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# SMALL- $x$ SINGLET STRUCTURE FUNCTIONS from the NONLINEAR GLR EQUATION

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## Abstract

The effect of absorptive corrections in the nonlinear GLR evolution equation is considered. A simple method how to estimate the corrections numerically is described. In the case of the parametrization based on semihard hadron phenomenology developed earlier a visible difference between linear and nonlinear evolution is expected at HERA energies.

1. The role of absorptive corrections in the small  $x$  behaviour of deep inelastic structure functions  $xG(x, q^2)$  and  $F_2^{sea}(x, q^2)$  is widely discussed now due to the new generation of accelerators: HERA, LHC, SSC. One can see for example the proceeding of "Small- $x$  DESY Meeting" [1,2], or one of the recent preprint [3].

But in all previous papers the parameters that characterize the absorption were chosen more or less arbitrarily. Here we are going to estimate the nonlinear corrections with the parametrization that was used before in semihard phenomenology to describe nucleon-nucleon interactions [4] and beauty production [5].

2. As it was shown in ref.[6] the nonlinear absorptive effects are essential at very small  $x$  only, and to the right of the critical line  $x = x_o(q^2)$  ( $\ln x_o = \frac{1}{12.7} \ln^2 \frac{q^2}{\Lambda^2}$ ) one can use the more simple linear equation that takes into account two types of logarithms:  $\alpha_s \ln q^2$  and  $\alpha_s \ln \frac{1}{x}$ . Moreover, in the region of interest ( $x > 10^{-6}$ ,  $q^2 < 10^5 \text{ GeV}^2$ ) the latter terms ( $\alpha_s \ln 1/x$ ) gives not a large contribution. In ref.[7] G. Marchesini and B.R. Webber compared the solutions of the standard one-loop evolution equation [8] (that sums up the  $(\alpha_s \ln q^2)^n$  terms) and the equation which includes the all-loop corrections to the anomalous dimension  $\gamma_N$  of the type  $\gamma_N = \Sigma (\frac{\alpha_s}{N-1})^n C_n$ , i.e. the terms  $(\alpha_s \ln \frac{1}{x})^n$ . In the whole kinematical region the results coincide within the accuracy of about 15-20 per cent.

Therefore we decided to use the standard GLAP evolution equation [8] at  $x > x_o(q^2)$ . But in this case we should add a new boundary condition on the line  $x = x_o(q^2)$ . As was shown in ref.[6] on this line the function  $xG(x, q^2) = \text{const} \cdot q^2$ .

At smaller  $x < x_o(q^2)$  we have used the semiclassical solution of the nonlinear master equation (eq. 2.41 and 2.108 of ref.[6]). The solution described in ref.[9] has the following form

$$xG(x, q^2) = [xG(x, q^2)]_{x=x_o(q^2)} \cdot \left[ (1 - 1/\sqrt{r}) \left( \frac{x}{x_o(q^2)} \right)^{\omega/2} + 1/\sqrt{r} \right]^{-2} \quad (1)$$

where:

$$r = k_o \ln(q^2/\Lambda^2); \quad \omega = \frac{\alpha_s(q^2)}{\pi} \cdot N_c \chi(k_o); \quad \alpha_s(q^2) = \frac{4\pi}{b \cdot \ln(q^2/\Lambda^2)};$$

$b = 11N_c/3 - 2n_F/3$ ,  $k_o = 0.63$ ,  $\chi(k_o) = 3.077$ , and  $N_c = 3$  is the number of colours,  $n_F = 4$  is the number of quarks.

The sea quark structure function in this approximation is given by the one loop relation

$$F_2^{sea}(x, q^2) = \Sigma e_F^2 \int x G\left(\frac{x}{z}, q^2\right) P_G^F(z) \frac{\alpha_s}{4\pi} \frac{dq^2}{q^2} dz \simeq \simeq c \Sigma e_F^2 x G(x, q^2) \frac{\alpha_s}{4\pi}, \quad (2)$$

and the coefficient  $c \simeq 1.4$  at  $x < x_0$ , as it was calculated in ref.[10].

3. Now we should fix the initial conditions. We choose

$$x G(x) = A(1-x)^3 \cdot x^{-\omega_0} \quad (3)$$

at  $q_0^2 = 4GeV^2$ . The coefficient  $A$  is fixed by the normalization

$$\int x G(x) dx = 0.55,$$

and

$$\omega_0 = \frac{N_c \alpha_s(q_0^2)}{\pi} 4 \ln 2 \quad (4)$$

corresponds to the QCD pomeron singularity given by the summation of leading-log contributions  $(\alpha_s \ln \frac{1}{x})^n$  [11].

For the sea quark structure function we use the same expression of the structure as in the small  $x$  region (2), but with additional factor  $(1-x)^4$

$$F_2(x) = \frac{5}{18} n_F (1-x)^4 x G(x, q_0^2) \frac{\alpha_s(q^2)}{4\pi} \cdot c \quad (5)$$

( $\frac{5}{18}$  is the mean quark's charge squared)

In the region of not very small  $x$  this function is close to the parametrization of D.W.Duke and J.F.Owens [13].

It should be stressed that even at "large"  $x > x_0(q^2)$  where we solve the linear evolution equation nonlinear absorptive corrections reveal itself due to the new boundary condition:

$$x G(x, q^2)_{x=x_0(q^2)} = a \cdot q^2 \quad (6)$$

The coefficient  $a = x_0 G(x_0, q^2)/q^2$  is fixed by the initial condition (3), and it turns out to be in agreement (within the accuracy of about 10 per cent) with our previous phenomenological estimate [4].

But if one tries to consider another initial condition, such as  $x G(x, q_0^2) = A'(1-x)^m$  one faces a contradiction. On the line  $x = x_0(q^2)$  an interval will

appear where the solution of the linear equation  $G^{A,P}(x, q^2)$  will be smaller than its boundary value  $x_0 G(x_0, q^2) = a \cdot q^2$  that takes into account screening. What happened? In order to obtain the condition (6), we have summed up the terms  $(\alpha_s \ln \frac{1}{x})^n$  in our modified nonlinear equation. Thus we have to keep these terms (that give us  $x^{-\omega_0}$ ) also in the initial condition.

4. The numerical solution of linear evolution equation

$$\frac{dG(x, q^2)}{d \ln q^2} = \frac{\alpha_s(q^2)}{4\pi} \left[ \int G\left(\frac{x}{z}, q^2\right) P_G^G(z) \frac{dz}{z} + \int F^{sea}\left(\frac{x}{z}, q^2\right) P_F^G(z) \frac{dz}{z} \right] \\ \frac{dF^{sea}(x, q^2)}{d \ln q^2} = \frac{\alpha_s(q^2)}{4\pi} \left[ \int G\left(\frac{x}{z}, q^2\right) 2n_F P_G^F(z) \frac{dz}{z} + \int F^{sea}\left(\frac{x}{z}, q^2\right) P_F^F(z) \frac{dz}{z} \right] \quad (7)$$

(terms proportional to  $\delta(1-z)$  are included into the kernels  $P_B^A(z)$ )

is trivial. We iterated it step by step using the computer.

The critical line  $x = x_0(q^2)$  where the new boundary condition (6) was added is known from theory in a LLA only. The nonleading terms were fixed from the phenomenological description of nucleon interactions[4]. It is better to write it in the form  $q_0^2 = q_0^2(x)$  (so that  $x_0(q_0^2) = x$ )

$$q_0^2(x) = Q_0^2 + \lambda^2 \cdot \exp(3.56 \sqrt{\ln(x_0/x)}) \quad (8)$$

$$Q_0^2 = 2GeV^2, \quad \lambda = 52MeV, \quad x_0 = 1/3 \quad (9)$$

This line is shown in fig.1 by the solid curve. It corresponds to a strong correlations between gluons inside a proton. Gluons group in a small "hot spots" with radius  $R_s \sim 0.2Fm$ . If gluons are distributed uniformly inside the proton the screening would be smaller and nonlinear effect reveals itself at smaller  $x$ . For this case ( $R_s \sim 0.7Fm \sim R_N$ ) we have drawn in fig.1 the dashed curve with  $x_0 = 0.0035$ .

The results are shown in figs.2 and 3 for  $q^2 = 10, 100$  and  $1000GeV^2$ . Solid lines are the solutions of the linear GLAP evolution equation with the initial conditions (3-5) at  $q^2 = q_0^2 = 4GeV^2$  only (without the boundary condition (6)). The long-dashed lines take into account the nonlinear effects (screening). It corresponds to our previous semihard phenomenology:  $x_0 = 1/3(R_s \simeq 0.2Fm)$ . The curves with  $x_0 = 0.0035$  (i.e.  $R_s \simeq R_N$ ) are shown by short-dashed lines.

Since in the leading-log approximation one can not guarantee that  $\lambda$  in eq.(8) is equal to  $\Lambda$  in the QCD coupling  $\alpha_s = \frac{4\pi}{\ln(q^2/\Lambda^2)}$ , we consider two variants: with  $\Lambda = \lambda = 52MeV$  (see fig.2a,3a) and with the more common

value of  $\Lambda = 200\text{MeV}$ (fig.2b,3b). But let us note that even at  $\Lambda = 52\text{MeV}$  one gets a reasonable value of  $\alpha_s(q^2) \approx 0.16$  at  $q^2 = 25\text{GeV}^2$ .

A slightly unpleasant feature of the case of  $\Lambda = 200\text{MeV} \neq \lambda$  is the break of the dashed line at  $x = x_c$ (the beginning of critical line in fig.1). This break is connected with the fact that calculating the coefficient  $a$  in eq.(6) for this line we have used  $\Lambda = \lambda$ . But this is not essential for our LLA estimates.

Another break one can see at  $x = x_o(q^2)$ , where the solution of the linear equation(7) is matched with the semiclassical solution eq.(1). Frankly speaking the semiclassical solution(1) is valid in the double-log approximation only, whereas the linear GLAP equation has the single log accuracy. It has been noted in ref.[6,14] that when matching the linear and nonlinear equations on the critical line  $x = x_o(q^2)$  one gets a jump of the derivative but a smooth behaviour of the structure function value. We are able to wash out the jump in the derivative, but this is beyond our LLA accuracy. Let us stress also that we see no sense to improve the accuracy of the semiclassical solution since we are not sure whether the nonlinear master equation is valid at  $x \ll x_o(q^2)$ . So far it has not been possible to estimate the contributions of the more complicated enhanced diagrams( which were not taken into account in GLR master equation) at  $x \ll x_o(q^2)$ .

5. As it can be seen from fig.2,3, there is no real chance to observe the nonlinear effects at  $x > 10^{-4}$  in the case of a homogeneous gluon distribution ( $R_s \sim R_N$ , short-dashed curves). The absorptive corrections are negligibly small. But if the radius of the so-called hot spots is small ( $R_s \sim 0.2Fm$ , as it follows from semihard phenomenology[4]) then we expect a visible effect (more than 30 per cent) already at  $x \leq 10^{-3}$  for  $q^2 = 10\text{GeV}^2$ .<sup>1</sup> So we hope that the nonlinear corrections(screening) in deep inelastic scattering will be observed in the coming years at HERA.

Certainly at one value of  $q^2$  one can reproduce the result of nonlinear GLR evolution changing the initial condition (at  $q_s^2$ ) in linear evolution equation. For example for the linear case we have chosen the new value of  $xG(x, q^2)$  (shown by dotted curve in fig.4) in such a way that the same structure functions as in the nonlinear equation are obtained at  $q^2 = 10\text{GeV}^2$ . But the evolution at larger  $q^2$  is still different. The new initial condition (dotted curve in fig.4) suppresses linear evolution (in a limited but rather large interval of  $q^2$ ) so strongly that in the available kinematical region the solution of linear GLAP equation turns to be even smaller than the nonlinear gluon GLR structure function (which takes into account the screening corrections).

<sup>1</sup>These statements are in agreement with the conclusions of the ref.[1-3].

Nevertheless at large  $q^2$  and  $x > x_o(q^2)$  (between 100 and 1000  $\text{GeV}^2$ , and  $10^{-3} > x > 10^{-4}$ ) the linear GLAP structure function increases faster than in nonlinear GLR case.

Finally, in fig.5 we show the behaviour of the ratio  $xG(x, q^2)/q^2$ , which is proportional to the inelastic cross section. In such a ratio the screening effects become even more visible. But please do not consider our curves at small  $q^2 < (1 - 2)\text{GeV}^2$  as a prediction. We are not sure that perturbative QCD is valid in this region, and we have drawn the curves here only in order to better clear up the effects.

6. The method we have used is very simple and effective. It does not take much of computer time and we can recommend it to those who are going to estimate the small  $x$  behaviour of deep inelastic structure functions in the framework of perturbative QCD.

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## Figure Captions

Fig.1.-: The form of critical line where the nonlinear absorptive effects force us to add a new boundary condition (6). Solid line- $x_o = 1/3$ . Dashed line- $x_o = 0.0035$ .

Fig.2.-: The  $x$ -dependence of gluon structure functions  $xG(x, q^2)$  at  $q^2 = 10, 100$  and  $1000 \text{ GeV}^2$  - curves 1,4,7; 2,5,8 and 3,6,9 respectively. Solid curves are the ordinary linear GLAP evolution; long-dashed curves take into account the absorptive corrections through the new boundary condition (6) at the critical line (8,9) ( $R_{\text{pot}} \sim 0.2Fm$ ); short-dashed is the same as long-dashed but for  $x_o = 0.0035$  ( $R_s \sim R_N$ ).  
a)  $\Lambda = 52 \text{ MeV}$ ; b)  $\Lambda = 200 \text{ MeV}$ .

Fig.3.-: The  $x$ -dependence of sea quark structure functions  $F_2(x, q^2)$ . Notations are the same as in fig.2.

Fig.4.-: The difference between linear (solid curve) and nonlinear (dashed curve) GLR evolution. The curves 1,4; 2,5 and 3,6 correspond to  $q^2 = 10, 100$  and  $1000 \text{ GeV}^2$ , respectively. The new initial condition for the linear case at  $q_s^2 = 4 \text{ GeV}^2$  is shown by the dotted curve. The old condition (eq.(3)) is shown by the dot-dashed curve.  
a)  $\Lambda = 52 \text{ MeV}$ ; b)  $\Lambda = 200 \text{ MeV}$ .

Fig.5.-: The behaviour of the ratio  $xG(x, q^2)/q^2$ , which is proportional to the in-elastic cross section.  $x = 10^{-2}, 10^{-3}$  and  $10^{-4}$  - curves 1,4,7; 2,5,8 and 3,6,9 respectively. Solid curves are the ordinary linear evolution[8]; long-dashed curves take into account the absorptive corrections through the new boundary condition (6) at the critical line (8,9) ( $R_{\text{pot}} \sim 0.2Fm$ ); short-dashed is the same as long-dashed, but for  $x_o = 0.0035$  ( $R_s \sim R_N$ ).  
a)  $\Lambda = 52 \text{ MeV}$ ; b)  $\Lambda = 200 \text{ MeV}$ .

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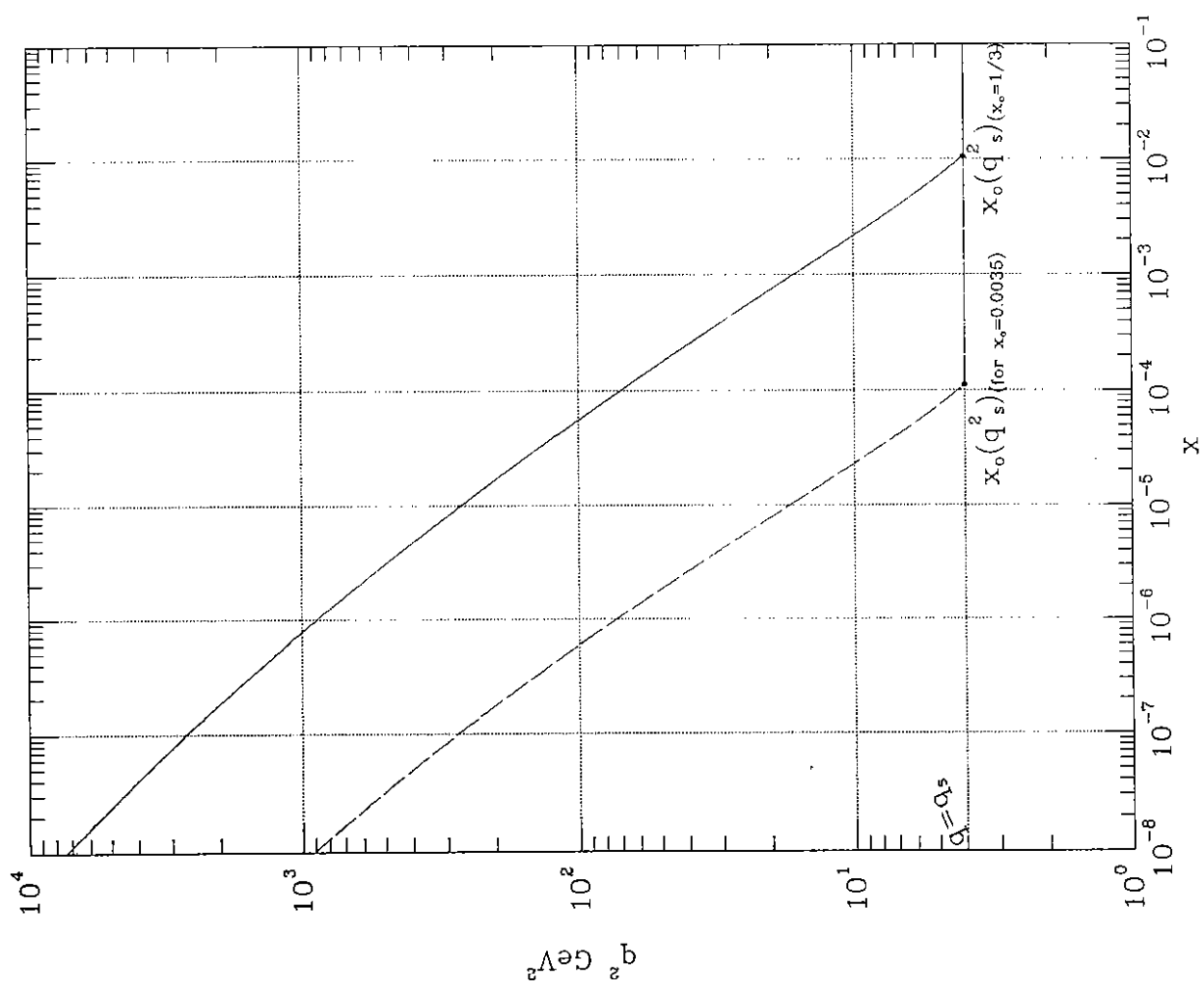


Fig.1.

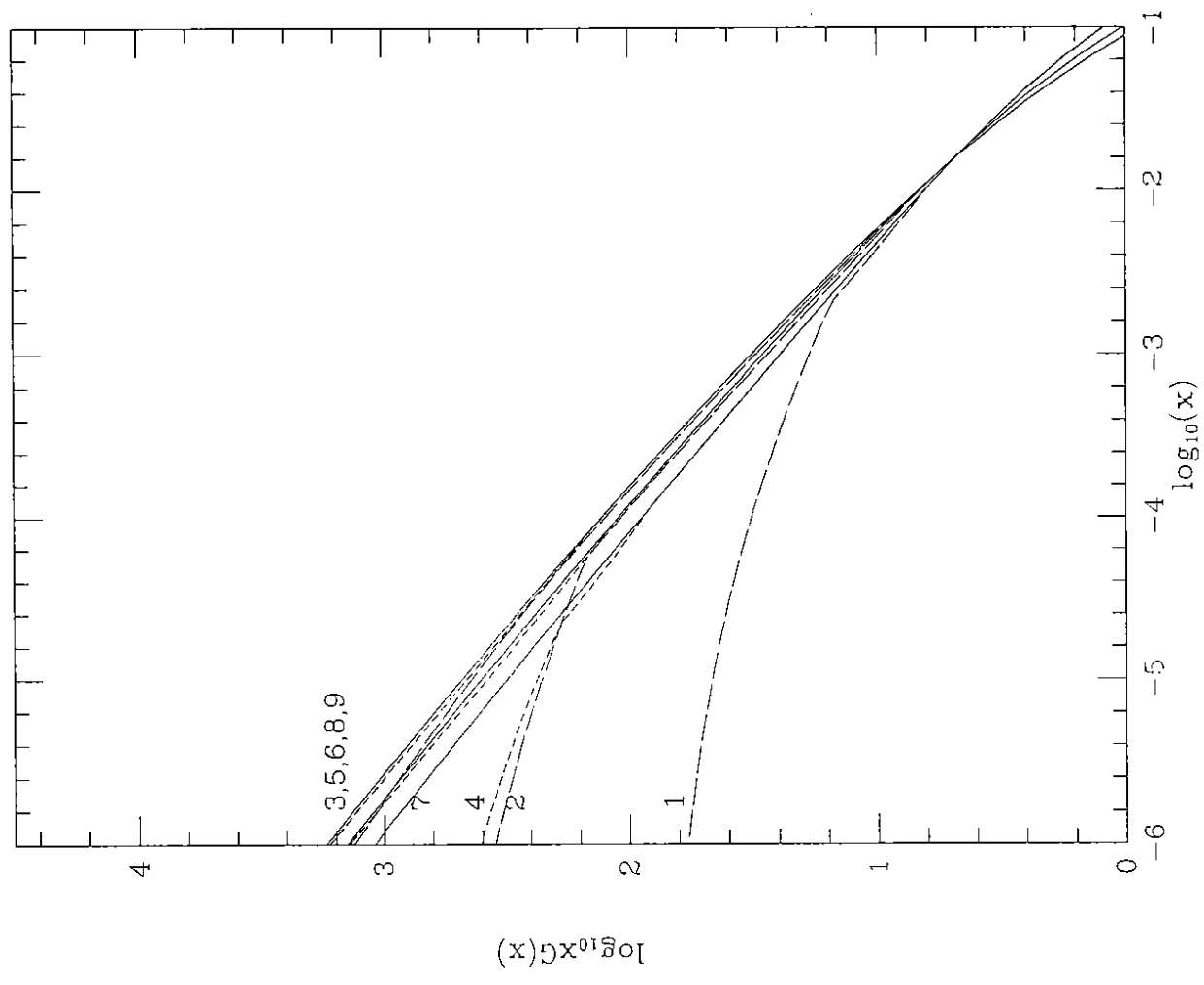
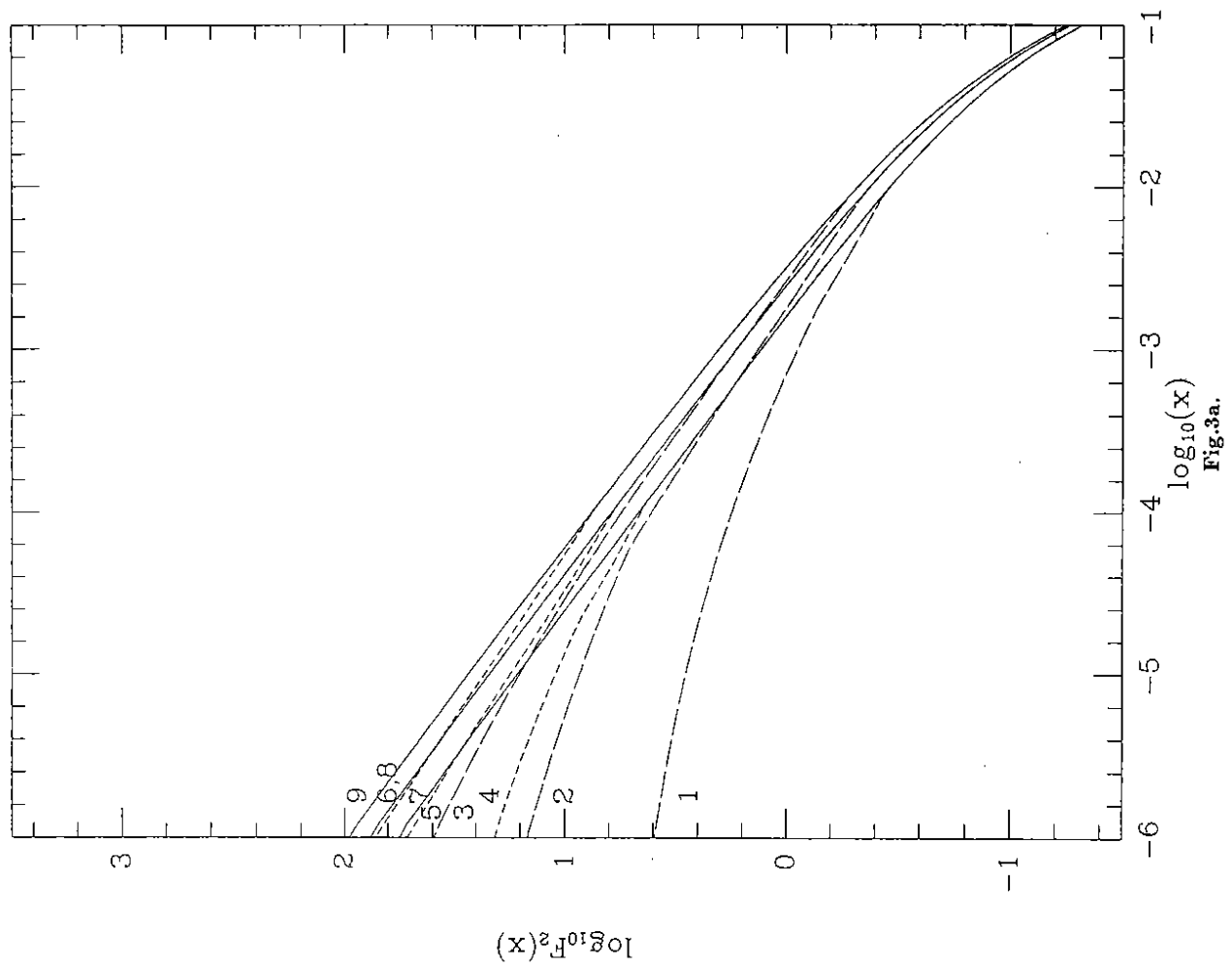
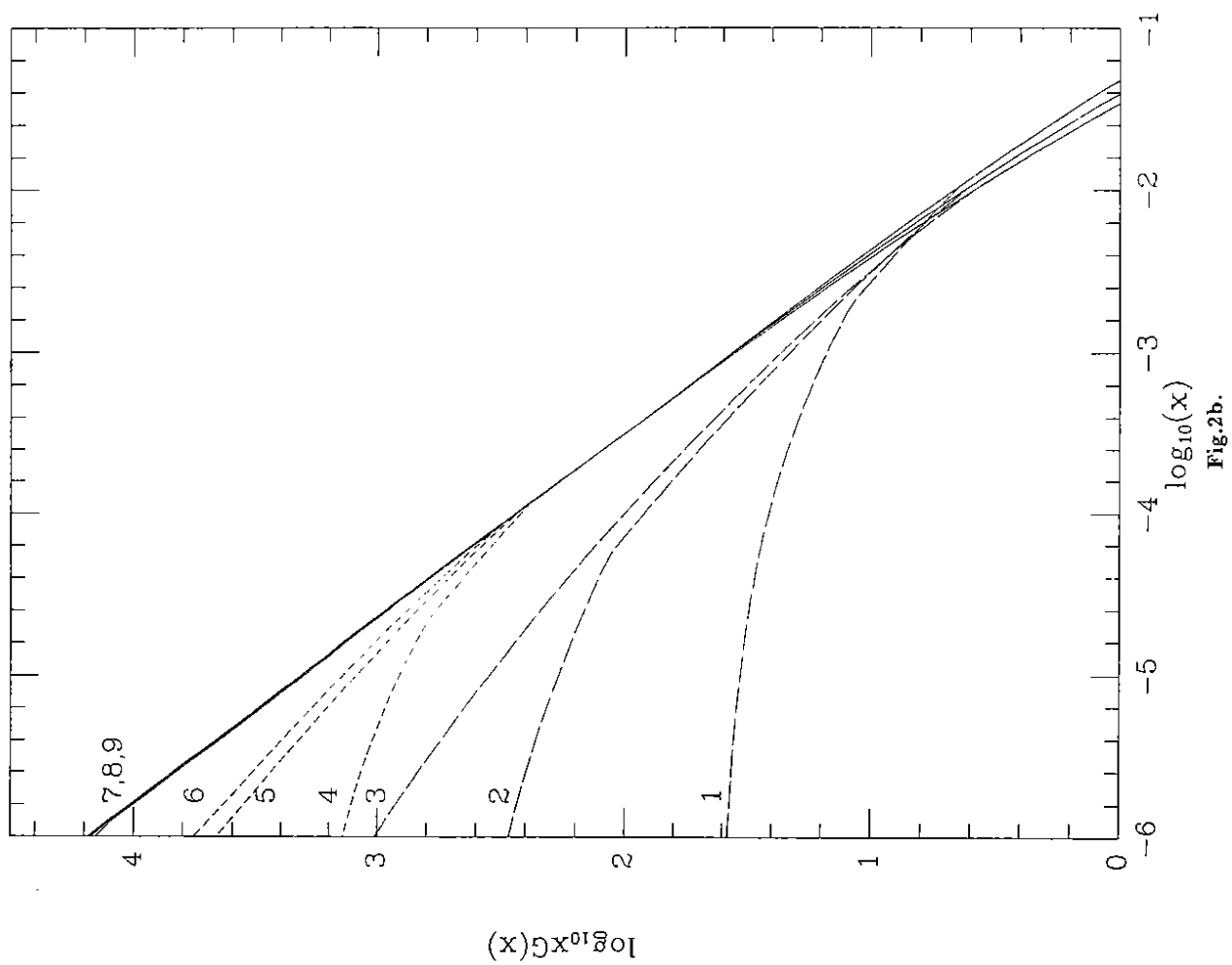
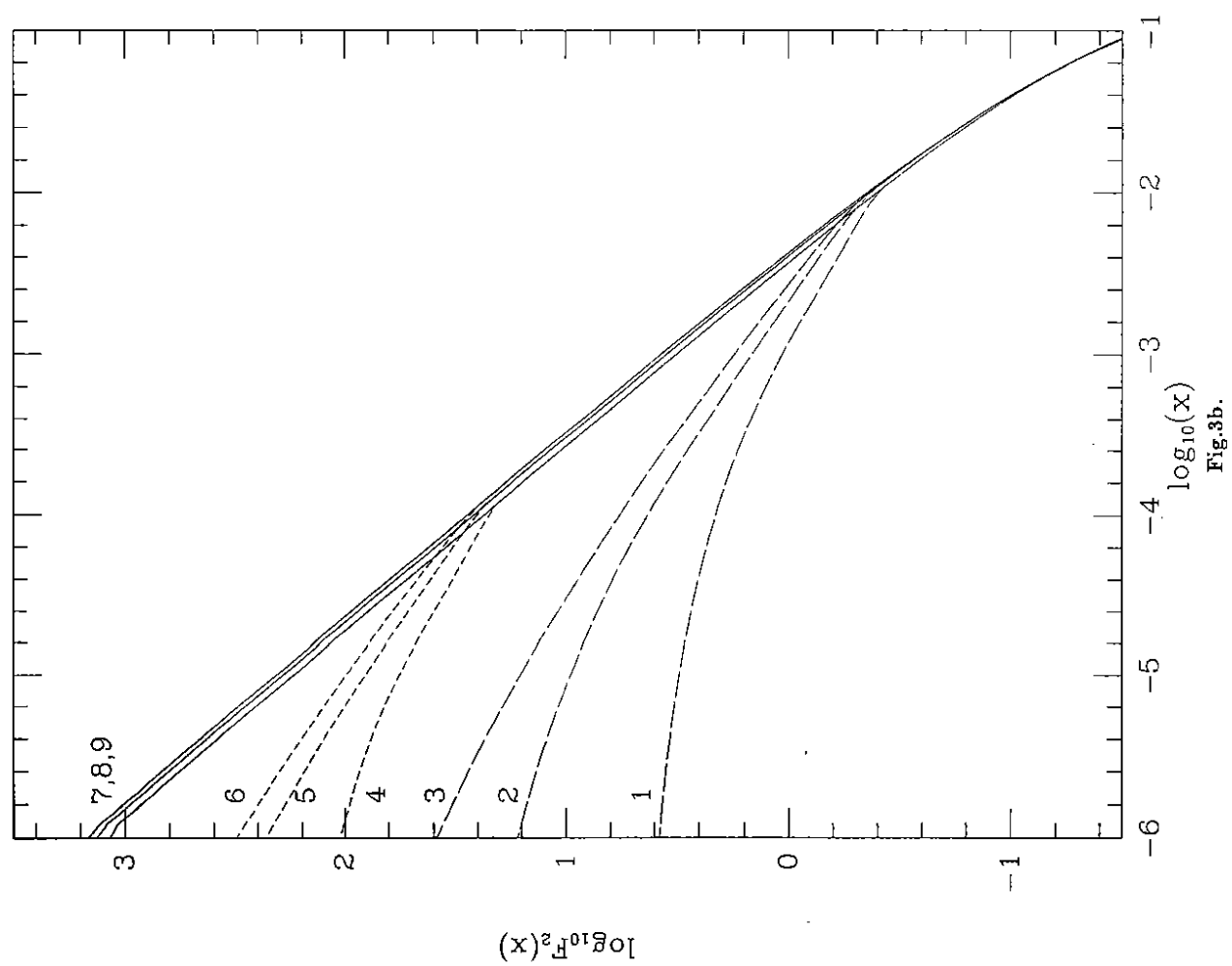
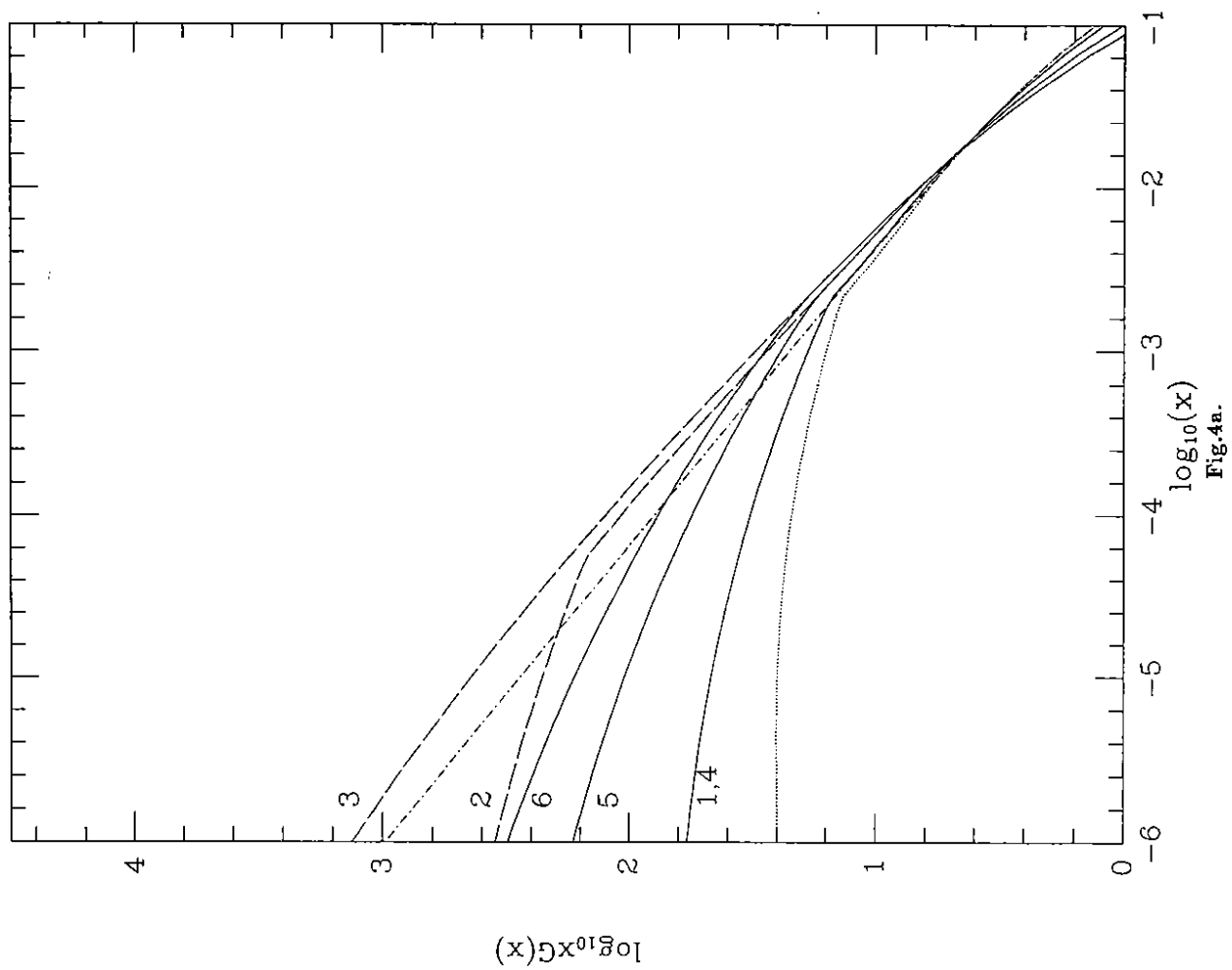


Fig.2a.







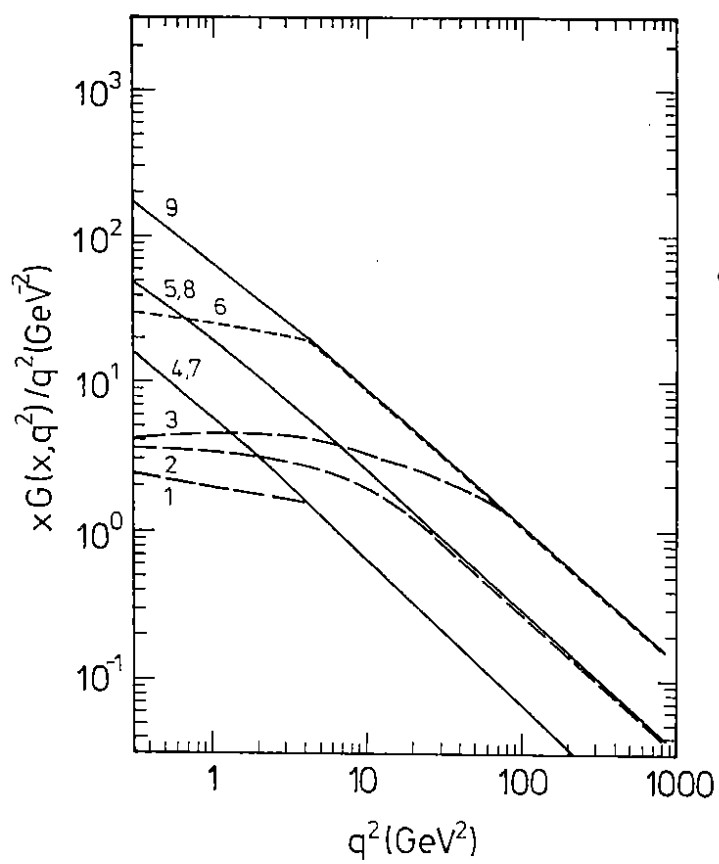


Fig. 5a

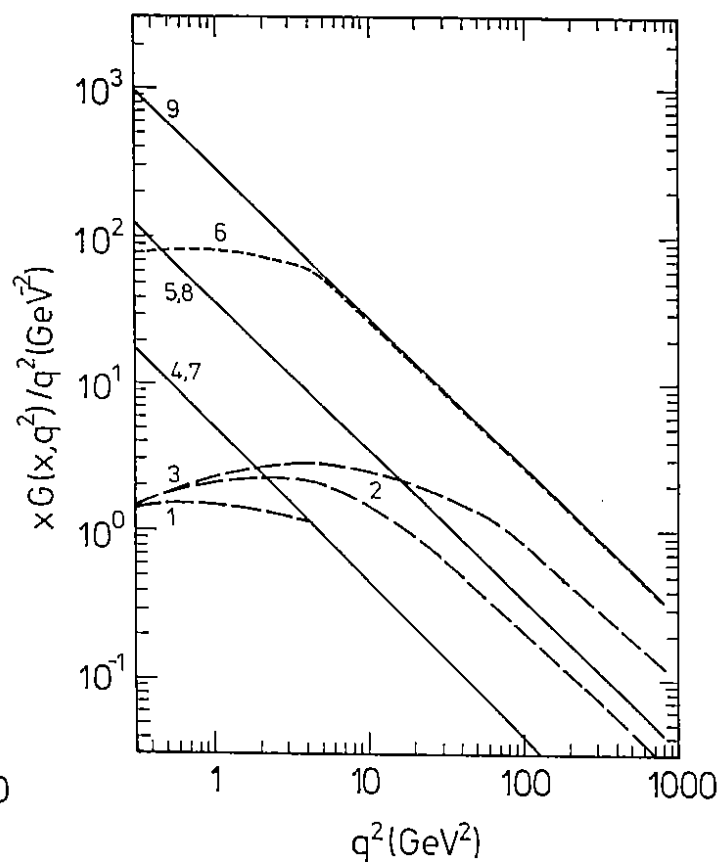


Fig. 5b

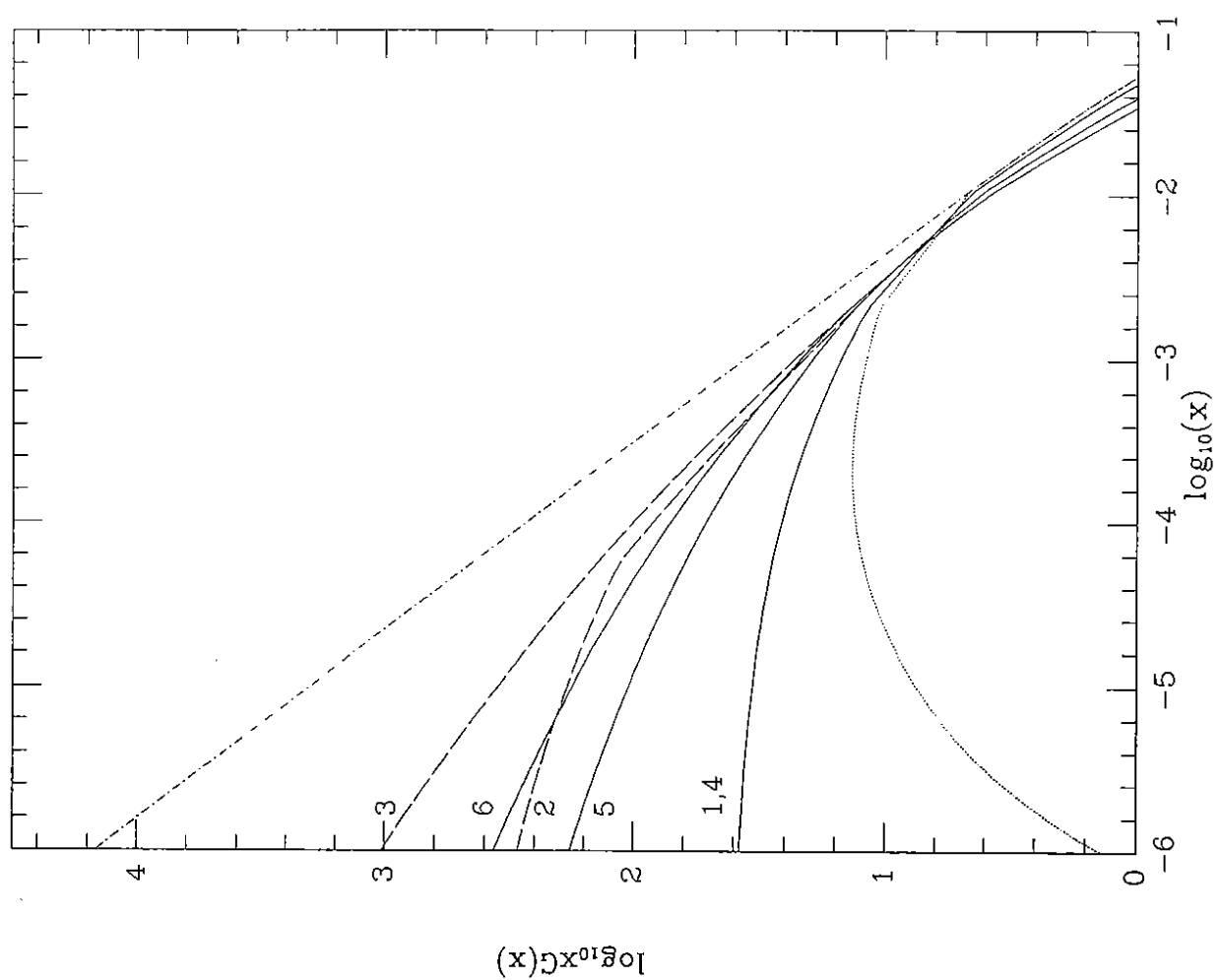


Fig. 4b.