Universität Hamburg

Faculty Economics
and Social Sciences
Chair for Economic Policy

TIM SIEVERS

A VECTOR-BASED APPROACH TO MODDELING KNOWLEDGE IN ECONOMICS

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University of Hamburg
Faculty Economics and Social Science
Chair for Economic Policy
Von-Melle-Park 5
D-20146 Hamburg | Germany
Tel +49 40 42838 - 4622
Fax +49 40 42838 - 6251
http://www.uni-hamburg.de/economicpolicy

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Tim Sievers
University of Hamburg
Faculty Economics and Social Science
Chair for Economic Policy
Von-Melle-Park 5
D-20146 Hamburg | Germany
Tel +49 40 42838 - 4622
Fax +49 40 42838 - 6251
hced@econ.uni-hamburg.de

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Tim Sievers

A Vector–based Approach to Modeling Knowledge in Economics

Abstract: This paper draws attention to two important characteristics of knowledge which so far have been left unexplored, and proposes a new method to capture them in economic modeling with the help of vectors. The direction of the vector represents the knowledge's complementarity with other knowledge and its norm represents its potential economic value.
Keywords: knowledge; innovation; innovative production; vector representation; complementarity
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1 Introduction

The characteristics of knowledge in innovative activity have been discussed at length in the literature (ARROW, 1962; COHEN & LEVINTHAL, 1989; DOSI, 1988; NELSON, 1959). This note draws attention to two important characteristics which so far have been left unexplored, and proposes a new method for modeling knowledge to capture these.

(1) The same new knowledge can be of different value to different firms.

Consider for instance a company specialised in the production of musical instruments which produced new knowledge about the characteristics of certain materials. The company is likely to recognise and use only those aspects of its new knowledge which fall in the area of its existing competences to improve the quality or reduce the production costs of its instruments. Yet the same knowledge might contain valuable insights for completely different products, for instance in the area of sports equipment, where similar materials might be used. This latter economic potential, however, is likely to not be exploited by the musical instruments company.

In what follows I shall refer to the degree of fit of a firm's new knowledge with its existing knowledge (comprising competences, routines, organisational structure, etc.) as the new knowledge's complementarity. This complementarity determines the *effective value* of knowledge for a company. I define the *potential value* of knowledge as comprising all its economically useful aspects, some of which its owner will not be able to recognise or exploit in case of less than perfect complementarity.

(2) The production of new knowledge involves uncertainty.

There are two types of uncertainty involved: firstly, about whether and how much new knowledge will be produced, and secondly, about what new knowledge exactly will be invented and how useful it will be. In the following I shall abstract from the first type of uncertainty by assuming that a certain amount of R&D expenditure will always produce an amount of knowledge of a certain potential value. However, uncertainty remains about the degree of complementarity of new knowledge and hence about its effective value for the inventor.

So far studies of innovative production have failed to take these knowledge-specific properties seriously. For instance, KLETTE & KORTUM (2004) explicitly take into account the importance of uncertainty, but their model fails to capture complementarity. In their model an invention adds the same amount to any firm's "knowledge capital" regardless of whether it is close to the firm's current activities or not.[1]

To capture the special properties of innovative production I propose to model knowledge as a vector characterised by a length, i.e. a scalar which reflects its potential value, and a relative direction, i.e. an angle which reflects its complementarity.

This type of representation also allows to deal with a conceptual problem which results from the importance of cumulativeness of knowledge: New knowledge builds on existing knowledge and often only becomes "inventable" on the basis

[1] Cobb-Douglas type innovative production functions suffer from the same limitation (e.g., KLETTE, 1996).

of what was known before. This entails for instance the “standing on the shoulders of giants” effect but works in reverse as well. New knowledge might allow to put previously existing knowledge to other uses and thereby lead to further commercial innovation. To capture the cumulativeness of knowledge requires the ability to model the outcome of the combination of two sets of knowledge. With a one-dimensional variable, however, this is difficult to handle because the importance of complementarity means that additivity does not hold for knowledge in a one-dimensional sense. The increase in effective knowledge due to any new knowledge is dependent on the degree of complementarity of the new knowledge.

2 Vector Representation of Knowledge

Let $f_i^t: \mathbb{R} \rightarrow \mathbb{R} \times (0°, 360°)$ be the knowledge production technology for *i* in period *t* such that

$$f_t^i: e_t^i \rightarrow \left\|\vec{n}_t^i\right\|, \beta\left(\vec{n}_t^i\right) \tag{1}$$

where e_t^i is a scalar denoting *i*'s R&D expenditure in period *t*. $\vec{n}_t^i$ is the vector of new knowledge. $\beta\left(\vec{n}_t^i\right) \in (0°, 360°)$ is a random variable following a known probability distribution denoting the direction of $\vec{n}_t^i$. $\left\|\vec{n}_t^i\right\|$ is the Euclidean norm of $\vec{n}_t^i$ and a known function $g_t^i: \mathbb{R} \rightarrow \mathbb{R}$ of e_t^i,

$$\left\|\vec{n}_t^i\right\| = g_t^i\left(e_t^i\right) \tag{2}$$

with $g_t^{i\,\prime} > 0$, $g_t^{i\,\prime\prime} > 0$. The interpretation of the scalar $\left\|\vec{n}_t^i\right\|$ is that it denotes the length and hence the potential value of $\vec{n}_t^i$, while $\beta\left(\vec{n}_t^i\right)$ is the basis for evaluating the relative direction and hence complementarity of $\vec{n}_t^i$. This allows to operationalise the notion of uncertainty in innovative activity.

Knowledge accumulation in this setting is captured with the help of simple vector addition

$$\vec{k}_t^i = \vec{k}_{t-1}^i + \vec{n}_i^i \tag{3}$$

where $\vec{k}_t^i$ is the total knowledge vector of firm *i* in period *t* which results from adding period *t*'s new knowledge, $\vec{n}_t^i$, to the inherited knowledge vector from the previous period, $\vec{k}_{t-1}^i$ (see Figure 1).

As the defining asset of an innovative firm one can think of $\vec{k}_{t-1}^i$ as a proxy for its inherited competences and organisation. The knowledge accumulation process above then reflects the evolution of the firm over time.

To the extent that it is exclusive (for instance, due to effective patent protection) new knowledge can be valuable for its owner because it affords a protected market position. Assume that the innovative firm's profit is increasing in the effective value of its new knowledge. Consider the following simple payoff function

$$\Pi_t^i = h\left[\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right)\right] \cdot \left\|\vec{n}_t^i\right\| \tag{4}$$

where $\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right) \in (0°, 180°)$ denotes the (smallest) difference between $\beta\left(\vec{n}_t^i\right)$ and $\beta\left(\vec{k}_{t-1}^i\right)$ and h is a continuously differentiable function monotonously decreasing in $\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right)$, with $h(0) = 1$.[2] As an inverse index of complementarity $\alpha\left(\vec{n}_t^i, \vec{k}_{t-1}^i\right)$ is an important determinant of the effective value of new knowledge, i.e. of how much of the knowledge's potential value a particular firm can realise.

3 Applications

The vector methodology offers promising ways in which to revisit some existing debates in knowledge economics. Even the simple payoff structure assumed above provides a theoretical basis to look at the case for efficiency enhancing reallocation of knowledge as a driver for knowledge transfers. The importance of complementarity and uncertainty in innovative activity can lead to a gap between the potential and the effective value of new knowledge for a specific agent. Reallocation of new knowledge to a firm with more complementary existing knowledge might become a profitable opportunity. This provides a market-based framework for the analysis of channels for knowledge transfers, e.g.

[2] An example for h is $h(x) = \cos(x)$.

through the M&A or the labour market. SIEVERS (2005) for instance uses the vector methodology to develop a model of clustering which does not rely on knowledge spillovers but derives from the importance of uncertainty and complementarity inherent in innovative activity and the cluster's role of alleviating informational imperfections.

Another possible extension applies to the "Schumpeterian" debate on the relationship between firm size and innovative success. This strand of the literature acknowledges the importance of uncertainty and complementarity and introduces the latter with a distinction between "drastic" and "incremental" innovation.[3] The restriction to a binary, discrete approach to complementarity and the lack of a framework for knowledge accumulation are major shortcomings. With vector representation the role of firm size for innovative success follows naturally from an alteration of the payoff structure, for instance to $\Pi_t^i = \|\vec{k}_t^i\| - \|\vec{k}_{t-1}^i\|$, without having to sacrifice the special characteristics of innovative production (Figure 2).

4 Conclusion

I propose to use vector representation as a simple yet powerful analytical tool to model knowledge more accurately and consistently, which has so far not been explored in the literature. This is useful for at least two reasons: The first is that the inherent properties of knowledge might be drivers of dynamics which – for lack of alternative explanations – have been interpreted as caused by other forces. An example that springs to mind is the role of knowledge spillovers as an explanation for clustering of innovative activity. The second reason is that inferences from any model involving knowledge as input or output and not taking into account its special characteristics emphasised in this paper must be interpreted with caution. The potential sensitivity of results with regard to the left out properties needs to be tested.

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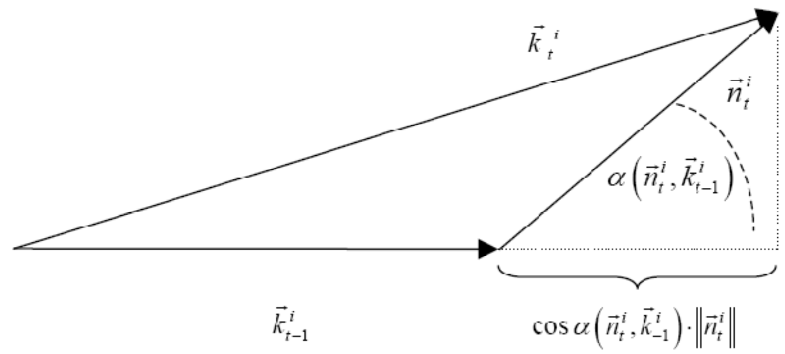
[3] See HENDERSON (1993) for an overview and extension.

5 Appendix

Fig. 1 Cumulativeness and Complementarity with Vector Representation

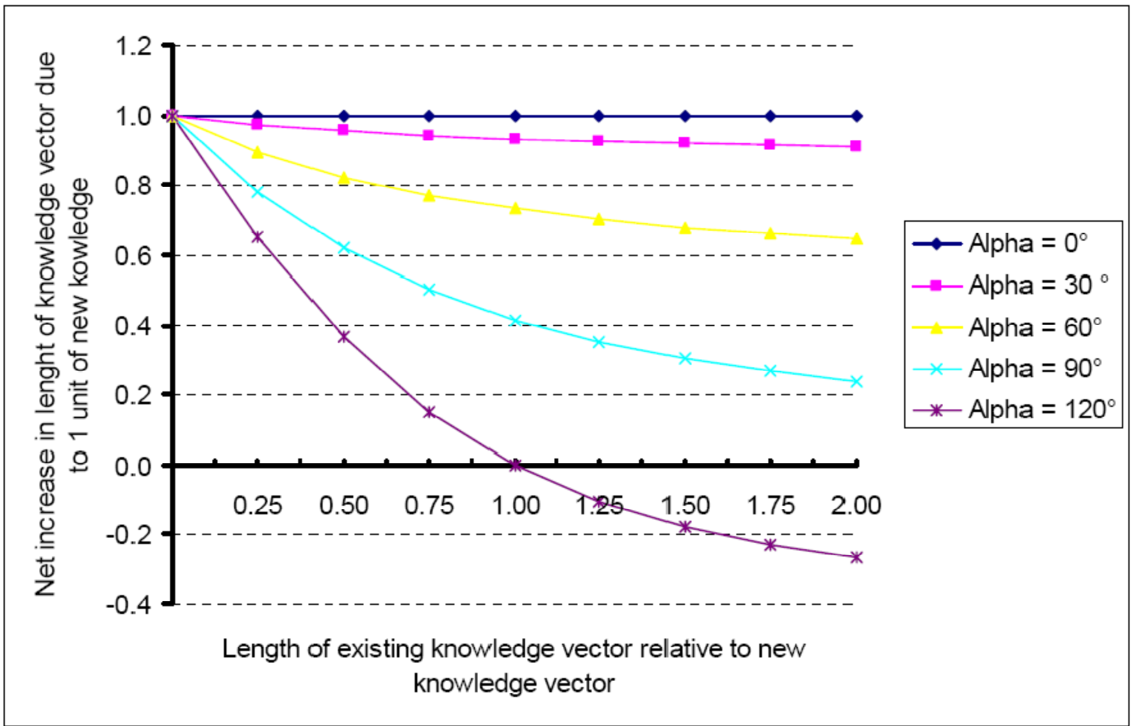


$\vec{k}_t^i = \vec{k}_{t-1}^i + \vec{n}_t^i$, and

$$\left\|\vec{k}_t^i\right\| = \sqrt{\left\|\vec{k}_{t-1}^i\right\|^2 + 2\cdot\left\|\vec{k}_{t-1}^i\right\|\cdot\left\|\vec{n}_t^i\right\|\cdot\cos\alpha(\vec{n}_t^i, \vec{k}_{t-1}^i) + \left\|\vec{n}_t^i\right\|^2} \leq \left|\vec{k}_{t-1}^i\right| + \left\|\vec{n}_t^i\right\|.$$

Source: Own illustration.

Fig. 2 Impact of New Knowledge as a Function of Non-complementarity (Alpha) and Existing Knowledge



Source: Own illustration.

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