DEUTSCHES ELEKTRONEN – SYNCHROTRON

DESY 91-020
April 1991

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ISSN 0418-9833

NOTKESTRASSE 85 · D-2000 HAMBURG 52

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An $SU(2)_L \otimes SU(2)_R$ Symmetric Yukawa Model in the Symmetric Phase

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Abstract

The lattice regularized $SU(2)_L \otimes SU(2)_R$ symmetric scalar fermion model with explicit mirror fermions is investigated in the phase with unbroken symmetry. In the present work numerical Monte Carlo calculations with dynamical fermions are performed on a $4^3 \cdot 8$ lattice near the expected perturbative Gaussian fixed point. The bare Yukawa coupling of the mirror fermion is fixed at zero. The numerical simulations are supported by lattice perturbation theory.

1. Introduction

Recently the Higgs-Yukawa sector in the Standard Model has attracted a lot of attention. Several groups have been studying various types of scalar-fermion models in order to study the mechanism of mass generation in the Standard Model in a non-perturbative framework (see [1] and references therein). With the present lower limit on the top quark mass being at 89 GeV [2], it is obvious that a non-perturbative treatment is of increasing interest.

One important issue in lattice studies of scalar-fermion models is "triviality". In our model, 1-loop perturbation theory tells us that the renormalization of the model is governed

by an infrared-stable fixed point at zero couplings and hence predicts that the theory is trivial in the sense that the renormalized scalar and Yukawa couplings vanish as the cutoff is removed. This in turn implies cutoff-dependent upper limits on the scalar and fermion masses in the phase with broken symmetry. Therefore, an interesting question for our lattice study is whether or not the 1-loop prediction is qualitatively correct.

One notorious problem associated with lattice formulations of chiral scalar-fermion models is species doubling [3]. Currently, there are mainly two ways of dealing with this problem, one of which was introduced by Smit and independently by Swift [4]. The other approach, which also forms the basis for this work makes use of the explicit inclusion of mirror fermions in the action [5]. In a series of previous papers a simplified $U(1)_L \otimes U(1)_R$ symmetric Yukawa model with explicit mirror fermions has been investigated in both phases [6]–[9]. There it was shown that the fermion doublers can be made heavy, and in the broken phase the mirror fermion can be given a large mass by the appropriate tuning of the two Yukawa couplings such that the light fermion spectrum is "chiral".

In this paper we investigate the renormalized couplings in the neighbourhood of the Gaussian fixed point in the symmetric phase of an $SU(2)_L \otimes SU(2)_R$ symmetric Yukawa model with explicit mirror fermions. We now work with an O(4)-symmetric ϕ^4-theory and fermion doublets, which represents a better approximation of the Standard Model. As before, the gauge fields are ignored because at the electroweak scale those couplings are weak. Our numerical simulations are supplemented by perturbative calculations of renormalized parameters, β-functions and tree unitarity upper limits on the Yukawa couplings. Since we want to compare our results to the previously studied U(1)-case, the structure of this paper is quite similar to [7] where also more details about the model can be found. Furthermore, as in [7] we set the Yukawa coupling of the mirror fermion $G_\chi = 0$ in the Monte Carlo simulations. As previous experience shows, this presumably does not influence the qualitative behaviour substantially.

In sect. 2 we discuss the lattice action and define the renormalized quantities for the Monte Carlo simulation. Sect. 3 is devoted to perturbation theory. We derive expressions for the renormalized parameters in bare lattice perturbation theory, and furthermore the perturbative β-functions and the tree unitarity upper limit on the Yukawa couplings are calculated. In sect. 4 the numerical results are presented, and the final section contains the discussion and summary of the results.

2. Actions and renormalization

2.1 Lattice actions and parameters

The lattice action of the $SU(2)_L \otimes SU(2)_R$ model with a general field normalization is

$$S = \sum_x \Big\{ \mu \phi_{Sx}\phi_{Sx} + \lambda(\phi_{Sx}\phi_{Sx})^2 - \kappa \sum_\mu \phi_{Sx+\hat\mu}\,\phi_{Sx}$$

$$+\mu_{\psi\chi}\left[(\bar{\chi}_x\psi_x)+(\bar{\psi}_x\chi_x)\right]-\sum_\mu\left[K_\psi(\bar{\psi}_{x+\hat\mu}\gamma_\mu\psi_x)+K_\chi(\bar{\chi}_{x+\hat\mu}\gamma_\mu\chi_x)\right]$$
$$+K_r\sum_\mu\left[(\bar{\chi}_x\psi_x)-(\bar{\chi}_{x+\hat\mu}\psi_x)+(\bar{\psi}_x\chi_x)-(\bar{\psi}_{x+\hat\mu}\chi_x)\right]$$
$$+G_\psi\phi_{Sx}(\bar{\psi}_x\Gamma_S\psi_x)+G_\chi\phi_{Sx}(\bar{\chi}_x\Gamma_S^+\chi_x)\Big\} \qquad (1)$$

where the summation over repeated O(4)-indices $S=0,\ldots,3$ is assumed. Here we denote the fermion doublet by ψ and its mirror fermion partner by χ, x is a lattice point and the sum $\sum_\mu$ runs over eight directions of the neighbours; $\hat\mu$ is the unit vector in the direction of μ. In the Yukawa couplings the $8\otimes 8$ matrices $\Gamma_S\,(S=0,1,2,3)$ are defined as $\Gamma_S=(1,-i\gamma_5\tau_s)$ where the τ's are the $2\otimes 2$ Pauli matrices. The ϕ_{Sx}, $S=0,\ldots,3$ are the four real components of the SU(2)-scalar field φ_x which is defined as

$$\varphi_x\equiv\begin{pmatrix}\phi_{0x}+i\phi_{3x} & \phi_{2x}+i\phi_{1x}\\ -\phi_{2x}+i\phi_{1x} & \phi_{0x}-i\phi_{3x}\end{pmatrix}. \qquad (2)$$

In this paper, we always use the following convenient normalization:

$$\mu_\phi=1-2\lambda;\qquad K_\psi=K_\chi\equiv K;\qquad K_r\equiv rK;\qquad \bar\mu\equiv\mu_{\psi\chi}+8rK=1. \qquad (3)$$

A different form of the fermionic lattice action which is particularly useful for the computation of the tree unitarity bound is obtained from the definition

$$\psi_A\equiv\psi_L+\chi_R,\qquad \psi_B\equiv\psi_R+\chi_L \qquad (4)$$

where the subscripts L,R denote the left and right handed field components, respectively. In our normalization the free fermionic action now reads

$$S_F=\sum_x\Big\{-K\sum_\mu\left[\bar\psi_{Ax+\hat\mu}\gamma_\mu\psi_{Ax}+\bar\psi_{Bx+\hat\mu}\gamma_\mu\psi_{Bx}\right]+\bar\mu\left[\bar\psi_{Ax}\psi_{Ax}+\bar\psi_{Bx}\psi_{Bx}\right]$$
$$-rK\sum_\mu\left[\bar\psi_{Ax+\hat\mu}\psi_{Ax}+\bar\psi_{Bx+\hat\mu}\psi_{Bx}\right]\Big\}. \qquad (5)$$

Defining

$$G_\alpha\equiv\frac{1}{2}(G_\psi+G_\chi),\qquad G_\beta\equiv\frac{1}{2}(G_\psi-G_\chi), \qquad (6)$$

one obtains the following expression for the Yukawa term

$$S_Y=\sum_x\left\{\bar\psi_{Ax}(G_\alpha-G_\beta\gamma_5)\varphi_x\psi_{Bx}+\bar\psi_{Bx}(G_\alpha+G_\beta\gamma_5)\varphi_x^+\psi_{Ax}\right\}. \qquad (7)$$

Note that in this formulation the free fermionic action is diagonal in the fields ψ_A,ψ_B, whereas the Yukawa coupling term involves only off-diagonal contributions.

In eq.(1), there is only one pair of fermion and its mirror partner. In order to be able to use the Hybrid Monte Carlo algorithm for dynamical fermions in the numerical simulations, the fermion spectrum has to be flavour-doubled just like in the U(1)-case. Throughout this paper we set the Wilson parameter $r=1$.

2.2 Renormalized quantities

The definitions of the renormalized quantities in the SU(2)-symmetric model are very similar to the previously studied U(1)-case [7] and require only slight modification. The fermion propagator matrix in the limit $p\to 0$ is

$$\widetilde{\Delta}_\Psi(p)\equiv\sum_x e^{ip\cdot(y-x)}\Delta^\Psi_{yx}=A-ip\cdot\gamma B+\mathcal{O}(p^2), \qquad (8)$$

and the inverse propagator for $p\to 0$ is

$$\widetilde{\Delta}_\Psi^{-1}=M+ip\cdot\gamma N+\mathcal{O}(p^2), \qquad (9)$$

where one has the following relations among the matrices A,B,M,N

$$M=A^{-1},\qquad N=A^{-1}BA^{-1}. \qquad (10)$$

The $16\otimes 16$ matrices A,B can be determined from measuring fermionic timeslices in a numerical simulation. The renormalization of fermionic quantities is defined in the same way as for the U(1)-model, and the reader is referred to [7] for further details.

For the definition of the renormalized Yukawa couplings we introduce fermion-fermion-scalar expectation values such as, e.g.

$$\langle\varphi^+\psi_L\bar\psi_R\rangle_0\delta_{\rho\sigma}\delta^{st}\equiv\frac{1}{L^3T}\sum_{x,y,z}e^{-i\frac{\pi}{T}(y_4-x_4)}\langle(\varphi_z^+\psi_{yL\rho})^s\bar\psi^t_{xR\sigma}\rangle, \qquad (11)$$

where ρ,σ are the spinor indices of the chiral components of fermion fields, and s,t are isospin indices. The renormalized Yukawa couplings $G_{R\psi}$ and $G_{R\chi}$ will be defined by eqs.(31) to (34) in [7] except that here we replace $\sqrt{2Z_\phi}$ everywhere by $\sqrt{4Z_\phi}$ and $\langle\phi^+\phi\rangle_0$ by $\langle\phi_S\phi_S\rangle_0$ where

$$\langle\phi_S\phi_S\rangle_0\equiv\frac{1}{L^3T}\sum_{x,y}\langle\phi_{Sx}\phi_{Sy}\rangle=\frac{4Z_\phi}{m_R^2}. \qquad (12)$$

3. Perturbation theory

3.1 Lattice perturbation theory and finite-volume effects

For the derivation of the 1-loop expressions for the renormalized parameters around the Gaussian fixed point it is convenient to rescale the fields such that the lattice action corresponds to the continuum expression. The bare SU(2)-field φ_0 and its real components ϕ_{0S}, $S=0,\ldots,3$ are defined as

$$\sqrt{\kappa}\,\varphi\equiv\varphi_0=\frac{1}{\sqrt2}\begin{pmatrix}\phi_{00}+i\phi_{03} & \phi_{02}+i\phi_{01}\\ -\phi_{02}+i\phi_{01} & \phi_{00}-i\phi_{03}\end{pmatrix}. \qquad (13)$$

In terms of the components ϕ_{0S} the scalar part of the action becomes

$$S_\phi=\sum_x\left\{\partial_\mu\phi_{0S}\,\partial_\mu\phi_{0S}+m_0^2\phi_{0S}\phi_{0S}+\frac{g_0}{6}(\phi_{0S}\phi_{0S})^2\right\} \qquad (14)$$

where ∂_μ denotes the finite difference lattice derivative, and the relation between the bare parameters (m_0, g_0) and (κ, λ) is

$$m_0^2 = \frac{1-2\lambda}{\kappa} - 8, \qquad g_0 = \frac{6\lambda}{\kappa^2}. \tag{15}$$

The free fermionic part is obtained by rescaling ψ, χ according to

$$\psi_0 = \sqrt{2K}\psi, \qquad \chi_0 = \sqrt{2K}\chi. \tag{16}$$

The free fermionic part of the action now becomes

$$S_F = \sum_x \Big\{ -\frac{1}{2}\sum_{\mu=1}^{4} \left[\left(\bar\psi_{0,x+\hat\mu} - \bar\psi_{0,x-\hat\mu}\right)\gamma_\mu \psi_{0,x} + \left(\bar\chi_{0,x+\hat\mu} - \bar\chi_{0,x-\hat\mu} - 2\bar\chi_{0,x}\right)\psi_{0,x} + (\psi \leftrightarrow \chi)\right]$$

$$+ \mu_0 \left(\bar\psi_{0,x}\chi_{0,x} + \bar\chi_{0,x}\psi_{0,x}\right) \Big\}, \tag{17}$$

and the Yukawa interaction is

$$S_Y = \sum_x \left[G_{0\psi}\phi_{0Sx}\left(\bar\psi_{0,x}\Gamma_S\psi_{0,x}\right) + G_{0\chi}\chi_{0Sx}\left(\bar\chi_{0,x}\Gamma_S^+\chi_{0,x}\right)\right] \tag{18}$$

where

$$\mu_0 = \frac{\mu_{\psi\chi}}{2K}, \qquad G_{0\psi} = \frac{G_\psi}{2K\sqrt{2\kappa}}, \qquad G_{0\chi} = \frac{G_\chi}{2K\sqrt{2\kappa}}. \tag{19}$$

With these definitions the Feynman rules can be easily derived. Now we consider vertex functions

$$\Gamma^{(n_B, 2n_F)}(p_a), \quad a = 1, \ldots, n_B + 2n_F, \tag{20}$$

for n_B bosons, n_F fermions and n_F antifermions. The relations between the bare and renormalized vertex functions are the same as for the U(1)-symmetric case, and we refer the reader to [7] for the details. Introducing a shorthand notation for the lattice momenta

$$\hat p_\mu = 2\sin\left(\frac{p_\mu}{2}\right), \qquad \bar p_\mu = \sin(p_\mu) \tag{21}$$

and the momentum sums

$$\int_p = \frac{1}{L^3 T}\sum_p = \int_0^{2\pi}\frac{d^4p}{(2\pi)^4} \qquad \text{for } L, T = \infty$$

we find that up to one loop the renormalized scalar mass squared is given by

$$m_R^2 = m_0^2 + g_0\int_p (\hat p^2 + m_0^2)^{-1} + 8N_f \int_p \frac{2G_{0\psi}G_{0\chi}\mu_p^2 - (G_{0\psi}^2 + G_{0\chi}^2)\bar p^2}{(\bar p^2 + \mu_p^2)^2} + m_0^2(2\kappa Z_\phi - 1) \tag{22}$$

where Z_ϕ is the wave function renormalization factor and $\mu_p = \mu_0 + \hat p^2/2$. The expression for Z_ϕ is too lengthy to be displayed here. Note that N_f is the total number of fermion-mirror doublet pairs and is equal to 2 in our flavour-doubled model. For the renormalized fermion mass one obtains

$$\mu_R = \mu_0 - 4G_{0\psi}G_{0\chi}\int_p \frac{\mu_p}{(\bar p^2 + \mu_p^2)(\hat p^2 + m_0^2)} - \mu_0(G_{0\psi}^2 + G_{0\chi}^2)\int_p \frac{\bar p^2}{(\bar p^2 + \mu_p^2)(\hat p^2 + m_0^2)^2}. \tag{23}$$

The renormalized scalar self-coupling g_R up to one loop is

$$g_R = g_0 - 2g_0^2\int_p (\hat p^2 + m_0^2)^{-2}$$

$$+48N_f\int_p (\bar p^2 + \mu_p^2)^{-4}\left[2G_{0\psi}^2G_{0\chi}^2\mu_p^4 - 4G_{0\psi}G_{0\chi}(G_{0\psi}^2 + G_{0\chi}^2 + G_{0\psi}G_{0\chi})\bar p^2\mu_p^2 + (G_{0\psi}^4 + G_{0\chi}^4)(\bar p^2)^2\right]$$

$$+ 2g_0(2\kappa Z_\phi - 1), \tag{24}$$

and the renormalized Yukawa couplings are determined as

$$G_{R\psi} = G_{0\psi} - 2G_{0\psi}^3\int_p \frac{\bar p^2}{(\hat p^2 + m_0^2)^2(\bar p^2 + \mu_p^2)}$$

$$+2G_{0\psi}^2\int_p \frac{G_{0\psi}\bar p^2 - G_{0\chi}\mu_p^2}{(\hat p^2 + m_0^2)(\bar p^2 + \mu_p^2)^2} + G_{0\psi}\frac{1}{2}(2\kappa Z_\phi - 1), \tag{25}$$

$$G_{R\chi} = (\psi \leftrightarrow \chi). \tag{26}$$

A simple observation is that all purely fermionic 1-loop contributions are twice as big as in the U(1)-case.

For sufficiently small bare couplings the perturbative formulae can be used to locate the critical points in parameter space, i.e. those points where the renormalized masses vanish, viz.

$$m_R = 0, \qquad \mu_R = 0. \tag{27}$$

For the special case $G_\chi = 0$, one derives the critical values for K and κ in a similar fashion to [7], i.e.

$$\mu_{0c} = 0, \qquad K_c = \frac{1}{8}, \tag{28}$$

and the quadratic equation for the determination of κ_c reads ($N_f = 2$)

$$\kappa_c^2 + \left[\frac{2\lambda - 1}{8} + 16G_\psi^2 I_2\right]\kappa_c - \frac{3\lambda}{4}I_1 = 0 \tag{29}$$

where

$$I_1 = \int_0^{2\pi}\frac{d^4p}{(2\pi)^4}(\hat p^2)^{-1} = 0.154933\ldots, \tag{30}$$

$$I_2 = \int_0^{2\pi}\frac{d^4p}{(2\pi)^4}\frac{\bar p^2}{[\bar p^2 + (\hat p^2/2)^2]^2} = 0.025703\ldots. \tag{31}$$

This should be approximately valid for small G_ψ^2. Solving for κ_c in eq. (26) one observes that at constant λ, κ_c decreases for increasing G_ψ, but here the decrease is much stronger than in the U(1)-case.

After we obtain the perturbative formulae expressing renormalized quantities in terms of bare parameters at 1-loop level, we can consistently invert those relations to get 1-loop renormalized lattice perturbation theory. This is particularly useful in the analysis of finite size effects. One can use nonperturbative input for the renormalized parameters used in the perturbative expressions for finite size effects which allows one to extrapolate the Monte Carlo results to infinite volume. Imposing the renormalization conditions at infinite volume (i.e. $L = T = \infty, \quad m_R = m_R^{(\infty)}, \quad g_R = g_R^{(\infty)}, \ldots$), we define the finite size difference δX_R for any renormalized quantity X_R as

$$\delta X_R \equiv X_R(L,T) - X_R(\infty,\infty). \tag{32}$$

Calculating δX_R essentially amounts to calculating the difference of the 1-loop integrals evaluated for different lattice sizes, viz.

$$\delta \int_p f(p) \equiv \frac{1}{L^3 \cdot T} \sum_p f(p) - \frac{1}{(2\pi)^4} \int d^4p\, f(p), \tag{33}$$

where $f(p)$ is some function of the lattice momenta. As an example we display the expression for the finite size effect on the fermion mass

$$\delta\mu_R = -4G_{R\psi}G_{R\chi}\,\delta \int_p \frac{\mu_{Rp}}{(\hat{p}^2+\mu_{Rp}^2)(\hat{p}^2+m_R^2)} - \mu_R(G_{R\psi}^2+G_{R\chi}^2)\,\delta\int_p \frac{\bar{p}^2}{(\bar{p}^2+\mu_{Rp}^2)(\hat{p}^2+m_R^2)^2} \tag{34}$$

where $\mu_{Rp} = \mu_R + \hat{p}^2/2$. Similar expressions can be derived for the other quantities. The finite size differences are evaluated numerically using suitable computer programs.

3.2 β-functions

The β-functions, which describe how the renormalized couplings change with varying cutoff if the bare couplings are held constant, are derived from lattice perturbation theory in 1-loop approximation. Their universal scaling parts read

$$16\pi^2\beta_g^{(1)} = 4\,g_R^2 + 16N_f g_R(G_{R\psi}^2 + G_{R\chi}^2) - 96N_f(G_{R\psi}^4 + G_{R\chi}^4), \tag{35}$$

$$16\pi^2\beta_\psi^{(1)} = 4N_f G_{R\psi}(G_{R\psi}^2 + G_{R\chi}^2), \tag{36}$$

$$16\pi^2\beta_\chi^{(1)} = 4N_f G_{R\chi}(G_{R\chi}^2 + G_{R\psi}^2). \tag{37}$$

The behaviours of the renormalized couplings as functions of the scale variable $\tau \equiv log(am)^{-1}$ (where am is some mass in lattice units) in the limit $\tau \to \infty$ are governed by the infrared structure of the β-functions. One can see that $g_R = G_{R\psi} = G_{R\chi} = 0$ is an infrared fixed point, therefore the continuum limit of the model is trivial unless there is some other nonperturbative nontrivial fixed point. In order to have an interacting continuum theory, one has to keep the cutoff at some finite scale higher than the typical mass scale of the theory. The renormalized couplings will depend on the cutoff scale and will go up as the cutoff is

decreased. When the cutoff is as low as the mass scale, the theory ceases being an effective theory due to large effects from the scaling violation terms. Therefore, one can get upper bounds on the renormalized couplings which are cutoff dependent.

It can also be easily seen that in the limit $\tau \to \infty$, the ratio $G_{R\chi}^2/G_{R\psi}^2$ can stay at any value at any energy scale. This arbitrariness of this ratio is important, because in the continuum limit in the spontaneously broken phase it allows to fix the mass ratio $\mu_{R\chi}/\mu_{R\psi}$. The 1-loop equations indicate that $G_{R\chi}$ is (approximately) zero once $G_{0\chi} = 0$.

The 1-loop formulae will break down and higher loop contributions should come in as the couplings get relatively strong. In order to estimate when the 1-loop formulae become invalid, we need at least to know the 2-loop β-functions.

The universal 2-loop contributions can be figured out from [10] in which the 2-loop β-functions of a general scalar-gauge-fermion model were worked out. The results are

$$(16\pi^2)^2\beta_g^{(2)} = -\frac{26}{3}g_R^3 - 32N_f g_R^2(G_{R\psi}^2+G_{R\chi}^2) - 64N_f g_R(G_{R\psi}^4+G_{R\chi}^4) + 768N_f(G_{R\psi}^6+G_{R\chi}^6), \tag{38}$$

$$(16\pi^2)^2\beta_\psi^{(2)} = (6-12N_f)G_{R\psi}^5 - 8N_f G_{R\psi}^3G_{R\chi}^2 - 4N_f G_{R\psi}G_{R\chi}^4 - 4g_R G_{R\psi}^3 + \frac{1}{6}g_R^2 G_{R\psi}, \tag{39}$$

$$(16\pi^2)^2\beta_\chi^{(2)} = (6-12N_f)G_{R\chi}^5 - 8N_f G_{R\chi}^3G_{R\psi}^2 - 4N_f G_{R\chi}G_{R\psi}^4 - 4g_R G_{R\chi}^3 + \frac{1}{6}g_R^2 G_{R\chi}. \tag{40}$$

Together with the 1-loop contributions, one can roughly estimate that the 1-loop β-functions become invalid when $G_{R\psi}^2, G_{R\chi}^2 \sim 30$ for $N_f = 2$. One can also see that, due to the alternating signs, some new zeros of the β-functions are generated at 2-loop level. It is possible that they are just 2-loop artifacts because the 2-loop formulae will also be invalid as the couplings become even stronger.

3.3 Tree Unitarity

The partial wave analysis of transition amplitudes at tree level allows the computation of upper bounds on the renormalized couplings, requiring the unitarity of the S-matrix. The calculations presented here are in the spirit of ref. [11]. For our purpose it is convenient to use the action defined in eqns. (5–7). Throughout this section we will work in continuous Minkowski space-time.

The interaction Lagrangian is

$$\mathcal{L}_Y = -\sqrt{2}\sum_{s,t}\left\{\bar{\psi}_{RA}^s(G_{R\alpha} - G_{R\beta}\gamma_5)\varphi_{Rst}\psi_{RB}^t + \bar{\psi}_{RB}^s(G_{R\alpha} + G_{R\beta}\gamma_5)(\varphi_R^\dagger)_{st}\psi_{RA}^t\right\} \tag{41}$$

where the sum is over the SU(2)-isospin indices and φ is the SU(2) matrix

$$\varphi = \frac{1}{\sqrt{2}}(\phi_0 1\!\!1 + i\tau_s\phi_s). \tag{42}$$

Here we use the renormalized fields and couplings throughout, since we calculate at tree level. The fermion propagator is

$$\widetilde{\Delta}_\Psi(p) = \langle \psi_A^s \bar{\psi}_B^t \rangle = \delta^{st}\delta_{AB}\frac{i}{\not{p}-\mu}\,, \tag{43}$$

and for the scalar propagator one gets

$$\widetilde{\Delta}_\varphi(p) = \langle \varphi_{st}(\varphi^+)_{uv} \rangle = \delta_{sv}\delta_{tu}\frac{i}{p^2-m^2}\,. \tag{44}$$

Now we consider the scattering process of a particle of type B and an antiparticle of type A according to

$$B^s(p_1,\lambda_1) + \bar{A}^t(p_2,\lambda_2) \longrightarrow B^u(p_3,\lambda_3) + \bar{A}^v(p_4,\lambda_4) \tag{45}$$

where the p_i's are the four-momenta, and $s,\ldots,v$ denote the SU(2)-isospin indices of the incoming and outgoing particles. The helicities are denoted by $\lambda_1,\ldots,\lambda_4$. The S-matrix element is determined as

$$\begin{aligned}&\langle \bar{A}^t\, B^s|S|\bar{A}^v\, B^u\rangle = \\ -\,\delta_{su}\delta_{tv}\langle \bar{A}^t\, B^s|\bar{\psi}^t_{RA}&(G_{R\alpha}-G_{R\beta}\gamma_5)\varphi_{Rts}\psi^s_{RB}\bar{\psi}^u_{RB}(G_{R\alpha}+G_{R\beta}\gamma_5)\varphi^+_{Ruv}\psi^v_{RA}|\bar{A}^v\, B^u\rangle\,.\end{aligned} \tag{46}$$

Hence the isospin indices of particle and antiparticle are left unchanged by the scattering process, and therefore the helicity amplitudes $T^{st}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ are the same for any choice of the indices s,t.

Now we go to the center-of-mass frame of the incoming particles by requiring

$$\vec{p}_1 + \vec{p}_2 = 0 \tag{47}$$

and choose

$$\vec{p}_1 = (p_1,0,0) = |\vec{p}|(1,0,0) \tag{48}$$

such that the momenta of the outgoing particles are

$$\vec{p}_3 = |\vec{p}|(\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi) = -\vec{p}_4 \tag{49}$$

where (θ,ϕ) are the polar and azimuthal angle, respectively. Furthermore one has the relations

$$s = (p_1+p_2)^2\,, \quad E_1 = \frac{\sqrt{s}}{2}\,, \quad |\vec{p}| = \sqrt{E_1^2-\mu_R^2}\,. \tag{50}$$

With these definitions, one obtains the helicity amplitudes $T^{st}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ for any choice of the isospin indices s,t up to a phase as

$$T^{st}_{++++} = -\frac{8}{s-m_R^2}\left(G^2_{R\alpha}(E_1^2-\mu_R^2) + G^2_{R\beta}E_1^2 - 2G_{R\alpha}G_{R\beta}E_1\sqrt{E_1^2-\mu_R^2}\right) \tag{51}$$

$$T^{st}_{----} = -\frac{8}{s-m_R^2}\left(G^2_{R\alpha}(E_1^2-\mu_R^2) + G^2_{R\beta}E_1^2 + 2G_{R\alpha}G_{R\beta}E_1\sqrt{E_1^2-\mu_R^2}\right) \tag{52}$$

$$\begin{aligned}T^{st}_{++--} &= \frac{8}{s-m_R^2}\left(G^2_{R\alpha}(E_1^2-\mu_R^2) - G^2_{R\beta}E_1^2\right) \\ &= T^{st}_{--++}\,.\end{aligned} \tag{53}$$

All other amplitudes are zero.

Expanding the above amplitudes in a partial wave series, one observes that only $J=0$ partial waves contribute since there is no dependence on θ,ϕ. Unitarity of the S-matrix requires [15]

$$\left|Re(\mathcal{T}^{J=0}_{\lambda_1\lambda_2\lambda_3\lambda_4})\right| \leq \frac{1}{2} \tag{54}$$

where $\mathcal{T}^{J=0}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ is the coefficient of the partial wave expansion. In the limit $s\to\infty$ we obtain the following bounds on the renormalized Yukawa couplings from eqns. (48–50)

$$G^2_{R\psi},\; G^2_{R\chi},\; G_{R\psi}G_{R\chi} \leq 4\pi\,. \tag{55}$$

From this result one expects the Yukawa couplings to be strong at $G_{R\psi}$, $G_{R\chi} \simeq$ 3–4. This value is slightly lower than the one derived from the analysis of the perturbative β-functions. In fact, tree unitarity yields the same result as in the U(1)-symmetric model [7] which is due to the fact that the isospin components are conserved during the scattering process.

4. Numerical simulations

We performed Monte Carlo simulations on the $4^3\cdot 8$ lattice with periodic boundary conditions in the spatial directions. Due to limited amount of computer time, we have not been able to move to larger lattices so far. In the (longest) time direction periodic boundary conditions were taken for the scalar field and antiperiodic ones for the fermions. The algorithm we use is the unbiased Hybrid Monte Carlo method [12]. This requires the flavour doubling of the fermion spectrum. In the molecular-dynamics step typically about 15000 trajectories per point were calculated, with about 10% at the beginning used for equilibration. The number of leapfrog steps per trajectory was chosen randomly between 3 and 10. The step size was tuned so that the acceptance rate for the trajectories was near 75%. The necessary inversions of the fermion matrix were done by the conjugate gradient iteration, until the residuum was smaller than 10^{-8}-times the length square of the input vector.

The Wilson-parameter was always chosen to be $r=1$. The bare Yukawa coupling of the mirror fermion field G_χ was fixed to zero, while G_ψ was changed between 0.1 and 1.0. The bare quartic coupling λ was chosen to be 10^{-4}, 1.0 and ∞. The remaining two bare parameters κ and K were tuned such that the system was in the symmetric phase and the masses of the scalar (m_R) and fermion (μ_R) were not too small to avoid large finite size effects. On the $4^3\cdot 8$ lattice, we let $m_R \simeq \mu_R \simeq 1.0$.

Our data are presented in tables I, II and III. With the present statistics, the renormalized scalar coupling g_R could not be determined accurately enough, and hence we do not include it in the tables. Furthermore, our results show that the fermion doublers can be easily made heavy, i.e. they receive masses greater than 2 in lattice units.

For the Yukawa couplings one can make the following statements: $G_{R\chi}$ is always substatially smaller than $G_{R\psi}$ as one expects from perturbation theory. The 1-loop expressions however, predict that $G_{R\chi}$ is exactly zero for $G_\chi = 0$. $G_{R\psi}$ is linearly rising with G_ψ (fig. 1) but comparing our results to the U(1)-case one observes that the rise is slightly steeper. This is also supported by the perturbative expressions, which for SU(2) involve a further positive contribution to $G_{R\psi}$. We see that at $G_\psi = 1.0$ the renormalized coupling $G_{R\psi}$ is twice as large as the tree unitarity limit. This indicates that the system is strongly interacting at that point, and we are therefore led to believe that in the broken phase the mirror fermion can be made heavy by appropriate tuning of G_χ, just like what we did in the $U(1)_L \otimes U(1)_R$ model [8].

Our results show only weak dependence on the value of the bare quartic coupling λ. However, at $\lambda = 10^{-4}$ and large values of G_ψ, one is forced to go to negative values of κ in order to stay at $m_R \simeq \mu_R \simeq 1.0$. Thus one ends up in a region of bare parameter space where reflection positivity cannot be proven, as was reported for the U(1)-case [8]. Plotting κ versus G_ψ, one observes that the values of κ are shifted parallely downwards for decreasing λ (see fig. 2). This suggests that at any value of λ and for large enough G_ψ the scalar hopping parameter κ becomes negative for fixed m_R, and hence the continuum limit is possibly ill-defined.

To get a hint on the dependence of the numerical results on the finite volume, we used them as input in the perturbative expressions for finite size effects (see sect. 3). Since g_R could not be determined accurately enough, we chose $g_R = 30$ as input, which corresponds roughly to the tree unitarity bound on g_R in the O(4)-symmetric ϕ^4-theory [15]. The finite size analysis shows that both masses decrease as one goes to the infinite lattice. The effect amounts up to 10% for m_R and less than 5% for μ_R. $G_{R\psi}$ is smaller by about 3% for the infinite volume. These are of course only rough estimates since there are no Monte Carlo results on larger lattices so far.

5. Discussion and summary

The behaviour of the renormalized Yukawa couplings was investigated in the symmetric phase of an $SU(2)_L \otimes SU(2)_R$ symmetric lattice Yukawa model with explicit mirror fermions. We started at small bare couplings ($\lambda = 0.0001, G_\psi = 0.1$) in the perturbative region near the Gaussian fixed point and then increased the couplings up to ($\lambda = \infty, G_\psi = 1.0$). The bare Yukawa coupling of the mirror fermion field was fixed to $G_\chi = 0$. In the numerical simulations only weak dependence of the renormalized Yukawa coupling $G_{R\psi}$ on the bare quartic coupling λ was observed. $G_{R\psi}$ rises approximately linearly in the whole range as a function of the bare Yukawa coupling G_ψ. The obtained values of the renormalized Yukawa coupling $G_{R\psi}$ at $G_\psi = 1.0$ are about twice the tree unitarity limit, whereas $G_{R\chi}$ is close to zero as predicted

by perturbation theory.

These findings are very similar to the previously studied U(1)-case. Thus, there is no qualitative difference to the results of ref. [7], and in particular the results imply that the upper limit on the renormalized Yukawa coupling, at a cutoff corresponding to masses in lattice units of about 0.5, is at least 2- or 3-times larger than the tree unitarity limit. This is different from pure ϕ^4-models, where the corresponding upper limit is roughly equal to the tree unitarity bound (for references see [13], and in particular for the symmetric phase [14,15,16]).

At the present stage we cannot make any statement concerning the renormalization of Yukawa couplings since we do not have any results on larger lattices. Thus we have not been able to confirm the linear rise of $G_{R\psi}$ with G_ψ on bigger volumes either. We leave this problem to a later study. Our data show that in order to fix the masses in the theory at, say $m_R \simeq \mu_R \simeq 1$ one is forced to negative values of κ as the bare Yukawa coupling is decreasing. This happens earlier for small λ than at $\lambda = \infty$. Hence one ends up in a region where reflection positivity cannot be proven. As a guideline for further studies this shows that in order to make predictions relevant for the continuum theory, one should restrict the analysis to medium values of the bare Yukawa couplings.

Acknowledgements

We thank I. Montvay for introducing this field to us. We are also grateful to him for very many helpful suggestions and discussions. H.W. likes to thank C. Greub for useful discussions. The Monte Carlo simulations for this paper have been performed on the CRAY Y-MP of HLRZ, Jülich.

Table I

The chosen points in the parameter space and the measured renormalized masses m_R and μ_R. All data are on the $4^3 \cdot 8$ lattice. Statistical errors in last numerals are in parenthesis.

label	λ	G_ψ	G_χ	κ	K	m_R	μ_R
A	∞	0.3	0.0	0.220	0.100	1.00(5)	1.06(2)
B	∞	1.0	0.0	0.042	0.100	1.07(11)	0.954(9)
C	1.0	0.1	0.0	0.205	0.100	1.04(1)	1.0733(4)
D	1.0	0.3	0.0	0.181	0.100	1.17(1)	1.058(1)
E	1.0	0.6	0.0	0.140	0.100	1.12(2)	1.009(3)
F	1.0	1.0	0.0	0.032	0.100	1.07(2)	0.917(5)
G	10^{-4}	0.3	0.0	0.096	0.100	0.99(2)	1.037(2)
H	10^{-4}	1.0	0.0	-0.055	0.100	0.87(3)	0.78(1)

Table II

Some global expectation values in the points with label defined in table I. l is the normalized link variable average. $|\varphi|$ denotes the magnitude of the average scalar field. The other notations are self-explaining. Statistical errors in last numerals are given in parenthesis.

	l	$\langle\chi_x\bar{\psi}_x\rangle$	$\langle\varphi_x^+\psi_{Lx}\bar{\psi}_{Rx}\rangle$	$\langle\varphi_x\chi_{Lx}\bar{\chi}_{Rx}\rangle$	$\langle\lvert\varphi\rvert\rangle$	$\langle\lvert\varphi_x\rvert\rangle$
A	0.144(2)	7.9313(8)	0.0201(7)	-1.1954(5)	0.115(5)	1.0
B	0.085(1)	7.848(2)	0.0405(8)	-3.919(1)	0.089(6)	1.0
C	0.1423(4)	7.942(1)	0.0073(8)	-0.4805(7)	0.119(1)	1.0649(2)
D	0.1276(3)	7.929(1)	0.0195(7)	-1.422(1)	0.106(1)	1.0593(2)
E	0.1189(4)	7.891(2)	0.0367(7)	-2.805(2)	0.107(2)	1.0561(2)
F	0.0896(5)	7.821(2)	0.0502(10)	-4.540(2)	0.104(2)	1.0476(3)
G	0.1369(5)	7.915(1)	0.0417(9)	-2.693(2)	0.168(3)	1.4135(5)
H	0.0592(7)	7.749(3)	0.078(2)	-8.01(1)	0.156(6)	1.362(1)

Table III

Renormalized couplings and Z-factors in the points with label defined in table I. Statistical errors in last numerals are given in parenthesis.

	$G_{R\psi}$	$G_{R\chi}$	Z_ϕ	Z_ψ	Z_χ
A	3.3(1)	-0.4(4)	1.7(1)	4.24(1)	4.44(1)
B	10.7(8)	-2.0(1)	2.0(2)	3.38(5)	4.29(1)
C	1.26(5)	-0.23(5)	2.24(4)	4.44(3)	4.41(3)
D	3.78(8)	-0.56(7)	2.22(3)	4.25(2)	4.43(2)
E	7.1(1)	-0.94(6)	2.05(4)	3.86(3)	4.35(2)
F	9.6(2)	-1.72(8)	1.76(4)	3.10(4)	4.22(2)
G	5.02(9)	-0.71(6)	4.03(7)	4.06(3)	4.41(3)
H	9.1(4)	-2.22(7)	2.6(1)	2.11(7)	4.05(3)

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Figure captions

Fig. 1. The renormalized Yukawa coupling $G_{R\psi}$ as a function of the bare coupling G_ψ at $\lambda = 10^{-4}$ (open circles), $\lambda = 1.0$ (open squares) and $\lambda = \infty$ (crosses) on an $4^3 \cdot 8$ lattice. At $G_\psi = 1.0$ the points are slightly shifted for a better display of the errorbars. For smaller values of G_ψ the errors are of size of the symbols.

Fig. 2. Values for the scalar hopping parameter κ versus the bare Yukawa coupling G_ψ at $m_R \simeq 1$ and for several values of λ. Open circles: $\lambda = 10^{-4}$; open squares: $\lambda = 1.0$; crosses: $\lambda = \infty$.

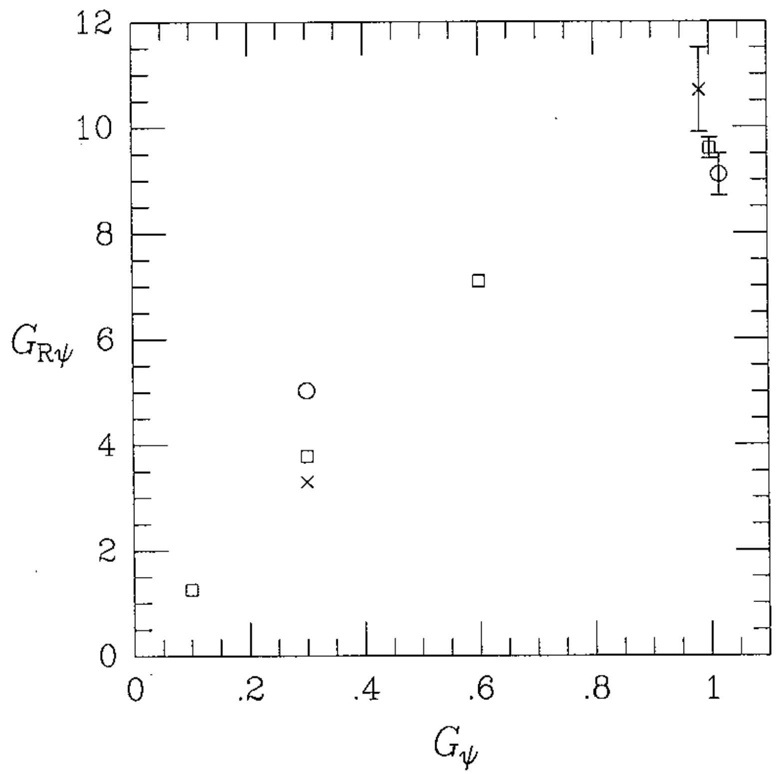

Figure 1

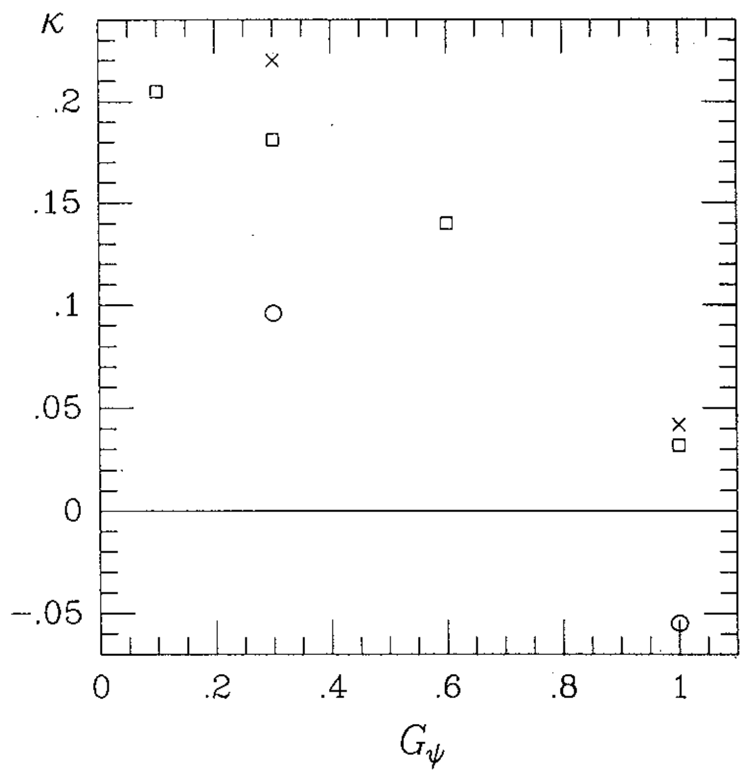

Figure 2