Land use decision modeling with dynamically updated soil carbon emission rates

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Abstract

Soil carbon can be sequestered through different land management options depending on the soil carbon status at the beginning of a management period. This initial status results from a given soil management history in a given soil climate regime. Similarly, the prediction of future carbon storage depends on the time sequence of future soil management. Unfortunately, the number of possible management trajectories reaches non-computable levels so fast that explicit representations of management trajectories are impractical for most existing land use decision models. Consequently, the impact of different management trajectories has been ignored.

This article proposes a computationally feasible mathematical programming method for integration of soil status dependent sequestration rates in land use decision optimization models. The soil status is represented by an array of adjacent status classes. For each combination of soil management and initial soil status class, transition probabilities of moving into a new or staying in the same status class are computed. Subsequently, these probabilities are used in dynamic equations to update the soil status level before and after each new soil management period. To illustrate the impacts of the proposed method, a simple hypothetical land use decision model is solved for alternative specifications.

Keywords

Soil carbon sequestration, Sink dynamics, Mathematical programming, Land use, Optimization, Agriculture, Forestry, Greenhouse gas mitigation

Introduction

Soil carbon sequestration through agriculture and forestry has been regarded as a potentially important option to lower greenhouse gas concentrations in the atmosphere (Schlesinger). It has been recognized that unlike other greenhouse gas emission abatement technologies, soil sequestration is very heterogeneous over location, time, and management. Local variations of soil management regimes have been studied by West and Post; Pautsch et al.; Lal; DeCara and Jayet; and McCarl and Schneider among many others. In addition, it has been recognized that soil carbon sequestration rates also vary over time (West et al.; Marland et al., Lal; Murray, McCarl, and Lee). Soil carbon sequestration encounters management specific capacity limits (Fix et al.). Continuous use of no-till agriculture for example will eventually lead to a point of saturation, where no additional storage can be achieved without a management change (Marland et al.).

Moreover, the direction and magnitude of soil carbon sequestration rates depend on the difference between initial soil carbon and equilibrium levels. If the initial soil carbon levels is substantially lower than the equilibrium level for a given soil management in a given location, than sequestration rates for this management will be positive and relatively high in magnitude. On the other hand, if the initial soil carbon level is above the equilibrium level, sequestration rates will be negative. For example, consider no-till wheat production. This management option can sequester substantial carbon amounts on fields that were intensively tilled during previous years. However, the same management may actually emit carbon if it is used on native grassland soils.

While the dynamic interaction between soil carbon levels and sequestration rates has been observed and verified in many experimental plots, it has been ignored in land use decision models. These models examine the adoption of alternative management decisions within a certain range of soil-climate regimes and are frequently used to find economic soil carbon sequestration potentials under various political scenarios. Spatial variations have been integrated in these land use models through coupling with bio-physical models (Antle et al.; McCarl and Schneider; Pautsch et al.) or through application of IPCC sequestration coefficients (De Cara and Jayet; Perez and Britz).

However, the dynamic nature of carbon sequestration rates has so far been ignored. Quite a few studies use static models in the beginning (Antle et al.; Pautsch et al.; McCarl and Schneider; De Cara and Jayet; Perez and Britz). Static models can only predict equilibrium changes between the initial and subsequent management. Dynamic models mostly employ non-dynamic sequestration rates (Sohngen and Mendelsohn; Murray, McCarl, and Lee). For example, the same sequestration rate is applied to no-till soils that were intensively tilled over many years and to soils which have not been tilled for some time. In reality, the sequestration rates should be high for the first and closed to zero for the second case.

The relatively simple carbon sequestration modeling approach adopted in existing land use decision models is caused by lack of data and computational restriction. Data wise it is much easier to establish a fixed coefficient for a given soil management in a given location than to estimate a set of carbon sequestration functions, which relate current sequestration rates not only to a given location but also to past management decision dynamics. Computationally, it is infeasible to optimize the decision over many soil management alternatives and time periods.

This study will propose an approach, which integrates the dynamic interaction between land use decisions, soil carbon levels, and sequestration rates within a mathematical programming framework. The approach is computationally feasible and provides the opportunity to trade computing time against accuracy on a continuous scale. The paper is structured as follows. The next section shows a theoretically ideal implementation of dynamic soil carbon sequestration rates within land use decision models. It will be demonstrated why this theoretically ideal approach is computationally infeasible. The following section develops the alternative approach. Subsequently, this approach is empirically tested. Finally, conclusions are given.

Land use decision models

Soil carbon sequestration results from the adoption of certain land use management. To analyze the economic potential of soil carbon storage, land use decision models are employed. These models are tools to understand, guide, and predict land use decisions. Mathematical programming is frequently used to determine optimal decisions when carbon sequestration efforts relate to substantial structural changes in the agricultural and forestry sector. These changes may include the introduction of new soil management practices and the implementation of new governmental policies. The approach presented here will be equally applicable to farm level, regional, and sector models, where land use decisions are optimized. Common to all land use optimization models is a predefined set of alternative land uses decision variables and an economic and/or ecologic objective to be optimized. In addition, most models include various biophysical or economic constraints. Let us denote alternative land use decisions by a nonnegative variable block $X_{t,r,i,u}$, where t = {1,…,T} denotes the set of time periods, r = {1,…R} the set of regions, i = {1,…,I} the set of soil type classes, and u = {1,…U} the set of land use classes.

The general structure of dynamic land use optimization models is displayed in equation block (1). The first line represents the objective function, where the total value V is a function of all possible land use decisions $X_{t,r,i,u}$. This potentially non-linear function may include both economic and environmental objectives. Line 2 generalizes restrictions on all decision variables.

$$\text{(1)} \qquad \begin{aligned} &\text{Maximize } V = v\left(X_{t=1,r=1,i=1,u=1},\cdots,X_{t=T,r=R,i=I,u=U}\right) \\ &\text{s.t.} \qquad g_j\left(X_{t=1,r=1,i=1,u=1},\cdots,X_{t=T,r=R,i=I,u=U}\right) \le 0 \end{aligned}$$

The purpose of this analysis is to illustrate how dynamic carbon sequestration rates can be modeled within a land use optimization framework. Without loss of generality, I use a simple linear system, where I maximize the sum of hypothetical net land use values over all time periods, regions, soil types, and management systems as shown in equation (2). The net value for a land use activity is the product of a constant per unit value $v_{t,r,i,u}$, a time period specific discount factor β_t, and the activity level $X_{t,r,i,u}$. The per unit value $v^M_{t,r,i,u}$ can be thought of as the gross margin of an activity, i.e. the difference of revenues minus costs of that activity. The second value coefficient $v^C_{t,r,i,u}$ would represent per unit cost or benefits related to the level of carbon net emissions. This type of objective function is used in land use models with constant input and output prices.

$$\text{(2)} \qquad \text{Max} \sum_{t,r,i,u}\left(\beta_t \cdot \left(v^M_{t,r,i,u} \cdot v^C_{t,r,i,u}\right) \cdot X_{t,r,i,u}\right)$$

We will also use a very simple restriction reflecting the fact that land use decisions are generally limited by the amount of land physically available. This restriction is incorporated through Equations (3). In particular, this equation requires the sum of all land use activities $X_{t,r,i,u}$ over all land use types u to be at or below a given natural land endowment $L_{t,r,i}$. Together, equations (2) and (3) yield a simple starting point for implementing dynamic soil carbon sequestration rates[1].

$$\text{(3)} \qquad \sum_u X_{t,r,i,u} \le L_{t,r,i} \qquad \forall t,r,i$$

[1] Note that a linear objective function subjected to a few linear constraints will produce a highly specialized solution.

Explicit representation of management trajectories

To accurately model soil carbon sequestration dynamics, the model has to be able to represent and track all different management decision paths. If we had 10 alternative land use opportunities in period 1, we would have 10^2 combinations of past and current land use decisions in period 2, 10^3 in period three and so forth. For our simple model, this implies that the land use decision variables must be modified to explicitly represent decisions from all past periods, i.e. $X_{t,r,i,u}$ change to X_{r,i,u^d}, where u^d denotes the index containing all possible management decision paths. Suppose, we have three land use alternatives u={1,2} and 3 time periods t={1,2,3}. The index u^d would then contain all possible management decision paths, i.e. u^d={111,112,121,122,211,212,221,222}. For each decision path, specific carbon and profit net present values would have to be generated. The objective would then be to maximize $\sum_{r,i,u^d}\left[\left(v^M_{r,i,u^d}+v^C_{r,i,u^d}\right)\cdot X_{r,i,u^d}\right]$, where v^M_{r,i,u^d} and v^C_{r,i,u^d} represent the net present market and carbon values for a given management decision path on land in a given region and soil type, respectively. The land use restriction from above would slightly change to $\sum_{u^d} X_{r,i,u^d} \le L_{r,i}$.

It can be easily seen that this approach is computationally prohibitive for models with numerous management alternatives and many time periods. Suppose, a dynamic model has 30 time periods and 20 different land management options resulting from combinations of different crop, tillage, fertilization, and irrigation systems. The total number of possible decision paths would equal 20^{30} = 1.07e+39 alternatives, clearly more than computers can handle. Having a model with multiple region and soil types, this number would be much higher.

Implicit management dynamics using separable soil classes

In this section, I will show an alternative approach, which employs dynamically updated carbon sequestration rates and is computationally feasible. The principal idea is to integrate management decision pathways indirectly through changes in soil organic matter. The following assumptions are used: i) a certain land management history is sufficiently identified by the current soil carbon status; ii) soil carbon can be treated of equal quality, and iii) the magnitude of soil carbon sequestration rates converges monotonically from the initial carbon level to the equilibrium level if a given soil management is continued forever in a given location. Depending on the initial carbon level, convergence can occur from above or below.

Soil status classes

To dynamically update carbon sequestration rates in our land use decision model, several modifications have to be made. First, the soil carbon level at the beginning of each management period has to be identified. To make this a feasible task, we group the possible range of soil carbon values into several classes denoted here by o = {1,…,O}. We append this index o to the land use decision variable, which changes from $X_{t,r,i,u}$ to $X_{t,r,i,u,o}$. Each land use decision is now associated with a certain soil carbon status, which by definition represents the status at the beginning of a management period. Note that the representation of the continuous soil carbon range into several discrete classes will cause approximation errors. However, the accuracy loss can be decreased by increasing the number of soil classes. This gives the modeler the opportunity to optimize the tradeoff between accuracy and computer time according to his preferences and resources.

Second, for each land management decision, regional, soil type, management, and soil status specific sequestration coefficients $s_{r,i,u,o}$ need to be pre-determined before the model is solved. Using the example from the previous section with 30 time periods and 20 soil management alternatives in each time period, the number of sequestration coefficients would equal 30 periods times 20 management alternatives times the number of soil classes plus one[2]. Suppose we have 50 soil classes, the total number of sequestration coefficients and associated land use decision variables would equal 30600, which is considerably lower than 1.07e+39.

Third, soil carbon states must be balanced after each management period. This is not straightforward because we have a limited number of soil carbon classes and a very heterogeneous set of carbon sequestration coefficients. To illustrate this, suppose the soil carbon level at the beginning of time period t is at $\tilde{o} \in \{1,...,O\}$ within the range [$\tilde{o}^{low}$, $\tilde{o}^{up}$].

Applying land management u in period t sequesters carbon in the amount of $s_{r,i,u,\tilde{o}} = \frac{\Delta C_{r,i,u,\tilde{o}}}{\Delta t}$[3], where $\Delta C_{r,i,u,\tilde{o}}$ denotes the carbon change in region r, soil class i, management u, and soil carbon status $\tilde{o}$ and Δt denotes the length of the time interval[4]. Thus, the carbon level at the end of a time period is somewhere in the interval $w_{r,i,u,\tilde{o}} = \left[\tilde{o}^{low} + s^{low}_{r,i,u,\tilde{o}}, \tilde{o}^{up} + s^{up}_{r,i,u,\tilde{o}}\right]$.

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[2] The addition of 1 to the number of soil classes is necessary because two coefficients are needed for each class representing both the lower and upper interval border. However, the upper interval border of a certain class is always equal to lower interval border of the class above it except for the last class. Thus, for all but the last soil carbon class, one coefficient is needed but for the last one two.

[3] The sequestration rate is a function of location r, soil type i, land management type u, and the initial soil carbon level $\tilde{o}$. Negative sequestration rates imply decreasing soil carbon levels.

[4] For convenience, we assume constant time steps. Many models use a one year time step because it reflects the time frame of land use decisions. The assumption simplifies the analysis because it eliminates the need to compute time step specific sequestration rates. If different time steps are used, the numbers sequestration coefficients would increase as many times as there are different period lengths.

Soil status transition probabilities

To accurately represent changes in soil carbon level, we propose to use a probability based approach. Let us define $\rho_{r,i,u,\tilde{o},o} \in [0,1]$ as the transition probability of moving the soil carbon status of soil type i in region r under land management u from class $\tilde{o}$ at the beginning of the management period to class o at the end. The clustering implies that soil carbon levels are not accounted for by their exact magnitude but rather by their membership within arbitrarily defined classes. Because sequestration rates and carbon clustering are completely independent, we can assume a uniform probability for the initial carbon level to be anywhere between the lower class boundary $\tilde{o}^{lo}$ and the upper class boundary $\tilde{o}^{up}$. Thus, the probability of moving from soil carbon class $\tilde{o}$ at the beginning of a time period to class o at the end can be simply calculated as the ratio of the probability range covering class o divided by the length of the total probability range[5]. Five general cases are possible, which result in different computations of $\rho_{r,i,u,\tilde{o},o}$. These cases are illustrated in Figure 1. The calculation of probabilities is given below.

Case I: If $\tilde{o}^{up} + s^{up}_{r,i,u,\tilde{o}} \geq o^{up}$ and $\tilde{o}^{low} + s^{low}_{r,i,u,\tilde{o}} \geq o^{low}$, then $\rho_{r,i,u,\tilde{o},o} = \dfrac{o^{up} - \left(\tilde{o}^{low} + s^{low}_{r,i,u,\tilde{o}}\right)}{w_{r,i,u,\tilde{o}}}$

Case II: If $\tilde{o}^{up} + s^{up}_{r,i,u,\tilde{o}} \leq o^{up}$ and $\tilde{o}^{low} + s^{low}_{r,i,u,\tilde{o}} \leq o^{low}$, then $\rho_{r,i,u,\tilde{o},o} = \dfrac{\left(\tilde{o}^{up} + s^{up}_{r,i,u,\tilde{o}}\right) - o^{low}}{w_{r,i,u,\tilde{o}}}$

Case III: If $\tilde{o}^{up} + s^{up}_{r,i,u,\tilde{o}} \geq o^{up}$ and $\tilde{o}^{low} + s^{low}_{r,i,u,\tilde{o}} \leq o^{low}$, then $\rho_{r,i,u,\tilde{o},o} = \dfrac{o^{up} - o^{low}}{w_{r,i,u,\tilde{o}}}$

Case IV: If $\tilde{o}^{up} + s^{up}_{r,i,u,\tilde{o}} \leq o^{up}$ and $\tilde{o}^{low} + s^{low}_{r,i,u,\tilde{o}} \geq o^{low}$, then $\rho_{r,i,u,\tilde{o},o} = 1$

Case V: If $\tilde{o}^{up} + s^{up}_{r,s,u,\tilde{o},o} \leq o^{low}$ or $\tilde{o}^{low} + s^{low}_{r,s,u,\tilde{o},o} \geq o^{up}$, then $\rho_{r,s,u,\tilde{o},o} = 0$

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[5] The probability approach presented here is well suited to integrate uncertainty about the true carbon sequestration function.

To check that the probability computations are correct, we can easily verify the validity of $\sum_{o} \rho_{r,i,u,\tilde{o},o} = 1$, which must hold for all regions, soil types, land uses, and initial carbon levels.

Soil status dynamics

Soil carbon class transition probabilities are then use in equation (4) to balance soil carbon levels. Particularly, the sum of all land falling in soil carbon class o at time t equals the transition probability weighted sum of all land use activities over all land uses u and initial soil carbon levels $\tilde{o}$ at the previous time.

$$(4) \qquad \sum_{u} X_{t,r,i,u,o} = \sum_{u,\tilde{o}} \left(\rho_{r,i,u,\tilde{o},o} \cdot X_{t-1,r,i,u,\tilde{o}} \right) \quad \forall t,r,i,o$$

Equation (5) is an accounting equation, which computes the carbon stock as the sum of the carbon stock at the beginning of period t plus $\Delta S_{t,r,i}$, the amount of carbon added or lost in period t.

$$(5) \qquad S_{t,r,i} = S_{t-1,r,i} + \Delta S_{t,r,i} \quad \forall t,r,i,\tilde{o}$$

The carbon stock change $\Delta S_{t,r,i}$ can be measured in two ways. It can be calculated as the sum of sequestration over all land use activities, i.e. $\sum_{u,\tilde{o}} \left(s_{r,i,u,\tilde{o}} \cdot X_{t,r,i,u,\tilde{o}} \right)$, or as the sum of soil carbon contents over all soil carbon level changes, i.e. $\sum_{u,o} \left(c_{r,i,u,o} \cdot X_{t,r,i,u,o} \right) - \sum_{u,o} \left(c_{r,i,u,o} \cdot X_{t-1,r,i,u,o} \right)$, where $c_{r,i,u,o}$ denotes the amount of contained soil carbon at the end of a period. Because of the approximation, the two measures will not be identical. However, as the width of the soil carbon classes converges to zero, so should the deviation between the two measures.

Empirical illustration

To illustrate the effects of dynamic carbon sequestration rates and isolate them from other effects, let us consider a simple land use decision model with only two management alternatives – labeled conventional tillage and zero tillage – on a 100 hectare field. The application to multiple regions and multiple soil types is straightforward and thus not needed here. Furthermore, we assume constant net profits of 33 for conventional tillage and 32 for zero tillage, a zero discount rate, a constant carbon price of 50 monetary units per carbon unit, and 30 time periods. Maximum and minimum soil carbon levels amount to 1.0 and 0.1 carbon units per hectare. The average initial soil carbon level ranges scenario specific between the minimum and maximum carbon value. Based on the average initial carbon value, an initial distributions of soil carbon classes is computed consisting of one or two adjacent classes such that the area weighted sum of the average carbon content of those carbon status classes equals the initial soil carbon level.

Using the above assumptions, and the equations described in the previous main section, this simple decision model was programmed in GAMS[6]. A reference scenario was established by solving the model for a very high number of soil carbon classes, i.e. 2000[7]. Subsequently, the model was solved over 1500 additional scenarios with 2, 4, 6, …, and 1000 soil carbon classes and three alternative assumptions about the initial soil carbon status. Major results for this exercise are given below.

Optimal management path

The optimal management path gives the best sequence of management decisions over time, where "optimal" relates to the specified objective function of the model in question and

[6] The full model is available from the author.

[7] Even for the small hypothetical model used here, it was not possible to portray all soil management trajectories explicitly and find the absolute maximum.

associated assumptions. Our illustrative model has a profit maximizing objective with total profit equaling the sum over all time periods of market profits plus carbon sequestration premiums minus carbon emission taxes. Consequently, the only dynamic element in our model is the dynamically updated soil carbon status, which leads to dynamically updated sequestration rates. If we would use constant carbon sequestration rates for the alternative management decisions instead, our simple model would become a sequence of repeated autarkic decisions. Thus, the optimal management in all periods would be identical. With dynamically changing carbon levels and sequestration rates, however, the profit of a soil management option in a certain period depends on the management decisions taken in all previous periods. Consequently, we might observe multiple changes in the optimal soil management decision over time.

Before examining the numerical results, some qualifications should be made. First, the number of possible management paths, which are implicitly represented is very high even for the small model we have adopted. Thus, there are probably many "good" paths, which have almost the same objective function value as the very best path. The differences in the objective function values between several very good paths may be of a negligible magnitude for practical purposes, i.e. they may amount to less than 1 cent per 100 hectares. Second, these almost perfect management paths can be noticeably different from the perfect path with the highest objective function value[8]. Small changes to the model such as a more detailed grouping of the soil carbon range may be enough to slightly change the objective function value of all management paths and make a different management path optimal.

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[8] Suppose we had a crop rotation specific model, where the best soil and crop management path consists of 20 years of crop rotation 1 under zero tillage followed by 10 years of crop rotation 2 under conventional tillage. Alternative soil and crop management paths yielding almost the same objective function value might involve the use of different crop rotations.

The above comments are graphically illustrated for different model solution properties. Figure 2 shows the optimal objective values for different numbers of soil carbon classes and for different assumptions about soil management flexibility. Two points are worth highlighting. First, as expected, if management can be changed in each period, the objective values are noticeably higher than otherwise. The difference between variable management and constant management can be interpreted as the dead weight loss from contracts, which restrict farmers to maintain a certain management over a certain time horizon. Second, as the number of soil carbon classes increases, the optimal objective values converge relatively fast to a stable value.

A more detailed indicator of the optimal soil management is given by the number of management changes between adjacent periods. Figure 3 illustrates both the effect of using few versus many soil carbon classes and the effect of different initial soil carbon levels on the number of management changes. Several observations can be made. First, the number of observed soil management changes decreases as the number of soil carbon classes increases. Actually, under the assumption of constant carbon prices and constant strategy profits, the number of soil management changes converges to one for a sufficiently high number of the soil carbon classes. Particularly, using 2000 soil carbon classes we find the optimal soil management path to consist of 25 periods conventional tillage followed by 5 periods of zero tillage.

Second, the convergence of the management change indicator is relatively slow and fluctuating compared to the convergence of the objective values. For example, if we consider the line representing a medium initial soil carbon level, we observe 3, 1, 4, 6, and 5 management changes at optimality using as many as 186, 188, 190, 192, and 194 soil carbon classes, respectively. This behavior confirms that disaggregated measures are more sensitive to model changes than aggregate ones. It also confirms the existence of alternative management paths, which yield almost identical objective values.

How different are the optimal management paths obtained by using more or less soil carbon classes in terms of the cumulated use of available management options over the whole time horizon? An answer is given in Figure 4. By using 40 or more soil carbon classes, we find a relatively stable division of the 3000 cumulative hectares in 2500 cumulative hectares of conventional tillage and 500 hectares of zero tillage. Thus, if soil carbon levels are represented at a sufficiently high resolution, the optimal soil management trajectory contains about 25 periods of conventional tillage and 5 periods of zero tillage. The implications of the optimal management path for soil carbon levels are given in the following section.

Soil carbon dynamics

Dynamically changing soil management decisions will affect the soil carbon emission and soil carbon stock levels. For the purpose of this article, we are interested in answering two questions. First, how sensitive is the soil carbon dynamics to the number of soil carbon classes? Second, how different are the two above described soil carbon measures?

To examine the first question, I computed for each time period the deviation between the total carbon sequestration of the reference scenario with 1000 soil carbon classes and other scenarios with considerably fewer classes. The whole process was repeated three times for three different assumptions about the initial carbon status. Figure 5 shows the sum of the squared deviation for different soil status classes and different initial carbon states. The assumption of a medium initial soil carbon status leads to the highest deviation. As many as 60 soil status classes are needed to ensure an accurate portrayal of the optimal carbon sequestration path. Under the assumption of either low or high initial carbon levels, convergence to the carbon sequestration path of the reference scenario is faster.

To suggest an answer to the second question, the deviation between the two alternative soil carbon measures is shown in Figure 6. We find that for more than 20 soil status classes, differences are relatively small compared with scenarios where less than 10 classes are used.

The initial carbon level does not seem to impact the speed of conversion between the two measures. Thus, the number of soil carbon classes needed for a consistent carbon accounting appears to be less than the number of classes needed to find the optimal carbon dynamics. One should note, however, if only very few soil carbon classes are used and soil carbon is accounted through net emission rates rather than changes in the distribution of soil carbon classes, cumulative sequestration effect may exceed the maximum or stay below the minimum soil carbon level.

Summary and Conclusions

The dynamic interactions between soil carbon sequestration rates, soil management decision paths, and soil carbon levels has been ignored in existing land use decision models. This seems unsatisfactory because the optimal soil management may involve multiple management changes in the future depending on the course of political, technological, and environmental developments. In addition, an efficient internalization of the carbon emission externality requires an accurate knowledge of actual net emission quantities. Carbon sequestration efforts should neither be over or underpaid relative to their impact on atmospheric carbon concentrations. Unfortunately, the consequences of using constant instead of dynamically updated sequestration rates are not always obvious. Variable soil management decision paths can also be simulated through other dynamic model components, i.e. dynamically changing resource endowments, factor prices, commodity demands, or carbon prices. However, while these conditions may produce dynamic output and hide the simplified representation of carbon sequestration rates, they do not justify it. Few people would accept a temperature model where the effect of atmospheric CO_2 concentrations on temperature is constant.

This paper proposes a new approach to integrate dynamically updated soil carbon sequestration rates in a manner that is computationally feasible. It can be implemented in any mathematical programming model, where land use decisions are optimized. To adopt this approach, carbon sequestration rates have to be established for all included locations, management alternatives, and discrete soil carbon classes. These rates can be based on estimated continuous functions or observed data. Biophysical field process models such as EPIC (Williams et al.) or CENTURY (Parton et al.) may be a good source to simulate a complete and consistent set of necessary sequestration rates for a large set of alternative locations and alternative management practices. The total range of soil carbon considered for the analysis is partitioned into several classes of even or uneven size. Subsequently, for each combination of location, soil management, and initial carbon level, transition probabilities will be computed that contain the probability of the soil carbon level staying in the same or reaching any other soil carbon class. These transition probabilities are used to dynamically update soil carbon levels before and after each management period. In turn, sequestration rates are adjusted corresponding to the change in soil carbon levels.

An explicit representation of all possible soil management trajectories is computationally infeasible for the majority of empirical models. The approach presented here uses an implicit representation of all soil management trajectories. The advantage of this implicit representation is that the vast majority of inferior trajectories is automatically disregarded by the mathematical programming solver during the optimization process. This makes this method computationally feasible for larger models. Computational feasibility, however, is inversely related to the accuracy of the model. As the number of soil carbon classes decreases, the model becomes easier to solve at the expense of accuracy. The simple empirical model employed here suggests a minimum of at least 20 soil status classes. However, this observation should not be generalized. Instead, a sensitivity analysis similar to the one conducted here could be used to determine the appropriate number of classes for individual models.

In general, the presented mathematical programming approach can be used to accurately estimate the time path of soil carbon sequestration, to examine incentives for land owners to adopt or abandon soil carbon sequestration practices at various times in the future, and to better analyze the impact of various future scenarios on the optimal soil management decision. These analyses could benefit policymakers, carbon credit brokers, and private decision makers. Additionally, this approach can be employed to represent the time path of other agro-environmental stock qualities such as soil erosion and soil nutrient level (phosphorous). A joint representation of soil carbon status, erosion, and soil nutrient level would require the computation of location and soil management specific transition probabilities for soil carbon, erosion, and nutrient level.

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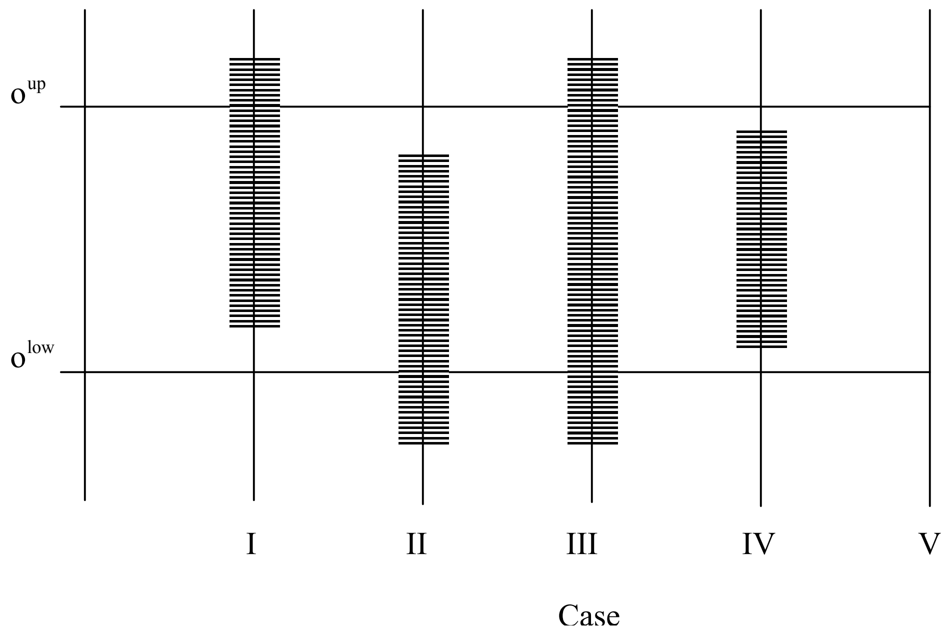

Figure 1 Generally possible cases of soil carbon change from initial soil carbon class $\tilde{o}$ to subsequent class o. The hatched rectangle represents the interval of width $w_{r,i,u,\tilde{o}} = \left(\tilde{o}^{up} + s^{up}_{r,i,u,\tilde{o}}\right) - \left(\tilde{o}^{low} + s^{low}_{r,i,u,\tilde{o}}\right)$.

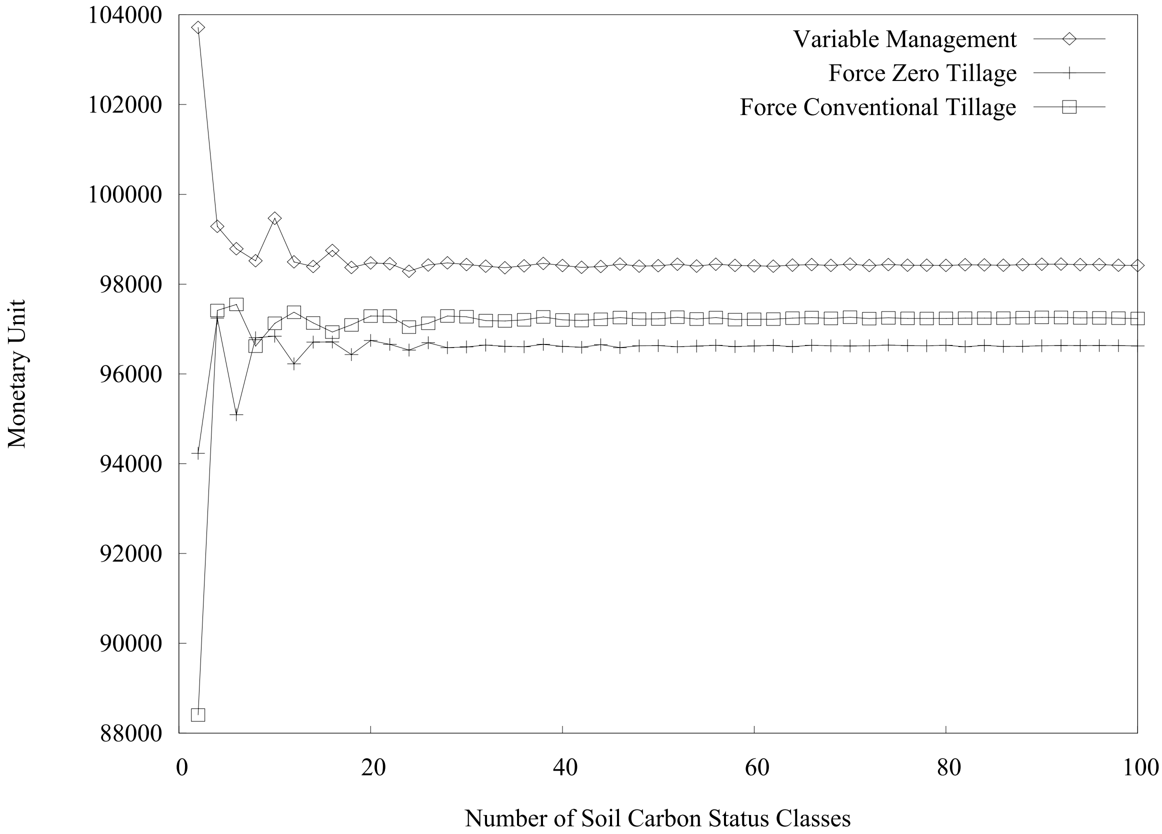

Figure 2 Objective function values at optimality for different numbers of soil carbon classes and different soil management restrictions

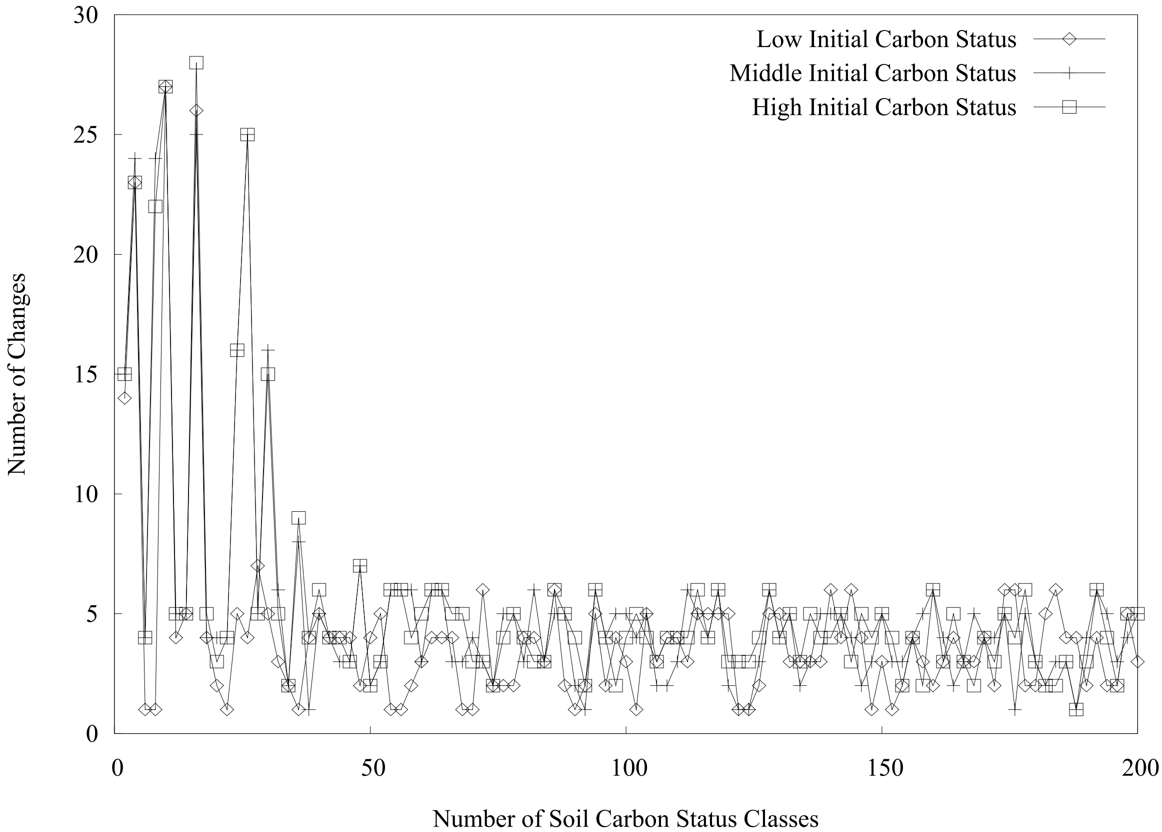

Figure 3 Number of soil management changes between adjacent periods summed over all periods. By definition, a management change exists if the allocation of management in a certain period differs from the previous period's allocation. No differentiation is made between the magnitude of change. For example, a change in management is already present if the chosen management in period t is 90.3 ha zero tillage and 9.7 ha conventional tillage while in period t+1 we find 90.5 ha zero tillage and 9.5 ha conventional tillage.

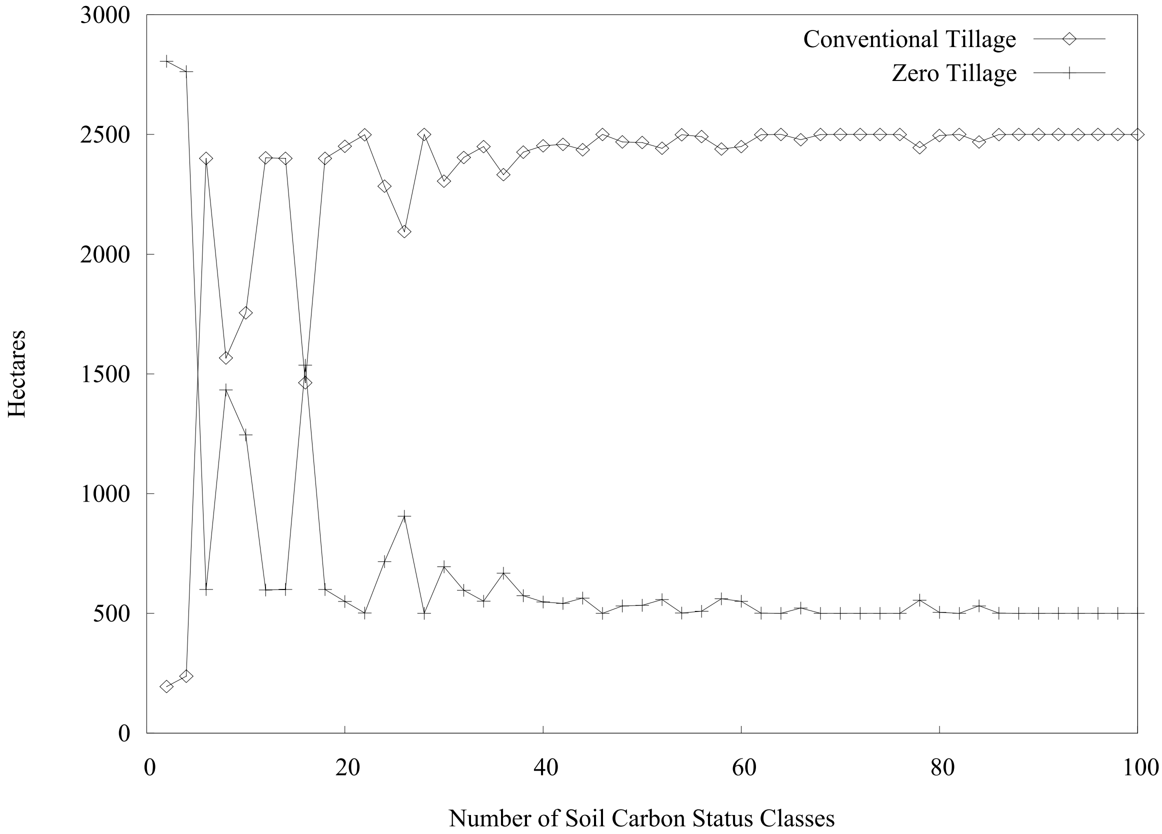

Figure 4 Area of each management alternative summed over all time periods

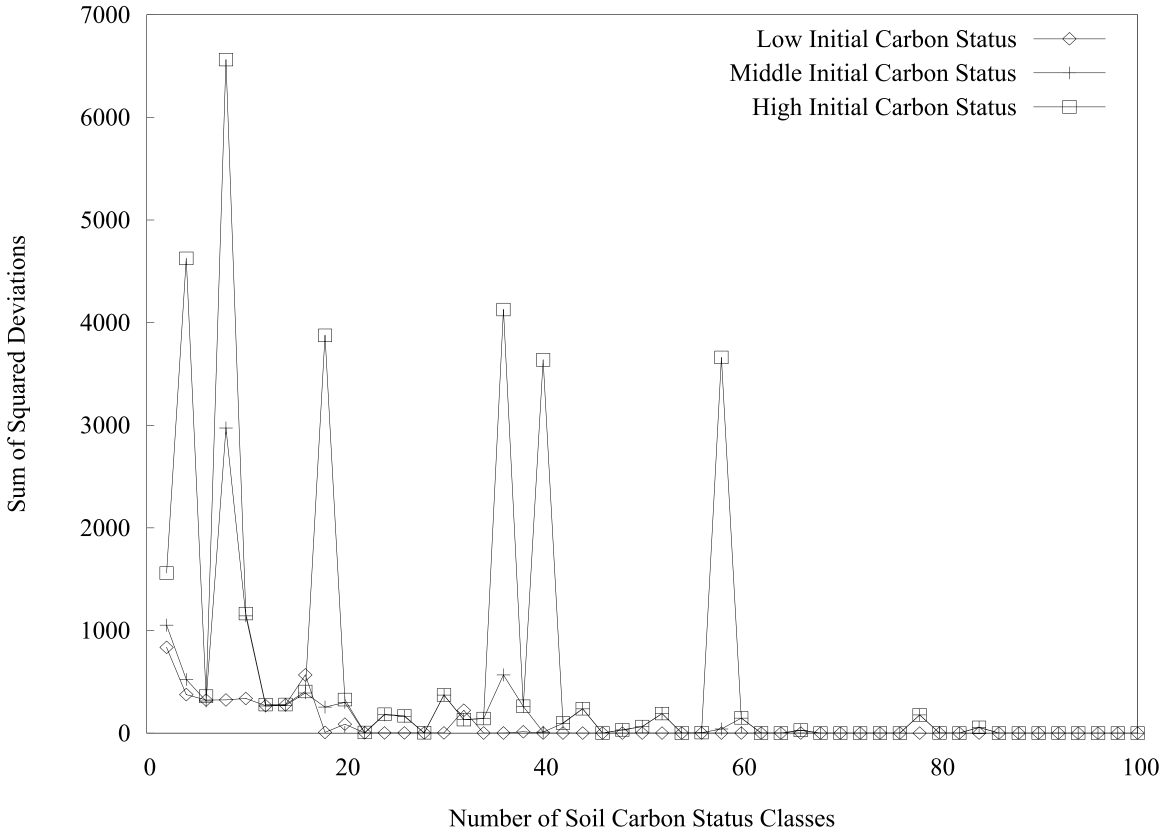

Figure 5 Deviations between optimal carbon sequestration path using a reduced number of soil carbon classes and the reference carbon sequestration path obtained by using 1000 soil carbon classes.

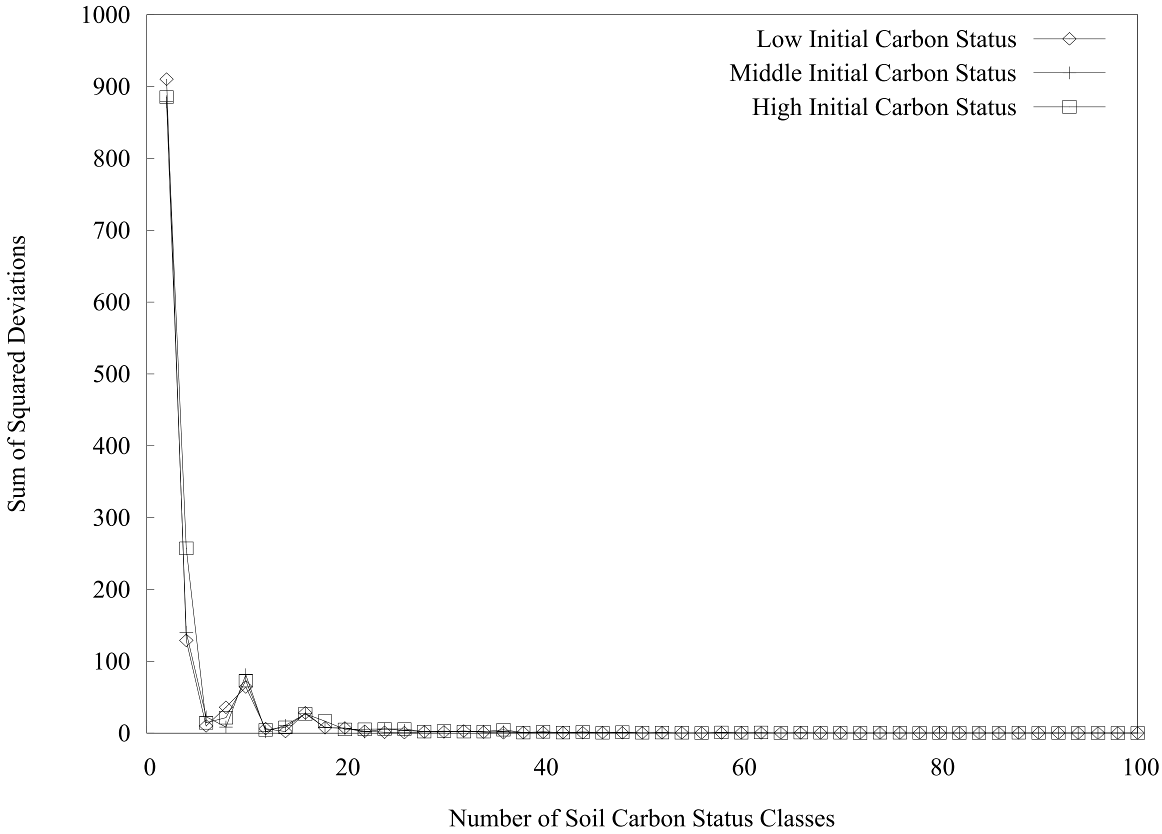

Figure 6 Deviation between two alternative soil carbon sequestration measures as function of the number of soil carbon classes used. The first measure is given by the area weighted sum of soil carbon class and soil management specific sequestration rates. The second measure is calculated as the difference in area weighted soil carbon class levels between the current and the previous period.