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A MECHANISM GENERATING AXION HAIR FOR KERR BLACK HOLES

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ABSTRACT

Recently Campbell et al. considered Kalb-Ramond axion fields with a Lorentz-Chern-Simons coupling to gravity. They find that in the background metric of a slowly rotating Kerr black hole the axion field strength develops a long range scalar "hair". We show that the same phenomenon also occurs with "ordinary" axions, which are not related to form-valued fields. If these scalars couple to the divergence of some anomalous axial vector current, the Hirzebruch signature density acts as a source for the axion. In a Kerr background this leads to an asymptotic $1/r^2$-fall off of the axion field, i.e., to a scalar "hair". Because of the chiral anomaly of the photon (a local version of the signature index theorem), this can happen even in purely bosonic field theory models.

The "no hair" conjecture of black hole physics states that the only long range fields present in the exterior black hole solutions are the field strengths associated with local gauge invariances. In this way general coordinate invariance and electromagnetic gauge invariance, respectively, are related to the mass, angular momentum and the charge of a black hole. These parameters (and, according to the conjecture, only these) can be measured at spatial infinity. The "no hair" conjecture has been proven in a number of special cases, but there exists no general proof which would apply to an arbitrary matter field sector. For pure gravity it is known that the Kerr metric is the unique stationary exterior solution, and similarly for gravity coupled to a U(1) gauge theory the Kerr-Newman solution is the only possibility. It describes a rotating black hole of definite mass, angular momentum and charge. In particular, for vanishing angular momentum one obtains the Schwarzschild and the Reissner-Nordström spacetimes, respectively. As for minimally coupled boson fields, it has been shown that exterior black hole solutions can not contain "hairs" due to scalars or to massive spin-one and spin-two bosons. Also minimally coupled spin one-half fermions have been ruled out. Furthermore, models involving non-abelian gauge fields or non-minimally coupled scalars were shown to possess some novel solutions which, however, turned out to be unstable against small perturbations. They are assumed to decay into the Schwarzschild solution. (See [1] for a detailed list of references.)

Recently Campbell et al. [1] considered Kalb-Ramond-type axion fields with a Lorentz Chern-Simons coupling to gravity. In the background of a Kerr black hole the Chern-Simons term provided a source for the axion field, which is found to give rise to a long range ($\propto r^{-2}$) axion hair for the field strength. It appears that this is the first known example of a stable non-gauge hair of a black hole. The kind of axion considered in ref. [1] naturally appears in the graviton multiplet of string theory [2]. Coupling bosonic or superstrings to external Yang-Mills and gravitational fields, for instance, anomaly cancellation requires the presence of a two-form field B whose (gauge invariant) field strength [3]

$$H = dB + \omega_L - \omega_{YM} \qquad (1)$$

contains the Lorentz and the Yang-Mills Chern-Simons three-forms ω_L and ω_{YM}, respectively. Under a gauge transformation B transforms according to the Chapline-Manton transformation law. In this context, Chern-Simons terms were first encountered in coupling the super-Yang-Mills multiplet to ten dimensional supergravity [4]. The pseudoscalar axion field a(x) arises as follows. The action

$$S = \int d^4x \sqrt{-g} \left(- \frac{1}{16\pi} R - H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \qquad (2)$$

for the coupled axion-graviton system implies the equation of motion $d(^*H)=0$ for the field strength three-form H. Thus, at least locally, $^*H$ can be represented by the exterior derivative of a scalar, the axion: $^*H=da$. Setting the Yang-Mills connection to zero, eq. (1) implies the Bianchi identity $dH=d\omega_L=\mathrm{Tr}(R\wedge R)$, where R is the curvature two-form. Thus the equation of motion for a becomes

$$D_\mu D^\mu a(x) = - \frac{1}{4!} \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} \qquad (3)$$

Campbell et al. [1] have investigated black hole solutions of eq. (3), together with Einstein's equation, in a perturbative manner. For slowly rotating black holes it is sufficient to solve eq. (3) in a fixed Kerr background metric, and to ignore the backreaction of the axion on the metric. To first order in the angular momentum parameter A one finds that for $r \gg 2M$ the axion field falls off as $a \propto Ar^{-2}$, i.e., for $A \neq 0$ the black hole has a long range "axion hair".

The purpose of this note is to point out that the phenomenon of long range axion fields surrounding Kerr black holes is much more general than one might expect from the above derivation. The scalar a(x) was introduced in order to represent the three-form field strength of a two-form field; for the effect to occur it was crucial that the field strength contains a Chern-Simons term whose presence was motivated by anomaly concellation in string theory. In the following we shall see that the same kind of scalar hair also can be obtained from a conventional pseudoscalar $\phi(x)$ which is not related to any form-valued field, but which is required to couple to the photon field A_μ via $\phi F_{\mu\nu}{}^{*}F^{\mu\nu}$. This coupling is inspired by the standard QCD axion, but for our purposes it is sufficient to consider the following toy model:

$$S = \int d^4x \sqrt{-g}\,\Big\{ -\frac{1}{16\pi} R + \frac{1}{2} D_\mu \phi D^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\lambda}{2} \phi F_{\mu\nu} {}^{*}F^{\mu\nu} \Big\} \tag{4}$$

To obtain an axion hair, we may treat the metric and the scalar as classical fields, but we have to take the (one-loop) vacuum fluctuations of A_μ into account. (At this level the non-renormalizability of the model is inessential.) The essential ingredient of our argument is the bosonic chiral anomaly discovered by Dolgov, Khriplovich and Zakharov [6]. These authors have shown that when an abelian gauge field is quantized in the background of a curved spacetime for which the Hirzebruch signature density $\epsilon^{\mu\nu\rho\sigma} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma}$ does not vanish, the vacuum expectation value of the chiral current

$$K^\mu = \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} A_\nu D_\rho A_\sigma \tag{5}$$

is not conserved, and the pseudoscalar $F_{\mu\nu}{}^{*}F^{\mu\nu}$ acquires a vacuum expectation value:

$$D_\mu \langle K^\mu \rangle \equiv \langle \tfrac{1}{2} F_{\mu\nu} {}^{*}F^{\mu\nu} \rangle = \frac{1}{192\pi^2} \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} \tag{6}$$

Equation (6) expresses the fact that quantum effects spoil the invariance of the classical theory under duality rotations [7]. For a free electromagnetic field this symmetry causes the difference of the numbers of right and left circularly polarized photons to be conserved. Hence, if the RHS of eq. (6) is non-zero, the gravitational field produces (chiral) photons from the vacuum. This is analogous to the anomalous fermion pair

creation by Yang-Mills fields as expressed by the relation $\Delta Q_5 = \frac{1}{8\pi^2}\int d^4x\, \mathrm{tr}(F_{\mu\nu}{}^{*}F^{\mu\nu})$. Here Q_5 is the chiral charge. In the case of the photon the chiral charge density $\psi^+\gamma_5\psi$ is replaced by K^0. Its space integral equals +1 (-1) for right- (left-) handed photons. It can be shown [7] that eq. (6) is a local version of the signature index theorem, and that similar anomalies also exist in higher dimensions.

From (4) we obtain the field equations

$$D_\mu D^\mu \phi = \frac{1}{2} \lambda F_{\mu\nu} {}^{*}F^{\mu\nu} \tag{7a}$$

$$D_\mu F^{\mu\nu} = \lambda \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} D_\mu \phi F_{\rho\sigma} \tag{7b}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi \{ T_{\mu\nu}(\phi) + T_{\mu\nu}(A) \} \tag{7c}$$

We are interested in solutions for which the metric is close to the Kerr metric. We also make the assumption that the black hole is rotating very slowly, i.e., that it is sufficient to retain only terms linear in the angular momentum parameter A. In this limit the Kerr metric in Boyer-Lindquist coordinates becomes

$$ds^2 = \Big(1 - \frac{2M}{r}\Big) dt^2 - \Big(1 - \frac{2M}{r}\Big)^{-1} dr^2 - r^2 \{ d\Theta^2 + \sin^2\Theta\, d\phi^2 \} - \frac{4MA \sin^2\Theta}{r} dt\, d\phi + \cdots \tag{8}$$

It gives rise to the Hirzebruch signature density

$$\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} = 1152 \frac{M^2 A \cos\Theta}{r^7} + O(A^2) \tag{9}$$

which vanishes for the Schwarzschild solution (A=0). Since we quantize only A_μ but keep ϕ and $g_{\mu\nu}$ as classical fields we have to replace (7a) by

$$D_\mu D^\mu \phi = \lambda \langle \tfrac{1}{2} F_{\mu\nu} {}^*F^{\mu\nu} \rangle = \frac{\lambda}{192\pi^2} \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} \tag{10}$$

This is essentially the same as eq. (3) for the Kalb-Ramond axion a(x). Because of the bosonic chiral anomaly of A_μ the Hirzebruch signature density acts as a source for ϕ. To first order in A, a solution of eqs. (7a,c) is given by the metric (8) together with a scalar field solving the equation

$$D_\mu D^\mu \phi = \frac{6\lambda}{\pi^2} \frac{M^2 A \cos\Theta}{r^7} \tag{11}$$

which follows from (10) with (9). This approximation is consistent because to lowest order the backreaction of ϕ and A_μ on the metric may be ignored. Similarly, the RHS of eq. (7b) can be neglected so that the photon couples to gravity only. (This was assumed in the derivation of (6).) In order to solve eq. (11) it is sufficient to construct the Green's function $(D_\mu D^\mu)^{-1}$ for a Schwarzschild background since the source term is already of order O(A). One finds [1]

$$\phi(r,\Theta) = -\frac{5\lambda}{384\pi^2} \cdot \frac{A\cos\Theta}{M^3}\left[\frac{4M^2}{r^2} + \frac{8M^3}{r^3} + \frac{72M^4}{5r^4}\right] = -\frac{5\lambda}{96\pi^2} \cdot \frac{A}{M} \cdot \frac{\cos\Theta}{r^2} + O\left(\frac{1}{r^3}\right) \tag{12}$$

Apart from the prefactor on the RHS of eq. (12) this is the same kind of scalar "hair" as the one found in ref. [1] for the Kalb-Ramond axion a(x). The scalar field shows a long range (r^{-2}) fall off, and a dipole form which implies that the total axion charge of the black hole vanishes, however. For a further discussion we refer to [1].

Up to now we only considered the coupling of the scalar to the divergence of the photonic chiral current K^μ. Similarly we could use an interaction of the form $\lambda'\phi\partial_\mu j_5^\mu$ where $j_5^\mu \equiv \bar\psi\gamma^\mu\gamma_5\psi$ is the axial vector current of some massless fermion. Now $\partial_\mu j_5^\mu$ replaces $F_{\mu\nu} {}^*F^{\mu\nu}$ as the source for ϕ. In presence of gravitational and electromagnetic fields we have the usual (fermionic) chiral anomaly so that the analogue of eq. (10) reads

$$D_\mu D^\mu \phi = \lambda' D_\mu \langle j_5^\mu \rangle = \frac{\lambda'}{384\pi^2} \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} - \frac{\lambda' e^2}{8\pi^2} \langle F_{\mu\nu} {}^*F^{\mu\nu} \rangle \tag{13}$$

Even if the fermion does not couple to A_μ, e=o, eq. (13) contains the Hirzebruch signature density. Therefore, in a Kerr background , the solution for $\phi(x)$ is again of the form (12). If e≠o, the expectation value $\langle F_{\mu\nu} {}^*F^{\mu\nu} \rangle$ has to be taken from eq. (6). In this way the bosonic chiral anomaly leads to a finite renormalization of the coefficient in front of the signature density:

$$D_\mu D^\mu \phi = \frac{\lambda'}{384\pi^2}\left(1 - \frac{e^2}{2\pi^2}\right) \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma} \tag{14}$$

(See also ref. [8] for a discussion of this two-loop result which amounts to a "reiteration of anomalies".) Thus the solution for $\phi(x)$ changes by an overall factor $(1-e^2/2\pi^2)$.

This completes our demonstration that also "ordinary" scalars, unrelated to p-form fields, can give rise to a scalar hair for Kerr black holes provided they have a coupling $\propto\phi\partial_\mu J_5^\mu$ to some axial vector current J_5^μ which develops an anomaly when coupled to gravity. As we have stressed, the standard fermionic anomaly of $J_5^\mu = j_5^\mu$ is not the only possibility here. Even a purely bosonic theory involving only axions and photons has this feature, because also the axial vector current $J_5^\mu = K^\mu$ of the photon is anomalous. In this paper we have considered a simple toy model in which the axion remains massless. For realistic theories where an axion mass is induced the above picture might change and the axion field could become short range [1]. This remark also applies to the Kalb-Ramond axion, however.

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