DEUTSCHES ELEKTRONEN – SYNCHROTRON

DESY 91-032
April 1991

New Universality Class for Superconducting Order Parameter

M.I. Dobroliubov

Inst. for Nuclear Research, Acad. Sci., Moscow

S.Yu. Khlebnikov

Deutsches Elektronen-Synchrotron DESY, Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · D-2000 HAMBURG 52

DESY 91-032
April 1991

ISSN 0418-9833

New Universality Class for Superconducting Order Parameter

M.I.Dobroliubov[1]

Institute for Nuclear Research of the Academy of Sciences of the USSR

60th October Anniversary prospect 7a, Moscow 117312 USSR

and

S.Yu.Khlebnikov[2]

Deutsches Elektronen-Synchrotron DESY, Notkestraße 85,

D-2000 Hamburg-52 FRG

PACS numbers: 74.65.+n

Abstarct

We present a model of superconductivity with pairing due to Aharonov-Bohm forces. The gap is proportional to the first power of the small papameter (in which the self-consistent perturbation scheme is developed), as opposed to the BCS class of models where the gap is exponentially suppressed with the small parameter.

[1]E-mail: mdobr@inucres.msk.su

[2]On leave from the Institute for Nuclear Research of the Academy of Sciences of the USSR, 60th October Anniversary prospect 7a, Moscow 117312 USSR. E-mail: sergei@inucres.msk.su

The challenge of high transition temperature in new superconductors has inspired searches for unconventional mechanisms of superconductivity. The absence of unambiguous isotopic effect as well as other considerations give support to non-phonon mechanisms. Some of these may be identified as "purely electronic" ones in a sense that all relevant interactions are present already in some strongly correlated electronic Hamiltonian[1]. When the pairing interactions in such systems are modelled by the effective boson (like paramagnon[2] or exciton[3]) exchange, the relations typical for BCS superconductors are generally violated. In particular, for magnetic mechanisms, the maximal frequency ω_D of the effective bosonic excitation is typically larger than the Fermi energy ϵ_F. Then the expansion parameter[4] of the BCS theory $\lambda\omega_D/\epsilon_F$, where λ is dimensionless coupling, may be kept small only for small λ, and this leads to the exponential suppression of the critical temperature. To explain the high values of T_c one may be tempted to ajust parameters in such a way that the interaction constant $\lambda\omega_D/\epsilon_F$ becomes large. Such philosophy is possible also with the phonon exchange.

In our opinion, the approach to explanation of high T_c based exclusively on the peculiarities of the strong coupling is unsatisfactory. If the expansion parameter characteristic for BCS is large in a real system, one should look for another approximation scheme which would be reliable under these circumstances. The exponential suppression of T_c in the BCS theory is in fact related to the renormalization properties of the dimensionless coupling λ. Indeed, the BCS solution for the gap,

$$\Delta \sim \omega_D \exp(-1/\lambda) \; , \tag{1}$$

may be viewed as a conventional formula of dimensional transmutation, where Δ is a physical (renorm-invariant) parameter, and ω_D plays a role of cutoff. To keep the physical parameter unchanged, λ should depend logarithmically on the cutoff, which is a kind of universal behaviour for dimensionless couplings.[5] In this respect, the problem of high transition temperature may be regarded as that of finding a new universality class for the gap parameter. Another way to identify the universality class is to consider the scaling properties of the gap $\Delta(p)$ as a function of momentum. For the BCS models the gap is essentially independent of the momentum.

The above considerations imply that to get solutions which would be parametrically larger than BCS, one has to consider a theory where, under quite general conditions, some dimensionless coupling does not acquire logarithmic renormalization. In this Letter we present an example of such theory provided by the Aharonov-Bohm (statistical) interaction in (2+1) dimensions. This interaction may be cast into the field-theoretical language by introducing the gauge field with topological (Chern-Simons) action. The topological gauge theories have attracted much attention recently, in particular, in connection with high-T_c superconductors. The appearence of the effective topological gauge field in the latter case may be related to a variant of the fractional quantum Hall state[6], or the flux phase[7] in two-dimensional spin systems (the viability of these scenarios remains, however, still far from being established). On the other hand, the field theoretical investigations[8] have shown, in many cases, the absence of ultraviolet divergencies in the dimensionless Chern-Simons coupling. Thus, the theory of electrons interacting via topological gauge field is a good candidate for a new universality class for superconducting order parameter.

A scenario for superconductivity based on the statistical gauge interactions is already widely known under the name of anyon superconductivity[9]. Let us stress that this is *not* what we are going to consider. The anyon superconductivity is a property of the systems where all particles have the same sign of charge with respect to the statistical gauge field. The corresponding mean-field solution describes

fermions moving in the uniform background statistical magnetic field, so that there is no obvious relation to the Fermi surface instability. Here we consider another type of the statistical gauge interactions, when the particles of opposite charges with respect to the statistical gauge field are present in equal amounts. Then the mean-field magnetic background is zero and the mean-field solution is the filled Fermi sphere. The superconducting solution we obtain in the present Letter is more like the BCS one, in particular, it results from the gap equation. This construction was previously considered in refs.[10, 11] motivated by the properties of elementary excitations in the FQH state; models with particles of opposite statistical charges were also proposed in refs.[12].

The primary purpose of the present Letter is to show that the pairing solution in the case of long-range potential may differ substantially from the BCS form, eq.(1). With this motivation, we will not be very concious about the precise origin of our model. Also, we will not account for the symmetry of lattice but simply consider an isotropic homogenous model. We will see that the Aharonov-Bohm interaction gives rise to the following asymptotics of the gap parameter,

$$\Delta \sim \frac{\epsilon_F}{\kappa}\,, \qquad (2)$$

where κ is the Chern-Simons coupling. The weak coupling corresponds to large κ. Thus, the gap proves to be proportional not to the ultraviolet cut-off but to the intrinsic energy scale, and the suppression in eq.(2) is only power-law, as opposed to the exponential suppression in eq.(2). This is in accord with the previous discussion of the renormalization properties of the coupling constant.

In fact, the form of the solution eq.(2) reflects not only the absence of the infinite renormalization of the coupling constant κ but also a possibility of its finite renormalization. This occurs for the pairing with non-zero momentum[13, 11] and arises, diagrammatically, from the RPA fermion bubbles in the gauge field propagator (the fermion Green function is modified by the pairing), leading to the renormalized coupling,

$$\kappa_{ren} = \kappa - \frac{l}{2\pi}, \tag{3}$$

where l is the orbital momentum of the pair. Thus, for small momenta along the gauge line, and large l, the system develops the strong pairing potential. We prove that, nevertheless, the system is in the weak coupling regime, in a sense that all higher order corrections to the gap equation, except the bubbles, are suppressed by $1/\kappa$ (up to logs of κ) at $l \sim \kappa$.

Thus, we start with the lagrangian of ref.[11]

$$L = \frac{\kappa}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \Psi^\dagger\left(i\partial_0 + \sigma_z A_0 - \epsilon\left(-i\frac{\partial}{\partial \mathbf{x}} - \sigma_z \mathbf{A}\right)\right)\Psi \ , \tag{4}$$

where the statistical coupling κ is taken positive and large, the components of the fermionic doublet $\Psi = (\psi_+, \psi_-)^T$ have opposite charges with respect to the statistical gauge field A, and $\epsilon(\mathrm{k})$ is the (quasi)particle dispersion law below taken to be $\epsilon(\mathbf{k}) = \mathrm{k}^2/2m - \epsilon_F$. The statistical gauge interaction is attractive for (quasi)particles of opposite statistical charges with non-zero relative orbital momentum l having the same sign as κ (Refs.[10, 11] and below). (The attraction exists also for equal charges but in that case the most long-range part of the potential will be screened by the combined action of Debye and Meissner effects for the statistical field, as explained after eq.(13) below, and the gap will be suppressed. So, we consider the former case in what follows.) The standard gap equation reads

$$\Delta_{\mathrm{p}} = -\frac{1}{2(2\pi)^2}\int U_{\mathrm{pp'}}\frac{\Delta_{\mathrm{p'}}}{\sqrt{\epsilon^2_{\mathrm{p'}} + |\Delta_{\mathrm{p'}}|^2}}d\mathbf{p}' \ . \tag{5}$$

In the tree approximation the pairing potential is

$$U(\mathrm{p},\mathrm{p}') = \frac{2i}{\kappa m}\frac{\varepsilon_{ij}p_i p'_j}{(\mathrm{p}-\mathrm{p}')^2} = -\frac{2i}{\kappa m}\frac{\sin\theta}{\frac{p}{p'} + \frac{p'}{p} - 2\cos\theta} \ , \tag{6}$$

where $\mathrm{p} = pe^{i\phi}$, $\mathrm{p}' = p'e^{i\phi'}$, $\theta = \phi - \phi'$. Consider pairing with the orbital momentum l,

$$\Delta_{\mathrm{p}} = \Delta(p)\, e^{il\phi} \ , \tag{7}$$

where $\Delta(p)$ may be taken real. Then the angle integration in eq.(5) may be carried out explicitly and yields, for $l \geq 1$, the integral equation,

$$\Delta(p) = \frac{1}{4\pi\kappa m}\int_0^\infty dp'p'\left[\theta(p'-p)\left(\frac{p}{p'}\right)^l + \theta(p-p')\left(\frac{p}{p'}\right)^l\right]\frac{\Delta(p')}{\sqrt{\epsilon^2(p') + \Delta^2(p')}} \ . \tag{8}$$

For $l = 0$ the potential vanishes due to antisymmetry, and for $l < 0$ the interaction is repulsive.

To find the scaling properties of the gap function with respect to the momentum, it is convenient to deduce the differential equation from eq.(8),

$$p^2\Delta'' + p\Delta' + \left(\frac{l}{2\pi\kappa}\frac{p^2}{m}\frac{1}{\sqrt{\epsilon^2(p) + \Delta^2}} - l^2\right)\Delta = 0 \ . \tag{9}$$

At lagre κ one can neglect the first term in the brackets compared to the second one everywhere except for the nearest vicinity of the Fermi level. Thus, we find the asymptotics of the regular solution,

$$\Delta(p) \simeq \Delta_0(p/p_F)^l \ \ , p < p_F \ ; \qquad \Delta(p) \simeq \Delta_0(p_F/p)^l \ \ , p > p_F \ , \tag{10}$$

which show that the gap indeed scales non-trivially, and the integral in eq.(8) converges. Nevertheless, in the tree approximation the magnitude of Δ is exponentially small in $1/\kappa$, by the standard argument. According to our general reasoning, this should imply that the tree approximation is insufficient in the pairing problem, and we have not accounted properly for renormalization of the statistical interaction. We now come to do so.

The pairing potential may be strongly enhanced, for large l and small transfer momenta, by the effect of fermion loops in the gauge field propagator, as discussed in ref.[11] and already mentioned above. We would now like to show that this enhancement leads to the drastic increase in the gap parameter, and moreover, that the emerging approximation scheme is self-consistent in a sense that the omitted contributions are suppressed by $1/l \sim 1/\kappa$. Thus, we substantiate the claim of the new universality class made in the beginning.

The enhancement of the pairing potential is due to the partial cancellation of the bare topological term κ by the induced one[13, 11], see eq.(3). Clearly, for large κ the system prefers pairing with large orbital momenta $l \sim 2\pi\kappa$. The induced term is operative only at small momenta $\mathbf{q}$ along the gauge line, $v_F|\mathbf{q}| \lesssim \Delta$, i.e. for the small momentum transfer in the scattering of two (quasi)particles. (For larger momenta the system behaves as the neutral plasma, for which the renormalization of the topological term is absent.) Now note that, with the potential of the form eq.(6), the integral in the gap equation (5) is saturated, at large l, in the narrow region of momenta $|\mathbf{p}' - \mathbf{p}| \lesssim p/l$. As verified aposteriori by the solution, see eq.(2), for $p \approx p_F$ this corresponds precisely to the region of momenta given above, where the most effective renormalization of κ takes place. Hence, one can understand the effect of this renormalization by simply substituting κ_{ren} instead of κ in the potential eq.(6). In this way we obtain

$$\Delta_0 \equiv \Delta(p_F) \sim \frac{\epsilon_F}{l} \exp(-2\pi\kappa_{ren}) \,, \qquad (11)$$

while the asymptotics eq.(10) remain unchanged. For small κ_{ren} the exponential suppression of the gap disappears. Thus, the enhancement of coupling in the narrow

region of momenta has profound consequences for the solution of the gap equation. Also, the calculation of the critical temperature from the equation for the four-fermion vertex in the ladder approximation[14], but now with the RPA improved gauge interaction, yields the essentially unsuppressed value of the same form as eq.(11), $T_c \sim \Delta_0$. For $l = 2\pi\kappa - 1$ (possible when $2\pi\kappa = integer$) we have numerically obtained the upper bounds $\Delta_0 \lesssim 3.1\epsilon_F/l$ and $2\Delta_0/T_c \lesssim 3.1$. The latter ratio is thus somewhat smaller than the BCS value 3.52.[14]

Now let us see that the previous analysis is indeed self-consistent, in a sense that all other contributions to the pairing potential, besides the tree/RPA ones, are supressed by $1/\kappa$. The omitted terms are of two types. First, there are non-tree non-RPA contributions to the potential. Second, the RPA graphs in the gauge field propagator induce other terms bilinear in the gauge field, in addition to the topological term. In fact, both types of contributions are suppressed for one and the same reason. It is that all these new interactions appear to be short-ranged as compared to the Aharonov-Bohm RPA improved one. Let us start with the second type of corrections since, if large, they would enter also non-RPA contributions of the first type. In the Coulomb gauge, the gauge field polarization operator $\hat{\Pi}$ is a 2×2 matrix in the space (A_0, f) with $A_i = \epsilon_{ij}\partial_j f$, and, as established previously[11], in the static long-range limit it reads

$$\hat{\Pi}(\mathbf{q}) = \begin{pmatrix} -A\mathbf{q}^2 + O(\mathbf{q}^4) & B_1\mathbf{q}^2 + B_2\mathbf{q}^4 + O(\mathbf{q}^6) \\ B_1\mathbf{q}^2 + B_2\mathbf{q}^4 + O(\mathbf{q}^6) & C\mathbf{q}^4 + O(\mathbf{q}^6) \end{pmatrix} , \qquad (12)$$

where[11, 15]

$$A = \frac{\epsilon_F}{6\pi}\,\frac{1}{\Delta^2}\,, \quad B_1 = \frac{l}{2\pi}\,, \quad B_2 = -\frac{l}{2\pi}\,\frac{v_F^2}{12\Delta^2}\,,$$
$$C = \frac{1}{4\pi m}\left(\frac{1}{3} + l^2\right) \,. \qquad (13)$$

The coefficient B_1 is the induced topological term entering eq.(3). It is seen from these expressions that at $l \neq 2\pi\kappa$ the pair interactions mediated by the (00) and (ij) components of the gauge field are both non-singular in $\mathbf{q} = \mathbf{p} - \mathbf{p}'$, that is they are $1/r^2$ in the coordinate space, while the Aharonov-Bohm interaction is $1/q$, that is $1/r$.[16] Note that for the pairing of equal statistical charges the diagonal terms in eq.(12) would be $O(1)$ (Debye screening) and $O(\mathbf{q}^2)$ (Meissner screening), respectively, so that neither of the propagator components (including the Aharonov-Bohm one) would be long-ranged.[17] As for the first type of corrections, the argument essentially follows that of Gell-Mann and Brueckner[18] showing that the non-RPA graphs are at most logarithmic in $|\mathbf{p} - \mathbf{p}'|$.

We now should explain why it is instructive to study the singularity of the various contributions to the potential in eq.(5) in the limit when $|\mathbf{p} - \mathbf{p}'|$ goes to zero. The point is that, as we have seen above, only the large angular momenta $l \sim \kappa$ are essential in the gap equation, and the Riemann-Lebesgue lemma[19] says that for any integrable function of angle θ between $\mathbf{p}$ and $\mathbf{p}'$, $\lim_{l\to\infty} \int V(\theta) e^{il\theta} d\theta = 0$. Hence, all non-singular (and logarithmic) contributions are suppressed. In fact, this suppression is $1/l$ up to logarithms. Note that the suppression takes place also for the tree/RPA result everywhere except the point $|\mathbf{p}| = |\mathbf{p}'|$. But at this point all angular harmonics indeed come with equal weight, as seen from eq.(8), and this leads to the uniformly unsuppressed solution for the gap.

In conclusion, we have shown that the presence of long-ranged pairing interaction combined with the non-renormalization property of certain couplings may lead to new patterns in weak-coupling asymptotics of the gap function and critical temperature. We have developed a self-consistent scheme for the particular case of Aharonov-Bohm $(1/r)$ interaction in (2+1) dimensions, the role of expansion parameter being played

by the inverse angular momentum of a pair. Though we do not claim for any concrete applications in the present paper, we hope that our results may form a new framework in studies of high-temperature superconductivity.

One of us (S.K.) would like to thank DESY Theory Group, where part of this work was done, for kind hospitality.

References

[1] P.W.Anderson, Science, **235**, 1196 (1987)

[2] See e.g. D.J.Scalapino, E.Loh and J.E.Hirsch, Phys. Rev. B, **34**, 8190 (1986) and references therein

[3] See e.g. V.L.Ginzburg and D.A.Kirzhnitz, Sov. Phys. Uspekhi, **30**, 671 (1987) and references therein

[4] G.M.Eliashberg, JETP, **11**, 696 (1960)

[5] It is more familiar to attribute the exponential suppression of the gap to the *infrared* logarithmic divergence in the gap equation at the Fermi level, for $\Delta = 0$. In fact, the ultraviolet and infrared scales in the BCS problem are related via the logarithmic renormalization precisely in the same way as, say, for the dimensional transmutation in the theory of massless scalars, cf. S.Coleman and E.Weinberg, Phys. Rev. D, **7**, 1888 (1973)

[6] V.Kalmeyer and R.B.Laughlin, Phys. Rev. Lett., **59**, 2095 (1987); Phys. Rev. B, **39**, 11879 (1989)

[7] I.Affleck and J.B.Marston, Phys. Rev. B, **37**, 3774 (1988); X.G.Wen, F.Wilczek and A.Zee, Phys. Rev. B, **39**, 11413 (1989);

[8] S.Coleman and B.Hill, Phys. Lett. **159B**, 184 (1985); G.W.Semenoff, P.Sodano and Y.-S.Wu, Phys. Rev. Lett. **62**, 715 (1989)

[9] A.Fetter, C.Hanna and R.B.Laughlin, Phys. Rev. B, **39**, 9679 (1989); Y.H. Chen, B.I.Halperin, F.Wilczek and E.Witten, Int.J. Mod. Phys. B, **3**, 1001 (1989); B.I.Halperin, J.March-Russel and F.Wilczek, Phys. Rev. B, **40**, 8726 (1989)

[10] Dam Thanh Son and S.Khlebnikov, unpublished

[11] S.Yu.Khlebnikov, *On the fractional quantum Hall scenario for two-dimensional spin systems*, Tokyo Inst. of Technology preprint TIT/HEP-162/COSMO-5, 1990; Phys. Rev. B, to be published

[12] N.Dorey and N.E.Mavromatos, Phys. Lett., **250B**, 107 (1990); G.W.Semenoff and N.Weiss, *ibid.*, p.117

[13] G.E.Volovik and V.M.Yakovenko, J.Phys., **C1**, 5263 (1989)

[14] A.A.Abrikosov, L.P.Gorkov and I.E.Dzyaloshinski, *Methods of quantum field theory in statistical physics*, Prentice, 1963, Sects. 33,34

[15] M.I.Dobroliubov, unpublished

[16] The case $l = 2\pi\kappa$ (possible when $2\pi\kappa = integer$) is somewhat exceptional. Then, the topological term in the effective action cancels completely, and both the (00) and (ij) components of the gauge proparator behave as $1/\mathbf{q}^2$ for small $\mathbf{q}$. But now their contribution to the gap equation (5) diverges at $\mathbf{p} = \mathbf{p}'$, leading to

the absence of any non-trivial solutions. Thus, the system chooses pairing with l close but not equal to $2\pi\kappa$.

[17] For the same reason, the electromagnetic response of the system (assuming equal electromagnetic charges of both quasiparticle species) contains no long-range parity-breaking component. This may even make the model compatible with the muon experiment of R.Kiefl et al., Phys. Rev. Lett., **64**, 2082 (1990)

[18] M.Gell-Mann and K.A.Brueckner, Phys. Rev., **106**, 364 (1957)

[19] E.T.Whittaker and G.N.Watson, *A course of modern analysis*, Cambridge, 1962, Sect. 9.41