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On Semileptonic B Decays into Longitudinally Polarized ρ Mesons

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Abstract

The heavy quark limit can not be taken directly for the decay $B \to \rho e\nu$ if the ρ meson is longitudinally polarized. We employ some recent ideas of a special realization of chiral symmetry to relate the above process to the decay $B \to \pi e\nu$ in the heavy quark limit, which corresponds to the chiral limit in this so called vector realization of chiral symmetry.

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1 Introduction

Heavy quark physics has attracted additional attention in the last two years, since a new, to a large extent model independent approach has been formulated [1]. These new ideas are based on the fact that in the static limit for the heavy quark (i.e. $m_Q \to \infty$, m_Q being the heavy quark mass) new symmetries emerge which allow to relate different processes in a completely model independent way.

One possible application of the heavy quark limit are the semileptonic decays of a heavy into a light quark [2,3]. In [3] a rather detailed study was carried through by employing the so called heavy flavor symmetry to relate B decays into light quarks to the corresponding D decays. In the heavy quark limit the additional symmetries do not reduce the number of independent form factors for individual decays; they only relate the form factors of different heavy to light transitions. Thus the rates for $b \to u$ exclusive semileptonic decays may be predicted from the corresponding $c \to d$ or $c \to s$ exclusive semileptonic modes using in addition the usual flavor $SU(3)$ symmetry.

In general the rates for the semileptonic decays of a heavy meson with mass m_H into a light meson with mass m_ℓ neglecting the lepton masses may be written as

$$\Gamma = G_F^2 m_H^5 f\left(\frac{m_\ell}{m_H}, \frac{\Lambda_{QCD}}{m_H}\right) \qquad (1)$$

where Λ_{QCD} is the scale parameter of QCD and G_F is the Fermi coupling constant. The dependence on the QCD scale parameters may be understood in the following way. The behavior of the form factors is determined by the resonances closest to the kinematically allowed region. If m_* is the mass of a resonance, the mass splitting to the heavy meson is $m_* - m_H = \mathcal{O}(\Lambda_{QCD})$.

The heavy quark limit is the limit $m_H \approx m_Q \to \infty$ taking into account that the rate scales with m_H^5 for dimensional reasons. It was shown in [3] that this limit may be directly used for the decays of heavy mesons into light pseudoscalars and transversely polarized vector mesons and that predictions for $B \to \pi e\nu$ and $B \to \rho_{trans} e\nu$ may be obtained from the experimental data on $D \to \pi e\nu$, $D \to K e\nu$ and $D \to K^* e\nu$.

However, for the longitudinal polarization of the light vector meson the limit $m_\ell \to 0$ is more complicated, since the longitudinal polarization vector ϵ_{long} behaves like

$$\epsilon^\mu_{long} \to \frac{p_\mu}{m_\ell} \quad \text{for} \quad m_\ell \to 0 \qquad (2)$$

where p is the momentum of the light vector meson.

On the other hand, it has been observed in [3] that cancellations between form factors may render the limit for the longitudinal rate finite[1]. This is also suggested by model calculations [4] where it was found that the longitudinal rate does not dominate the transverse one in the decay $B \to \rho e\nu$. To decide whether the longitudinal rate remains finite one needs knowledge about how the limit $m_\ell \to 0$ is approached in the function f of (1).

The purpose of the present paper is to obtain this additional piece of information by using chiral symmetry. We use a realization of the symmetry which should be appropriate

[1] In fact, in [3] an esimate for the total rate of $B \to \rho e\nu$ was obtained by assuming that the longitudinal rate vanishes in the heavy quark limit. This assumption is valid if the ρ-meson is coupled like a gauge boson and adding a mass term by hand.

below the chiral symmetry breaking scale. As the B mass is larger than the chiral symmetry breaking scale the procedure is rather questionable. On the other hand, we are dealing with semileptonic decays which have a three body final state. Thus the light vector meson has a much smaller energy than the B meson mass in large regions of the phase space and thus one may hope to get at least an estimate by employing chiral symmetry.

In the next section we shall briefly collect the necessary relations for chiral symmetry in the vector realization. In section 3 we give the relation between the form factors involved in the semileptonic heavy to light transitions implied by chiral symmetry. From this we arrive at predictions for $B \to \rho_{long} e\nu$ and $B \to \rho e\nu$ using the results from [3]. Finally we discuss possible breaking terms and conclude.

2 Vector Realization of Chiral Symmetry

It was recently argued [5] that chiral symmetry may be realized in a somewhat unconventional way. In this realization the chiral symmetry $SU(3)_L \otimes SU(3)_R$ is in fact unbroken; still the axial vector current A_μ creates an octett of pions out of the vacuum

$$< 0|A^a_\mu(0)|\pi^b(p) >= \delta^{ab} f_\pi p_\mu \tag{3}$$

where p is the momentum of the pion, a and b are $SU(3)_{L+R}$ indices and f_π is the usual pion decay constant.

An unbroken $SU(3)_L \otimes SU(3)_R$ symmetry has the immediate consequence that the vector current creates an octett of scalar particles from the vacuum

$$< 0|V^a_\mu(0)|s^b(p) >= \delta^{ab} f_\pi p_\mu \tag{4}$$

The relations (3) and (4) hold in the chiral limit; off the chiral limit the octett of scalar particles disappears from the spectrum and becomes the longitudinal components of the light vector mesons via the Higgs mechanism. Thus the chiral limit in this realization of chiral symmetry corresponds to the limit $m_\ell \to 0$ for the light vector mesons which is assumed to exist as a limit of QCD[2]. We also assume that the QCD scale parameter Λ_{QCD} does not vanish in this limit, so that there are massive excitations and their masses scale with Λ_{QCD}. From this considerations we see that this so called vector limit differs from the heavy quark limit, since we have from (1)

$$\Gamma \to G_F^2 m_H^5 f\left(0, \frac{\Lambda_{QCD}}{m_H}\right) \quad \text{(Vector limit)} \tag{5}$$

$$\Gamma \to G_F^2 m_H^5 f(0,0) \quad \text{(Heavy quark limit)} \tag{6}$$

The vector limit allows some interesting predictions. The mass of the ρ meson is generated by a Higgs mechanism breaking a larger symmetry down to $SU(3)_L \otimes SU(3)_R$ [5]; thus this mass may be expressed in terms of the vacuum expectation value (in this case the pion decay constant) and the coupling constant $g_{\rho\pi\pi}$ of the transverse components of the ρ meson to the pion. This yields a prediction for the ρ mass of the form [5]

$$m_\rho = 2 g_{\rho\pi\pi} f_\pi \approx 1120 \text{ MeV} \tag{7}$$

[2] In [5] a scenario for this limit is given which is based on the $1/N_c$ expansion, where N_c is the number of colors.

which is off by a factor of $\sqrt{2}$ from the usual KSRF II relation [6] which works quite well. This already indicates that reality is not particularly close to the vector limit.

Various other predictions have been made using the vector limit [7,8,9,10]; in general it was found that predictions from this limit are off by about the same amount as the prediction (7). Thus we expect that the predictions of the vector limit may be off at worst by about 40 %.

3 Chiral Relations between Form Factors

In this section we shall use the vector limit to establish relations between the form factors relevant in the semileptonic B decays into a light meson. The group theory of vector symmetry for the semileptonic decays was worked out already in [7] and we shall briefly expose it.

3.1 The Group Theory of Vector Symmetry for Semileptonic Heavy to light decays

To be definite, we look at $\overline{B^0} \to \pi^+ e^- \bar\nu$ which corresponds to a quark transition $(b\bar d) \to (u\bar d) + e^- + \bar\nu$. In the vector limit the $\overline{B^0}$ has a chiral partner $\overline{B^0}_S$ which is a heavier scalar meson[3]. From these two we may form linear combinations which transform simply under the chiral group $SU(3)_L \otimes SU(3)_R$

$$\overline{B^0}_L = \overline{B^0} + \overline{B^0}_S \sim (\bar 3, 1) \tag{8}$$

$$\overline{B^0}_R = \overline{B^0} - \overline{B^0}_S \sim (1, \bar 3) \tag{9}$$

Note that the b quark transforms as a singlet under the chiral group and thus the transformation properties of the B mesons are determined by its light quark content.

Similarly we form from the pion and the scalars (or, equivalently, the longitudinal components of the ρ meson) combinations which transform simply under the chiral group

$$\pi_L^+ = \pi^+ + s^+ \sim (8, 1) \tag{10}$$

$$\pi_R^+ = \pi^+ - s^+ \sim (1, 8) \tag{11}$$

The semileptonic transitions involve the left handed current with one heavy quark, which transforms like a triplett under the left handed part of the chiral group

$$L_\mu = \bar u \gamma_\mu (1 - \gamma_5) b \sim (3, 1) \tag{12}$$

From (8) to (12) we may form the following matrix elements

$$\mathcal{A}_\mu = < \pi_L^+ | L_\mu | \overline{B^0}_L > \sim (8,1) \otimes (3,1) \otimes (\bar 3, 1) \neq 0 \tag{13}$$

$$\mathcal{B}_\mu = < \pi_L^+ | L_\mu | \overline{B^0}_R > \sim (8,1) \otimes (3,1) \otimes (1, \bar 3) = 0 \tag{14}$$

$$\mathcal{C}_\mu = < \pi_R^+ | L_\mu | \overline{B^0}_L > \sim (1,8) \otimes (3,1) \otimes (\bar 3, 1) = 0 \tag{15}$$

$$\mathcal{D}_\mu = < \pi_R^+ | L_\mu | \overline{B^0}_R > \sim (1,8) \otimes (3,1) \otimes (1, \bar 3) = 0 \tag{16}$$

[3] Note that the chiral partner of the B meson is not the longitudinal component of the B^* since there is no chiral symmetry for the heavy quark.

In particular, the last two equations imply

$$\begin{aligned} 0 &= \frac{1}{2}(\mathcal{C}_\mu + \mathcal{D}_\mu) = < \pi_R^+|L_\mu|\overline{B^0}> \\ &= <\pi^+|L_\mu|\overline{B^0}> - <s^+|L_\mu|\overline{B^0}> \end{aligned} \tag{17}$$

or

$$<\pi^+|L_\mu|\overline{B^0}> = <\rho^+_{long}|L_\mu|\overline{B^0}> \tag{18}$$

Eq.(18) is the desired relation between the decays involving a light pseudoscalar and the ones with a light vector meson. In the next subsection we shall give the relations between the form factors implied by (18).

3.2 Relations between Form Factors

The matrix element of a left handed current between a heavy and a light pseudoscalar meson involves two form factors $F_\pm$

$$<\pi^+(p)|L_\mu|\overline{B^0}(P = m_B v)> = (P_\mu + p_\mu)F_+ + (P_\mu - p_\mu)F_- \tag{19}$$

where the form factors

$$F_\pm = F_\pm(r, \lambda, x) \tag{20}$$

in general depend on

$$r = \frac{m_\ell}{m_H}, \quad \lambda = \frac{\Lambda_{QCD}}{m_H}, \quad x = \frac{2(v\cdot p)}{m_B} \quad 2r \le x \le 1 - r^2 \tag{21}$$

Similarly, for the decay of a heavy into a light vector meson we have four form factors which depend on the same variables as $F_\pm$

$$\begin{aligned} &<\rho^+(p,\epsilon)|L_\mu|\overline{B^0}(P = m_B v)> = m_B \Big[\epsilon_\mu F_1^A + v_\mu(v\cdot\epsilon)F_2^A \\ &+ \frac{1}{m_B}(v\cdot\epsilon)(P_\mu - p_\mu)F^q + \frac{i}{m_B}\epsilon_{\mu\alpha\beta\gamma}\epsilon^\alpha v^\beta p^\gamma F^V\Big] \end{aligned} \tag{22}$$

In the heavy quark limit all the form factors depend only on the one variable x with $0 \le x \le 1$ [2,3].

To establish a relation between the form factors using the vector limit we start from the longitudinal polarization vector which may be expressed in a covariant way by using the velocity of the B meson as an auxiliary vector

$$\epsilon^\mu_{long} = \frac{1}{m_\ell\sqrt{(v\cdot p)^2 - m_\ell^2}}\left((v\cdot p)p^\mu - m_\ell^2 v^\mu\right) \to \frac{p_\mu}{m_\ell} \quad \text{for} \quad m_\ell \to 0 \tag{23}$$

Inserting this into (22) we have, replacing the polarization vector by its limit as $m_\ell \to 0$

$$\begin{aligned} <\rho^+(p,\epsilon_{long})|L_\mu|\overline{B^0}(P)> &= \frac{m_B}{m_\ell}\Big[p_\mu F_1^A + v_\mu(v\cdot p)F_2^A + \frac{1}{m_B}(v\cdot p)(P_\mu - p_\mu)F^q \\ &\quad + \mathcal{O}\left(\frac{m_\ell^2}{(v\cdot p)}\right)\Big] \end{aligned} \tag{24}$$

Rearranging terms to compare with (19) we have

$$\begin{aligned} <\rho^+(p,\epsilon_{long})|L_\mu|\overline{B^0}(P)> &= \frac{m_B}{2m_\ell}\Big[(P_\mu + p_\mu)\left\{F_1^A + \frac{x}{2}F_2^A\right\} \\ &+ (P_\mu - p_\mu)\left\{\frac{x}{2}F_2^A - F_1^A + \frac{x}{2}F^q\right\} + \mathcal{O}\left(\frac{m_\ell^2}{(v\cdot p)}\right)\Big] \end{aligned} \tag{25}$$

From (25) and (19) we may extract some useful information. First of all, assuming that the vector limit exists we have

$$F_1^A(0,\lambda,x) = -\frac{x}{2}F_2^A(0,\lambda,x) \tag{26}$$

$$F_1^A(0,\lambda,x) = \frac{x}{2}F_2^A(0,\lambda,x) + \frac{x}{2}F^q(0,\lambda,x) \tag{27}$$

These relations for $\lambda = 0$ where already given in [3] where the existence of the heavy quark limit was assumed for the longitudinal polarization of the light vector mesons.

However, from the vector limit we also have additional information, namely how the limit $m_\ell \to 0$ is approached. By comparing (19) with (25) we have the additional relations

$$\begin{aligned} F_+(0,\lambda,x) &= \lim_{r\to 0}\frac{1}{2r}\left(F_1^A(r,\lambda,x) + \frac{x}{2}F_2^A(r,\lambda,x)\right) \\ &= \frac{1}{2}\frac{\partial}{\partial r}\left(F_1^A(r,\lambda,x) + \frac{x}{2}F_2^A(r,\lambda,x)\right)_{r=0} \end{aligned} \tag{28}$$

$$\begin{aligned} F_-(0,\lambda,x) &= \lim_{r\to 0}\frac{1}{2r}\left(\frac{x}{2}F_2^A(r,\lambda,x) - F_1^A(r,\lambda,x) + \frac{x}{2}F^q(r,\lambda,x)\right) \\ &= \frac{1}{2}\frac{\partial}{\partial r}\left(\frac{x}{2}F_2^A(r,\lambda,x) - F_1^A(r,\lambda,x) + \frac{x}{2}F^q(r,\lambda,x)\right)_{r=0} \end{aligned} \tag{29}$$

In these relations we may now also take the limit $\lambda \to 0$ and obtain the relations valid in the heavy quark limit. In particular eqs.(28) and (29) may be used to give a prediction for the rate.

3.3 Relations between Decay Rates

In the heavy quark limit the differential rate for $\overline{B^0} \to \pi^+ e^- \bar\nu$ is given in terms of the variables from (21) by [3]

$$\frac{d\Gamma}{dx} = \frac{G_F^2}{192\pi^3}|V_{ub}|^2 m_B^5 (x^2 - 4r^2)^{3/2}|F_+|^2 \tag{30}$$

Similarly, the rate for the decay into a longitudinally polarized ρ meson was found to be [3]

$$\frac{d\Gamma_{long}}{dx} = \frac{G_F^2}{768\pi^3}|V_{ub}|^2 m_B^5\sqrt{x^2 - 4r^2}\frac{1}{r^2}\left|(x - 2r^2)F_1^A + \frac{1}{2}(x^2 - 4r^2)F_2^A\right|^2 \tag{31}$$

In these two expressions we may now insert the relations between the form factors (28,29) and get a relation between the spectra for the semileptonic decays.

$$\frac{d\Gamma(\overline{B^0} \to \pi^+ e^- \bar\nu)}{dx} = \frac{d\Gamma(\overline{B^0} \to \rho^+_{long} e^- \bar\nu)}{dx} \tag{32}$$

In principle we only need the vector limit to establish the relation (32). However, in order to have a model independent input for $B \to \pi e \bar{\nu}$ we shall employ the heavy quark limit and use the numbers given in [3].

Before we actually insert numbers some caveats are in order. First of all, the kinematic range for the variable x is universal only in the heavy quark or vector limit; for finite masses the range is $2r \leq x \leq 1 - r^2$. The ratio of the masses is $r = 2.5\%$ for $B \to \pi e \bar{\nu}$ and $r = 14.5\%$ for $B \to \rho e \bar{\nu}$. Thus one expects (32) to hold in the region $0.4 \leq x \leq 0.9$. In addition, since (32) is based on chiral symmetry we would expect a better prediction if the light meson has – in the rest frame of the decaying B meson – an energy $E \leq \Lambda_{\chi sb} \approx 2$ GeV, where $\Lambda_{\chi sb}$ is the chiral symmetry breaking scale. This gives annother restriction to the region $x \leq 0.75$.

With these caveats in mind we may integrate (32) and find

$$\Gamma(\overline{B^0} \to \pi^+ e^- \bar{\nu}) = \Gamma(\overline{B^0} \to \rho^+_{long} e^- \bar{\nu}) \tag{33}$$

Using the numbers from the heavy quark limit [3] for the decay of a heavy into a light pseudoscalar meson

$$\Gamma(\overline{B^0} \to \pi^+ e^- \bar{\nu}) = |V_{ub}|^2 \cdot 2 \cdot 10^{-11} \text{ GeV} \tag{34}$$

and into a transversely polarized light vector meson

$$\Gamma(\overline{B^0} \to \rho^+_{trans} e^- \bar{\nu}) = |V_{ub}|^2 \cdot (2 \cdots 3) \cdot 10^{-11} \text{ GeV} \tag{35}$$

we arrive at the predictions

$$\Gamma(\overline{B^0} \to \rho^+_{long} e^- \bar{\nu}) = |V_{ub}|^2 \cdot 2 \cdot 10^{-11} \text{ GeV} \tag{36}$$

$$\Gamma(\overline{B^0} \to \rho^+ e^- \bar{\nu}) = |V_{ub}|^2 \cdot (4 \cdots 5) \cdot 10^{-11} \text{ GeV} \tag{37}$$

$$\frac{\Gamma(\overline{B^0} \to \rho^+_{long} e^- \bar{\nu})}{\Gamma(\overline{B^0} \to \rho^+ e^- \bar{\nu})} = 0.4 \cdots 0.5 \tag{38}$$

The present prediction in combination with the ones given in [3] are in reasonable agreement with the present experimental data [12]

$$\Gamma_{expt}(\overline{B}^0 \to \rho^+_{long} e^- \bar{\nu}) = (3.2 \pm 1.0 \pm 0.7) \cdot 10^{-16} \text{ GeV} \tag{39}$$

which corresponds to a value

$$2.0 \cdot 10^{-3} \leq |V_{ub}| \leq 3.3 \cdot 10^{-3} \tag{40}$$

4 Conclusions: Vector Symmetry Breaking Terms and Caveats

We have used the vector realization of chiral symmetry to relate semileptonic B decays into light, longitudinally polarized vector mesons to the corresponding decays into light pseudoscalar mesons. Using the predictions of the heavy quark limit for $B \to \pi e \bar{\nu}$ from [3] we arrive at predictions for $B \to \rho e \bar{\nu}$.

The use of chiral symmetry, in particular in its vector realization, may be criticized in two ways. As already mentioned in the introduction the mass of the B meson is more than a

factor of two larger than the chiral symmetry breaking scale $\Lambda_{\chi sb} \approx 2$ GeV, a value already at the upper limit of the commonly accepted values for this scale. On the other hand, detailed investigations of the τ decays [9] show, that such a large value does not yield unreasonable results. We think that we may still employ chiral symmtry in the present case since the semileptonic B decays have a three body final state and the energy E of the light meson in the rest frame of the B meson ranges between $m_l \leq E \leq (m_H^2 - m_l^2)/(2m_H)$ which is in most of the phase space smaller than the chiral symmetry breaking scale.

The second caveat concerns the fact, that reality seems not very close to the vector limit and thus chiral symmetry breaking terms may yield substantial corrections. For instance, in the vector limit the transverse components of the vector mesons should decouple which means that the rate for the B decay into transversely polarized light vector mesons should be suppressed compared to the decays into longitudinally polarized ones. Thus one would at least expect that the longitudinal rate should be dominant, if reality were close to the vector limit. The transverse rate is determined by different form factors which should vanish faster when the vector limit is approached.

Corrections to the vector limit have been studied for various cases in [5,9,10]. In particular the τ decays examined in [9] are a nice testing ground since a lot of experimental information is availiable. It turns out that the predictions of the vector limit are off by about the same amount as the prediction for the ρ mass, i.e. the vector limit is roughly correct within a factor of two. The same uncertainties have been found in the predictions of vector symmetry for nonleptonic D decays [10]. In total it seems plausible that the present estimate also should hold within a comparable margin.

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References

[1] H. Politzer and M. Wise, Phys. Lett. **B206** (1988) 681; Phys. Lett. **B208** (1988) 504; M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. **45** (1987) 292; Sov. J. Nucl. Phys. **47** (1988) 511; E. Eichten and B. Hill, Phys. Lett. **B234** (1990) 511.; H. Georgi, Phys. Lett. **B240** (1990) 447; B. Grinstein, Nucl. Phys. **B339** (1990) 253; A. Falk, H. Georgi, B. Grinstein and M. Wise, Nucl. Phys. **B343** (1990) 1.

[2] N. Isgur and M. Wise, Phys. Lett. **B232** (1989) 113, Phys. Lett. **B237** (1990) 527.

[3] G. Kramer, T. Mannel and G. Schuler, DESY Report DESY 90-168, to appear in Z. Phys. **C**.

[4] J. Körner and G. Schuler, Z. Phys. **C 38** (1988) 511.

[5] H. Georgi Phys. Rev. Lett. **63** (1989) 1917); Nucl. Phys. **B 331** (1990) 311).

[6] K. Kawarbayashi and M. Suzuki, Phys. Rev. Lett. **16** (1966) 255); Riazuddin and Fayyazuddin, Phys. Rev. **147** (1966) 1071).

[7] H. GEORGI and F. UCHIYAMA, Phys. Lett. **B 238** (1990) 395)

[8] A. FALK and M. LUKE, Nucl. Phys. **B 337** (1990) 49).

[9] T. MANNEL Phys. Lett. **B 244** (1990) 502)

[10] H. GEORGI and F. UCHIYAMA, Phys. Lett. **B 247** (1990) 394)

[11] H. GEORGI, *Weak Interactions and Modern Particle Theory*, Benjamin Cummings, Menlo Park 1984

[12] H. SCHROEDER (ARGUS Collaboration), Talk given at La Thuille, March 91.