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Abstract

We compare the results of calculations of heavy quark production cross section in leading log QCD to those in parton model. The formulae of parton model are obtained from QCD via several subsequent simplifications. The main result is that QCD and parton model calculations for total cross section of b-quark production at Tevatron energies coinsides within 30 per cent accuracy while the parton model predicts faster decrease of the cross section with p_T.

In this paper we compare two approaches to the production of the heavy quarks in high energy hadron-hadron collisions. One of them is so called parton approach in which the heavy quark production can be expressed as a product of the probability to find two partons by the hard cross section of heavy quark production from their interaction.The second approach is based on QCD in the region of small x. The related calculations were presented in original papers [1-4].Here we would like to give the simple explanation what is the origin for the difference between selfconsistent QCD approach and the obvious at fist sight parton idea about factorization properties for heavy quark production.

The heavy pair production is described in the lowest order of QCD as an elementary process given by the sum of three graphs in Fig.1 in both approaches. In the parton model all particles involved are assumed to be on mass shell and the cross section is averaged over two transverse polarizations of the gluons. The virtualities q^2 of the initial partons are taken into account through their densities. The latter are calculated in leading logarithm approximation (LLA) collecting the terms of the form $(\alpha_s \ln q^2)^n$ and resulting in the well-known Gribov-Lipatov-Altarelli-Parisi evolution equation. The probabilistic picture of noninteracting partons underlies this way of proceeding.

As example of selfconsistent approach we will use the theory of semihard processes [1],which deals with the region where the value of Bjorken variables x is very small. It is just the region that dominates in the heavy quark production at high energies $\sqrt{s}$ since the characteristic value of x is $x \sim 2m_T/\sqrt{s}$ ($m_T^2 = m_Q^2 + p_T^2$ is transverse mass of the quark). The drastical growth of the parton density in this domain leads to the fact that parton-parton interaction effects very significantly on the value of cross section for heavy quark production. For the correct description of these phenomena it is necessary to sum up in. Feynman diagrams not only the terms of the form $(\alpha_s \ln q^2)^n$ but also the terms $(\alpha_s \ln 1/x)^n$ and $(\alpha_s \ln q^2 \ln 1/x)^n$. The difference between leading $\log q^2$ approximation and leading $\log x$ ones reveals itself in the following properties of the gluon distributions at low x :

1.The drastical increase of the gluon deep inelastic structure function.which behaves as some power of x ($xG(x, q^2) \sim x^{-\omega_0}$ at $x \to 0$).

2.The typical transverse momentum of gluons also increases,namely, $ln^2 \frac{q_T^2}{\lambda^2} \propto \alpha_s ln\frac{1}{x}$. . So q_T could be even larger than m_{QT} at very small x and in this case m_{QT} could not define the scale for our hard processes.Thus we have to integrate over transverse momenta of gluons more carefully than in the parton approach.

3.In evolution equation only one polarization of gluons works,namely,each gluon turns out to have 100 per cent alighnment along its transverse momentum.This fact becomes very important at low x because the hard cross section depends crucially on gluon polarization.

The above properties become even more important at low x since in this region of x the absorption correction starts to play remarkable role.Namely the screening (absorption)corrections stop the growth of the cross section and restore the unitary. As a result the gluon structure function $xG(x,q^2)$ becomes proportional to q^2R^2 at relatively small virtuality $q^2 \leq q_0^2(x)$ and the cross section is $\sigma \sim (1/q^2)xG(x,q^2) \sim R^2$. Here $R = const$ is a new phenomenological parameter with dimension of $(\text{mass})^{-1}$ and the value of $q_0(x)$ can be considered as a new typical transverse momentum of partons in the parton cascade of the hadron which leads to natural infrared cut-off in semihard processes (see ref. [1] for details).

The main contribution to the cross section at small x is known to come from gluons. Their distribution over x and transverse momenta q_T in hadron is given in semihard theory by function $\varphi(x,q^2)$. It differs from the usual function $G(x,q^2)$:

$$\alpha_s(q^2)\, xG(x,q^2) = \frac{1}{4\sqrt{2}\,\pi^3}\int^{q^2} \alpha_s(q_1^2)\varphi(x,q_1^2)dq_1^2. \tag{1}$$

Such definition of $\varphi(x,q^2)$ makes possible to treat correctly the effects arising from gluons virtualities. The exact expression for this function can be obtained as a solution of the evolution equation which, contrary to the parton model case, is nonlinear due to interactions between the partons in small x region.

Here we have used the following parametrization [1] for the gluon distribution (see ref. [2] for discussion):

$$\varphi(x,q_T^2) = \varphi_0(1-x)^5 f_1(q_T^2,x) \tag{2}$$

$$f_1(q_T^2,x) = \begin{cases} 1 & q_T^2 < q_0(x) \\ \left[\frac{q_0^2(x)}{q_T^2}\right]^2 & q_T^2 \geq q_0^2(x), \end{cases} \tag{3}$$

where $\varphi_0 = 170\ mb$, the infrared cut-off $q_0(x)$, which specifies the border of the saturation of the parton density at each x value, is

$$q_0^2(x) = Q_0^2 + \Lambda^2 \exp\left(3.56\sqrt{\ln x_0/x}\right) \tag{4}$$

[1] Frankly speaking at very high q_T^2, $q_T^2 \gg q_0^2(x)$ we have $f(x,q_T^2) \sim 1/q_T^2$, but due to pre-exponent factor $F(\ln q_T^2/q_0^2)$ the function $f(x,q_T^2)$ decrease faster than q_T^{-2} in the region $q_T^2 \sim q_0^2(x)$ [1]. In the calculation of heavy quark production cross section the main contribution gives the region $q_T^2 \sim (1-2)q_0^2(x)$ where the parametrization $f(x,q_T^2) \sim q_0^2[q_0^2(x)+q_T^2]^2/(4q_T^6)$ or $(q_0^2(x)/q_T^2)^2$ seems to be more adequate. The numerical estimates show that the variation of the values of heavy quark production cross sections for these two parametrizations does not exceed $(10-20\%)$.

and $Q_0^2 = 2\ GeV^2$, $\Lambda = 52\ MeV$, $x_0 = 1/3$ are the phenomenological parameters.

The cross section of heavy quarks production is given schematically by the graph of Fig.2 where upper and lower blocks show the functions $\varphi(x,q^2)$ and the side ones include the sum of the diagrams shown in Fig.1. In what follows we shall use Sudakov presentation for quarks' momenta $p_{1,2}$ through the momenta of colliding hadrons p_A and p_B ($p_A^2 = p_B^2 \simeq 0$) and transverse ones $p_{1,2T}$:

$$p_{1,2} = x_{1,2}p_B + y_{1,2}p_A + p_{1,2T}. \tag{5}$$

The differential cross section of heavy quark production has the form

$$\frac{d\sigma}{dy_1^* dy_2^* d^2p_{1T}d^2p_{2T}} = \frac{1}{(2\pi)^8}\frac{1}{(s)^2}\int d^2q_{1T}d^2q_{2T}\delta(q_{1T}+q_{2T}-p_{1T}-p_{2T})$$

$$\times \frac{\alpha_s(q_1^2)}{q_1^2}\frac{\alpha_s(q_2^2)}{q_2^2}\varphi(q_1^2,y)\varphi(q_2^2,x)|M|^2. \tag{6}$$

Here $s = 2p_Ap_B, y_{1,2}^*$ are the quarks' rapidities in c.m.s.,

$$x_1 = \frac{m_{1T}}{\sqrt{s}}e^{-y_1^*},\quad x_2 = \frac{m_{2T}}{\sqrt{s}}e^{-y_2^*},\quad x = x_1 + x_2$$
$$y_1 = \frac{m_{1T}}{\sqrt{s}}e^{y_1^*},\quad y_2 = \frac{m_{2T}}{\sqrt{s}}e^{y_2^*},\quad y = y_1 + y_2. \tag{7}$$

$q_{1,2T}$ are the gluons' transverse momenta. $|M|^2$ is the square of the matrix element. The explicit expression for $|M|^2$ is rather bulky and we do not present it here (one can find it in ref. [2]).

In the axial gauge $p_B^\mu A_\mu = 0$ the gluon propagator takes the form $D_{\mu\nu}(q) = d_{\mu\nu}(q)/q^2$,

$$d_{\mu\nu}(q) = \delta_{\mu\nu} - (q^\mu p_B^\nu + q^\nu p_B^\mu)/(p_Bq). \tag{8}$$

For the gluons in $t-$channel the main contribution comes from the so called 'nonsense' polarization $g_{\mu\nu}^n$, which can be picked out by decomposing the numerator into longitudinal and transverse parts:

$$g_{\mu\nu}(q) = 2(p_B^\mu p_A^\nu + p_A^\mu p_B^\nu)/s + \delta_{\mu\nu}^T \approx 2p_B^\mu p_A^\nu/s \equiv g_{\mu\nu}^n. \tag{9}$$

One can easy check that for the simplest Born graph in Fig.3 other terms are suppressed by the powers of s. Generally this substitution for $d_{\mu\nu}$ is true if the difference of the rapidities between the points of emission and absorption of a gluon is large. This is the case of $t-$channel gluons in ladder kinematics (for $\ln q^2$ and $\ln 1/x$ terms).

Thus the situation considered here seems to be quite opposite to the parton model since in this approach we deal with off-mass shell gluons carrying longitudinal polarization. There is a certain limit case in which our formulae can be transformed into parton model ones. To show it we note firstly that in the ladder kinematic $q_1 = yp_A + q_{1T}$, $q_2 = xp_B + q_{2T}$. These equations and the transversality condition for the ends of the gluon line in Fig.2 enables one to replace p_A^μ by $-q_{1T}^\mu/y$ in expression for $g_{\mu\nu}^n$. Thus we get

$$d_{\mu\nu}(q) \approx -2\,\frac{p_B^\mu q_T^\nu}{y\,s} \tag{10}$$

or

$$d_{\mu\nu}(q) \approx 2\,\frac{q_T^\mu q_T^\nu}{xys} \tag{11}$$

if we do such a trick for vector p_B too. Both these equations for $d_{\mu\nu}$ can be used but for the form (10) one has to modify the gluon vertex slightly (to account for several ways of gluon emission — see ref.[2]). Let us assume now that the characteristic values of quarks' momenta p_{1T} and p_{2T} are much larger than the values of gluons' momenta q_{1T}, q_{2T} and one can keep only lowest powers of q_{1T}, q_{2T}. It means that we can put $q_{1T} = q_{2T} = 0$ everywhere in M except the vertices. Introducing the polar coordinates

$$d^2 q_{1T} = \frac{1}{2}\, dq_{1T}^2 d\theta_1 \tag{12}$$

(and the same for q_{2T}) and performing angular integration with the help of the formula

$$\int_0^{2\pi} d\theta_1 q_{1T}^\mu q_{1T}^\nu = \pi\, q_{1T}^2 \delta_T^{\mu\nu} \tag{13}$$

we obtain

$$\int d\theta_1 \int d\theta_2 |M|^2 = 2\pi^2 \frac{q_{1T}^2 q_{2T}^2}{(x\,y)^2}\,|M_{part}|^2. \tag{14}$$

Here M_{part} is just the matrix element in the parton model since the result is the same as that calculated for the real (mass shell) gluons and averaged over transverse polarizations (it clearly follows from eq.(13)). Then we obtain the cross section (6) in the form

$$\frac{d\,\sigma}{dy_1^* dy_2^* d^2 p_{1T}} = \int |M|^2 \frac{d\theta_1 d\theta_2}{2\pi^2 q_1^2 q_2^2 (s)^2} \int \frac{\alpha_s(q_1^2)\varphi(y, q_{1T}^2)}{4\sqrt{2}\pi^3} \frac{\alpha_s(q_2^2)\varphi(x, q_{2T}^2)}{4\sqrt{2}\pi^3} dq_{1T}^2 dq_{2T}^2$$

$$= |M_{part}|^2 \frac{1}{(\hat{s})^2} \int \frac{\alpha_s(q_1^2)\varphi(y, q_{1T}^2)}{4\sqrt{2}\,\pi^3} \frac{\alpha_s(q_2^2)\varphi(x, q_{2T}^2)}{4\sqrt{2}\,\pi^3} dq_{1T}^2 dq_{2T}^2 \tag{15}$$

where $\hat{s} = xys$ is the mass square of $\bar{Q}Q$ pair and $q_i^2 = -q_{i,T}^2$ $(i = 1, 2)$. Using eq.(1) we can rewrite eq.(15) in the partonic form

$$\frac{d\sigma}{dy_1^* dy_2^* d^2 p_{2T}} = |M_{part}|^2 [\frac{\alpha_s(4m_T^2)}{\hat{s}}]^2 xG(x, 4m_T^2) \cdot yG(y, 4m_T^2) \tag{16}$$

It should be stressed that we used the factorization properties of gluon distributions in derivation of eq.(16).It means that we assumed that $x(y)$ does not depends on $q_{1T}^2(q_{2T}^2)$ respectively.It is not true due to kinematics (see eq.(7)).The correct kinematics leads to difference from eq.(16) but not too large.Namely,it turns out that the cross section of b-quark production, $d\sigma/dp_{1T}^2$ calculated by formula (16) increases in the region $p_{1T}^2 \ll m_b^2$ about 20 per cent at $\sqrt{s} = 1.8\ TeV$ and by $1.5 - 2$ times at $\sqrt{s} = 18\ TeV$ in comparison with eq.(15).

The main difference between correct expression (6) and parton formula (15) is caused by change of the matrix element squared $|M|^2$ by its limit $q_{1T}^2 q_{2T}^2 [M/(q_{1T}^2 q_{2T}^2)]_{q_{1T}=q_{2T}=0}$. In order to show the errors that we have made in such a procedure we present in Fig. 4 the values of $|M|^2$ averaged over azumuthal angle between gluon (say q_{1T}) and heavy quark (p_{1T}) as the functions of $q_{1T} - q_{2T} = \Delta q$ at three values of b-quark ($m_b = 4.7\,GeV$) transverse momenta p_{1T} = 1GeV, 7GeV, 13 GeV,respectively. $\sqrt{s} = 1.8\,TeV$,$y_1^* = 0.5$, $y_2^* = 0$.Even slight glance at Fig.4 shows us that we can evaluate the integral (6) by the value of $|M|^2$ at $\Delta q = 0$ only for sufficiently small values of p_{1T}.Of course, the values of $\Delta q \geq 10\,GeV/c$ don't contribute really to the total cross section due to the fast decrease of the gluon distributions of eq.(3).As seen from Fig.4, the value of $|M|^2$ decreases with Δq at small p_{1T}. Thus eq. (15) overestimates the cross section in this region. But on the other hand at sufficiently large p_T we have even minimum at $\Delta q \to 0$ and maximum of $|M|^2$ which moves to the value of $\Delta q \approx p_T/2$.So as a result we have not a big difference between correct calculation and parton approach (see Fig.5) especially at $p_T \geq 5 - 7GeV/c$. The numerical calculations show that the cross sections given by eqs.(6) and (15) coincide approximately at p_{1T} =$5 - 7\ GeV/c$. At larger values of p_T the cross section of eq.(15) becomes smaller. The difference is, however, not so large and does not vary with p_{1T} (see fig.5). The reason is that the maximum value of $|M|^2$ at high p_T is achieved at large values of Δq which give very small contribution to the cross section. The most important is the different p_T dependence of the two eqs. (6) and (15). For example, the cross section ratios at $p_{1T} = 0$ to $p_{1T} = 10\,GeV/c$ are about 1.5 times different.

The next problem which we are going to discuss is the value of the argument in α_s QCD constant. As it was discussed earlier (see, e.g., refs.[5,6]) the theory cannot fix the value of this argument in Born Approximation.Eq.(6) corresponds to direct calculation of the diagrams of Fig.1 and here it seems to be the most natural to use $\alpha_s(q_1^2)\cdot\alpha_s(q_2^2)$(see Appendix).In our calculation the essential values of q_1^2 and q_2^2 in eq. (6) are of the order of $q_0^2(x)$, which is equal to several GeV^2 and increase very slowly with initial energy [1]. Thus the essential values of α_s are of the order of $\alpha_s \sim 0.2$. In the parton model the value of mass of heavy quark is used usually as an argument of α_s as has been seen from eq.(16).However, in the parton model calculation the cross section could be rewritten only through the gluon structure function but not $\phi(x,q^2)$.To illustrate how large is the influence of α_s argument in such a formulas we present in Fig.5 the dotted curve that corresponds to eq.(17).

Namely,

$$\frac{d\sigma}{dy_1^* dy_2^* d^2p_{1T}} = |M_{part}|^2 \left[\frac{\alpha_s(4m_T^2)}{\hat{s}}\right]^2 \int \frac{\varphi(y,q_{1T}^2)}{4\sqrt{2}\pi^3}\frac{\varphi(x,q_{2T}^2)}{4\sqrt{2}\pi^3} dq_{1T}^2 dq_{2T}^2 \qquad (17)$$

It is seen from Fig.5 that eq.(16) gives substantially smaller cross section .This example shows us how carefully we should be with the argument of running α_s.It should be mention once more that functions φ and xG were defined in a such way that they carry different arguments of α_s (see eq.(1).

To summarize, we obtained that eq.(6) differs from the predictions of parton model in the case of heavy quark production by decades of per cents.Let us resume:

1. The main difference of QCD approach at low x from previuos ones [5,6] is the value of the gluon structure function $xG(x,q^2)$ in the region of small x.We took $xG(x,q^2) \propto x^{-\omega_0} \sim \frac{1}{\sqrt{x}}$ which originated from summation of $(\alpha_s ln\frac{1}{x})^n$ terms in LL(x)A in perturbative QCD. Different papers [2-4] of course,use different parametrization but all of them were based on LL(x)A in the region of small x.The fact that the value of the total cross section for heavy quark production depends mainly on normalization of gluon structure function is the main result of this paper.The accuracy of this statement is about 20 per cent as seen from Fig.5.

2.In the region of small x turns out that the gluon polarization is very essential for our " hard " heavy quark production. Numerically,this difference was illustrated in Fig.4 but we would like to stress that it is even more important in double inclusive cross section $\frac{d^2\sigma}{d^3p_1 d^3p_2}$ (see ref. [2] for some estimates).

3.The dependence of the cross section on the choice of the argument in running α_s is crucial.Of course the final answer on this question one can get only after calculation of $O(\alpha_s^3)$ correction in the region of small x. We are far even from the fist attempt of such a calculation.But in Appendix we'll get a

strong argument in a favour that α_s dependence is controlled by the value of t-channel gluon virtuality q_1^2 and q_2^2 but not by the value of heavy quark mass $(q_1+q_2)^2 \geq 4m_T^2$ for our master formula (6).

4.We would like also to draw your attention to the fact that in the region of $q_0(x) \geq m_{QT}$ we should be very carefull with using operator expansion arguments in support of parton model approach since in this kinematical region the "hardest" cell may be not the gluon-gluon fusion in heavy quarks but some gluon cell of our "ladder".The situation really depends on kinematical region of rapidities of detected heavy quarks.However even in the case when the hardest cell is still gluon -gluon fusion to heavy quarks the value of $q_0(x)$ but not m_{QT} is responsible for "hardness" of heavy quark production.

We hope that the qualitative results should be true also in the case of accounting for higher α_s corrections. We use the concrete parametrization of gluon distributions, eqs. (2) and (3), however the significant variation should result in contradiction in description of the data on high p_T production of jets and hadrons.

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Appendix

The simplest way to understand what argument controls the behaviour of coupling constant α_s is to follow the light quark contribution to Gell-Mann-Low function (see ref. [7]).Due to renormalization group all other contributions to invariant charge α_s should lead to the same Q^2 dependence as light quark ones.

In the first order (after the Born Approximation) one can insert the new quark loop in each gluon propagator as well as in the triple gluon vertex(see Fig.6),but only two of them (Figs.6 a and b) could depend on heavy quark momentum $p_1+p_2 = q_1+q_2 \sim m_{QT}$. It turns out that in axial gauge used in ref. [2] [2]the diagrams Figs.6 a and b give the same leading $ln\frac{m_Q^2}{\Lambda^2}$ contribution but with opposite sign. Indeed in our gauge the polarization vectors of gluons q_1 and q_2 are $\mathbf{e}_{1\nu} \propto q_{1T\nu}$ and $\mathbf{e}_{2\sigma} \propto q_{2T\sigma}$.The s-channel (q_1+q_2) gluon polarization $\mathbf{e}_{3\mu}$ is parallel to $(q_1-q_2)_\mu$.So the trace in the light quark loop is equal to

$$tr[\hat{q}_{2T}\hat{k}\hat{q}_{1T}(\hat{k}+\hat{q}_1)\gamma_\mu(\hat{k}-\hat{q}_2)]e_{3\mu} =$$

$$= 4e_{3\mu}(q_{1T}q_{2T})\{(q_1-q_2)_\mu(k_T^2-k^2) + k_\mu[2k_T^2 - k^2 + (k(q_1-q_2))]\}$$

So the factor in front of the effective triple gluon vertex (Fig.1c)

$$f^{abc}\Gamma_{\mu\nu\sigma}e_{1\nu}e_{2\sigma}e_{3\mu} = (q_{1T}q_{2T})(q_1-q_2)_\mu e_{3\mu}f^{abc}$$

is equal to

$$n_F\frac{g^2}{16\pi^4}\int^\Lambda \frac{d^4k}{k^2(k+q_1)^2(k-q_2)^2}\cdot$$

$$\{(k_T^2-k^2) + \frac{(ke_3)}{((q_1-q_2)e_3)}[2k_T^2 - k^2 + (k(q_1-q_2))]\} =$$

$$= \frac{\alpha_s}{4\pi}ln\frac{\Lambda^2}{(q_1+q_2)^2}[-\frac{2}{3}n_F]$$

The last factor is exactly the same as the famous part $\frac{2}{3}n_F$ of coefficient in front of log in Gell-Mann-Low function $b = \frac{11}{3}N_c - \frac{2}{3}n_F$ but with the oposite signh.

[2]In this gauge the polarizations of t-channel gluons q_1 and q_2 are parallel to $\mathbf{q_1}$ and $\mathbf{q_2}$ vectors respectively.

Thus, the above two $ln\frac{(q_1+q_2)^2}{\Lambda^2}$ contribution from triangle and gluon self energy diagram (gluon with momentum q_1+q_2) cancel each other.Therefore, the whole amplitude does not depend on $ln\frac{m_Q^2}{\Lambda^2}$ and the final result is proportional to $\alpha_s(q_{1T}^2)\alpha_s(q_{2T}^2)$.The last logs are related to loops in Figs,6 c and d.

1 Figure Captions

Fig.1.-: The diagrams, taking into account in calculation of heavy quark production via gluon-gluon collision.

Fig.2.-: Inclusive cross section of $\bar{Q}Q$ pair production in hadron collisions.

Fig.3.-: Simplest Born diagram for quark-quark interaction.

Fig.4.-: Dependences of the matrix element square of $gg \to \bar{Q}Q$ reaction versus the gluon transverse momenta Δq at different values of the heavy quark transverse momentum p_{1T},(p_{1T} = 1 GeV, 7 GeV and 13 GeV for curves 1,2 and 3 , respectively). The values of another variables are $\sqrt{s} = 1.8\ TeV$, $y_1^* = 0.5$, $y_2^* = 0$, $m_Q = 4.7\,GeV$. y_1* and y_2* are rapidities of the produces heavy quarks.

Fig.5.-: The calculated dependences of heavy quark production cross section versus its transverse momenta in the LLA QCD, eq.(2) (solid curve), in the parton model approximations of eq. (15) (dashed curve) .The dotted curves was calculated accordind to eq. (17). The values of another variables are $\sqrt{s} = 1.8\ TeV$, $m_Q = 4.7\ GeV$, $y_1^* = 0$.

Fig.6-: The light quark insertion in the amplitude of $gg \to \bar{Q}Q$

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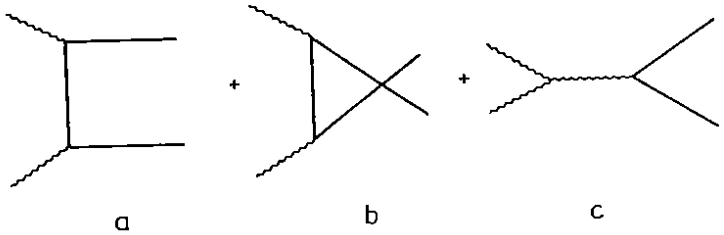

Fig.1

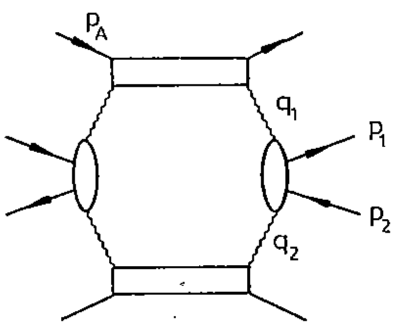

Fig.2

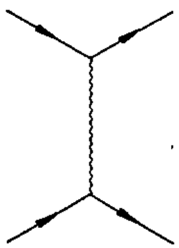
Fig.3

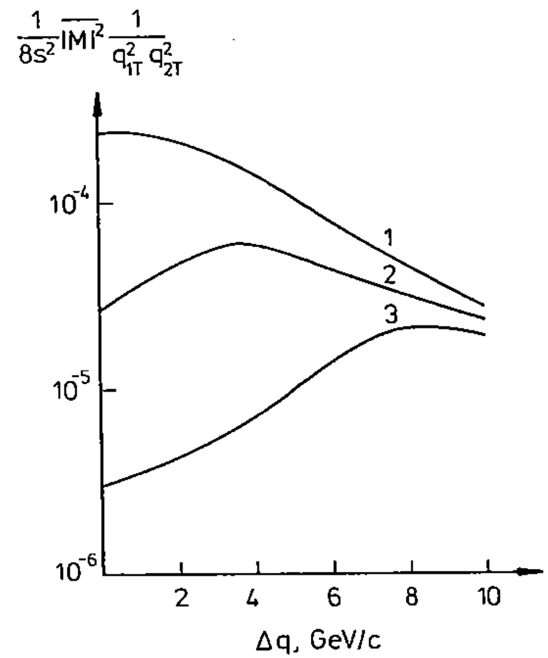

Fig.4

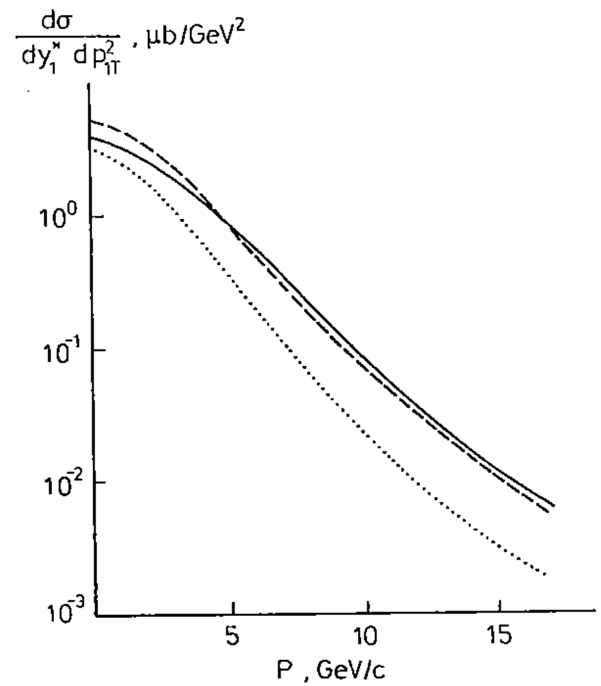
$\frac{d\sigma}{dy_1^* \, dp_{1T}^2}$, μb/GeV^2
10^0
10^{-1}
10^{-2}
10^{-3}
5
10
15
P , GeV/c

Fig.5

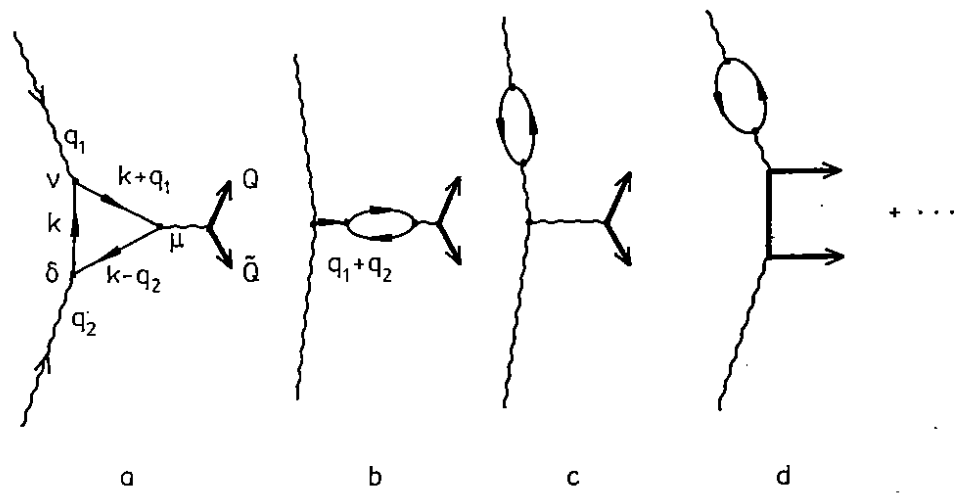
q_1
ν
k
δ
q_2
$k+q_1$
$k-q_2$
μ
Q
$\tilde{Q}$
q_1+q_2
+ . . .
a
b
c
d

Fig.6