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Optimal Social Security Taxation in Spain

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Optimal Social Security Taxation in Spain

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Abstract

We use a dynamic general equilibrium model with overlapping generations of long-lived agents facing lifetime uncertainty, population growth and a Pay-As-You-Go social security system, in order to analyze the optimal tax structure of the Spanish social security system, as it currently stands and in the light of the forecasted demographic changes.

We characterize and solve the equilibrium of the model for different demographic scenarios and relevant parameter values, making use of the concept of optimality of social security taxes developed by İmrohoroğlu, İmrohoroğlu and Joines (1995).

Our main findings indicate that the optimal (average utility maximizing) taxes need not necessarily be equal to zero. Under plausible parameterizations we find positive tax rates, albeit lower than the current ones, supporting the hypothesis that PAYG systems may increase economic welfare. Dynamic inefficiency plays a key role in this outcome.

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1 Introduction[1]

Most industrialized and many developing countries have expanded considerably their social security systems in the postwar period. The greatest share of social security expenditure in these economies comes from contribution-related retirement benefits. In most OECD countries, these are financed through a Pay-As-You-Go (PAYG) system so that current workers, who pay the social security tax (contribution), finance the pension benefit of those retired in the same year.

In light of the expected demographic changes, these countries will face "the desire to find a way out of the Social Security System's long-run financing problem", in words of Kotlikoff (1996).[2] The key factor underlying this problem is the prospect of a change in the demographic structure of the population in the medium and in the long run. This change basically consists of a decrease in both fertility and mortality rates that will bring about a considerable increase in the dependency ratio of pensioners to workers.[3]

In the case of Spain, demographic projections show that, if the trend in the fertility and mortality rates remains constant, the potentially active population will remain close to stable until the early 2000s. The old-dependency ratio will increase progressively thereafter. From the year 2021 onwards, it will rise considerably due to the entry of the cohorts who were born in the fifties to early sixties, the so-called Baby-Boomers, into the group of people older than 65. This situation will become worse in the following years and will start to ease only towards the year 2050. Thus, demographic projections for Spain imply that in the medium and long run, there will be a sharp increase in the old-dependency ratio (see Figure 1 below).

A Beneficial Role for PAYG Social Security Systems?

Several authors have focused on the possible effects that PAYG social security systems could have on the savings behaviour of individuals (see, for instance,

[1] This paper is based on the second chapter of the author's PhD dissertation, "A Dynamic General Equilibrium Analysis of the Spanish Social Security System" defended at the EUI in June 2000.

[2] See Arjona (2000b) for a survey of the main micro-simulation studies and dynamic general equilibrium models dealing with the issue of pension reform.

[3] The old-dependency ratio is defined as the ratio that relates the number of people aged 65 and more to those which are currently working.

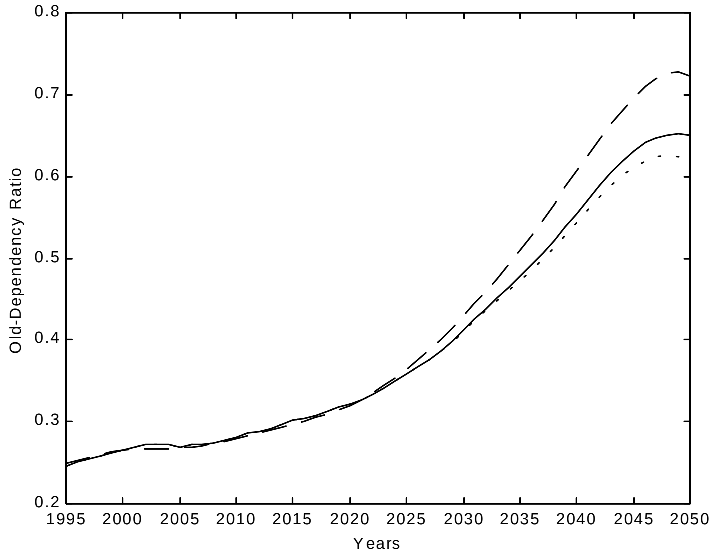

Figure 1: Old-Dependency Ratio (1995-2050). Dashed (upper) line: United Nations medium-growth variant; Continuous line: Spanish Insitute of Economics and Geography (IEG) low-growth scenario; Dotted (lower) line: IEG high-growth scenario.

Feldstein and Pellechio (1979) or Blinder and Gordon (1981), for U.S. microeconomic evidence on the issue). Most of these studies have concluded that social security reduces individual saving.[4] Keeping this in mind, a stream of the literature has moved on to analysing whether an unfunded social security system could have a beneficial role in the economy, i.e. whether the "optimal" (average utility maximizing) level of social security replacement rates could be different from zero and under which circumstances that may happen.[5] The

[4] Others, nevertheless, claimed opposite results (see Barro's (1978) contribution).

[5] Although this paper does not examine the issue of optimal taxation *à la Ramsey*, it seems necessary to briefly comment on some key contributions to that strand of the literature.

A classic problem in the economic literature is the one of financing a stream of government expenditure when lump-sum taxation is not available, the so-called "second-best" approach to optimal taxation, whose seminal contribution dates back to Ramsey's (1927) and Mirrlees' (1971) work. Optimal Ramsey policies are derived as solutions to the "Ramsey problem" in which the government chooses a set of taxes such that the associated allocations chosen by individual agents and firms solve the competitive equilibrium and the social welfare function is maximized. That is, the government in the economy chooses the policy which maximizes its objective function, using a certain intergenerational discount factor, taking the response of individual agents to this policy as given.

Several papers have analyzed optimal taxation in dynamic general equilibrium models (see, for instance, Ambler (1999), Chari, Christiano and Kehoe (1995), or Judd (1999), for output of these studies can be summarized in two main lines of argument:

1. On the one hand, a strand of the literature found no positive role for such a scheme, concluding that a replacement rate of zero (or close to zero) is the "optimal" one. Feldstein (1985) concludes that, unless there is a large number of myopic agents, the optimal level of social security benefits is relatively low. Hubbard and Judd (1987) and Auerbach and Kotlikoff (1987) show that, in life-cycle economies, one can find a substantial reduction in the steady state capital stock and welfare when payroll taxes are used to finance the social security system.[6]

2. On the other hand, further research carried by İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995), who examine an overlapping generations economy in which agents face mortality risk, individual income risk and borrowing constraints, found a beneficial role for a PAYG social security system, with utility-maximizing replacement rates in the positive range.

   This beneficial role is partly generated by the fact that such a scheme substitutes for the missing annuities markets, reducing savings and hence capital overaccumulation, and helping consumers in taking more desirable consumption/saving decisions in the face of uncertain lifetimes (it also helps the economy as a whole in choosing a more efficient allocation of aggregate consumption). Therefore, a PAYG social security system

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an analysis of the main predictions and a survey of the literature). Most of these studies have centered on the standard neoclassical growth model with infinitely-lived agents and aimed at supporting or challenging the Chamley-Judd results. Chamley (1986) and Judd (1985) show that the optimal income tax policy is to tax capital at arbitrarily high rates in the short-run. Capital income taxes would then converge to zero in the long-run (the supply of capital is inelastic in the short-run and, hence, taxing it at confiscatory rates brings the economy close to a 'first-best' setting in which lump-sum taxes are available).

A few other studies have focused on the Ramsey problem in life-cycle models. Among them are Atkinson and Sandmo (1980), Park (1991) and Erosa and Gervais (1998). What differentiates optimal taxation results for overlapping generations models from the results for representative agent models is the existence of positive capital income tax levels in the steady state. Both types of models can lead to positive tax rates on labour income in the long run.

The author is currently working on the optimality of social security designs within the framework of the literature on optimal taxation in life-cycle models. For that purpose he is analzying the Ramsey problem in a large overlapping generations model with an unfunded PAYG social security system.

[6]Feldstein (1985) examines a model in which agents' expectations mechanism departs from perfect foresight. Hubbard and Judd (1987) use a life-cycle economy in which agents face borrowing constraints and lifetime uncertainty. Auerbach and Kotlikoff's (1987) overlapping generations model is populated by agents which can borrow against lifetime income and face no lifetime uncertainty.

in the model lowers the capital stock, and may increase aggregate consumption (if the replacement rate is below a certain "optimal" level, or decrease it thereafter, offsetting the benefits of this more efficient allocation).[7] The authors obtain the maximum benefit of social security at an "optimal" replacement rate of around 30% for their U.S. calibration.[8]

* * *

In this paper we use a dynamic general equilibrium model with overlapping generations to analyze the optimal structure of the Spanish PAYG social security system, as it currently stands and in the light of the forecasted changes that will occur in the demographic structure in the medium to the long run. We focus on the contribution-related pension benefits of the General Regime, which are the key element of the social security expenditure.[9] Its aim is to analyze and quantify the social security (PAYG) taxes, and associated replacement rates, which maximize average utility for the current and forecasted demographic structure of Spain.

We focus on the concept of optimality outlined above and build a model economy, based on the Auerbach and Kotlikoff (1987) framework and consisting of overlapping generations of long-lived agents who face lifetime uncertainty, following the approach of Hubbard and Judd (1987) and Imrohoroglu, Imrohoroglu and Joines (1995). The demographic structure, in the form of a stable population (see Rios-Rull (1994) or Cubeddu (1996)), is introduced exogenously into the model using two datasets provided by the Spanish Institute

[7] Because their benchmark economy is "dynamically inefficient" in the absence of an unfunded social security system (in the sense Diamond (1965)), an unfunded social security arrangement lowers the steady-state capital stock and provides a higher rate of return than physical capital.

[8] In a further contribution, İmrohoroğlu, İmrohoroğlu and Joines (1999) incorporate a fixed factor of production in the model. The existence of this new factor rules out any possible dynamic inefficiency and impedes the overaccumulation of capital.

In such a setup, as the capital stock increases, its rate of return decreases and the implied decline in the discount rate makes the price of land rise. This increase in the price absorbs the savings of workers and gives capital gains to the owners of this factor (land). Because the price of land can increase without bound, overaccumulation of capital is ruled out and the introduction of an unfunded social security system lowers aggregate consumption and welfare.

It is worth noting, however, that this mechanism holds insofar as the discount rate is below a certain level (1.032 for their calibration). The authors argue that this value is higher than the empirical estimates which are widespread in the literature, and therefore the market value of land does not exceed the value of capital for all reasonable discount rate levels.

[9] See Arjona (2000a) for a description of the Spanish social security system in place.

of Economics and Geography (IEG) and the United Nations (UN). In total, three different demographic scenarios are used.

Agents in the model retire after having worked for 45 years. From the mandatory retirement age until they die, at age I, they receive pension benefits. In order to finance those pension benefits, which are calculated according to the standard rules, the government has established a corresponding social security contribution rate such that thegovernment budget constraint is balanced each period. The model is calibrated to fit the forecasted demographic patterns and Spanish microeconomic evidence.

The main findings indicate that when we compute the average utility-maximizing taxes, and associated replacement rates, for calibrations in which the discount factor is less than unity, the optimal social security tax (and replacement rate) is equal to zero, eliminating any possible beneficial role for a PAYG social security system.[10] These results hold for different demographic scenarios and hence, population growth rates and are mainly due to the fact that the economy is dynamically efficient at a zero tax (replacement rate). Results change dramatically, however, when the chosen discount factor is larger than unity, the case we are most interested in. The average utility-maximizing (optimal) replacement rates rise to 40 percent (for the initial steady state) and 10 percent (for the final steady state). In the absence of social security, this economy is dynamically inefficient. Hence, the introduction of social security not only provides a higher rate of return than physical capital but also reduces private savings, and a replacement rate of 40 percent for the initial steady state or 10 percent for the final steady state eliminates any possible overaccumulation of capital. A newly born agent would prefer a social security system with a replacement rate of 40 percent over any other possible alternative arrangements. The optimal tax rates, associated with these replacement rates are 10.42 percent in the initial steady state and 7.13 percent in the final steady state, both of them considerably lower than the current Spanish social security payroll rate.

The paper is organized as follows. Section 2.2 describes the model economy. Section 2.3 characterizes the consumer problem, derives the optimality conditions and describes the measures of utility and welfare used. Section 2.4 describes the sets of experiments which we run in the paper, as well as the parameterization, calibration and computational procedures we use in order to obtain the equilibrium of the model. Section 2.5 summarizes the results.

[10] An additional result, however, is that the tax rates associated to a replacement rate of 100 percent (not average utility-maximizing) practically double (become 90 higher) between the initial and final steady states, as a result of the expected demographic changes.

Finally, section 2.6 concludes.

2 The Model

The model economy builds on the Auerbach and Kotlikoff (1987) setup and consists of overlapping generations of agents who live for a maximum of I periods, after which death is certain, where ages will be denoted by subindex $i \in \mathcal{I} \equiv \{1, ..., I\}$. Time will be denoted by subindex t, $t \in \mathcal{T} \equiv \{1, ..., T\}$. A new generation is born at each point in time. Agents belonging to the same generation are identical. There is single consumption good.

At each point in time, agents face a probability of survival between age i and age $i + 1$, which is denoted by s_i, being $s_I = 0$. The unconditional probability of reaching age i is therefore $s^i = \prod_{k=1}^{i} s_{i-k+1}$. The population in the model grows at a constant rate, n, each period. Cohort shares, $\{\mu_i\}_{i \in I}$, are calculated by $\mu_i = \frac{\mu_{i-1}\, s_i}{(1+n)}$, where $\sum_{i=1}^{I} \mu_i = 1$, generating the so-called "stable population" structure. Hence, cohort shares are time invariant. Aggregate quantities in the economy will be weighted averages of individual quantities, where cohort shares will act as weights.

2.1 Preferences

Agents in the economy derive their utility from consumption and leisure. Each agent is assumed to have preferences which can be represented by the following discounted utility function:

$$E \sum_{i=1}^{I} \beta^{i-1} \, s^i \, U\left(c_{t',i}, x_{t',i}\right) \tag{1}$$

$c_{t',i}$ stands for consumption in period $t' = t + i - 1$ of an age i agent born in t. Correspondingly, $x_{t',i}$ stands for leisure, $1-$ $l_{t',i}$ ($l_{t',i}$ stands for labour supply), and β^i is the subjective discount factor. The utility function satisfies the standard properties, i.e. it is strictly concave, increasing in consumption and leisure, and satisfying the Inada conditions.

The budget constraint of an age i agent in period t is as follows:

$$
\begin{aligned}
c_{t',i} + s_i\, a_{t'+1,i+1} &= (1+r_{t'})\; a_{t',i} + y_{t'} \qquad (2)\\
\text{where } y_{t'} &= \begin{cases} w_{t'}(1-\tau_{t'})\,\varepsilon_i\, l_{t',i} & \text{if } i \leq I_R \\ b_{t'} & \text{if } i > I_R \end{cases}\\
a_{t,1},\; a_{t,I+1} &= 0
\end{aligned}
$$

where, for an individual of age i, $a_{t',i}$ stands for the asset holdings (accumulated net wealth), $a_{t'+1,i+1}$ is next period's asset holdings ($s_i\, a_{t'+1,i+1}$ is gross savings), $\tau_{t'}$ stands for the social security payroll tax and $b_{t'}$ for the retirement benefit.

A unit of time of an agent aged i, can be transformed into one unit of leisure or ε_i exogenous age-specific efficiency units of labour input, as in Auerbach and Kotlikoff (1987). The gross interest rate is $R_{t'} = (1 + r_{t'})$ and the real wage rate per efficiency unit of labour in terms of the single consumption good is $w_{t'}$.

Agents are endowed with one unit of time, which they allocate to leisure or labour. They choose how much labour to supply between age 1 and age I_R and must retire and supply no labour thereafter, relying exclusively on private savings and social security benefits in order to finance their consumption.

We also impose, as in Auerbach and Kotlikoff (1987), a non-negativity constraint on the labour supply decision, i.e. if the agent were to demand more than one unit of leisure in a given period, the individual would have to retire and supply no labour. This is represented by the following constraint:

$$l_{t',i} > 0 \qquad (3)$$

In each period agents receive capital income, $(1+r_{t'})\; a_{t',i}$, and alternatively labour income (if $i \leq I_R$), $w_{t'}\,(1-\tau_{t'})\varepsilon_i\, l_{t',i}$ or retirement benefit (if $i > I_R$), $b_{t'}$, and they decide how much to allocate to savings, $s_i\, a_{t+1',i+1}$, and leisure, $x_{t',i}$, on the basis of the life cycle model of behavior.[11] Agents accumulate assets to smooth their consumption over time.

There exists a market for private annuities where individuals insure against mortality risk. Here, as in Rios-Rull (1994), it is assumed that agents of the same age cohort sign a contract stipulating that survivors will share the assets

[11] Notice that with this specification of (2) any liquidity constraint is disregarded and households can borrow against lifetime income, including retirement benefits, at the going market rate of interest.

(or debts) of agents that die so that the next period's asset holdings (debts) are this period's savings (borrowings) divided by the probability of surviving. Agents are born with zero non-human wealth, $a_{t',1} = 0$ and leave no intended bequests, $a_{t',I+1} = 0$.

2.2 Technology

The economy produces a single good from aggregate capital and labour according to a standard production function with constant returns to scale:

$$Y_t = f\ (K_t,\, L_t) \tag{4}$$

where K_t is the aggregate capital stock and L_t is the aggregate labour input. Output can be used for consumption in the same period as production takes place or can be used for increasing next period's capital stock. Capital depreciates at rate δ. Aggregate output is represented by Y_t. This function has the property that $\lim_{K_t \to \infty} f_K\,(K_t, L_t) = 0$, $\forall\ t$, and $L_t > 0$.

Firms hire physical capital and effective labour until factor prices equal marginal products:

$$r_t = f_K\,(K_t,\, L_t) - \delta \tag{5}$$
$$w_t = f_L\,(K_t,\, L_t) \tag{6}$$

2.3 Social Security System

There is a government in the model which manages the retirement insurance program. The government collects the social security payroll tax and makes the corresponding transfers in form of retirement benefits to the agents once they retire. This retirement insurance program adopts the form of a publicly administered pure PAYG system. This implies that government revenue must equal government expenditure in every period. The government uses the social security contributions to finance its expenditure flows at every point in time.

In this system, agents contribute a certain percentage of their income ($\tau_{t'}\,\varepsilon_i\, w_{t'}\, l_{t',i}$), the social security contribution, to the system during I_R years of working time. Government's revenue from taxation is therefore given by:

$$T_t = \tau_t \, w_t \sum_{i=1}^{I_R} \mu_{t,i} \, \varepsilon_i \, l_{t,i} \tag{7}$$

Define average earnings over the last f years of the working life of an age $i \in [I_R + 1, I]$ agent, born in $t,$ as:

$$z_t = \frac{1}{f} \sum_{i=I_R-f+1}^{I_R} \varepsilon_i \, l_{t'-I_R+i,i} \, w_{t'-I_R+i} \, (1 - \tau_{t'-I_R+i}) \tag{8}$$

After their retirement, for $i \in [I_R + 1, ..., I]$, agents receive benefits according to the following formula:

$$b_t = \rho \, z_t \tag{9}$$

where ρ is the social security replacement rate.

The total government expenditure in retirement benefits is therefore:

$$G_t = \sum_{i=I_R+1}^{I} \mu_{t,i} \, b_t \tag{10}$$

The budget constraint of the government is therefore balanced period by period, $T_t = G_t$:

$$\tau_t \, w_t \sum_{i=1}^{I_R} \mu_{t,i} \, \varepsilon_i \, l_{t,i} = \sum_{i=I_R+1}^{I} \mu_{t,i} \, b_t \tag{11}$$

3 The Optimal Social Security Tax

In the following sections, we will compute the optimal social security tax as the social security tax level that maximizes a measure of average utility, which we describe in section 3.1.2.

In this section we formulate and characterize the solution to the consumer's problem, and define the set of measures of utility and welfare that we will subsequently use in examining the optimality of social security tax rates.

3.1 The Consumer's Problem

All agents maximize (2.1) subject to (2.2), i.e. they solve the following optimization problem:

$$\max\ E\sum_{i=1}^{I}\beta^{i-1}\ s^{i}\,U\left(c_{t',i},x_{t',i}\right) \tag{12}$$

$$\begin{aligned}
&s.t.\\
&c_{t',i}+s^{i}\,a_{t+1',i+1}=(1+r_{t'})\ a_{t',i}+y_{t'} \\
&\text{where } y_{t'}=\begin{cases} w_{t'}(1-\tau_{t'})\,\varepsilon_i\,l_{t',i} & \text{if } i\leq I_R\\ b_{t'} & \text{if } i>I_R\end{cases}\\
&a_{t,1},\ a_{t,I+1}=0
\end{aligned} \tag{13}$$

3.1.1 Consumer's Optimality Conditions

Let $\lambda_{t',j}$ denote the Lagrange multiplier associated to the consumer's budget constraint (2). The first order conditions of the consumer's problem are as follows:[12]

$$\beta^{i-1}\,s^{i}\,U_{c_{t',i}}-\lambda_{t',i}=0 \tag{14}$$

$$\beta^{i-1}s^{\iota}\,U_{l_{t',i}}+\lambda_{t',i}w_{t'}(1-\tau_{t'})\varepsilon_i=0 \tag{15}$$

$$-\lambda_{t',i}s^{i}+\beta\lambda_{t'+1,i+1}(1+r_{t'})=0 \tag{16}$$

$$a_{t',1},\ a_{t',i+1}=0 \tag{17}$$

Combining (14) and (15), we obtain:

$$\frac{U_{l_{t',i}}}{U_{c_{t',i}}}=-w_{t'}(1-\tau_{t'})\varepsilon_i \tag{18}$$

[12] Let us denote the derivative of the utility function with respect to $c_{t',j}$ and $l_{t',j}$ as $U_{c_{t',j}}$ and $U_{l_{t',j}}$, respectively. Notice that the functional specification of the utility function guarantees that $c_{t',j}>0$ and $l_{t',j}>0$, $\forall j,t$.

3.1.2 Measures of Utility and Welfare

In order to proceed with a comparison of alternative social security arrangements, we first need a measure of *average utility*. We borrow the concept from İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995), who define average utility as the expected discounted utility a newly born individual derives from the lifetime consumption policy function under a given social security arrangement, Ω, i.e.

$$W(\Omega) = E \sum_{i=1}^{I} \beta^{i-1}\, s^i\, U\left(c_{t',i}, x_{t',i}\right)$$

Besides, we need a measure to help us quantify the welfare benefits (costs) of alternative social security arrangements.[13] For that purpose, we define a baseline economy, i.e. our benchmark calibration with the initial population structure (see Tables 1 and 2), and compute the *compensating variation* (a lump-sum consumption compensation) necessary to make an individual indifferent between the baseline arrangement in which he receives the additional compensation and the one under analysis. The compensating variation is expressed relative to output in the baseline economy.

4 Simulations

In this section we start by describing the different sets of experiments which we run for the model economy. We then move on to calibrating and parameterizing the model, using Spanish microeconomic evidence, so that it mimics some important features of the Spanish economy during the 1990s. We conclude the section by describing the computational strategy used in order to solve the model.

4.1 Experiments

We focus our analysis on solving the consumer's problem for replacement rates varying from zero to one-hundred percent, and determining the average utility

[13] As in İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995), Erosa and Gervais (1998) or DeNardi, İmrohoroğlu, and Sargent (1999).

maximizing social security taxes, and associated replacement rates, under different parameterizations and demographic scenarios (for both the initial and final steady state) emphasizing "dynamic efficiency" considerations. We also compute the measures of average utility and welfare presented in 3.1.2, in order to be able to compare the different social social security arrangements.

4.2 Parameterization and Calibration

In the model, agents are born into adulthood at age 21 and can live up to age 85, after which death is certain. They supply labour until retirement age, which is imposed at age 65. This implies that $I_R = 45$ and $I = 65$.

Table 1: Calibrated Parameter Values[14](Baseline Case)

β	σ^c	σ^l	A	α	δ	ε	f	n
0.985	2	6.5	1	0.40	6.50	ECBC (1991)	15	0.015

4.2.1 Preferences

The utility function is parameterized as:

$$u\left(c_{t',i}, x_{t',i}\right) = \frac{c_{t',i}^{1-\sigma^c}}{1-\sigma^c} + \frac{\left(1-l_{t',i}\right)^{1-\sigma^l}}{1-\sigma^l} \tag{19}$$

so that it is additive separable.

In order to calibrate preferences, the parameters β, σ^c and σ^l must be chosen. In overlapping generations economies, theory does not impose any restriction on the size of the discount factor, as pointed out by Rios-Rull (1996) and İmrohoroğlu, İmrohoroğlu and Joines (1995). Traditionally, authors have used a subjective discount factor less than unity, in accordance with representative agent models. However, empirical studies which introduce lifetime uncertainty in the framework of a life-cycle model find values greater than unity for the discount factor in the majority of cases (ranging from the 1.011 of Hurd (1989)

[14]The values of the calibrated parameters in Table 1 are for yearly model periods (i.e. a reference model with 70 generations). In order to run the simulations we have chosen a model period of 5 years (the model has 14 generations) and adapted the values of the parameters accordingly.

to values over 1.020 in Hotz et al. (1988)).[15] The discount factor, β, is set to be 0.9852, as in Auerbach and Kotlikoff (1987) in the baseline case. We will also use a discount factor of 1.011 as in İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995), matching the uncertain lifetime hump-shaped profiles of effective discount factors, as obtained by Hurd (1989), see Figure A3 in Appendix II.[16]

We choose $\sigma^c = 2$, in accordance with the value selected in the literature for large overlapping generation models (İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995), Hugett and Ventura (1998)). We choose a value of 6.5 for σ^l, so that the time devoted to work by an individual agent is on average 38 hours per week, following the argument put forward in Auerbach and Kotlikoff (1987).

4.2.2 Technology

The economy produces a single good from aggregate capital and labour according to a Cobb-Douglas production function:[17]

$$Y_t = f\left(K_t^d,\, L_t^d\right) = A\left(K_t^d\right)^{\alpha}\left(L_t^d\right)^{1-\alpha} \tag{20}$$

A is a scaling constant and α is a parameter which measures the capital share in income.

The problem of the representative firm is therefore:

$$\max_{K_t^d,\, L_t^d} A\left(K_t^d\right)^{\alpha}\left(L_t^d\right)^{1-\alpha} - w_t\, L_t^d - (r_t + \delta)\, K_t^d \tag{21}$$

In equilibrium, factors' markets clear so that labour demand equals labour supply, $L_t^d = L_t^s$, and capital supply equals capital demand, $K_t^d = K_t^s$. Firms

[15] See İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995) for a discussion of the role of subjective discount factor bigger than unity. See Hotz et al. (1988), Hurd (1989) or Hansen and Singleton (1993) for empirical negative estimates of rates of time preference (most studies focus on the US). See Davies (1981), İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995), Rios-Rull (1996), or Hugett and Ventura (1999) for life-cycle models including this feature.

[16] Figure A3 shows the sequence of effective discount factors, , for $\beta = 0.985$ and $\beta = 1.011$. The effective discount factor for $\beta = 0.985$ is more than proportionally decreasing, as age increases. For $\beta = 1.011$, the effective discount factor shows a slight increase up to retirement age, followed by a sharp decrease the probabilities of survival become low.

[17] The empirical evidence that factor shares and capital-output ratios have remained roughly constant over time (see Licandro et al. (1995)) while the ratio $\frac{w}{r}$ has increased suggests the use of this functional specification.

maximize profits taking factor and output prices as given. They hire physical capital and labour until factor prices equal marginal products, so that:

$$r_t = A\theta \left(\frac{K_t}{L_t}\right)^{\theta-1} - \delta \tag{22}$$

$$w_t = A(1-\theta) \left(\frac{K_t}{L_t}\right)^{\theta} \tag{23}$$

In order to calibrate the production function we need the values for the following parameters: A, α, δ, and ε.

Production function constant: The choice of the scaling parameter representing total factor productivity, A, depends on the units chosen for output. It is set to be a constant value of $A = 1.00$.

Capital intensity parameter: For a Cobb-Douglas production function, this parameter represents the share of capital on income. The historical share of capital on national income in the National Accounts suggests a value of around 0.5. Ríos-Rull (1994) and Bailén and Gil (1996), among others, consider that this value is far too high because labour income in the National Accounts excludes a part of the income of self-employed workers which is in turn included in capital income. We will choose a value of $\alpha = 0.40$.

Depreciation rate: The rate of depreciation in the National Accounts is around 10 % of output. We have chosen the value of δ such that the steady state capital-output ratio in the initial steady state (ISS) matches the one of the economy, which is the procedure in standard literature on overlapping generations. The chosen capital-output value lies in the range between the empirical estimates obtained by Licandro et al. (1995), 2.35, and the value of the capital-output ratio reported by King and Levine (1994) to be 2.65. The parameter δ is set to 6.5 %.

Efficiency units profile: This profile is exogenous and age-specific, as in Auerbach and Kotlikoff (1987). It determines relative wages by age. We have chosen a profile derived using data obtained from the Encuesta Sobre Conciencia y Biografía de Clase of the National Statistics Institute (Instituto Nacional de Estadística, 1991).The resulting profile is in line with the findings of Hansen (1993).

The efficiency units profile is plotted in Figure A2 (in Appendix II). That figure has the clear implication that workers with a higher work experience achieve a higher efficiency level. Wages double between ages 21 and 40 and from then onwards they do not alter widely until retirement age.

4.2.3 Social Security System

The parameter f, in equation (8), which represents the number of periods involved in the rule which is used to calculate pension benefits, is set to 15, as in the current system.

4.2.4 Demographics

We have calibrated the growth rates of the model population to match the old-dependency ratios of three different sets of demographic data.[18] Two of these have been provided by the Spanish Institute of Economics and Geography (IEG), including a high population growth and low population growth variant. An additional dataset has been obtained from the medium population growth variant of the United Nations (UN) demographic projections database. Both of them include population projections until year 2050, with an age breakdown. Figure 1 in page 3 presents the evolution of the old-dependency ratios in the three different scenarios for the period 1991-2050.

As pointed out above, we calibrate the growth rate of the model population, n, to match IEG's and UN's series for the old-dependency ratio, for a given set of probabilities of survival obtained from the same sources.[19] The initial steady state demographics are parameterized using a rate of growth of the population of $n = 0.019$, generating an old-dependency ratio of 24.63 %.[20] In order to compute the population structure for the final steady state of the model, we use all three different demographic scenarios (denoted by IEG-high, IEG-low and UN-medium). The following table summarises the chosen values for n:

[18] The old-dependency ratio is defined as the ratio that relates the number of people aged 65 and more to those who are currently working.

[19] For that purpose, we built an iterative procedure on the basis of the equations characterizing the stable population structure presented in page 7 (section 2.2). We iterated until convergence was achieved, and the resulting dependency ratio matched the reported one.

[20] See Figure A1 in Appendix II for a plot of the simple probabilities of survival for the initial and final steady state population structure, common to all demographic scenarios, and provided by the IEG.

Table 2: Population Growth Rates

	IEG-high	IEG-low	UN-medium
n	-0.0063	-0.0077	-0.0108

4.3 Computation Procedure

As there is no detailed data available on individual asset holdings by age cohort, we must approximate the initial assets distribution. From the set of available options (see Rios-Rull (1994)), we choose to calibrate a set of parameters so that the steady state mimics the main macroeconomic features of the Spanish economy in the 1990s. We then take this outcome as initial conditions[21].

In order to find the initial and final steady state equilibria of the model, for a given social security arrangement, we must solve a complicated set of non-linear equations that specify the optimization behaviour of individual agents, firms and governments.[22] For that purpose, we chose to solve the systems of non-linear equations using a non-linear least squares algorithm to find the root (zero) of the system.[23]

[21] See Rios-Rull (1994), where the author presents a version of a large overlapping generations model with a procedure to generate initial conditions for the distribution of wealth by age groups. In order to do so, he models demographic dynamics with the aid of univariate stochastic processes. The model is simulated until the age distribution of the population resembles the one of the current economy (see also footnote 8 in Chapter 1).

[22] For that purpose, Auerbach and Kotlikoff (1987) developed a Gauss-Seidel algorithm which starts with guesses about one of the endogenous variables of the model. This makes the system easier to solve for the endogenous variables, including the one for which the guess was made. When the solution for this initially guessed endogenous variable equals the guess itself, a true solution to the system is found. Otherwise a new guess which is obtained as a combination of the two sets of values from the previous iteration must be tried. The procedure is repeated until the true solution is found.

An alternative procedure is presented by İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995). An agent in their model faces a finite-horizon, finite-state dynamic programming. The value functions and the decision rules for each age i can be computed working backward from the last period of life, I.

See Arjona (2000a) and Arjona (2000b) for a detailed description of both algorithms and an application of the methods to the Spanish economy.

[23] For that purpose, we use the algorithm provided with MATLAB 5.2. The line-search algorithm included in this procedure is a safeguarded mixed quadratic and cubic polynomial interpolation and extrapolation method, requiring fewer function but more gradient evaluations than standard algorithms. Due to the fact that the software can calculate gradients relatively inexpensively, it becomes very efficient.

5 Results

This section presents the main simulation results (in Appendix I and II we have included the complete set of tables and figures corresponding to the different experiments) with a view to providing insight on the properties of the model for the computation of optimal taxes and replacement rates.

Each row in Tables 3 and 4 stands for a different social security arrangement (given by its replacement rate, ρ). The tables are divided into two subsections. Subsection ISS (Initial Steady-State) reports the values of the social security payroll-tax (τ), the capital-output ratio ($\frac{K}{Y}$), the rate of return on capital (r) and the measure of average utility (U) for the initial steady state calibration (see Tables 1 and 2 for details on the parameterization). The second subsection of the table, IEG-Low (Spanish Institute of Econmics and Geography, low variant), reports values for the same variables plus the compensating variation with respect to the ISS case (k). See tables A1-A4 (for $\beta = 0.985$) and B1-B4 (for $\beta = 1.011$) in Appendix I to obtain a detailed outcome for the each of the different demographic scenarios presented.

Table 3: Main Results of Experiment I for $\beta = 0.985$

	ISS				SID-Low				
ρ	τ	$\frac{K}{Y}$	r	U	τ	$\frac{K}{Y}$	r	U	k
0.00	0.0000	3.1936	0.0602	$\boxed{-72.1955}$	0.0000	3.8854	0.0380	$\boxed{-70.5491}$	0.0204
0.10	0.0293	3.0761	0.0650	-73.6598	0.0720	3.5966	0.0462	-74.6520	-0.0114
0.20	0.0562	2.9780	0.0693	-75.1298	0.1325	3.3852	0.0532	-78.6829	-0.0372
0.30	0.0812	2.8942	0.0732	-76.5948	0.1844	3.2208	0.0592	-82.6283	-0.0582
0.40	0.1043	2.8212	0.0768	-78.0490	0.2296	3.0878	0.0645	-86.4902	-0.0756
0.50	0.1259	2.7568	0.0801	-79.4897	0.2694	2.9770	0.0684	-90.2746	-0.0900
1.00	0.2164	2.5176	0.0939	-86.4632	0.4149	2.6076	0.0884	-108.2508	-0.1326

Several conclusions can be extracted from Table 3 presenting the simulation results for an economy with population growth and lifetime uncertainty, where the discount factor is 0.985. First, an increase in the replacement rate induces a monotonic reduction in the capital-output ratio and an increase in the rate of return on capital. With a replacement rate of 100 percent, the economy generates a capital-output ratio of 2.51, matching the estimates for the Spanish economy. Second, individual welfare, as measured by average utility, worsens as the replacement ratio increases. The possible beneficial role of social security as an insurance against lifetime uncertainty is outweighed by the costs in terms of a lower capital stock. Third, because this economy is dynamically efficient at a zero replacement rate, social security offers a lower steady-state rate of return than physical capital. The fact that average individual welfare declines the higher the replacement rate implies that there is no beneficial role for social security.

Table 4: Main Results of Experiment I for $\beta = 1.011$

	ISS				SID-Low				
ρ	τ	$\frac{K}{Y}$	r	U	τ	$\frac{K}{Y}$	r	U	k
0.00	0.0000	6.1757	-0.0002	-154.8894	0.0000	7.6387	-0.0126	-156.9818	-0.0084
0.10	0.0294	5.7986	0.0040	-151.7278	0.0713	6.8586	-0.0077	$\boxed{-156.6436}$	-0.0216
0.20	0.0564	5.4966	0.0078	-150.0759	0.1314	6.2896	-0.0022	-158.5901	-0.0372
0.30	0.0812	5.2481	0.0112	-149.3490	0.1830	5.8552	0.0026	-161.6835	-0.0534
0.40	0.1042	5.0390	0.0144	$\boxed{-149.2247}$	0.2279	5.5112	0.0070	-165.4049	-0.0678
0.50	0.1257	4.8601	0.0173	-149.5115	0.2674	5.2309	0.0110	-169.4891	-0.0810
1.00	0.2149	4.2392	0.0294	-154.0409	0.4118	4.3468	0.0270	-191.8348	-0.1236

Table 4 presents an economy with the same features as the one examined above, but where the discount rate is slightly greater than unity, $\beta = 1.011$ (see section 2.4.2 for a discussion on the issue). The conclusions that can be drawn from this table are substantially different from the ones arising from Table 3.

In this economy, the capital-output ratio also decreases as the replacement rate increases, while the rate of return on capital increases. However, the resulting capital-output ratios are far higher than the ones obtained for the baseline calibration (70 percent higher for a replacement rate of 100 percent).

The average utility-maximizing (henceforth, optimal replacement rate) in this economy is 40 percent for the initial steady state calibration. In the absence of social security, this economy is dynamically inefficient. Hence, the introduction of social security not only provides a higher rate of return than physical capital but also reduces private savings, and a replacement rate of 40 percent eliminates any possible overaccumulation of capital. A newly born agent would prefer a social security system with a replacement rate of 40 percent over any other possible alternative arrangements.

In the final steady state calibration, a lower social security replacement rate (of 10 percent) is enough to eliminate dynamic inefficiency. This is due to the fact that dynamic inefficiency, which could be measured by the difference between the rate of return on capital and population growth, is lower in the final steady state because the population grows less than in the initial steady state. Hence, a smaller replacement rate is required to eliminate it and the optimal replacement rate is 10 percent. In fact, if we examine Table B4 in Appendix I, we can observe that any beneficial role for social security vanishes for scenario UN-Medium.

Figures A4-A6, in Appendix II, plot the age-assets, age-labour and age-consumption profiles for the baseline economy with a replacement rate of 100 percent. The asset profiles generated by the model and displayed in Figure A4 show the standard features of life-cycle models, i.e. because agents are not liquidity constrained when they are young and therefore they may borrow in early age, the asset holdings may display negative values during that period; as they move towards retirement age, they accumulate assets, which they start to deplete only after retirement. We can also observe from Figure A4 that the demographic variations between the initial steady state and the final one, lead to a lower age-asset profile for all ages (the tax level in the final steady state is around 90 percent higher than in the initial one for the baseline case). As it can be observed from Figure A10, this life-cycle pattern is altered according to the intensity of the PAYG social security in place. A higher replacement rate, smooths the individual asset accumulation, while a lower replacement rate, makes the individuals more motivated to save to provide for old-age consumption.

Figure A5 displays the labour-supply profiles of agents in the baseline parameterization with a replacement rate of 100 percent. The labour profile varies from the initial to the final steady state. In fact, agents supply more labour at all ages in the final steady state. Figure A6 presents the consumption profile of individuals for the same parameterization and replacement rate level. It shows that, accordingly, consumption is lower in the final steady state for all ages. The paths of labour-supply and consumption also alter considerably with movements in the replacement rate. A higher replacement rate induces more labour-supply when young and less when old, as well as more consumption when old and less when young (see Figures A11 and A12).

Figures A7-A9 display similar patterns to the ones presented above. However, because the discount factor is bigger than unity, the patterns of asset, consumption and labour are different. Agents tend to accumulate more assets and at a higher rate of growth than with a discount factor lower than unity, and they do so until slightly earlier than retirement age. From then onwards, they start depleting their asset holdings. Labour supply tends to be higher the younger the individuals (see Figures A13 and A14). Consumption is higher with a larger discount factor (see Figure A15).

6 Conclusions

In this paper we used a dynamic general equilibrium model with overlapping generations of long-lived agents facing lifetime uncertainty and population growth. We analyzed the optimal structure of the Spanish PAYG social security system, as it currently stands and in light of the forecasted demographic changes that will occur in the medium to the long run, focusing on the contribution-related pension benefits of the General Regime and adapting the dependency ratio forecasts of the two datasets at hand, provided by the Spanish Institute of Economics and Geography (IEG) and the United Nations (UN) and containing three different population scenarios.

For that purpose, we have defined optimal taxes and replacement rates as those which maximize average utility (in section 3.1 we characterized the solution to the problem of an agent in the economy and described the measures of utility and welfare) in an overlapping generations economy incorporating an unfunded (PAYG) system.

We calibrated the model to fit the forecasted demographic patterns and Spanish microeconomic evidence and computed the optimal taxes and replacement rates. When we compute the average utility-maximizing taxes and replacement rates we find that, for calibrations in which the discount factor is less than unity, the optimal social security replacement rate is equal to zero. This eliminates any possible beneficial role for a PAYG social security system. These results hold for different demographic scenarios and hence, population growth rates. They are mainly due to the fact that the economy is dynamically efficient at a zero replacement rate and the possible beneficial role of social security as an insurance against lifetime uncertainty is outweighed by the costs in terms of a lower capital stock, supporting the argument put forward by İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995). Besides, the calculated tax rate practically doubles (becomes 90 higher) between the initial and final steady states, as a result of the expected demographic changes. The capital-output ratio increases and the interest rate decreases, accordingly.

These results change dramatically, however, when the chosen discount factor is larger than unity, the case we are most interested in. As a result of that change, the average utility-maximizing (optimal) replacement rates rise to 40 percent (for the initial steady state) and 10 percent (for the final steady state). In the absence of social security, this economy is dynamically inefficient. Hence, the introduction of social security not only provides a higher rate of return than physical capital but also reduces private savings, and a replacement rate of 10 percent in the initial steady state or 40 percent in the final steady state eliminates any possible overaccumulation of capital. A newly born agent would prefer a social security system with such a replacement rate over any other possible alternative arrangements.The optimal tax rates, associated with these replacement rates are 10.42 percent in the initial steady state and 7.13 percent in the final steady state, both of them considerably lower than the current Spanish social security payroll rate.[24] In fact, when we compare results for the initial and final steady state, for scenario UN-Medium, we can observe that the associated tax rates for the final steady state are equal to zero. This is due to the fact that the greater negative population growth rates make dynamic inefficiency vanish.

[24] Note, however, that these rates cannot be fully compared with the Spanish ones, as the choice of a discount factor bigger than unity raises the capital-output ratio and lowers the interest rate, with respect to the baseline calibration.

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Appendix I: Tables

Table A1: Results for Baseline Calibration

($\beta = 0.985$, $\sigma = 2$ and Initial Population Structure)

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	0.0602	0.0890	3.1936	$\boxed{-72.1955}$	0.0000
0.10	0.0293	0.0650	0.0868	3.0761	-73.6598	0.0000
0.20	0.0562	0.0693	0.0849	2.9780	-75.1298	0.0000
0.30	0.0812	0.0732	0.0833	2.8942	-76.5948	0.0000
0.40	0.1043	0.0768	0.0819	2.8212	-78.0490	0.0000
0.50	0.1259	0.0801	0.0807	2.7568	-79.4897	0.0000
1.00	0.2164	0.0939	0.0760	2.5176	-86.4632	0.0000

Table A2: Results for SID-High

($\beta = 0.985$, $\sigma = 2$)

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	0.0390	0.1007	3.8444	$\boxed{-70.6605}$	0.0186
0.10	0.0690	0.0471	0.0958	3.5683	-74.5678	-0.0102
0.20	0.1274	0.0539	0.0921	3.3645	-78.4111	-0.0342
0.30	0.1777	0.0598	0.0892	3.2052	-82.1764	-0.0540
0.40	0.2216	0.0650	0.0868	3.0758	-85.8643	-0.0702
0.50	0.2605	0.0698	0.0847	2.9676	-89.4798	-0.0840
1.00	0.4038	0.0886	0.0777	2.6050	-106.6641	-0.1254

Table A3: Results for SID-Low

($\beta = 0.985$, $\sigma = 2$)

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	0.0380	0.1014	3.8854	$\boxed{-70.5491}$	0.0204
0.10	0.0720	0.0462	0.0963	3.5966	-74.6520	-0.0114
0.20	0.1325	0.0532	0.0925	3.3852	-78.6829	-0.0372
0.30	0.1844	0.0592	0.0895	3.2208	-82.6283	-0.0582
0.40	0.2296	0.0645	0.0870	3.0878	-86.4902	-0.0756
0.50	0.2694	0.0694	0.0849	2.9770	-90.2746	-0.0900
1.00	0.4149	0.0884	0.0777	2.6076	-108.2508	-0.1326

Table A4: Results for UN-Medium

($\beta = 0.985$, $\sigma = 2$)

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	0.0355	0.1031	3.9791	$\boxed{-70.2915}$	0.0234
0.10	0.0790	0.0443	0.0975	3.6601	-74.8576	-0.0132
0.20	0.1444	0.0516	0.0934	3.4309	-79.3312	-0.0432
0.30	0.1999	0.0579	0.0901	3.2549	-83.7015	-0.0672
0.40	0.2478	0.0635	0.0875	3.1138	-87.9736	-0.0864
0.50	0.2896	0.0685	0.0853	2.9971	-92.1561	-0.1026
1.00	0.4397	0.0881	0.0799	2.6128	-111.9987	-0.1482

Table B1: Results for Baseline Calibration

($\beta = 1.011$, $\sigma = 2$ and Initial Population Structure)

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	-0.0002	0.1381	6.1757	-154.8894	0.0000
0.10	0.0294	0.0040	0.1325	5.7986	-151.7278	0.0000
0.20	0.0564	0.0078	0.1278	5.4966	-150.0759	0.0000
0.30	0.0812	0.0112	0.1239	5.2481	-149.3490	0.0000
0.40	0.1042	0.0144	0.1206	5.0390	$\boxed{-149.2247}$	0.0000
0.50	0.1257	0.0173	0.1178	4.8601	-149.5115	0.0000
1.00	0.2149	0.0294	0.1075	4.2392	-154.0409	0.0000

Table B2: Results for SID-High

($\beta = 1.011$, $\sigma = 2$)

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	-0.0121	0.1580	7.5553	-157.0002	-0.0090
0.10	0.0684	-0.0062	0.1473	6.8031	$\boxed{-156.3799}$	-0.0204
0.20	0.1264	-0.0010	0.1393	6.2513	-158.0210	-0.0348
0.30	0.1763	0.0036	0.1329	5.8280	-160.8113	-0.0498
0.40	0.2200	0.0078	0.1277	5.4917	-164.2382	-0.0636
0.50	0.2586	0.0117	0.1234	5.2169	-168.0376	-0.0756
1.00	0.4007	0.0270	0.1093	4.3457	-189.0788	-0.1170

Table B3: Results for SID-Low

$(\beta = 1.011,\ \sigma = 2)$

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	-0.0126	0.1592	7.6387	-156.9818	-0.0090
0.10	0.0713	-0.0067	0.1481	6.8586	$\boxed{-156.6436}$	-0.0216
0.20	0.1314	-0.0014	0.1398	6.2896	-158.5901	-0.0372
0.30	0.1830	0.0033	0.1333	5.8552	-161.6835	-0.0534
0.40	0.2279	0.0076	0.1280	5.5112	-165.4049	-0.0678
0.50	0.2674	0.0115	0.1237	5.2309	-169.4891	-0.0810
1.00	0.4118	0.0270	0.1093	4.3468	-191.8348	-0.1236

Table B4: Results for UN-Medium

$(\beta = 1.011,\ \sigma = 2)$

ρ	τ	r	w	$\frac{K}{Y}$	U	k
0.00	0.0000	-0.0139	0.1618	7.8280	$\boxed{-156.9198}$	-0.0084
0.10	0.0782	-0.0077	0.1499	6.9825	-157.2682	-0.0240
0.20	0.1432	-0.0022	0.1411	6.3739	-159.9458	-0.0426
0.30	0.1984	0.0026	0.1342	5.9139	-163.7594	-0.0612
0.40	0.2459	0.0070	0.1287	5.5526	-168.1777	-0.0774
0.50	0.2874	0.0110	0.1241	5.2601	-172.9346	-0.0918
1.00	0.4365	0.0270	0.1093	4.3478	-198.3569	-0.1380

Appendix II: Figures

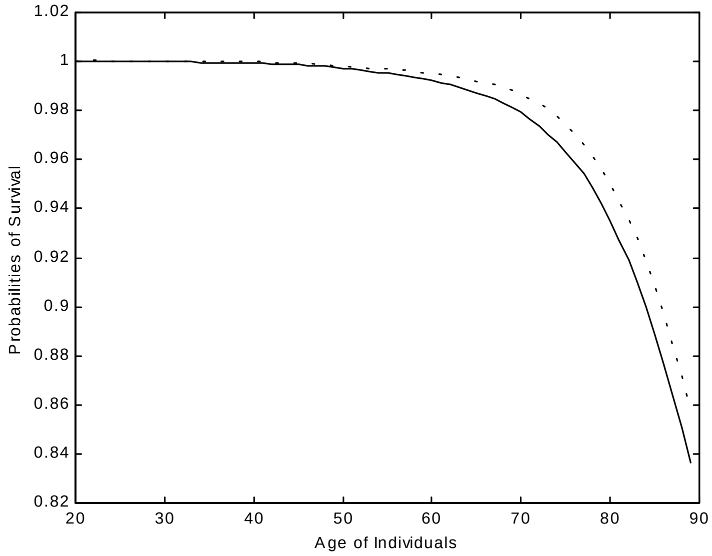

Figure A1: Simple Probabilities of Survival. Continuous line: Initial Profile. Dotted line: Final Profile.

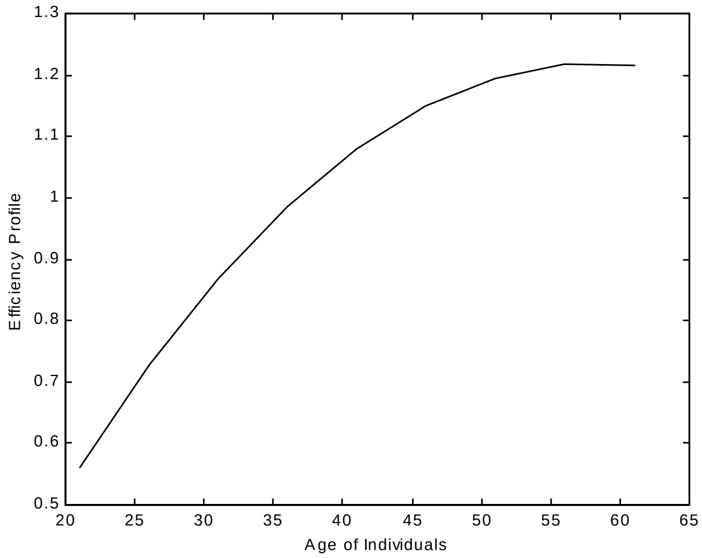

Figure A2: Smoothed Efficiency Units Profile

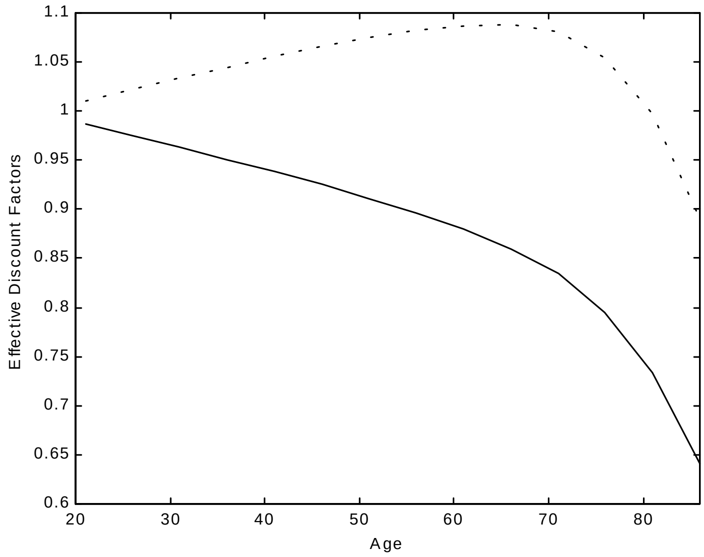

Figure A3: Effective Discount Factors. Upper curve: $\beta = 1.011$; lower curve: $\beta = 0.985$.

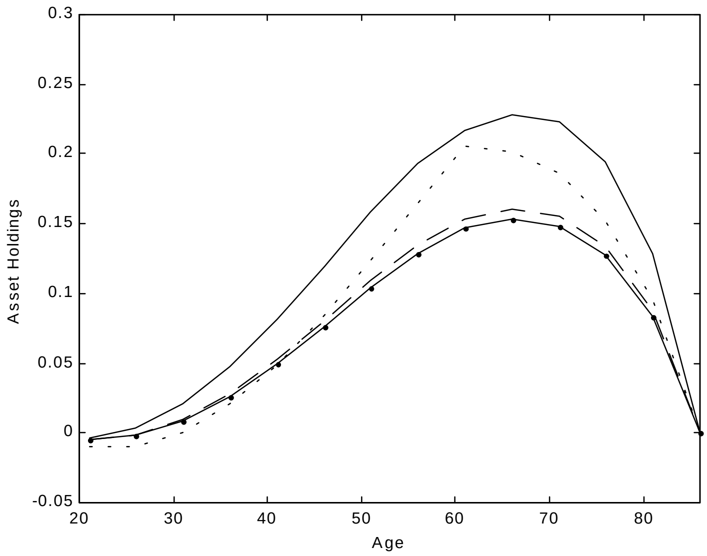

Figure A4: Asset Profiles in Steady States (baseline parameterization with $\rho = 1$). Upper: ISS. Dotted: IEG-High. Dashed: IEG-Low. Dash-dotted: UN-Medium.

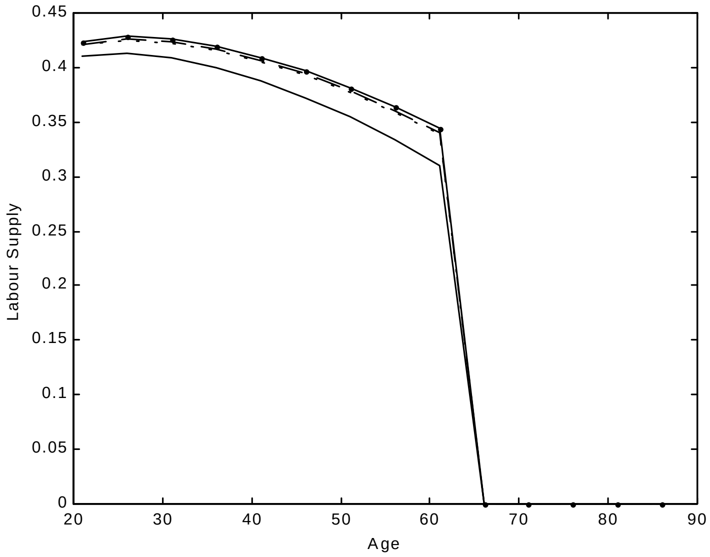

Figure A5: Labour Supply Profiles in Steady State (baseline parameterization with $\rho = 1$). Upper: ISS. Dotted: IEG-High. Dashed: IEG-Low. Dash-dotted: UN-Medium.

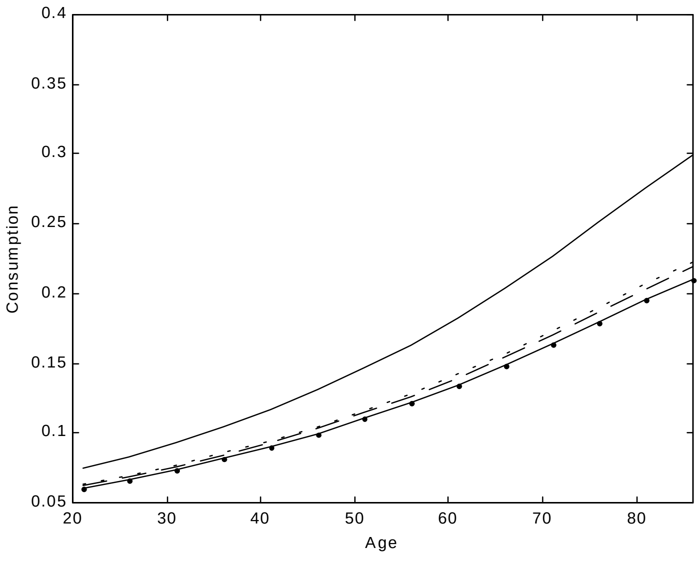

Figure A6: Consumption Profiles in Steady State (baseline parameterization with $\rho = 1$). Upper: ISS. Dotted: IEG-High. Dashed: IEG-Low. Dash-dotted: UN-Medium.

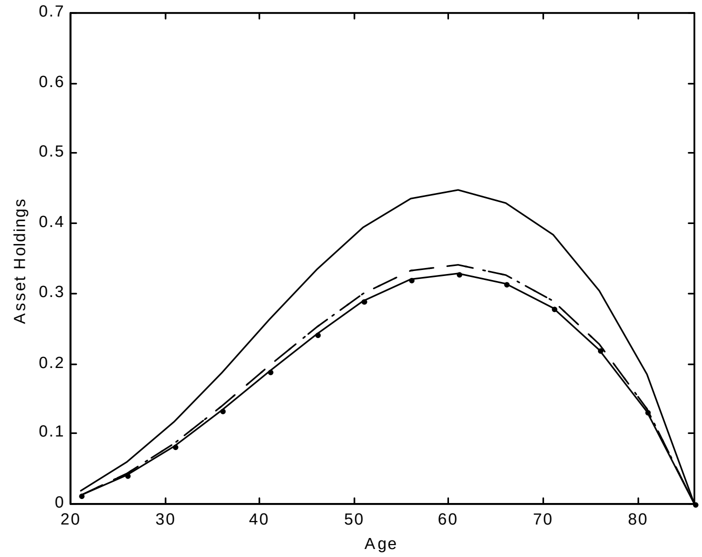

Figure A7: Asset Profiles in Steady State ($\beta = 1.011$ and $\rho = 1$). Upper: ISS. Dotted: IEG-High. Dashed: IEG-Low. Dash-dotted: UN-Medium.

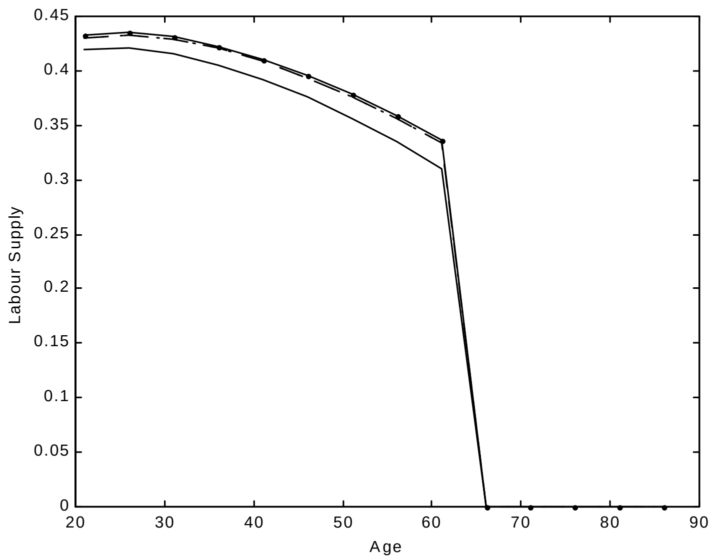

Figure A8: Labour Supply in Steady State ($\beta = 1.011$ and $\rho = 1$). Upper: ISS. Dotted: IEG-High. Dashed: IEG-Low. Dash-dotted: UN-Medium.

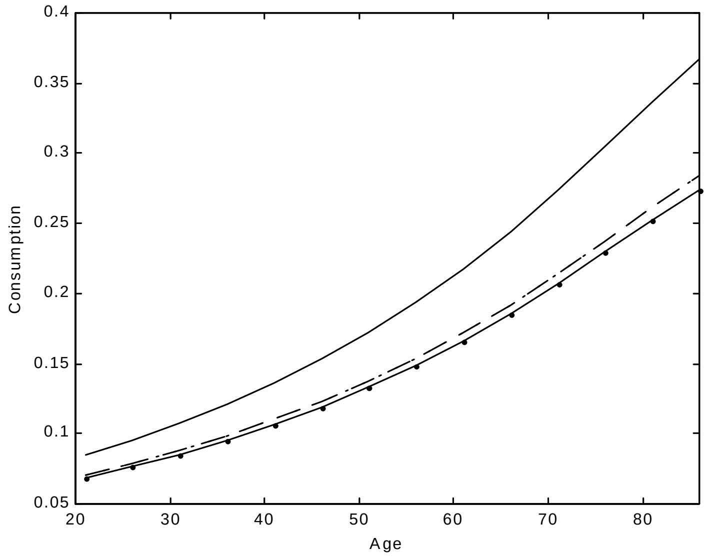

Figure A9: Consumption Profile in Steady State ($\beta = 1.011$ and $\rho = 1$). Upper: ISS. Dotted: IEG-High. Dashed: IEG-Low. Dash-dotted: UN-Medium.

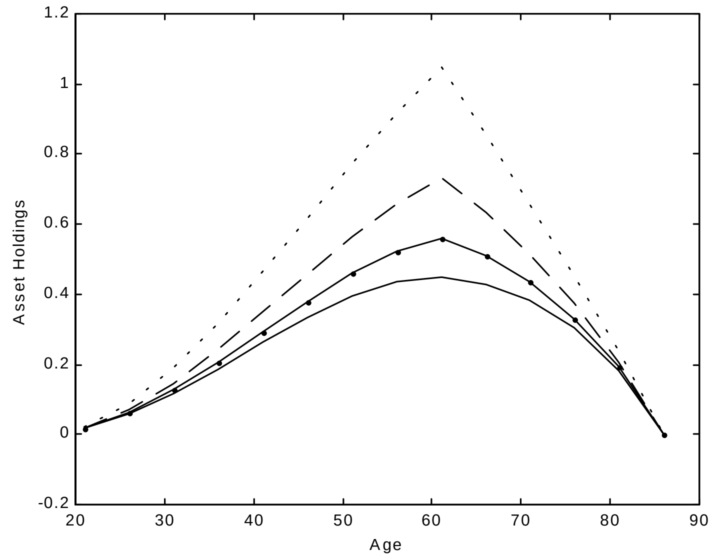

Figure A10: Asset Profiles in initial steady state for $\beta = 1.011$ and $\rho = \{0,\ 0.30,\ 0.60,\ 1.00\}$. Continuous: highest ρ. Dotted: lowest ρ.

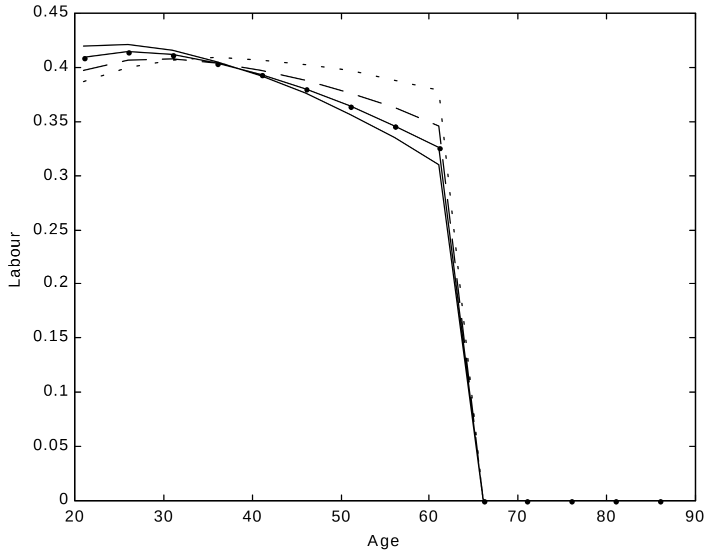

Figure A11: Labour Supply Profiles in initial steady state for $\beta = 1.011$ and $\rho = \{0,\ 0.30,\ 0.60,\ 1.00\}$. Continuous: highest ρ. Dotted: lowest ρ.

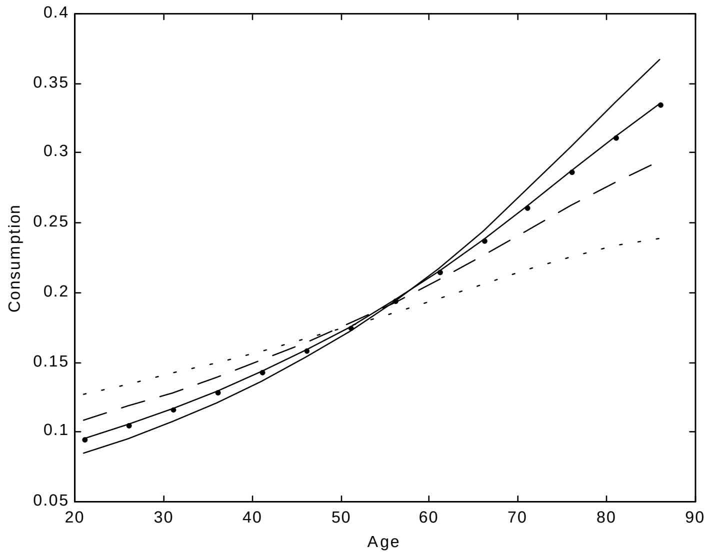

Figure A12: Consumption Profiles in initial steady state for $\beta = 1.011$ and $\rho = \{0,\ 0.30,\ 0.60,\ 1.00\}$. Lower curve: highest ρ. Continuous: highest ρ. Dotted: lowest ρ.

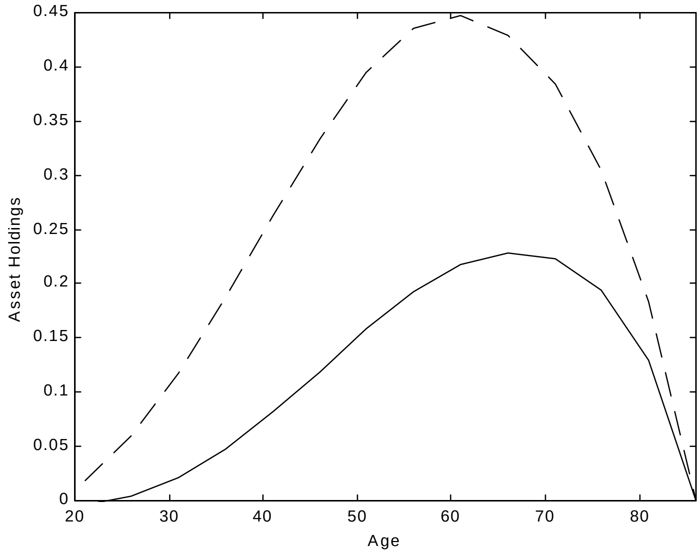

Figure A13: Asset Profiles in Steady State ($\rho = 1$). Comparision between outcomes for $\beta = 0.985$ and $\beta = 1.011$. Continuous: $\beta = 0.985$. Dashed: $\beta = 1.011$.

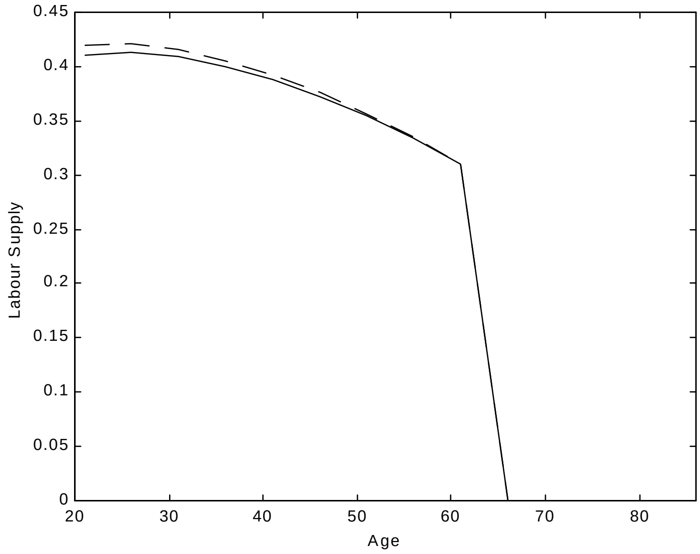

Figure A14: Labour-Supply in Steady State ($\rho = 1$). Comparision between outcomes for $\beta = 0.985$ and $\beta = 1.011$. Continuous: $\beta = 0.985$. Dashed: $\beta = 1.011$.

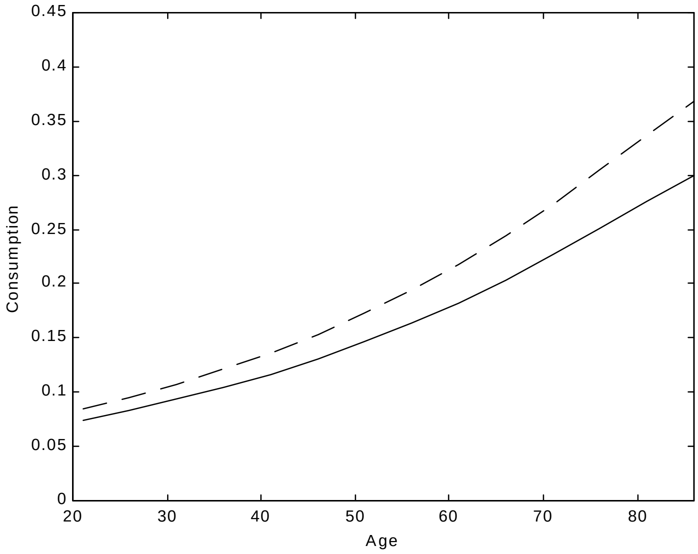

Figure A15: Consumption in Steady State ($\rho = 1$). Comparision between outcomes for $\beta = 0.985$ and $\beta = 1.011$. Continuous: $\beta = 0.985$. Dashed: $\beta = 1.011$.