The formal content of Trairūpya doctrine, Dignāga's Hetucakra, and Uddyotakara's extension

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Introduction

While ancient Indian as well as European logic have been developed in a setting where mathematical formulas were not in use, it is today tempting to utilise modern formal methods for analyzing the ancient texts. For contemporary research on ancient logic, formal methods are a valuable tool. But, formal logic should be used very carefully and with the constant awareness that these methods in itself are not independent of all philosophical presuppositions. Thus there arises the danger of imposing ideas and interpretations which are totally alien to the source texts.

It should also be taken into account that today there is not only one "correct" logic which may serve as a neutral and ideal instrument for any task whatsoever. There is a whole "zoo" of modern logics – paraconsistent, modal, temporal etc. - , and one has to decide which one to apply in which situation.

Formal methods may even result in an inundation of pseudo problems which are not inherent in the ancient logic itself but which have been artificially generated by the careless application of modern logic.[1] Therefore, one should try to utilize only a minimal amount of formalization, always being aware of the critical side effects of such a procedure.

In the present article, we will employ such an approach for taking a fresh look onto the doctrine of *Trairūpya* and its connection to Dignāga's *Hetucakra* and Uddyotakara's extension of Dignāga's ideas. We try to utilize formulas as little as possible by concentrating instead on formal methods (tables, illustrations) which could – but must not - be translated into formulas. For sake of completeness, we will also employ the standard predicate logical formulation and compare it to the results given by the other strategies.

In the last part of the paper I will discuss the question of the construction of a "different formal logic" which might be better suited to mediaeval Indian logic, than Western predicate logic. This can only be realised after a thorough discussion of the desired semantics of such a formal system. This semantics, of course, has to depend on the ontology of the specific Indian logic in question. Thus, before presenting a formal syntax, a formal semantics must be developed and, in the last part of my paper, I will suggest a starting point – not yet a solution - for this endeavour.

Part of what follows is a translation into English of my German paper on the *Hetucakra* of 1999[2].

1. The doctrine of Trairūpya

The problem of Nyāya logic

The standard setting in the *vāda* tradition in Indian logic is the following: Someone, let us call him the **proponent**, presents to his **opponent** a **thesis** of the following form:

Thesis: "There is a property (*dharma*) S in a property-possessor (*dharmin*) P."

Such a thesis, for example, might be " The Self (*ātman*) is eternal." Or, in order to adjust this to the abovementioned structure: "There is eternity in the Self."
Having stated this, and assuming that the opponent does not agree to the thesis, the proponent is obliged to give a **reason** for his statement and, afterwards, to show that it is a **valid** reason, i.e. a reason which in fact proves his argument.

[1] An extreme example of this type of creation of pseudo problems is Oetke's monography "Studies on the Doctrine of Trairūpya" (1994), where interesting and original results disappear behind a lot of uninteresting anwers to artificial questions.
[2] Das Rad der Gründe – Der Hetucakradamaru von Dignaga. Mitt. Math. Ges. Hamburg 18 (1999), 75 – 104.

Traditionally, such a reason is given by the presentation of another *dharma* H, the *hetu*, which acts as a sign (*liṇga*), indicating that the *dharma* S to be proven is indeed present in the *dharmin* P:

Reason: "Because of the *dharma* H."

Announcement of Thesis and Reason are the first two steps in the ancient rhetorical scheme of "Five-membered syllogism"[i]. We have already mentioned step 1 (Thesis) and step 2 (Reason); steps 3 and 4 consist in showing that the reason is "valid", i.e. that it proves the thesis (by checking that existence of *hetu* in a location implies existence of *sādhya* – step 3 -, and of the existence of *hetu* in *pakṣa* – step 4).

The *Trairūpya* doctrine provides three conditions which ensure that these steps 3 and 4 can be performed.

The theory of *Trairūpya* , the "three marks of a good reason", is ***the*** sophisticated central tool in ancient Indian logic. The origin of this theory is not known, but there are different early references to it, for example in the Chinese *Shun-Yhong-lun*[3] attributed to *Asanga* (ca. 395 - 470 A.C.), see Katsura (1985)[4] for a detailed exposition of the development of the *Trairūpya* formula.

The three conditions of the Trairūpya doctrine

Let now return to the fictitious situation of a discussion between proponent and opponent, and let us assume that the proponent has made the statements described above, which lay the basis for a dispute with the opponent on the validity of the reason, and, in consequence, of the thesis:

Thesis: "The *dharma* S (called the *sādhya*) occurs in the *dharmin* P (called the *pakṣa*)."
Reason: "Because of the *dharma* H (the *hetu*)."

As an example, let us imagine that the proponent says: "This object is a manmade, because it is impermanent." (*pakṣa:* this object*; sādhya*: manmade; *hetu*: impermanent).

The *Trairūpiya* doctrine gives three conditions to be checked:

T1 *dharmin* H, the *hetu*, occurs in *dharmin* P, the *pakṣa*.
T2 There is a *dharmin* in which S occurs **and** in which H occurs.
T3 There is **no** *dharmin* in which H occurs, and in which S does **not** occur.

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[3] Taisho Vol. 30, 1565, p. 42 a
[4] On Trairūpya Formula. In: Buddhism and its relation to other religions, Kyoto 1985

By common reason, T3 implies that, whenever H occurs in a *dharmin* P, i.e., if T1 holds, then necessarily S occurs in P.– because by T3 it is *not* possible that H occurs and S does *not* occur. Thus T1 and T3 together imply the occurrence of S in P, i.e., the validity of the thesis presented by the proponent.

The role of T2 is a kind of 'seriousness test' to be delivered by the proponent, ensuring to exclude a blatant nonsensical argumentation: Trying to convince the opponent of the fact that H is a sign for S, the proponent has to adduce *at least one* example where - agreed to by both parties - , the two properties H and S (the sign and the property to be proved) occur together.

Let us look at the example mentioned above.

Thesis and Reason: "This object is manmade, because it is impermanent."

In this case, the property H="impermanent" and S="manmade" satisfy T2, because everyone has already seen a broken manmade object before. But T3 does not hold – it is contradicted by any impermanent thing which is not manmade, for example, lightning. In this case, we might say that, in spite of H being a "serious" sign for S, it is not a *valid* sign.

The discussion between proponent and opponent might be agreed upon to proceed due to the following "protocol"[5]:

0. The proponent states the thesis and the reason.
1. The two parties agree upon **T1** – if not, the thesis has not been proven[6].
2. Proponent is obliged to give an example according to **T2**. If he fails to give such an example, his thesis has not been proven.
3. Proponent claims **T3**; if the opponent does not agree, he (the opponent) has to procure a counterexample. If he does, the thesis has not been proven, otherwise it has to be accepted.

The *Trairūpya* doctrine, imbedded in these rules of dispute, defines a sophisticated basis for deciding the result of a fair discussion of a controversial thesis[7]. The mediaeval philosopher **Dignāga** developed this method further into a means of strictly proving arguments by rational reasoning. He transformed this tool of the *vāda* (dispute) tradition into an instrument of inference, i.e. for the acquisition and transmission of knowledge. By means of his *Hetucakra,* he laid a firm theoretical basis for his theory.

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[5] I am not aware of any textual basis for this "protocol". Its purpose is to illustrate the power of the *Trairūpya* doctrine for deciding discussions of a certain type as well as the logical function which "simple examples" assume in such a situation.

[6] In the example above, T1 would have been met in case the proponent points to a pot while making the statement of his thesis.

[7] The protocol of a fair dispute given above is of course only a skeleton. For practical use, it must be supplemented by additional rules; for example, by rules which describe how to proceed in case one of the parties does not accept the example of the other side.

The Trairūpya scheme

Let us assume for the following, that the first condition, T1, of the *Trairūpya* conditions has been met, and let us concentrate on the conditions T2 and T3 which concern the relation between *sādhya* S and *hetu* H.

The *Trairūpya* doctrine, depending on these two conditions, may be put into the following table (which is just another representation of the original doctrine):

	T3 not met	T3 met
T2 met	NOT VALID	VALID
T2 not met	NOT VALID	NOT VALID

Table 1.1: *Trairūpya* conditions 2 1nd 3

The *Trairūpya* doctrine allows us also to specify the "not valid" reasons of Table 1.1, which results in the following table[ii].

	T3 not met	T3 met
T2 met	TOO WIDE	VALID
T2 not met	CONTRADICT.	SPECIAL

Table 1.2: Trairūpya conditions 2 and 3

Here, "contradictory" means that H proves non-S, the opposite of the proposition to be proved. "Too wide" signifies that the reason proves neither S nor non-S. The fourth case, called "special", requires extra considerations to be discussed later on.

Dignāga's Hetucakra

In his early work, the *Hetucakraḍamaru*, Dignāga found a refinement of the *Trairūpya* scheme. He split the two alternatives of the conditions T2 and T3 into three cases each:

T2 met:	1.	H occurs **in all** *dharmin*s in which S occurs
	2.	H occurs **in some** *dharmin*s in which S occurs
T2 not met:		H occurs **in no** *dharmin* in which S occurs.
T3 not met:	1.	H occurs **in all** *dharmin*s in which S does not occur.
	2.	H occurs **in some** *dharmin*s in which S does not occur.
T3 met:		H occurs **in no** *dharmin* in which S does not occurs.

This classification is a subdivision of the conditions T2 and T3, thus the scheme of Table 1.2 remains intact - only the first row and the first colum of the scheme have to be subdivided into two each. Therefore, the valid reasons are the two cases in the upper right corner.

	All non-S are H	**Some** non-S are H	**No** non-S are H
All S are H **Some** S are H	TOO WIDE		VALID
No S are H	CONTRADICT.		SPECIAL

Table 1.3: From *Trairūpya* to *Hetucakra*

Inserting Dignāga's order (from D1 to D9) of the nine cases into this table, we obtain the following scheme:

	ALL	**SOME**	**NO**
ALL	D1	D3	D2
SOME	D7	D9	D8
NO	D4	D6	D5

VALID: D2, D8
CONTRAD.: D4, D8
TOO WIDE: D1, D3, D7, D8
SPECIAL: D5

Table 1.4: The nine cases of *Hetucakra*

This is in total agreement with Dignāga's "Wheel of Reason", which merely uses a different ordering of rows and columns, making the table look more like a wheel[iii]:

"vipakṣa – side"

„sapakṣa - side"

	ALL	**NO**	**SOME**
ALL	D1	D2	D3
NO	D4	D5	D6
SOME	D7	D8	D9

Table 1.5: *Dignāga's Hetucakra*

Uddyotakaras extension

Uddyotakara extended the *Hetucakra* by adding seven new cases to Dignāga's scheme. This may be seen as an additional split of the *Trairūpya* scheme, this time affecting the last row and the last column of Table 1.3.

Splitting of the last row refers to the following process:

Consider the "**No-case**"of a *hetu* H (last line of Table 1.3). Out of these cases we select those where there is **no *dharmin* at all**, in which S occurs. This will add a new row to Dignāga's scheme, which is a special case of the original "No"- row. We denote this new row by "Sa=∅" and retain the heading "No" for the remaining cases of a *hetu* which does not occur in a *dharmin* where the *sādhya* S occurs. We then perform the same procedure for the last column of this scheme, thus splitting off the cases in which there is no *dharmin* in which non-S occurs (we denote this by "Vi = ∅" and obtain

	All non-S are H	**Some** non-S are H	**No** non-S are H	**Vi** =∅
All S are H	TOO WIDE		VALID	
Some S are H				
No S are H	CONTRADICT.		SPECIAL	
Sa=∅				

Table 1.6: Final splitting of the *Trairūpya* table

The result is a table containing 16 cases where – **according to the *Trairūpya* doctrine** – the *hetu-sādhya* pair is of one of the four types:

- **four valid** cases (right upper corner)
- **four contradictory** cases
- **four "too wide"** cases
- **four "special"** cases which, according to the *Trairūpya* doctrine, the reason is not valid.

Thus we obtain the following scheme, which, concerning the cases D1 to D9, and except for the change in order and enumeration[iv], agrees exactly with Dignāga's classification, Table 1.4. According to the Trairūpya doctrine, which is the basis for our classification, the "special cases" U6, U12, U15, and U16 **all represent INVALID cases**.

	ALL	**SOME**	**NO**	**VI=∅**
ALL	D1,U1	D3,U2	D2,U3	U10
SOME	D7,U7	D9,U8	D8,U9	U11
NO	D4,U4	D6,U5	D5,U6	U12
SA=∅	U13	U14	U15	U16

Table 1.7: Classification according to the *Trairūpya* doctrine

Case U15 is of special interest, because *Uddyotakara* claimed it to be a **valid reason**. The method of his proof is open to dispute, as it differs completely from the proof of all the other cases. We will come back to this in the second part of our paper.

This Table 1.7 is the main result of the first part of this paper. It shows how Dignāga's and Uddyotakara's ideas are connected to the ancient *Trairūpya* doctrine. One result is that Dignāga's *Hetucakra* can be regarded as direct consequence of the *Trairūpya* doctrine without using formulas at all. It shows also, that Uddyotakara's system is not in total accordance with the *Trairūpya* doctrine, as it differs in one of the 16 cases.

2. The Hetucakra of Dignāga

The text

Dignāga's *Hetucakraḍamaru* is a very short treatise. It consists of two parts: The first one gives a set of 9 *sādhya / hetu* pairs[v], see R. P. Hayes (1980)[8]. The second one is a list of 9 sets of examples[vi], which belong to the *sādhya / hetu* pairs with corresponding numbers.

There is a concise way to depict these two sets of information within one table. *Dignāga* himself probably had in mind a scheme similar to that, when he described how to set up his 9 cases into a "wheel". The Tibetan translation of the *Hetucakra* contains also such a "matrix" (see Table 2.1).

Sound is permanent Because it is knowable Like space Unlike a pot	Sound is impermanent Because it is produced Like a pot Unlike space	Sound is manmade Because it is impermanent Like a pot Not like lightning and space
Sound is eternal Because it is produced Like space Unlike a pot	Sound is permanent Because it is audible Like space Unlike a pot	Sound is permanent Because it is manmade Like space Unlike a pot and lightning
Sound is not manmade Because it is impermanent Like lightning and space Unlike a pot	Sound is impermanent Because it is manmade Like a pot and lightning Unlike space	Sound is permanent Because it is incorporeal Like space and atom, Unlike action and a pot

Table 2.1: Table form of *Hetucakra*

Thesis and reason follow the scheme which we have seen in the discussion of the *Trairūpya* doctrine. Thus, for example, in the first case in the left upper corner, we have *sādhya* S="permanent" and *hetu* H="knowable", etc. Relating the examples to the *sādhya / hetu* - pairs in the table above, we see that the examples relate to the *dharmas* in the following way (where we take, as an example, Case 1 in the upper left corner):

space	is		*permanent*	and	*knowable*
pot	is	not	*permanent*	and	*knowable*.

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[8] Diṇṇāga's View of Reasoning (svārthānumāna). J. Indian Phil. 8 (1980), 219-277. See also R. S. Y. Chi, Diṇṇāga and Post-Russell Logic. Buddhist Logic and Epistomology, pp. 107-115, 1986.

There are no examples given for the combinations *permanent / not knowable* and *not permanent / not knowable*, respectively. **We will hereafter interprete this omission of examples as signifying that no such examples exist.**

This situation may be assembled into the following scheme invented by the logician Lewis Carroll (1897)[9], the autor of "Alice in Wonderland":

	knowable	not knowable
permanent	space	
not permanent	pot	

Table 2.2: "Carroll frame" of example-set for Case 1

The logical content

We will now arrange the examples into a special scheme, where each set of examples appears in its corresponding Carroll-frame:

D1

perma nent	knowable	
	space	
	pot	

D2

imper man.	manmade	
	pot	
		space

D3

manm ade	impermanent	
	pot	
	lightn.	space

D4

perma nent	produced	
		space
	pot	

D5

perma nent	audible	
		space
		pot

D6

perma nent	manmade	
		space
	pot	lightn.

D7

not manm ade	impermanent	
	lightn.	space
	pot	

D8

Imper man.	manmade	
	pot	lightn.
		space

D9

perma nent	uncorporeal	
	space	atom
	action	pot

Table 2.3: "Wheel" of Carroll-frames

In this table, the main information is *not* hidden in the nature of the special examples but in the distribution of existing and nonexisting examples. Therefore, we will make use of an abbreviated form of the Carroll frame. In the following Table 2.4 which resumes Table 2.3, the *existing* examples have been depicted as a "+", the *not existing* ones by "-":

[9] Lewis Carroll: Symbolic Logic, Part I: Elementary. Mac Millan and Co., Ltd., London 1897

	H	¬H
S	+	-
¬S	+	-

Table 2.4: Short form of Carroll frame, case 1.

All Carroll frames will be arranged into the scheme of Dignāga's "wheel" in Table 2.5.

		ALL		**NO**		**SOME**	
		H	¬H	H	¬H	H	¬H
ALL	S	+	-	+	-	+	-
	¬S	+	-	-	+	+	+
		D1		D2		D3	
		H	¬H	H	¬H	H	¬H
NO	S	-	+	-	+	-	+
	¬S	+	-	-	+	+	+
		D4		D5		D6	
		H	¬H	H	¬H	H	¬H
SOME	S	+	+	+	+	+	+
	¬S	+	-	-	+	+	+
		D7		D8		D9	

Table 2.5: Wheel with examples

This table contains the total logical content of Dignāga's *Hetucakra*. Its significance becomes clear especially in connection with the *Trairūpya* – table, Table 1.5, which classifies Dignāga's 9 cases into valid and (different types of) invalid ones.

The meaning of "ALL"

The distribtion of "+" and "-" marks seems to be very irregular. But this changes if we depict the upper and lower parts of the Carroll frames separately (Table 2.6 and Table 2.9):

				H	¬H		
ALL		+	-	+	-	+	-
NO	S	-	+	-	+	-	+
	¬S						
SOME		+	+	+	+	+	+

Table 2.6: Upper part of Carroll frames

	ALL		NO		SOME	
	+	-	-	+	+	+
			H	¬H		
S						
¬S	+	-	-	+	+	+
	+	-	-	+	+	+

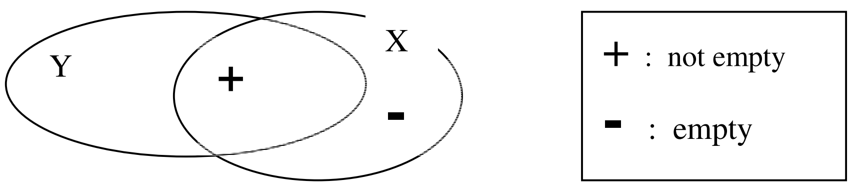
Table 2.7: Wheel of lower part of Carroll frames

Tables 2.6 and 2.7 show what Dignāga's "quantor" ALL signifies. Let us abbreviate "All X are Y" by A(X,Y). Then, in terms of Carroll frames, this can be interpreted as

	Y	¬Y
X	+	-
¬X		

Table 2.8: The meaning of "A(X,Y)"

An empty box signifies that there is **no condition present** for that case. Brought into the shape of a Venn- diagram[10], this says



This may also be performed for the "No"- and "Some"- quantors[vii].

"Proof" of the Hetucakra

[10] Anticipating the last Section, this may be expressed in predicate logic as
$$A(X,Y)= \exists x(Xx \textbf{ and } Yx) \textbf{ and } \neg\exists x(Xx \textbf{ and } \neg Yx) = \exists x(Xx \textbf{ and } Yx) \textbf{ and } \forall x(Xx \rightarrow Yx).$$

In our interpretation of the *Hetucakra*, there is still one important step left: How to interprete the "valid" cases in terms of the quantors A, U, and N.

The valid cases are D2 and D8, for which we repeat the Carroll diagrams here:

	H	¬H
S	+	-
¬S	-	+

	H	¬H
S	+	+
¬S	-	+

Table 2.9: The valid cases D2 and D8 of Table 2.5

At this place, we will use a device which looks like a trick, but which is a main formal tool for our whole argumentation. Let us remember that the central Table 2.5 has been constructed in order to capture the formal content of the *Trairūpya* doctrine. The propositions which have been "coded" into the shape of Carroll frames are of the type

"All (No, Some) S are H" or "All (No, Some) ¬S are H".

The valid cases D2 and D8, which are the cases of pervasion of H by S, are those, for which

"All H are S",

holds true. Thus , in the conclusion, an exchange of S and H has to be made. We will perform this exchange by arranging the contents of the Carroll frames in a different form, where we interchange S and H while retaining the logical content of the frames. This means that the contents of the frames will be reflected at the main diagonal. Let us perform this reflection, as an example, for the important "valid" case D8. **The logical content of the frame will not change** by this transformation, because the four entries change corresponding to the column- and row-headings:

	H	¬H
S	+	+
-S	-	+

→

	S	¬S
H	+	-
¬H	+	+

Table 2.10: The valid case D8 of Table 2.5

We see that for this valid cases D8,

A(H,S)

holds true according to Table 2.10; **this is the formal reason why case D8 has to be considered as valid**.

If we perform the same operation to all other frames of Table 2., i.e. if we interchange S and H in each Carroll frame by retaining the "logical content" of the frames, we get the following diagram:

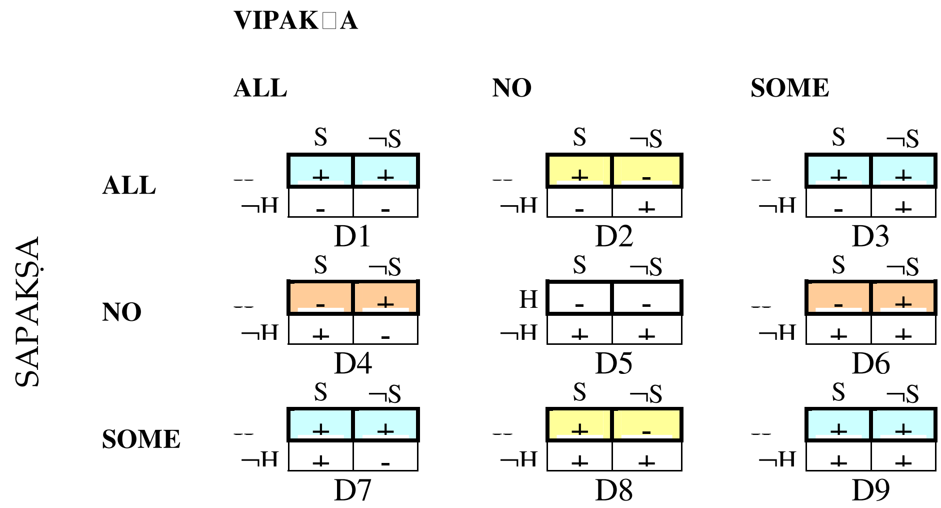


Table 2.11: The wheel

Here the special role of D5 has to be noted: The "No/No" case D5 in the middle of the wheel gives rise to a new "quantor", which we will call Y. It has the shape

	S	¬S
H	-	-
¬H		

Table 2.12: Carroll frame for Y(H,S)

In this case, the sets HS as well as H¬S are empty[11]. Taking this new quantor into account, the content of Table 2.11 may be rewritten in an "algebraic" form as follows:

	A	N	U
A	U	A	U
N	N	Y	N
U	U	A	U

Table 2.13

[11] Implying that there is no x such that Hx.

Uddyotakara's extension

The quantor "Y" of Table 2.13 does not appear in the premises of the 9 cases of the *Hetucakra*, but is generated by the coming together of N(S,H) and N(¬S,H). It was Uddyotakara's idea to enlarge the *Hetucakra* by using the new quantor Y also in the **premises** of his scheme. We will construct this extension as follows.

Let us start with Table 2.5, where we add one new row at the bottom, containing the quantor Y(S,H):

		H	¬H		H	¬H		H	¬H
Y	S	-	-	S	-	-	S	-	-
	¬S	+	-	¬S	-	+	¬S	+	+

Table 2.14: New row for Y

The second rows of the three single Carroll frames have been filled like the ones in the existing rows in Table 2.5. - We also add a new *column* for Y, where, in each Carroll frame, the lower row (in this case, the "¬S" – row, because we are dealing with the *vipakṣa*) has been changed to

	H	¬H
S		
¬S	-	-

Table 2.15

Finally, we add the Carroll frame in the lower right corner, in which the two Y-quantors "meet" to produce

	H	¬H
S	-	-
¬S	-	-

Table 2.16: Right lower corner frame

The result of this whole process of

- adding one row and one column
- reflecting each Carroll/ frame at its main diagonal
-exchanging colums 2 and 3 and rows 2 and 3, respectively

is the following extension of the *Hetucakra:*

		A: All		***U: Some***		***N: No***		***Y: Vi=∅***	
		S	¬S	S	¬S	S	¬S	S	¬S
A: All	H		⊥	⊥	⊥	⊥	-	⊥	-
	¬H	-	-	-	⊥	-	⊥	-	-
		U1,D1		U2,D3		U3,D2		U10	
		S	¬S	S	¬S	S	¬S	S	¬S
U: Some	H	⊥	⊥	⊥	⊥	⊥	-	⊥	-
	¬H	⊥	-	⊥	⊥	⊥	⊥	⊥	-
		U7,D7		U8,D9		U9,D8		U11	
		S	¬S	S	¬S	S	¬S	S	¬S
N: No	H		+	-	+	-	-	-	-
	¬H	+	-	+	+	+	+	+	-
		U4,D4		U5,D6		U6,D5		U12	
		S	¬S	S	¬S	S	¬S	S	¬S
Y: Sa=∅	H	-	+	-	+	-	-	-	-
	¬H	-	-	-	+	-	+	-	-
		U13		U14		U15		U16	

Table 2.17 Uddyotakara's 16 cases

Again we see that, from the standpoint of *Trairūpya* doctrine, the cases U3, U10, U9, and U11 are the only valid ones[viii]. Uddyotakara gave a "proof" for the validness of his case U15, too. Although this is not consistent with the *Trairūpya* doctrine, arguments can be found supporting Uddyotakara's claim. This will be dealt with in the next Section.

There is also an "algebraic" version of Uddyotakara's table, being the extension of Table 2.13:

	A	**U**	**N**	**Y**
A	U	U	A	A
U	U	U	A	A
N	N	N	Y	Y
Y	N	N	Y	Y

Table 2.18 Algebraic version

3. *On using formulas*

Trairūpya – implication and alternatives

In order to understand the defining property of a valid reason from a formal standpoint, one has to specify the meaning of "*hetu* implies *sādhya*". We will now discuss several different possibilities to accomplish this task. The first definition is based on the *Trairūpya* doctrine; it is exactly the relation which we have been using in the previous Section 2 for identification of the valid cases. The new notation ("Trairūpya implication => ") is just a matter of convenience:

"*hetu* T-implies *sādhya*" = **H=>S**
= $\exists x(Hx \textit{ and } Sx) \textit{ and } \neg\exists x(Hx \textit{ and } \neg Sx)$
= A(H,S).

Exactly as we have done in Section 2, we call a *hetu* (reason) to be valid for a *sādhya* S, if H=>S (= A(H,S)) holds[12]. Here, A has been defined as in Table 2.8 and, in addition,

$$N(H,S)= \neg\exists x(Hx \text{ and } Sx) \text{ and } \exists x(Hx \text{ and } \neg Sx).$$

$$U(H,S)= \exists x(Hx \text{ and } Sx) \text{ and } \exists x(Hx \text{ and } \neg Sx).$$

In Uddyotakara's extension of the *Hetucakra*, we have to define an additional relation between two *dharmas* X und Z, which we denote by Y(X,Z). According to Uddyotakara's intention of treating the cases of void *sapakṣa* and *vipakṣa*, we define (compare Table 2.12):

$$\mathbf{Y(S,H)} = \neg\exists x(Sx \text{ and } Hx) \textit{ and } \neg\exists x(Sx \textit{ and } \neg Hx) = \neg\exists x(Sx)$$

And, consequently,

$$\mathbf{Y(\neg S,H)} = \neg\exists x(\neg Sx \textit{ and } Hx) \text{ and } \neg\exists x(\neg Sx \textit{ and } \neg Hx) = \neg\exists x(\neg Sx).$$

As result we find – after some standard predicate logical calculations - that this logic, together with taking the "Trairūpya implication " => as the criterion for a valid, reason, reproduces exactly the valid and invalid cases as depicted in Table 1.7 of Section 1 and 2.18 of Section 2, which we obtained by using the *Trairūpya* doctrine. The reason behind this agreement is the fact that we have chosen all predicate logical

---

[12] H=>S should be read as "*H is a Trairūpya proof of S*". We will call "=>" the *Trairūpya* implication.

formulas in a way that they reproduce exactly the *Trairūpya* doctrine (see the following Table 3.1).

	ALL	SOME	NO	VI=∅
ALL	U1	U2	U3	U10
SOME	U7	U8	U9	U11
NO	U4	U5	U6	U12
SA=∅	U13	U14	U15	U16

Table 3.1: Results by Trairūpya theory: Four valid cases

The "modern" interpretation

Let us now try out another formulation which is not based on *Trairūpya* doctrine but on the modern notion of the "All"- quantor, where no existential condition will be imposed. Thus let us, as a criterion for a valid reason, replace H=>S by

$$H \rightarrow S \quad = \quad \forall x(Hx \rightarrow Sx)$$
$$= \quad \neg\exists(Hx \text{ and } \neg Sx)$$

We have to check all the 16 cases of Uddyotakara's scheme and find out which combinations of the premises imply $H \rightarrow S$.

After some calculations, we obtain the following result[ix]

	ALL	SOME	NO	VI=∅
ALL	U1	U2	U3	U10
SOME	U7	U8	U9	U11
NO	U4	U5	U6	U12
SA=∅	U13	U14	U15	U16

Table 3.2: Results by standard predicate calculus: Seven "valid" cases

There is still another modern interpretation by means of set theory. This reproduces the same results as predicate logic except for the most exotic case (16), which set theory renders a valid reason (and a contrary at the same time)[13]:

	ALL	SOME	NO	VI=∅
ALL	U1	U2	U3	U10
SOME	U7	U8	U9	U11
NO	U4	U5	U6	U12
SA=∅	U13	U14	U15	U16

Table 3.3: Results by set theory: 8 valid cases

[13] (16) is the case of an empty universe which is, in set theory, a well defined object.

In contrast to this modern treatment of the 16 cases, Uddyotakara gave the following result[14], where case (15) is a valid case.

	ALL	SOME	NO	VI=∅
ALL	U1	U2	U3	U10
SOME	U7	U8	U9	U11
NO	U4	U5	U6	U12
SA=∅	U13	U14	U15	U16

Table 3.4: Uddyotakara's result: 5 valid cases

Okazaki (2003)[15] has shown that Uddyotakara's result can be reproduced by predicate logic in the following way: Let "All", "Some", "No" be defined just like in the *Trairūpya* doctrine. For deciding which cases are valid, Okazaki defines

$$H\text{Ł}\ S = H \Rightarrow S \textit{ or } (\neg S \Rightarrow \neg H \textit{ and not } S \Rightarrow \neg H)$$

which, if split into its "elementary building blocks", is the following formula:

$$
\begin{aligned}
H\text{Ł}\ S = \quad & \exists x(Hx \textit{ and } Sx) \textit{ and } \neg\exists x(Hx \textit{ and } \neg Sx) \\
\textit{or}(\quad & \exists x(\neg Hx \textit{ and } \neg Sx) \text{ and } \neg\exists x(\neg Sx \textit{ and } Hx) \\
& \textit{and} \quad \neg(\exists x(\neg Hx \textit{ and } Sx) \textit{ and } \neg\exists x(Sx \textit{ and } Hx)) \\
). &
\end{aligned}
$$

Okazaki proved that his formula captures exactly the valid cases of Uddyotakara's Table 3.7. While it is possible to find different formulas reproducing Uddyotakara's results[x], Okazaki's formula has the advantage of being based on a philological analysis of Uddyotakara's text.

Problems of Predicate logic

In the preceding part we have seen that predicate logic can be successfully utilized for the interpretation of the connection between *Trairūpya* doctrine and the *Hetucakra*. We also saw that the results produced by this tool are comparable to the results obtained by relatively straightforward graphical methods like those presented in Section 1 and 2. In this case, predicate logic did not increase our understanding of the *Trairūpya* doctrine, the *Hetucakra*, or Uddyotakara's results but served as a kind of formal justification.

[14] The classification of the cases U1 to U9 is the one given by Dignāga.
[15] Asādhāraṇa-hetvābhāsa and Uddyotakara's vyatirekin. Nagoya Studies in Indian Culture and Buddhism: Saṃbhāṣā 23, 2003.

In addition to that, predicate logic may sometimes lead to misunderstandings and misinterpretations because of the special ontological background on which this formalism has been developed.

In order to highlight this problem, let us consider the most prominent example, the formulation of the pervasion of two *dharmas* H and S. The fact that H is pervaded by S is usually written as

$$\forall x(Hx \rightarrow Sx),$$

which, by the laws of predicate logic, is equivalent to

$$\neg\exists x(Hx \text{ and } \neg Sx).$$

This formula can be smoothly interpretated as *avinābhāva:*

"There is nothing which has the property H, and which does not have the property S."

While this, at first sight, sounds reasonable, a short reflection shows that things are not so easy as they seem. The question is: For what *kind of entities* stands the "x" in the formula above? Concerning predicate logic, the answer is clear: The "x" refers to individuals (particulars) in an underlying universe; preceded by a quantor, x "runs" through the whole universe.

But do the ancient Indian texts really define the pervasion of two *dharma*s by referring to a universe of particulars? I have some doubts about that, and to me it seems that the relation between the two *dharma*s is meant as a *direct* one: for example, S= "impermanence" resides (is located in) H="being a product". This locus/locatee relation is the central relation, and it is not restricted to a relation between a particular and a predicate. Quite on the contrary, Dignāga (in *PSV)* explicitely states that " a *dharma* proves a *dharma*, not a *dharmin*".

So we should better look for a formulation which directly connects two *dharma*s without explicitely referring to their residence in particulars. This proposal has been made by several authors during the last decades, for example, by Matilal (2001)[16]. Matilal proposes to formalize this locus/locatee-relation of two predicates by

$$Q(H,S),$$

signifying that S is located in H. In this formulation, obviously no particulars are being involved, and it is not necessary to define this relation of two *dharma* by means of predicate logic.

While giving much textual evidence to his proposal, Matilal did not work out his idea[17]. In our next chapter we will take up Matilal's proposal of constructing a formal

[16] Logic,Language and Reality: An Introduction to Indian Philosophical Studies, Delhi, 1985.

[17] In fact, he obscured it by writing down formulas which are well intentioned but do not make much sense.

system of logic which, as a building block, has the locus/locatee-relation. The main step in attacking this task will be the designing of a formal semantics, which has to be in accord to the ontology underlying the ancient texts. Only if this design has been achieved, a formal syntax can be developed which then will fit the semantics and will go nicely with the ontology on which Indian logic is based on.

Sketching a different semantics

Constructing a formal languge, i.e., a syntax for a certain field of reasoning, requires to deal with two different tasks:

- formulating semantics (formal ontology)
- constructing the formal system.

The semantical domain of a logical system may be seen as a formal ontology, where the structure of the underlying philosophical ontology has been put into a formal framework, using simple formal concepts like sets or graphs. The aim of constructing a formal ontology or semantics is, to have at hand a precisely defined "world" where the formal language, to be constructed later on, can be applied to. Thus, semantics is "half way" between ontology and syntax.

In the case of medieaval Indian logic, we propose that the semantic domains should be *directed graphs*, consisting of *nodes* which are conneted by *arrows*. The *nodes* refer to ontological units like substances, qualities, and universals, and the *arrows* describe the mutual connections of the nodes. An arrow leading from an node A to an element B may, for example, signify that a quality A resides in a substance B.

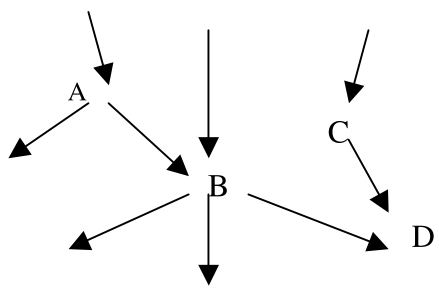


Fig. 3.5: Semantic domain

It may be difficult or even impossible to imagine an all-embracing ontology being pressed into such a scheme, but this is not the task of a formal ontolgy, which only aims at depicting the *structure* of such a domain.

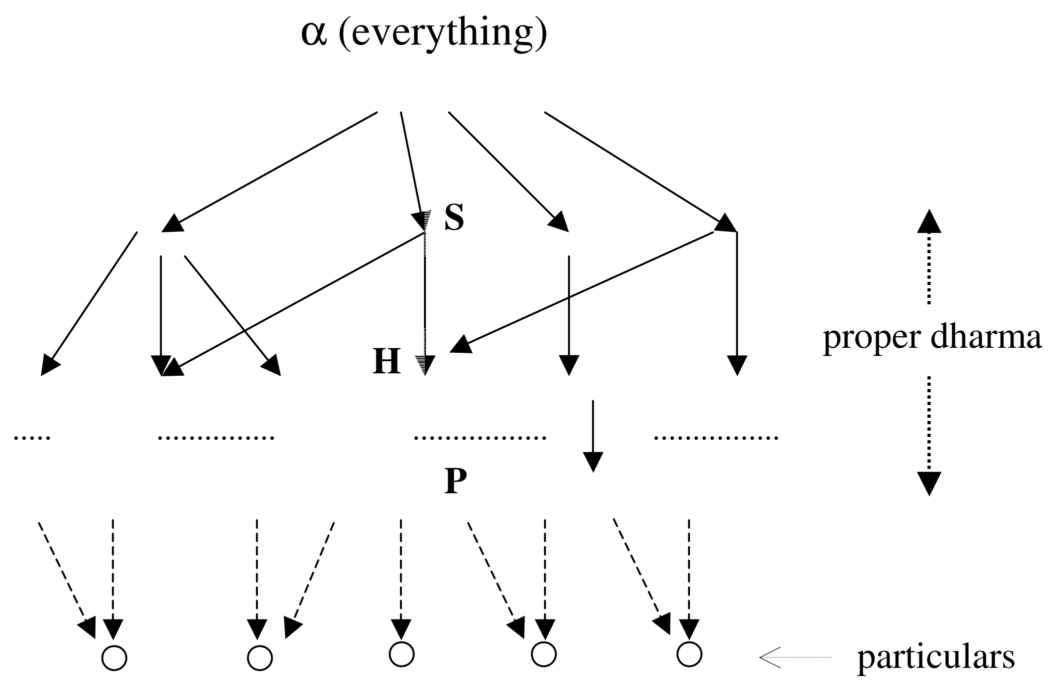
The syntax of the symbolic logic to be designed, will be required to "hold" in every such domain. From a logical point of view the semantic domain consists of *dharmas*. The relation represented by an arrow between two *dharmas* A and B, shows the inheritence relation between two *dharmas*: A is located in B.

The definition of the semantic domain is, at this stage, not complete. We have to add how a *dharma* and its "negative" relate to each other in such a model (like permanent and impermanent). It must be ensured that a *dharma* and its negative do not both reside in another *dharma*. The details are a bit technical and will therefore be omitted here.

It is open to discussion to which extent such structures can truly model the ontology of, let us say, the Nyāya – Vaiśeṣika school. It is certainly a drastic simplification, but it is not more simplifying than imagining the whole world as to be divided into sets of particulars. The main advantage of the semantic domains described above is that their focus lies on the *relation* of different *dharmas*, not on their constitution.

There is no principal difficulty of including *particulars* into this scheme: They are the bottommost nodes in each semantic domain, and they have no outgoing nodes. Whether one should include particulars into the semantics or not will depend on the underlying ontology.

Here is an overall picture of "admissible" semantic graphs>



There is a topmost element, called α ("everything"), and, as bottommost elements, the particulars. Between these limits, the dharmas are located, and they are connected by a location- relation, represented by an arrow starting at the "more general" *dharma* to the one in which it is located (the more special one).

The central element in the *syntax* belonging to this class of admissible graphs is the formula Q(X,Y), which represents the "location" – relation "X is located in Y". Q(X,Y) signifies that there is a sequence of arrows, all pointing into the same direction, starting in X and ending in Y.

The additional “quantors” N and U will be defined by means of Q by

$$N(X,Y):=Q(X,\neg Y)$$
$$U(X,Y):= \neg N(X,Y).$$

The calculus has to be enriched by rules of double negation of terms and formulas as well as the “law of contraposition”. The i*nference rule* is

$$Q(X,Y), Q(Y,Z) \mapsto Q(X,Y) \qquad \text{(transitivity of Q).}$$

which allows one to reason by “syllogism”.

The details of this system of “natural deduction” which, at the syntax- side, has similarities with Corcoran’s deduction system for Aristotelian logic, will be published in a forthcoming paper.

Acknowledgement

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Notes

[i] The five-membered "Indian syllogism" consists of 5 steps:

Step	Name of step	Example
1	pratijñā	parvatovahnimān
2	hetu	dhūmāt
3	udāharaṇa	yatra yatra dhūmastatra tatrāgniḥ yathā mahānase
4	upanaya	thathā cāyam
5	nigamana	tasmāttathā

[ii]. Let us consider the case in which T2 as well as T3 are both *not met*.

Because non-occurrence of S means occurrence of non-S, we may replace "S occurs" in T2 by "non-S does not occur", and, in T3, "S does not occur" by "non-S occurs". Thus, we obtain the following equivalent formulations of the two *Trairūpya* conditions:

T2 : There **is** a *dharmin* in which H occurs and non-S does not occur.
T3 : There **is no** *dharmin* in which H occurs and in which non-S occurs.

If these conditions are **not** met, the negated forms of T2 and T3 hold true:

T2′: There **is no** *dharmin* in which H occurs and non-S does not occur.
T3′: There **is** a *dharmin* in which H occurs and in which non-S occurs.

Comparing T2′ and T3′ with T2 and T3, we see that

- T2′ is identical to T3 after having replaced S by non-S
- T3′ is identical with T2 after having replaced S by non-S.

Thus, in case T2 and T3 are both **not met**, the last two *Trairūpya* conditions are met for proving **non-S**[ii]. In this case, H is therefore called a **contradictory** reason.

There is another observation, concerning the reason in the left upper corner, where T2 is met, and where T3 is not met: In this case, the reason H occurs, by definition of T2 and T3, in a *dharmin* where S holds as well as in *dharmin* where S does not hold. Thus the reason is a sign of neither S or non-S, and the argument cannot lead to a conclusion – it is inconclusive, or "too wide".

It is not so easy to give an interpretation of the fourth case in the lower right corner, therefore we leave that for later, and we will call this invalid reason "special".

[iii] In this table we have made use of the standard notion of "*sapakṣa*" and "*vipakṣa*". *Sapakṣa* is the set of all *dharmin* of the Universe, for which property S holds; *vipakṣa* is the complementary set: all *dharmin* in which S does not hold. The three rows then signify the cases "*hetu* is located in All/No/Some *sapakṣa*", and the rows have been arranged as "hetu is located in All/No/Some vipakṣa".

[iv]

Uddyotakara choose a different enumeration for the cases already dealt with by *Dignāga*, and he himself did not present his findings in a "wheel". In order to relate his findings to *Dignāga's* and to the *Trairūpya* theory, we will insert the numbers of his 16 cases into our *Trairūpya* scheme, Table 1.6.

We will denote Dignāga's case numbers by D1 to D9, and Uddyotakara's case numbers by U1 to 16. His first 9 cases are not exactly the same as *Dignāga's* numbers 1 to 9. The correspondence is as follows:

Dignāga	D1	D2	D3	D4	D5	D6	D7	D8	D9
Uddyotakara	U1	U3	U2	U4	U6	U5	U7	U9	U8

Notation of *Dignāga* and *Uddyotakara*

[v]

	sādhya S	***hetu H***
1	permanent	knowable
2	impermanent	produced
3	manmade	impermanent
4	eternal	produced
5	permanent	audible
6	permanent	manmade
7	not manmade	impermanent
8	impermanent	manmade
9	permanent	incorporeal

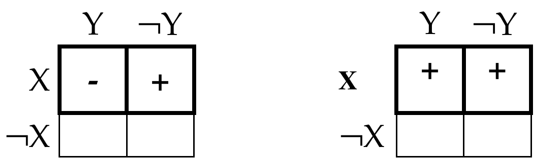
***Dignāga*'s *sādhya* / *hetu* pairs**

[vi] This is a list of 9 sets of examples, which belong to the *sādhya* / *hetu* pairs with corresponding numbers:

	H and S	H and not-S	not-H and S	not-H and not-S
1	space	pot	-	-
2	pot	-	-	space
3	pot	lightning	-	space
4	-	pot	space	-
5	-	-	space	pot
6	-	pot	space	lightning
7	lightning	pot	space	-
8	pot	-	lightning	space
9	space	action	atom	pot

***Dignāga*'s set of examples**

[vii]



The meaning of "No" (left) and "Some" (right)

[viii]

[ix] Case U16 is not a valid case because of a special property of predicate logic:

Y(S,H) AND Y(¬S,H) = ¬∃X(SX AND HX) AND ¬∃X(SX AND ¬HX)

and	$\neg\exists x(\neg Sx \textit{ and } Hx)$ and $\neg\exists x(\neg Sx \textit{ and } \neg Hx)$
=	$\neg\exists x(Sx) \textit{ and } \neg\exists x(\neg Sx)$.

There is no "empty model"; i.e. each model contains at least one element. Let us choose a model and one arbitrary element **m** of this model. Then, either S**m** or ¬S**m** holds true, which implies that $\neg\exists x(Sx)$ *and* $\neg\exists x(\neg Sx)$ is not a tautology . Thus, case U16 is invalid (for any H and any S).

[x] The following table showsthe conclusions can be drawn with respect to the "Trairūpya implication", ⇒ .

	A: All	***U: Some***	***N: No***	***Y: Vi=∅***
A: All	S ⇒ H ¬S⇒H U1,D1	S⇒H ¬H⇒¬S U2,D3	H⇒S ¬H⇒¬S U3,D2	H⇒S S⇒H U10
U: Some	¬H⇒S ¬S⇒H U7,D7	- - U8,D9	H⇒S ¬S⇒¬H U9,D8	H⇒S ¬H⇒S U11
N: No	H⇒¬S ¬H⇒S U4,D4	H⇒¬S S⇒H U5,D6	S⇒¬H ¬S⇒¬H U6,D5	¬H⇒S S⇒¬H U12
Y: Sa=∅	H⇒¬S ¬S⇒H U13	H⇒¬S ¬H⇒¬S U14	¬H⇒¬S ¬S⇒¬H U15	- - U16

Again we see that, from the standpoint of Trairūpya doctrine, the cases U3, U10, U9, and U11 are the only valid ones: These are the cases which imply "**H⇒S**". If one wants to include U15 into the set of valid cases, one could modify the criterion of a valid reason H for a sādhya S from H ⇒ S into (H ⇒ S and ¬S⇔¬H). Okazaki's criterion H ⇒ S or (¬S⇒¬H and not S⇒¬H) can also be read off the table. The term not S⇒¬H serves as a means for differentiating between U6 and U15, for both satisfy ¬S⇒¬H.