner in Proceedings of the International Symposium on Production and Decay of Heavy Hadrons, Heidelberg 1986, K.R. Schubert, R. Waldi Eds., DESY, Hamburg.

[6] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C34 (1987) 103, M. Wirbel, B. Stech, M. Bauer, Z. Phys. C29 (1985) 637

[7] M. Shifman. A. Vainshtein, V. Zakharov, Sov. Phys. JETP 45 (1977)670, Nucl. Phys. B129 (1977) 316

[8] J. M. Gerard, W.-S. Hou, Phys. Rev. D43 (1991) 2909

[9] M. Tanimoto, K. Goda, K. Senba, Phys. Rev. D42 (1990) 3741 and the earlier papers given there

[10] A. N. Kamal, N. Sinha, R. Sinha, Z. Phys. C41 (1988) 207

[11] M. Wanninger, L. M. Seghal, Z. Phys. C50 (1991) 47

[12] Ling-Lie Chau et al., Phys. Rev. D43 (1991) 2176 and the earlier literature given there

[13] G. Valencia, Phys. Rev D 39 (1989) 3339

[14] I.I. Bigi et al. in CP Violation, Editor C. Jarlskog, Advanced Series on Directions in High Energy Physics, Vol. 3, World Scientific, Singapore, New Jersey, London, Hong Kong, 1989 and the earlier literature quoted there.

[15] B. Kayser et al., Phys. Lett. B237 (1990) 508

[16] I. Dunietz et al., Phys. Rev. D43 (1991) 2193

[17] T. Dugan, B. Grinstein, Harvard Univ. preprint HUTP-90/AO71, D. Politzer, M.B. Wise, CALTEC preprint CALT-68-1696

[18] H. Albrecht, et al. Z. Phys. C48 (1990) 543, A. Hölscher, DESY, Interner Bericht, F15-90-06

[19] K. Berkelman, S.L. Stone, Cornell preprint CLNS 91-1044 (To be published in Annals of Nuclear and Particle Science)

[20] A. J. Buras, J.-M. Gerard, R. Rückl, Nucl. Phys. B268 (1986) 16

[21] W. A. Bardeen, A. J. Buras, J.-M. Gerard, Phys. Lett. B180 (1980) 133

[22] N. Desphande, M. Gronau, D. Sutherland, Phys. Lett. B90 (1980) 431

[23] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945

[24] M. Lusignoli, et al., Universita Roma "La Sapienza " preprint n.792, March 6, 1991

[25] H Albrecht et al., Phys. Lett. B197 (1987) 452, D. Bortoletto et al., Phys. Rev. Lett. 63 (1989) 554, D. Antreasyan et al., Z. Phys. C48 (1990) 553

[26] R. Fulton et al.,Phys. Rev. Lett. 64 (1990) 16 H. Albrecht et al., Phys. Lett. B197 (1990) 409

[27] I. Dunietz, A. Snyder, Phys. Rev. D43 (1991) 1593

[28] M. Gronau, D. London, DESY report 90-125

[29] J. G. Körner, G. R. Goldstein, Phys. Lett. B87 (1979) 105

[30] H. Schröder, talk presented at the ECFA Working Group Meeting, DESY, April 1991 (unpublished), H. Albrecht et al. DESY report 90-157 (1990)

[31] H. Albrecht et al., DESY 89-096 (1989)

[32] P. Avery et al., Phys. Lett. B223 (1989) 470

[33] H. Albrecht et al., Phys. Lett. B241 (1990) 278

[34] D. Bortoletto, F. Stone, Phys. Rev. Lett. 65 (1990) 2951

[35] J. Rosner, Phys. Rev. D42 (1990) 3732

[36] T. Mannel, W. Roberts, Z. Rysak, Phys. Lett. B259 (1991) 485

[37] M. Jacob, G. C. Wick, Ann. Phys. 7 (1959)404

Table Captions

Table 1: Comparison of various models for the amplitude coefficients. The eight independent combinations in the first column take the indicated values in various models.

Table 2: Given for the channels in the first column are: branching ratios (column 2), $\bar{B}^0 B^0$ mixing parameter (column 3), transverse to total decay rate ratio (column 4), and coefficients of the indicated azimuthal terms in the angular distribution, where ϕ is the angle between the V_1 and V_2 decay planes (columns 5 and 6). The input CKM parameters are those of reference [24](ρ positive solution.) The model for the effective Hamiltonian coefficients is $O(\alpha_s)$ QCD with Fierz terms. These channels have no penguin diagrams.

Table 3: Given for the channels in the first column are: branching ratios (column 2), $\bar{B}^0 B^0$ mixing parameter (column 3), transverse to total decay rate ratio (column 4), and coefficients of the indicated azimuthal terms in the angular distribution, where ϕ is the angle between the V_1 and V_2 decay planes (columns 5 and 6). The input CKM parameters are those of reference [24](ρ positive solution.) The model for the effective Hamiltonian coefficients is renormalization group improved QCD with Fierz terms. These channels have no penguin diagrams.

Table 4: Given for the channels in the first column are: branching ratios (column 2), $\bar{B}^0 B^0$ mixing parameter (column 3), transverse to total decay rate ratio (column 4), and coefficients of the indicated azimuthal terms in the angular distribution, where ϕ is the angle between the V_1 and V_2 decay planes (columns 5 and 6). The input CKM parameters are those of reference [24](ρ positive solution.) The model for the effective Hamiltonian coefficients is renormalization group improved QCD without Fierz terms. These channels have no penguin diagrams.

Table 5: Given for the channels in the first column are: branching ratios (column 2), $\bar{B}^0 B^0$ mixing parameter (column 3), transverse to total decay rate ratio (column 4), and coefficients of the indicated azimuthal terms in the angular distribution, where ϕ is the angle between the V_1 and V_2 decay planes (columns 5, 6, 7, and 8). The input CKM parameters are those of reference [24](ρ positive solution.) The model for the effective Hamiltonian coefficients is $O(\alpha_s)$ QCD with Fierz terms. These channels can have penguin diagrams.

Table 6: Given for the channels in the first column are: branching ratios (column 2), $\bar{B}^0 B^0$ mixing parameter (column 3), transverse to total decay rate ratio (column 4), and coefficients of the indicated azimuthal terms in the angular distribution, where ϕ is the angle between the V_1 and V_2 decay planes (columns 5, 6, 7, and 8). The input CKM parameters are those of reference [24](ρ positive solution.) The model for the effective Hamiltonian coefficients is renormalization group improved QCD with Fierz terms. These channels can have penguin diagrams.

Table 7: Given for the channels in the first column are: branching ratios (column 2), $\bar{B}^0 B^0$ mixing parameter (column 3), transverse to total decay rate ratio (column 4), and coefficients

of the indicated azimuthal terms in the angular distribution, where ϕ is the angle between the V_1 and V_2 decay planes (columns 5, 6, 7, and 8). The input CKM parameters are those of reference [24](ρ positive solution.) The model for the effective Hamiltonian coefficients is renormalization group improved QCD without Fierz terms. These channels can have penguin diagrams.

Table 8: Given for the channels in the first column are: branching ratios (column 2), $\bar{B}^0 B^0$ mixing parameter (column 3), transverse to total decay rate ratio (column 4), and coefficients of the indicated azimuthal terms in the angular distribution, where ϕ is the angle between the V_1 and V_2 decay planes (columns 5, 6,7,and 8). The input CKM parameters are those of reference [24](ρ negative solution.) The model for the effective Hamiltonian coefficients is renormalization group improved QCD without Fierz terms. These channels can have penguin diagrams.

Table 1.

QCD CLASSIFICATION – BSW Vs. QCD Coefficients of Factorized $\Delta b = 1$ Amplitudes						
Amplitude Coefficient $N_c = 3$	No QCD Value		BSW Model Fit to B Decays	$O(\alpha_s)$ Model	QCD Value (This Work)	
	$N_c = 3$	$N_c = \infty$		$N_c = 3$	$N_c = 3$	$N_c = \infty$
$\frac{2}{3}c_+ + \frac{1}{3}c_-$	1	1	$a_1 = 1.1$	1	1.07	1.22
$\frac{2}{3}c_+ - \frac{1}{3}c_-$	$\frac{1}{3}$	0	$a_2 = -0.24$	$\frac{1}{3}$	-0.041	-0.45
$\frac{c_1}{6} + \frac{c_2}{2}$	1	1	$a_1 = 1.1$	1	1.09	1.16
$\frac{c_2}{6} + \frac{c_1}{2}$	$\frac{1}{3}$	0	$a_2 = -0.24$	$\frac{1}{3}$	0.17	-0.22
$\frac{c_3}{6} + \frac{c_4}{2}$	0	0	0	-0.10	-.036	-0.042
$\frac{c_4}{6} + \frac{c_3}{2}$	0	0	0	0	0.005	0.019
$\frac{16}{9}c_5 + \frac{1}{3}c_6$	0	0	0	-0.20	-0.10	-0.005
$\frac{c_6}{2}$	0	0	0	0	-0.005	-0.005

Table 2.

MODEL: $O(\alpha_s)$ QCD Coefficients *with* Fierz Terms (ρ *positive*)					
Channel	Br(%)	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_T}{\Gamma}$	α_1 $[cos\phi]$	α_2 $[cos2\phi]$
Channels without Penguins: $\Delta s = 0,\ \Delta b = -\Delta c = 1\ (H_1)$					
$\bar{B}^0 \to \omega + D^{*0}$	0.0362	-0.657	0.284	-0.511	0.0402
$\bar{B}^0 \to \rho^0 + D^{*0}$	0.0371	-0.659	0.280	-0.509	0.0395
$\bar{B}^0 \to \rho^- + D^{*+}$	1.03	-0.787	0.126	-0.426	0.0402
$B^- \to \rho^- + D^{*0}$	1.65	—	0.153	-0.448	0.0417
Channels without Penguins: $\Delta b = \Delta s = -\Delta c = 1\ (H_4)$					
$\bar{B}^0 \to K^{*-} + D^{*+}$	0.0549	-0.778	0.161	-0.471	0.0519
$\bar{B}^0 \to \bar{K}^{*0} + D^{*0}$	0.00448	-0.665	0.304	-0.537	0.0549
$B^- \to K^{*-} + D^{*0}$	0.0903	—	0.189	-0.490	0.0538
Channels without Penguins: $\Delta b = \Delta c = \Delta s = 1\ (H_5)$					
$\bar{B}^0 \to \bar{K}^{*0} + \bar{D}^{*0}$W	0.00104	0.428	0.304	-0.537	0.0549
$B^- \to K^{*-} + \bar{D}^{*0}$	0.00104	—	0.304	-0.537	0.0549
Channels without Penguins: $\Delta s = 0,\ \Delta b = \Delta c = 1\ (H_6)$					
$\bar{B}^0 \to \rho^+ + \bar{D}^{*-}$	0.000401	-0.432	0.281	-0.509	0.0396
$\bar{B}^0 \to \rho^0 + \bar{D}^{*0}$	0.0000222	-0.432	0.280	-0.509	0.0395
$\bar{B}^0 \to \omega + \bar{D}^{*0}$	0.0000217	-0.431	0.284	-0.511	0.0402
$B^- \to \rho^- + \bar{D}^{*0}$	0.0000445	—	0.280	-0.509	0.0395
$B^- \to \rho^0 + D^{*-}$	0.000401	—	0.281	-0.509	0.0396

Table 3.

MODEL: QCD Coefficients *with* Fierz Terms (ρ *positive*)					
Channel	Br(%)	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_T}{\Gamma}$	α_1 [$cos\phi$]	α_2 [$cos2\phi$]
Channels without Penguins: $\Delta s = 0,\ \Delta b = -\Delta c = 1\ (H_1)$					
$\bar{B}^0 \to \omega + D^{*0}$	0.000531	-0.657	0.284	-0.511	0.0402
$\bar{B}^0 \to \rho^0 + D^{*0}$	0.000543	-0.659	0.280	-0.509	0.0395
$\bar{B}^0 \to \rho^- + D^{*+}$	1.19	-0.787	0.126	-0.425	0.0402
$B^- \to \rho^- + D^{*0}$	1.12	—	0.122	-0.421	0.0398
Channels without Penguins: $\Delta b = \Delta s = -\Delta c = 1\ (H_4)$					
$\bar{B}^0 \to K^{*-} + D^{*+}$	0.0631	-0.778	0.161	-0.471	0.0519
$\bar{B}^0 \to \bar{K}^{*0} + D^{*0}$	0.0000657	-0.665	0.304	-0.537	0.0549
$B^- \to K^{*-} + D^{*0}$	0.0592	—	0.157	-0.468	0.0514
Channels without Penguins: $\Delta b = \Delta c = \Delta s = 1\ (H_5)$					
$\bar{B}^0 \to \bar{K}^{*0} + \bar{D}^{*0}$	0.0000152	0.428	0.304	-0.537	0.0549
$B^- \to K^{*-} + \bar{D}^{*0}$	0.0000152	—	0.304	-0.537	0.0549
Channels without Penguins: $\Delta s = 0,\ \Delta b = \Delta c = 1\ (H_6)$					
$\bar{B}^0 \to \rho^+ + \bar{D}^{*-}$	0.000461	-0.432	0.281	-0.509	0.0396
$\bar{B}^0 \to \rho^0 + \bar{D}^{*0}$	0.000000326	-0.432	0.280	-0.509	0.0395
$\bar{B}^0 \to \omega + \bar{D}^{*0}$	0.000000318	-0.431	0.284	-0.511	0.0402
$B^- \to \rho^- + \bar{D}^{*0}$	0.000000651	—	0.280	-0.509	0.0395
$B^- \to \rho^0 + D^{*-}$	0.000461	—	0.281	-0.509	0.0396

Table 4.

MODEL: QCD Coefficients *without* Fierz Terms (ρ *positive*)					
Channel	Br(%)	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_T}{\Gamma}$	α_1 [$cos\phi$]	α_2 [$cos2\phi$]
Channels without Penguins: $\Delta s = 0,\ \Delta b = -\Delta c = 1\ (H_1)$					
$\bar{B}^0 \to \omega + D^{*0}$	0.0653	-0.657	0.284	-0.511	0.0402
$\bar{B}^0 \to \rho^0 + D^{*0}$	0.0669	-0.659	0.280	-0.509	0.0395
$\bar{B}^0 \to \rho^- + D^{*+}$	1.54	-0.787	0.126	-0.425	0.0402
$B^- \to \rho^- + D^{*0}$	0.788	—	0.0818	-0.372	0.0344
Channels without Penguins: $\Delta b = \Delta s = -\Delta c = 1\ (H_4)$					
$\bar{B}^0 \to K^{*-} + D^{*+}$	0.0820	-0.778	0.161	-0.471	0.0519
$\bar{B}^0 \to \bar{K}^{*0} + D^{*0}$	0.00808	-0.665	0.304	-0.537	0.0549
$B^- \to K^{*-} + D^{*0}$	0.0395	—	0.112	-0.425	0.0455
Channels without Penguins: $\Delta b = \Delta c = \Delta s = 1\ (H_5)$					
$\bar{B}^0 \to \bar{K}^{*0} + \bar{D}^{*0}$	0.00188	0.428	0.304	-0.537	0.0549
$B^- \to K^{*-} + \bar{D}^{*0}$	0.00187	—	0.304	-0.537	0.0549
Channels without Penguins: $\Delta s = 0,\ \Delta b = \Delta c = 1\ (H_6)$					
$\bar{B}^0 \to \rho^+ + \bar{D}^{*-}$	0.000598	-0.432	0.281	-0.509	0.0396
$\bar{B}^0 \to \rho^0 + \bar{D}^{*0}$	0.0000401	-0.432	0.280	-0.509	0.0395
$\bar{B}^0 \to \omega + \bar{D}^{*0}$	0.0000392	-0.431	0.284	-0.511	0.0402
$B^- \to \rho^- + \bar{D}^{*0}$	0.0000801	—	0.280	-0.509	0.0395
$B^- \to \rho^0 + D^{*-}$	0.0000273	—	0.281	-0.509	0.0396

Table 5.

Channel	Br(%)	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_2}{\Gamma}$	α_1 $cos\phi$	α_2 $cos2\phi$	$\beta_1(10^{-4})$ $[sin\phi]$	$\beta_2(10^{-4})$ $[sin2\phi]$
MODEL: $O(\alpha_s)$ **QCD Coefficents** *with* **Fierz Terms** (ρ *positive*)							
Channels with Penguins: $\Delta c = 0,\ \Delta b = -\Delta s = 1\ (H_2)$							
$\bar{B}^0 \to \bar{K}^{*0} + \omega$	0.00339	-0.717	0.109	-0.336	0.00900	-18.8	1.79
$\bar{B}^0 \to \bar{K}^{*0} + \rho^0$	0.00212	-0.823	0.106	-0.333	0.00911	23.0	-2.01
$\bar{B}^0 \to K^{*-} + \rho^{+}$	0.00271	-0.506	0.106	-0.333	0.00892		
$\bar{B}^0 \to \bar{K}^{*0} + \psi$	0.423	-0.674	0.429	-0.621	0.123	—	—
$\bar{B}^0 \to D^{*+} + D_s^{*-}$	1.38	-0.732	0.477	-0.665	0.184	—	—
$B^- \to K^{*-} + \omega$	0.00120	—	0.108	-0.335	0.00895	-36.3	3.49
$B^- \to K^{*-} + \rho^0$	0.00152	—	0.105	-0.332	0.00903	31.2	-2.74
$B^- \to K^{*-} + \psi$	0.424	—	0.428	-0.621	0.123	—	—
$B^- \to D^{*0} + D_s^{*-}$	1.39	—	0.477	-0.664	0.183	—	—
Channels with Penguins: $\Delta s = \Delta c = 0,\ \Delta b = 1\ (H_3)$							
$\bar{B}^0 \to \omega + \rho^0$	0.0000244	-0.0427	0.0874	-0.304	0.00706	-64.0	0.462
$\bar{B}^0 \to \omega + \psi$	0.00918	-0.663	0.394	-0.599	0.0987	—	—
$\bar{B}^0 \to \rho^0 + \rho^0$	0.000224	-0.522	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to \rho^0 + \psi$	0.00943	-0.664	0.388	-0.597	0.0969	—	—
$\bar{B}^0 \to \omega + \omega$	0.000328	-0.902	0.0867	-0.303	0.00707	—	—
$\bar{B}^0 \to \rho^+ + \rho^-$	0.00449	-0.714	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to D^{*+} + D^{*-}$	0.0744	-0.769	0.456	-0.660	0.172	—	—
$B^- \to \omega + \rho^-$	0.0326	—	0.0857	-0.302	0.00695	3.61	-0.0268
$B^- \to \rho^0 + \rho^-$	0.00387	—	0.0838	-0.299	0.00676	-0.0317	0.00142
$B^- \to \rho^- + \psi$	0.0188	—	0.389	-0.597	0.0970	—	—
$B^- \to D^{*0} + D^{*-}$	0.0745	—	0.456	-0.660	0.172	—	—

Table 6.

Channel	Br(%)	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_2}{\Gamma}$	α_1 $cos\phi$	α_2 $cos2\phi]$	$\beta_1(10^{-4})$ $[sin\phi]$	$\beta_2(10^{-4})$ $[sin2\phi]$
MODEL: QCD Coefficients *with* **Fierz Terms** (ρ *positive*)							
Channels with Penguins: $\Delta c = 0,\ \Delta b = -\Delta s = 1\ (H_2)$							
$\bar{B}^0 \to \bar{K}^{*0} + \omega$	0.00527	-0.786	0.0854	-0.314	0.0113	0.518	-0.0492
$\bar{B}^0 \to \bar{K}^{*0} + \rho^0$	0.0120	-0.782	0.0902	-0.318	0.0104	-0.174	0.0152
$\bar{B}^0 \to K^{*-} + \rho^{+}$	0.000145	0.911	0.106	-0.333	0.00892	—	—
$\bar{B}^0 \to \bar{K}^{*0} + \psi$	0.00635	-0.674	0.429	-0.621	0.123.	—	—
$\bar{B}^0 \to D^{*+} + D_s^{*-}$	1.85	-0.733	0.477	-0.665	0.184	—	—
$B^- \to K^{*-} + \omega$	0.00847	—	0.0899	-0.319	0.0108	-33.1	3.18
$B^- \to K^{*-} + \rho^0$	0.00946	—	0.0881	-0.315	0.0104	32.5	-2.86
$B^- \to K^{*-} + \psi$	0.00636	—	0.428	-0.621	0.123	—	—
$B^- \to D^{*0} + D_s^{*-}$	1.85	—	0.477	-0.664	0.183	—	—
Channels with Penguins: $\Delta s = \Delta c = 0,\ \Delta b = 1\ (H_3)$							
$\bar{B}^0 \to \omega + \rho^0$	0.00000274	0.0155	0.0749	-0.284	0.00619	25.0	-0.181
$\bar{B}^0 \to \omega + \psi$	0.000137	-0.659	0.394	-0.599	0.0987	—	—
$\bar{B}^0 \to \rho^0 + \rho^0$	0.000214	-0.219	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to \rho^0 + \psi$	0.000140	-0.660	0.388	-0.597	0.0969	—	—
$\bar{B}^0 \to \omega + \omega$	0.000145	-0.262	0.0867	-0.303	0.00707	—	—
$\bar{B}^0 \to \rho^+ + \rho^-$	0.00473	-0.570	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to D^{*+} + D^{*-}$	0.0938	-0.745	0.456	-0.660	0.172	—	—
$B^- \to \omega + \rho^-$	0.0155	—	0.0855	-0.301	0.00693	5.42	-0.0402
$B^- \to \rho^0 + \rho^-$	0.00196	—	0.0837	-0.298	0.00676	-0.406	0.0183
$B^- \to \rho^- + \psi$	0.000280	—	0.389	-0.597	0.0970	—	—
$B^- \to D^{*0} + D^{*-}$	0.0939	—	0.456	-0.660	0.172	—	—

Table 7.

MODEL: QCD Coefficients *without* Fierz Terms (ρ *positive*)							
Channel	Br(%)	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_T}{\Gamma}$	α_1 [$cos\phi$]	α_2 [$cos2\phi$]	$\beta_1(10^{-4})$ [$sin\phi$]	$\beta_2(10^{-4})$ [$sin2\phi$]
Channels with Penguins: $\Delta c = 0,\ \Delta b = \Delta s = 1\ (H_2)$							
$\bar{B}^0 \to \bar{K}^{*0} + \omega$	0.00783	-0.746	0.0855	-0.315	0.0113	4.53	-0.431
$\bar{B}^0 \to \bar{K}^{*0} + \rho^0$	0.0141	-0.753	0.0905	-0.318	0.0104	-1.93	0.168
$\bar{B}^0 \to K^{*-} + \rho^+$	0.00279	-0.439	0.106	-0.333	0.00892	—	—
$\bar{B}^0 \to \bar{K}^{*0} + \psi$	0.716	-0.673	0.429	-0.621	0.123	—	—
$\bar{B}^0 \to D^{*+} + D_s^{*-}$	2.40	-0.733	0.477	-0.665	0.184	—	—
$B^- \to K^{*-} + \omega$	0.00608	—	0.0832	-0.312	0.0115	-55.5	5.33
$B^- \to K^{*-} + \rho^0$	0.0190	—	0.0924	-0.320	0.0101	16.9	-1.49
$B^- \to K^{*-} + \psi$	0.717	—	0.428	-0.621	0.123	—	—
$B^- \to D^{*0} + D_s^{*-}$	2.40	—	0.477	-0.665	0.183	—	—
Channels with Penguins: $\Delta s = \Delta c = 0,\ \Delta b = 1\ (H_3)$							
$\bar{B}^0 \to \omega + \rho^0$	0.000000326	0.482	0.0636	-0.240	0.00525	957	-7.09
$\bar{B}^0 \to \omega + \psi$	0.0165	-0.663	0.394	-0.599	0.0987	—	—
$\bar{B}^0 \to \rho^0 + \rho^0$	0.000824	-0.925	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to \rho^0 + \psi$	0.0163	-0.674	0.389	-0.597	0.0969	—	—
$\bar{B}^0 \to \omega + \omega$	0.000663	-0.923	0.0867	-0.303	0.00707	—	—
$\bar{B}^0 \to \rho^+ + \rho^-$	0.00658	-0.690	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to D^{*+} + D^{*-}$	0.127	-0.733	0.456	-0.660	0.172	—	—
$B^- \to \omega + \rho^-$	0.00108	—	0.0831	-0.298	0.00677	-18.2	0.135
$B^- \to \rho^0 + \rho^-$	0.00111	—	0.0837	-0.298	0.00676	-0.753	0.0339
$B^- \to \rho^- + \psi$	0.0326	—	0.389	-0.597	0.0970	—	—
$B^- \to D^{*0} + D^{*-}$	0.122	—	0.456	-0.660	0.172	—	—

Table 8.

MODEL: QCD Coefficients *without* Fierz Terms (ρ *negative*)							
Channel	Br(%)	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_T}{\Gamma}$	α_1 [$cos\phi$]	α_2 [$cos2\phi$]	$\beta_1(10^{-4})$ [$sin\phi$]	$\beta_2(10^{-4})$ [$sin2\phi$]
Channels with Penguins: $\Delta c = 0,\ \Delta b = \Delta s = 1\ (H_2)$							
$\bar{B}^0 \to \bar{K}^{*0} + \omega$	0.00864	-0.208	0.0859	-0.315	0.0113	2.16	-0.206
$\bar{B}^0 \to \bar{K}^{*0} + \rho^0$	0.0149	-0.216	0.0903	-0.318	0.0104	-0.962	0.0839
$\bar{B}^0 \to K^{*-} + \rho^+$	0.00611	-0.0748	0.106	-0.333	0.00892	—	—
$\bar{B}^0 \to \bar{K}^{*0} + \psi$	0.718	-0.200	0.429	-0.621	0.123	—	—
$\bar{B}^0 \to D^{*+} + D_s^{*-}$	2.40	-0.217	0.477	-0.665	0.184	—	—
$B^- \to K^{*-} + \omega$	0.00394	—	0.0769	-0.305	0.0119	-45.2	4.34
$B^- \to K^{*-} + \rho^0$	0.0256	—	0.0938	-0.321	0.0100	6.60	-0.580
$B^- \to K^{*-} + \psi$	0.719	—	0.428	-0.621	0.123	—	—
$B^- \to D^{*0} + D_s^{*-}$	2.40	—	0.477	-0.664	0.183	—	—
Channels with Penguins: $\Delta s = \Delta c = 0,\ \Delta b = 1\ (H_3)$							
$\bar{B}^0 \to \omega + \rho^0$	0.00000182	0.0560	0.0759	-0.286	0.00627	90.3	-0.669
$\bar{B}^0 \to \omega + \psi$	0.0165	-0.197	0.394	-0.599	0.0987	—	—
$\bar{B}^0 \to \rho^0 + \rho^0$	0.000144	-0.716	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to \rho^0 + \psi$	0.0155	-0.207	0.388	-0.597	0.0969	—	—
$\bar{B}^0 \to \omega + \omega$	0.0000351	-0.825	0.0867	-0.303	0.00707	—	—
$\bar{B}^0 \to \rho^+ + \rho^-$	0.00355	0.580	0.0837	-0.298	0.00674	—	—
$\bar{B}^0 \to D^{*+} + D^{*-}$	0.127	-0.218	0.456	-0.660	0.172	—	—
$B^- \to \omega + \rho^-$	0.00240	—	0.0846	-0.300	0.00687	-4.30	0.0319
$B^- \to \rho^0 + \rho^-$	0.00263	—	0.0838	-0.299	0.00676	-0.166	0.00749
$B^- \to \rho^- + \psi$	0.0310	—	0.389	-0.597	0.0970	—	—
$B^- \to D^{*0} + D^{*-}$	0.115	—	0.456	-0.660	0.172	—	—