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Gradually Capitalizing the Spanish Retirement Pension System

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Gradually Capitalizing the Spanish Retirement Pension System

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Abstract

The aim of this paper is to analyze the magnitude of the transition and long-run costs of changing the current Spanish Pay-As-You-Go (PAYG) system into a partially funded system, using a gradual capitalization strategy, with a view to assessing its impact on the intergenerational distribution of welfare. The analysis is conducted in light of the expected demographic changes in Spain.

For that purpose, we build a stochastic dynamic general equilibrium model with overlapping generations of long-lived agents facing mortality and idiosyncratic employment risk and expand the computational procedure of İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995) to examine transition dynamics between steady states with a changing population structure.

Our main findings indicate that gradual capitalization strategies are appealing mechanisms which generate viable transitions to partially

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funded social security systems. Dynamic general equilibrium effects play an important role in this outcome, especially during the transition process. We find that all future generations are better off under the proposed gradual capitalization schemes, while older generations suffer a welfare loss. We complement the analysis by computing the optimal threshold for the gradual capitalization percentage.

1 Introduction¹

Most industrialized and many developing countries have expanded considerably their social security systems in the postwar period. While some of these countries have decided to shift them, partially or completely, toward a private pension system,² there is still a large set of countries that have a Pay-As-You-Go (PAYG) social security system. In a PAYG (or unfunded) system, current workers, who pay the social security tax (contribution), finance the pension benefit of those retired in the same year. The greatest share of social security expenditure in these economies comes from contribution-related retirement benefits. These countries will face, in words of Kotlikoff (1996), "the desire to find a way out of the Social Security System's long-run financing problem". The key factor underlying this problem is the prospect of a change in the demographic structure of the population in the medium and in the long run, consisting basically of a decrease in both fertility and mortality rates.

Several recent studies³ have focused on the possible welfare gains in the U.S.A. arising from privatization strategies of the current PAYG system. These gains arise mainly from both: (1) the rate of return differential between the low rate of return of the mature PAYG system and the higher real rate of return on capital, and (2) the elimination of distortions on the labour supply decisions. Using different strategies and models, these studies conclude that a transition

¹This paper is based on the third chapter of the author's PhD dissertation, "A Dynamic General Equilibrium Analysis of the Spanish Social Security System" defended at the EUI in June 2000.

²As it is the case of Argentina, Australia, Chile, U.K. or Mexico.

³See Feldstein (1996), Feldstein and Samwick (1996), Gustman and Steinmeier (1995), and Mitchell and Zeldes (1996) for micro simulation models, and Huang, İmrohoroğlu and Sargent (1996), Kotlikoff (1995, 1996), Hugett and Ventura (1999) and Conesa and Krueger (1999) for dynamic general equilibrium life-cycle models.

from a PAYG system towards a fully funded (FF) one would increase overall welfare.

Nevertheless, as pointed out by Feldstein and Samwick (1996), each country involved in a transition to a private funded plan "faces a unique problem, reflecting the demographic and economic situation of that country and the promises and expectations embedded in existing law". For the case of Spain, indeed, several studies have recently tried to assess the future viability of the Spanish PAYG system⁴. Using a micro simulation framework, these studies basically conclude that the current Spanish PAYG social security system will undergo financial difficulties, as the demographic changes start to take place⁵.

Other sets of studies (which mainly includes Barea et al. (1995), Piñera (1996) and Bailén and Gil (1996)) have tried to examine and quantify possible reforms of the system in order to surmount the financial problems. Using a similar micro simulation framework they analyze the costs and implications of the transition to a funded system. Both the studies of Piñera (1996) and Bailén and Gil (1996) conclude that this transition is feasible.

The aim of this paper is to analyze the magnitude of the transition and long-run costs of changing the current Spanish Pay-As-You-Go (PAYG) system into a partially funded system using a gradual capitalization strategy, with a view to assessing its impact on the intergenerational distribution of welfare. The analysis is conducted in light of the expected demographic changes in Spain.

For that purpose, we analyze, in a stochastic dynamic general equilibrium framework, the economic and welfare effects of implementing a Mandatory Individual Retirement Account (MIRA) system in Spain, exclusively for workers that enter the social security system for the first time, and only for a given percentage of their income.

We use a model based on the İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995 and 1998) setup and expand their computational procedure to examine the transition dynamics between steady states with a changing demographic

⁴See Arjona (2000a) for a more detailed description of these studies and a summary of their results.

⁵See also Arjona (2000a) and Arjona (2000b) for a dynamic general equilibrium analysis of the impact of demographic changes on the fiscal balance of the Spanish social security system. Using different demographic scenarios, those studies conclude that the impact of those projected demographic changes on the fiscal balance of the Spanish social security system will be large.

structure.6

The model incorporates an exogenous time-path for fertility change, as in Auerbach and Kotlikoff (1984), Auerbach and Kotlikoff (1987) and Auerbach, Kotlikoff, Hagemann and Nicoletti (1989). We also use time-varying conditional survival probabilities, as in De Nardi, İmrohoroğlu and Sargent (1999) and Arjona (2000a). The demographic projections which are introduced exogenously into the model cover the period 1995 to 2051. For the following period 2052 onwards we have expanded the projections using criteria in accordance with those covering the period until 2051.

The economy consists of overlapping generations of long-lived agents who face two different types of risks: lifetime uncertainty and income risk (i.e. during their working period, agents face stochastic employment opportunities which they can only incompletely insure), following the approach of Hubbard and Judd (1987) and İmrohoroğlu, İmrohoroğlu and Joines (1995, 1999). Agents are also liquidity constrained. Hence, they are heterogeneous with respect to age, employment status and asset holdings.

1.1 Micro-Based Simulation Studies

Using a micro simulation model, Feldstein and Samwick (1996) analyze the transition from a PAYG social security system to a system of Mandatory Individual Retirement Accounts (MIRAs)⁷. They argue that the low implicit return on contributions, together with the adverse effect of social security on national saving⁸ are the key factors that make the strategy of privatization desirable. Privatizing would benefit from the rate of return differential between the real rate of return on capital and the implicit rate of the mature PAYG system, and reduce the deadweight loss generated by the distortive payroll tax. According to the authors, a transition to a fully funded program can be articulated such that its costs are moderately low and the long-run benefits are

⁶İmrohoroğlu, İmrohoroğlu and Joines (1995 and 1999) present a dynamic general equilibrium model with overlapping generations of long-lived agents. The main features of their model are uncertain lifetimes and endowments. It also includes borrowing constraints. They focus on the steady state properties of the model and analyze the optimal social security replacement rate and its associated benefits.

⁷Employers or employees would make contributions that would be invested in the market (stocks or bonds) and therefore would earn a real rate of return equal to the pretax marginal product of capital.

⁸See also Feldstein (1995) for an empirical assessment of the impact of social security on savings in the U.S.A.

high. The authors also analyze a phasing-in method of shifting from the current system to the MIRA system. Their method basically consists of starting with a partial privatization for every agent and then expanding the share until it completely substitutes for the PAYG program. In the first year, individuals are required to contribute to the MIRA in such a way that the accumulated returns will yield as much as necessary to replace $\frac{1}{4}$ of the corresponding PAYG benefits. From that year onwards, the share increases by 3% annually, until it reaches 100% over a 25 years phase-in period. This proposal is in line with the one of Piñera (1996), which is examined below.

Mitchell and Zeldes (1996) propose a structure for the analysis of privatization. They suggest the use of a two-pillar plan: both a "demogrant" (i.e. a small indexed pension equal for all retirees who contributed to the system for at least over a full lifetime of work) and "fully-funded individual-defined contribution accounts", financed by payroll taxation and managed by financial institutions, under the direction of the participants. There would also be a need to compensate the participants in the mandatory system by issuing recognition bonds, in an approach similar to that followed by Chile⁹. They conclude that such a system would reduce political risks, increase the portfolio choice of agents and improve work incentives. The side-effects would be a reduction in redistribution and risk pooling and an increase in administrative costs.

Gustman and Steinmeier (1996) propose a generic policy to privatize the system, with an opt-out mechanism and no changes in the basic benefit-tax structure. The scheme is articulated as follows. An individual may opt out of social security in each year. If that is the case, neither the agent nor the employer pay any social security payroll tax but $\frac{1}{35}$ th of the benefits to be received by the agent in the future are lost. Instead, the amount to be payed as contribution is invested in an Individual Retirement Account (IRA) on behalf of the worker. The authors examine the incentive effects from the proposed privatization scheme using a simple micro simulation model with constant wage growth. They conclude that labour market participation rates are not largely affected by the privatization, even if that generates large variations in benefits or taxes.

Several studies have focused on the future viability of the Spanish social security system and attempted to quantify transition strategies from the current PAYG system towards a new system with a higher degree of linkage for individual agents between the contributions that they have actually payed over time and benefits they will receive when they retire. These studies are based on projections of social security revenue and expenditure flows over time and use

⁹See Diamond and Valdés-Prieto (1994) for a detailed analysis of the Chilean reform.

a micro-based simulation framework together with demographic projections for the period under analysis, as described earlier.

Herce et al. (1995) use a simulation model to analyze the impact of different reform measures on the Spanish social security system. The main measures the authors analyze are: (1) a reduction in the amount of the pension benefit through modifying the formula involved in the calculation, (2) an increase in the parameter "number of years" involved in the calculation of the pension benefit, and (3) an increase in the mandatory retirement age. They conclude that the Spanish social security system will encounter severe financial troubles in the medium-run and the risk of a worrisome acceleration of these from year 2025 onward. Implementing these measures as single isolated policies would not significantly improve the functioning of the system, however the implementation of a self-contained package including a (sub)set of these measures may indeed improve the overall situation in the long-run. Of course, these would necessarily imply a redistribution of costs among the population.

Barea et al. (1995) make a proposal for restructuring the social security system (both "regimes") which basically consists of a two-tier partial capitalization scheme in which, for the General Regime (Régimen General), new workers contribute with 1% of the total contribution rate to a publicly administered MIRA, keeping the rest of the contribution on a PAYG basis. Moreover, those workers willing to increase their capitalization share may provide funds voluntarily to a privately administered IRA. For the latter, the government would provide fiscal incentives. For the Special Regimes (Regimenes Especiales), the system would basically switch to a capitalization one.

Piñera (1996) uses a simulation model to evaluate a gradual transition to a capitalization system, in which workers below 45 years of age are given the choice of remaining in the PAYG system or switching to a privately administered IRA system. Those who enter the workforce afresh would join the capitalization system directly. Those workers who are older than 30 and decide to change receive a recognition bond from the government which compensates, fully or partially, for the contributions that they made in the past. Those who decide not to switch to the capitalization system, those whose age is above 45 and the current pensioners will receive pension benefits according to the current PAYG rules. The author assumes that 10% of the workers aged 45 and less will move to the capitalization system each year (starting in 1997), until 60% of them have moved. The author concludes that the implementation of the aforementioned reform is viable from a financial point of view and it will bring substantial benefits in the medium- to the long-run.

Bailén and Gil (1996) use a reduced version of both a neoclassical growth

and endogenous growth model to estimate the effects of a substitution of the current PAYG system for another one financed through mandatory savings. The strategy presented obliges all workers under 40 to switch to a fully funded system (MIRA), whereas old workers continue in the old PAYG system. During the transition period, the reform generates a deficit because of the loss of contributions by young workers. Focusing on the output effects, transition costs and the change in pension benefits, the authors conclude that the greater capital accumulation process during the transition leads to a sustainable path for the transition.

1.2 DGE Models of Pension Reform

Both Kotlikoff (1995, 1996) and Huang, İmrohoroğlu and Sargent (1996) analyze the transition path of changes in social security. For Kotlikoff (1995, 1996), who uses the Auerbach and Kotlikoff (1987) model with identical agents within generation (but a labour-leisure choice), privatization can generate large increases in the output and living standards. Kotlikoff (1995) proposes the use of a "Personal Security System". This strategy would privatize the total contribution to the U.S.A. Old Age Insurance program by investing it in the so-called Private Retirement Accounts (PRAs) for agents of age below 62. Employers and employees would still contribute to social security for the survivor and disability insurances, where the benefits would be calculated on the basis of the earning history of the worker. At the time of the reform those workers below 62 would receive benefits to the extent that they had contributed to the system. The phasing-out of retirement benefits is therefore gradual, as the workers below age 62 at the moment of the reform will still receive benefits when they retire. The efficiency gains (i.e. the welfare improvement) after having compensated the initial generations, are also large.

Huang, İmrohoroğlu and Sargent (1996) use a dynamic general equilibrium model to examine the impact that fully funding social security has on the intergenerational distribution of consumptions. Their model has heterogeneous agents but no labour-leisure choice. They perform two experiments. In the first one, social security payments are finished suddenly but entitled generations are compensated with a one-time large increase in government debt, close to three times real GDP, financed through an increase in labour income tax during the first fourty years of the transition. In their second experiment, a government run scheme, social security benefits stay untouched but the government increases temporarily the tax on labour income in order to foster private physical capital accumulation which, in turn, generates more revenue to

finance the social security payments. This latter scheme yields higher efficiency gains than the first one (privatization) due to the fact that it provides insurance both against life span risk and income volatility.

Hugett and Ventura (1999) use a life-cycle dynamic general equilibrium model to analyze the distributional effects of a proposal to reform the US social security system (Boskin proposal, 1986), relative to what would occur under current arrangements. The Boskin proposal separates an annuity or insurance part of the social security programs from a welfare or transfer part. The annuity part would be financed from proportional taxes either through a PAYG or a FF system, while the transfer part would be financed out of general revenues. Under this proposal, benefits are proportional to the maximum of an accumulated value of individual tax payments or a floor benefit. They focus on the steady-state equilibrium, and find out that the outcome of the reform gives similar values to those under the current system. They examine whether there is a superior allocation of consumption and labour over the life cycle stemming from the implementation of this proposal. The authors find out that the aggregate gains from this implementation are never positive. There is a positive effect on the average hours worked due to the fact that benefits are proportional to the future value of taxes paid after the reform, whereas currently they depend on average indexed earnings.

De Nardi, Îmrohoroğlu and Sargent (1999) use a dynamic general equilibrium model with overlapping generations of long-lived agents, in the tradition of Auerbach and Kotlikoff (1987), and examine transitions across steady states arising from time-varying demographic patterns and survival probabilities. They extend the İmrohoroğlu, İmrohoroğlu and Joines (1995) setup to incorporate demographic variations over time. Their main findings are that it will be costly to maintain pension benefits at the current levels. A high increase in the social security payroll tax and large welfare losses will take place if the system is to be maintained. However, policies aiming at reducing benefit through their taxation or postponing retirement eligibility contribute to a significant reduction of the fiscal adjustment required to cope with the ageing process of the populations. Policies with similar long-run outcomes can have different intergenerational distribution implications during the transition. Those policies that partially reduce benefits or phase them out over the transition yield welfare gains for future generations but make current ones worse off. Sustainability of reforms require reduced distortions in the labour/leisure and consumption/savings choices and some transition policies to compensate current generations.

2 The Model

The model is based on the İmrohoroğlu, İmrohoroğlu and Joines (1995) setup and consists of an overlapping generations model of long-lived agents with the following features:

- 1. Mortality risk,
- 2. incomplete insurance against idiosyncratic employment risk, 10 and
- 3. borrowing constraints.

Individuals are heterogenous with respect to age, employment status and asset holdings. During their working period, they face stochastic employment opportunities. Agents are liquidity constrained. We use time-varying probabilities of survival, together with time-varying population growth rates (see De Nardi, İmrohoroğlu and Sargent (1999) or the model presented in Arjona (2000a)). There is no market for private annuities.

2.1 Demographics

Time is discrete and it is denoted by subindex t while age is denoted by subindex j. At time t, agents face a probability of surviving between age j and age j+1, which is denoted by $\psi_{t,j}$, being $\psi_{t,J}=0$. The unconditional probability of reaching age j is therefore $\psi_t^j=\prod_{k=1}^j \psi_{t-k,j-k}$.

At a date t, a cohort $N_{t,1}$ of new workers is born. Let $N_{t,j}$ denote the number of people of age j alive at time t, and let n_t be the rate of growth of new agents. Then for an agent of age j, the following law of motion holds:

$$N_{t,j} = \psi_{t,j} \, N_{t-1,j-1}$$

while for newborn agents:

$$N_{t,1} = (1 + n_t) N_{t-1,1}$$

¹⁰With a view to a more in-depth presentation of the model, we will include this feature in the model description. However, when calibrating and computing the equilibrium of the model, we will exclude the possibility of being unemployed, without loss of generality.

Let $\eta_t = \prod_{k=1}^t (1 + n_k)$. Then, as shown by De Nardi, İmrohoroğlu and Sargent (1999), the fraction of people of age j alive at time t is:

$$\mu_{t,j} = rac{\psi_t^j \, \eta_t}{\sum\limits_{k=0}^j \psi_t^j \, \eta_{t-j}}$$

The aggregate population is given by $N_t = \sum_{j=1}^J N_{t,j}$. We take the paths $\{n_t\}^{t\in\mathcal{T}}$ and $\{\psi_{t,j}\}^{t\in\mathcal{T}}$ as parameters.

2.2 Preferences

The (ex-ante identical) agents in the economy derive their utility from consumption, maximizing the following discounted lifetime utility:

$$E \sum_{j=1}^{J} \beta^{j-1} \psi_{t'}^{j} U \left(c_{t',j} \right) \tag{1}$$

 $c_{t',j}$ is consumption in period t' = t + j - 1 of an age-j agent born in t, and β is the subjective discount factor.

Individuals below mandatory retirement age, j_R , face a stochastic employment opportunity. Denote by $s \in S = \{e, u\}$ the employment opportunities state. The transition function for the individual earnings state is denoted by Π , such that: $\Pi(s', s) = [\pi_{xy}], x, y = e, u$, where π_{xy} is the probability of being in state y during age j + 1 conditional of being in state x during age j, i.e. $\pi_{xy} = \Pr\{s_{j+1} = y | s_j = x\}$. If the agent is employed $(j \leq j_R \text{ and } s = e)$, he will supply ε_j exogenous age-specific efficiency units of labour input. If he is unemployed $(j \leq j_R \text{ and } s = u)$, he will supply no labour and receive unemployment benefit. If the agent is retired $(j > j_R)$ he will receive a pension benefit.

The budget constraint of an age j agent in period t' is:

$$c_{t',j} + a_{t',j} = (1 + r_{t'}) \ a_{t'-1,j-1} + y_{t',j} + \zeta_{t'} \tag{2}$$

where:

$$y_{t',j} = \begin{cases} \left(1 - \tau_{t'}^l - \tau_{t'}^u\right) w_{t'} \varepsilon_j & \text{if } j \in [1, j_R] \text{ and } s = e \\ \phi w_{t'} \varepsilon_j & \text{if } j \in [1, j_R] \text{ and } s = u \\ b_{t'} & \text{if } j \in (j_R, J] \end{cases}$$
(3)

and:

$$a_{t',j} \ge 0$$
 (4)
 $a_{t',J} = 0$ (5)

$$a_{t',J} = 0 (5)$$

where, for an individual of age j, $a_{t'-1,j-1}$ stands for the asset holdings (accumulated net wealth), $a_{t',j}$ is next period's asset holdings, $\tau_{t'}^l$ stands for the social security payroll tax, $\tau_{t'}^u$ for the tax rate to finance the unemployment benefit, and $b_{t'}$ for the retirement benefit (see section 3, below). The gross interest rate is $R_{t'} = (1 + r_{t'})$ and the real wage rate per efficiency unit of labour in terms of the single consumption good is $w_{t'}$. $\zeta_{t'}$ stands for a lumpsum transfer of accidental bequests received by an agent. $y_{t',i}$ is the disposable income of an age-j individual. The assets accumulated by those who die before age J (accidental bequests) are redistributed to all agents which remain alive in a lump-sum fashion.¹¹

In each period agents receive capital income, $(1+r_{t'})$ $a_{t'-1,j-1}$, and alternatively labour income (if $j \leq j_R$ and s = e), $(1 - \tau_{t'}^l - \tau_{t'}^u) w_{t'} \varepsilon_j$, unemployment benefit, $\phi w_{t'} \varepsilon_j$, (if $j \leq j_R$ and s = u), or retirement benefit (if $j > j_R$), $b_{t'}$, and they decide how much to allocate to savings, on the basis of the life cycle model of behavior. Agents accumulate assets to smooth their consumption over time.

The agents in this economy are liquidity constrained and therefore are not allowed to borrow and have no access to private annuities/insurance markets, (4). They may leave no intended bequests 12 , (5).

¹¹There are other possible redistribution schemes. For instance, these accidental bequests could be distributed to the newborns in a lump-sum fashion. They could also be disposed without providing use to any agent. Or they could be distributed to the survivors of the same cohorts. As pointed out by Imrohoroğlu, Imrohoroğlu and Joines (1995), this latter redistribution scheme is equivalent to an annuity contract that allows individuals to insure against lifetime uncertainty, and its implementation may alter significantly the quantitative

 $^{^{12}}$ This is an implication arising from the fact that there are borrowing constraints and death is certain after age J.

2.3 Technology

The economy produces a single good from aggregate capital and labour according to a standard constant returns to scale production function:

$$Y_t = f(K_t, L_t) (6)$$

where K_t is the aggregate capital stock and L_t is the aggregate labour input. Output can be used for consumption in the same period as production takes place or can be used for increasing next period's capital stock. Capital depreciates at rate δ . Aggregate output is represented by Y_t .

Firms hire physical capital and effective labour until factor prices equal marginal products:

$$r_t = f_K(K_t, L_t) - \delta \tag{7}$$

$$w_t = f_L(K_t, L_t) \tag{8}$$

3 Social Security System

There is a government in the model which manages the retirement PAYG program. It collects the social security payroll tax and pays the corresponding social security benefits to retired agents. This publicly administered PAYG social security system is balanced every period.

3.1 Pay-As-You-Go System

Define average earnings over the last f years of the working life of an age $j \in [j_R + 1, J]$ agent, born in t, as:

$$z_t = \frac{1}{f} \sum_{j=j_R-f+1}^{j_R} \varepsilon_j \, w_{t'-j_R+j} \tag{9}$$

After their retirement, for $j \in [j_R + 1, ..., J]$, agents receive benefits according to the following formula:

$$b_t = \rho z_t \left(1 - \tau_{t'}^l \right) \tag{10}$$

where ρ is the social security replacement rate.

The total government expenditure in retirement benefits is:

$$G_t = \sum_{j=j_R+1}^{J} \mu_{t,j} \, b_{t-j} \tag{11}$$

In this system, agents contribute a certain percentage of their income $(\tau_{t'} \varepsilon_j w_{t'})$, the social security contribution, to the system during j_R years of working time. Government's revenue from taxation is therefore given by:

$$T_{t} = \tau_{t}^{l} \left(w_{t} \sum_{j=1}^{j_{R}} \sum_{a} \mu_{t,j} \lambda_{t,j} \left(a, s = e \right) \varepsilon_{j} + \rho z_{t} \sum_{j=j_{R}+1}^{J} \mu_{t,j} \right)$$
(12)

where $\lambda_{t,j}$ stands for the distribution of agents (see section 5.3 in page 22).

The budget constraint of the government is therefore balanced period by period, $T_t = G_t$:

$$\tau_t^l \left(w_t \sum_{j=1}^{j_R} \sum_{a} \mu_{t,j} \lambda_{t,j} \left(a, s = e \right) \, \varepsilon_j + \rho \, z_t \sum_{j=j_R+1}^{J} \mu_{t,j} \right) = \sum_{j=j_R+1}^{J} \mu_{t,j} b_{t-j} \quad (13)$$

3.2 Unemployment Benefits Program

The unemployment insurance benefits program is assumed to be self-financing, i.e.:

$$\sum_{j=1}^{j_R} \sum_{a} \mu_{t,j} \lambda_j (a, s = u) (\phi w_t \varepsilon_j) = \sum_{j=1}^{j_R} \sum_{a} \mu_{t,j} \lambda_j (a, s = e) \tau_t^u w_t \varepsilon_j$$
 (14)

3.3 Gradual Capitalization Strategy

From period t^* onwards (year 2000), a gradual (phasing-out) capitalization strategy leading towards a partially funded system is introduced. The chosen capitalization strategy has the following features. Newborn agents will pay their social security PAYG payroll tax solely on (1-x%) of their income (henceforth, gradual capitalization percentage, GCP), while the remaining x% will go untaxed. As these individuals retire, they will receive only (1-x%) of their average earnings as social security benefit, such that in the final steady-state equilibrium, as the economy will enjoy a higher degree of capitalization (hence, a smaller PAYG system), all agents will be paying social security taxes on (1-x%) of their income and receiving social security benefits equal to (1-x%) of their average earnings in the last f years.¹³

However, as the transition takes place, the fiscal balance of the social security system will be severely affected by the capitalization strategy. On top of the fiscal burden generated by the ageing population, the social security system will have to cope with the fact that the revenue generated by the working age population will be decreasing over time, as they gradually move into the partially funded scheme, while the old-age population will still receive pensions equal to 100 percent of their average earnings.

In fact, expenditure is expected to be larger than under the PAYG scheme until all newborn agents (born in moment t^* and onwards) receive their pension benefits according to the new rule. In period $t^* + j_R$ the position of the social security system will be most critical, due to the fact that all old-age pensions will be paid at a 100 percent rate, while working age population will be taxed only on (1 - x%) of their income. From then onwards, the situation will gradually stabilize, becoming self-sustainable from year $t^* + J$ ahead, when the final steady-state equilibrium will be reached.

During the transition from the initial to the final steady-state, the government disposes only of social security payroll taxation as a means to finance the additional fiscal burden generated by the demographic variations and the capitalization strategy.¹⁴ Hence, taxes are expected to increase during the transitions.

¹³The implementation of the possible different gradual capitalization strategies brings about a similar level of social security payroll tax in the long-run, by construction. Both the pension benefits and the taxed income level are lowered by the same percentage, and hence the tax levels remain practically unaltered in the final steady state of the model.

¹⁴The use of government debt as an instrument to finance the transition (see Huang, Imrohoroglu and Sargent (1996) among others) spreading its cost over time is excluded from the model.

sition in comparison to the situation in the baseline scenario with a PAYG system. The amount by which they will increase will depend, in turn, on the intensity of the general equilibrium effects that moving towards a partially funded system may bring about (i.e. mainly, the capital-output ratio is expected to increase, the rate of return on capital to decrease and the wage rate to increase).

4 Equilibrium

Definition 1 An **Equilibrium** for a given sequence of government policy arrangements $\left\{\rho, \phi, x, \tau_t^l, \tau_t^u\right\}^{t \in T}$ is a collection of value functions $\left\{V_{t,j}\left(a,s\right)\right\}_{j \in J}^{t \in T}$, individual decision rules $\left\{A_{t,j}\left(a,s\right), C_{t,j}\left(a,s\right)\right\}_{j \in J}^{t \in T}$, distribution of agent types $\left\{\lambda_{t,j}\left(a,s\right)\right\}_{j \in J}^{t \in T}$, a set of factor prices $\left\{r_t, w_t\right\}^{t \in T}$, and a sequence of lump-sum transfers $\left\{\xi_t\right\}^{t \in T}$ such that for all t:

- 1. All agents maximize (1) subject to (2)-(5), i.e. the individual decision rules $\{C_{t,j}\}_{j\in J}$, $\{A_{t,j}\}_{j\in J}$ solve the individuals' dynamic programming.
- 2. Factor markets clear:

$$K_{t} = \sum_{i} \sum_{a} \sum_{s} \mu_{t,j} \lambda_{t,j} (a,s) a_{t-1,j-1}$$
(15)

$$L_t = \sum_{i} \sum_{a} \mu_{t,j} \lambda_{t,j} (a, s = e) \varepsilon_j$$
 (16)

- 3. Prices are competitively determined, i.e. firms maximize profits, so that factor prices equal the marginal productivities of the factors of production, as in (7) and (8).
- 4. Goods market clears, so that the following feasibility condition is satisfied:

$$\sum_{j} \sum_{a} \sum_{s} \mu_{t,j} \lambda_{t,j} (a,s) C_{t,j} (a,s) + (1+n) K_{t}$$

$$= f(K_{t}, L_{t}) + (1-\delta) \sum_{j} \sum_{a} \sum_{s} \mu_{t,j} \lambda_{t,j} (a,s) a_{t-1,j-1}$$
(17)

which can be rewritten as:

$$Y_t = C_t + (1+n) K_t - (1-\delta) K_{t-1} = C_t + I_t$$

5. The collection of age-dependent distribution of agents follows the law of motion:

$$\lambda_{t,j}(a',s') = \sum_{s} \sum_{a:a'=A_j(a,s)} \Pi(s',s) \lambda_{t-1,j-1}(a,s)$$
 (18)

- 6. The budget constraint of the social security system, (13), is fulfilled.
- 7. The unemployment insurance benefits program is self-financing, (14).
- 8. The lump-sum distribution of the accidental bequests is determined by the following rule:

$$\xi_{t} = \sum_{j} \sum_{a} \sum_{s} \mu_{t,j} \lambda_{t,j} (a,s) \left(1 - \psi_{t,j+1} \right) A_{t,j} (a,s)$$
 (19)

Definition 2 A Stationary Equilibrium is an equilibrium that does not change over time and which consists of a given set of government policy arrangements $\{\rho, \phi, x, \tau^l, \tau^u\}$, a collection of value functions $\{V_j(a, s)\}_{j \in J}$, individual decision rules $\{A_{t,j}(a, s)\}_{j \in J}$, distribution of agent types $\{\lambda_j(a, s)\}_{j \in J}$, a set of factor prices $\{r, w\}$, and a lump-sum transfer ξ .

5 Computational Method

Agents are born into adulthood at age 1, the age of 21 (j = 1), and can live up to age 85 (j = J), after which death is certain. The mandatory retirement age is 65 $(j = j_R = 65)$. The chosen model period is 5 years, i.e. 14 generations (reported parameters display the one-year period equivalent values).

5.1 Parameterization and Calibration

We need to choose specific functional forms and particular values for the parameters of the model, in order to obtain numerical solutions to the model.

Calibrated Transition Demographics The demographic data which is exogenously fed into the model has been obtained from an original set of demographic projections for the period 1995 - 2051 provided by the Spanish Institute of Economics and Geography (IEG) and extended into the future up to year 2150.¹⁵ The resulting dependency ratio is plotted in Figure 1 below:

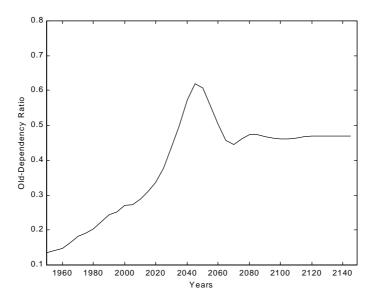


Figure 1: Old-Dependency Ratio (1950-2150)

Departing from a value of 0.246 in 1995, the old-dependency ratio reaches the value of 0.625 in year 2048 due to the entry of the Baby-Boom population into the stock of old people. As these cohorts move along the age distribution, the situation progressively improves, stabilizing at a level of 0, 486 in the long run (see Figure 1, where the SID old-dependency ratio has been complemented by United Nations data back to 1950). ¹⁶

In order to obtain this dependency ratio, the underlying methodology (based on the De Nardi, Imrohoroglu and Sargent (1999) technique) amounts to calibrating the rates of growth of the model population to match the Life

¹⁵The IEG projections that we have used assume that the average number of children per woman departs from a value of 1.17 in year 1995 and grows at an average yearly rate of around 3% until year 2012, after which it starts to decline, reaching a value of 1.8 in year 2025 and stabilizing thereafter. Besides, life expectancy is increasing, from a value of 81,16 for women and 74,13 for men, in 1995 to values of 84,65 and 78,21 in 2051, respectively.

¹⁶The UN data used for Figure 1 has been obtained from the United Nations' publication "World Population Prospects 1950-2050 (The 1998 Revision)".

Tables and the path of the dependency ratio implicit in the IEG projections.¹⁷ Those demographic projections have been extended into the future assuming a constant population growth rate for the period 2051 - 2150 equal to the SID average growth rate during the period 1995 - 2051.

We calibrate and compute an initial steady state, associated with constant pre-1995 values of the demographic parameters, using the 1995 IEG life table (i.e. the age distribution for the probabilities of survival). As in De Nardi, Imrohoroglu and Sargent (1999), we assume that prior to 1995, the economy was in a steady state and agents were experiencing the survival probabilities of the 1995 IEG life table. In 1995, they suddenly realize that the tables are changing over time and switch to using the correct ones. When the conditional survival probabilities stabilize in year 2051, the demographic structure of the population will still adjust for J+1 years, such that a new steady state is reached in period 2051+(J+1), where it will be essentially equal to its long-run structure.

Preferences The utility function is parameterized as:

$$u(c_{t',j}) = \frac{c_{t',j}^{1-\sigma} - 1}{1-\sigma}$$
 (20)

so that it is of the constant relative risk aversion class (CRRA), where σ is the coefficient of relative risk aversion.

In order to calibrate preferences, the parameters β and σ must be chosen. We choose $\sigma=2$ in accordance with the value selected in the literature for large overlapping generations models (see İmrohoroğlu, İmrohoroğlu and Joines (1995) or Hugett and Ventura (1999)). We set the discount factor to $\beta=0.985$ as in Auerbach and Kotlikoff (1987).¹⁸

Technology The economy produces a single good from aggregate capital and labour according to a Cobb-Douglas production function¹⁹:

 $^{^{17}}$ The IEG Life Tables at our disposal run only for the period 1995-2051. Prior to 1995 it is assumed that the age distribution of the probabilities of survival is equal to the one in 1995. From year 2051 onwards, a similar assumption is made and the figures for year 2051 prevail.

¹⁸See İmrohoroğlu, İmrohoroğlu and Joines (1995), Rios-Rull (1996) or the model in Chapter 2 for an analysis of a dynamic general equilibrium model with overlapping generations in which the discount factor greater than unity.

¹⁹The empirical evidence that factor shares and capital-output ratios have remained roughly constant over time (see Licandro et al. (1995)) while the ratio $\frac{w}{r}$ has increased

$$Y_t = f\left(K_t^d, L_t^d\right) = B\left(K_t^d\right)^{\theta} \left(L_t^d\right)^{1-\theta} \tag{21}$$

B is a scaling constant and θ is a parameter which measures the capital share in income.

The problem of the representative firm is therefore:

$$\max_{K_t^d, L_t^d} B\left(K_t^d\right)^{\theta} \left(L_t^d\right)^{1-\theta} - w_t L_t^d - (r_t + \delta) K_t^d$$
(22)

In equilibrium, factors' markets clear so that labour demand equals labour supply, $L_t^d = L_t^s$, and capital supply equals capital demand, $K_t^d = K_t^s$. Firms maximize profits taking factor and output prices as given. They hire physical capital and labour until factor prices equal marginal products, so that:

$$r_t = B\theta \left(\frac{K_t}{L_t}\right)^{\theta-1} - \delta \tag{23}$$

$$w_t = B(1-\theta) \left(\frac{K_t}{L_t}\right)^{\theta} \tag{24}$$

In order to calibrate the production function we need the values for the following parameters: B, θ , δ and ε .

The choice of the *scaling parameter* representing total factor productivity, B, depends on the units chosen for output. It is set to unity.

The capital intensity parameter represents the share of capital on income when the production function is of the Cobb-Douglas type²⁰. We choose a value of $\theta = 0.4$.

The rate of depreciation in the National Accounts is around 10 % of output. We have chosen the value of δ such that the steady state capital-output ratio in the 1990s matches the one of the economy, which is the procedure in standard literature on overlapping generations. For a value of the capital-output ratio

suggests the use of this functional specification.

²⁰The historical share of capital on national income in the National Accounts suggests a value of 0.5. Ríos-Rull (1994) and Bailén and Gil (1996), among others, consider that this value is far too high because labour income in the National Accounts excludes a part of the income of self-employed workers which is in turn included in capital income. They choose a value of 0.4.

in the range of these reported by Licandro et al. (1997) and King and Levine (1994), 2.35 - 2.65, the parameter δ is set to 6.5%.

The efficiency units profile is exogenous and age-specific, as in Auerbach and Kotlikoff (1987). It determines relative wages by age. We have chosen a profile derived from data obtained in the Encuesta Sobre Conciencia y Biografía de Clase²¹. The resulting profile is in line with the findings of Hansen (1993).

Asset Grid The discrete grid for asset values, $D = \{d_1, d_2, ..., d_m\}$ is determined by $d_1 = 0$, $d_m = 15$, and m = 601, with an even distribution of points.²² As in Imrohoroğlu, Imrohoroğlu and Joines (1995), we change d_m in the computations (when necessary) such that its value is never binding.

Social Security System We set the social security payroll tax rate, τ_t^l such that the replacement rate is approximately 100%. The parameter f, in equation (9), which represents the number of periods involved in the rule which is used to calculate pension benefits, is set to 15, as in the current system.

Unemployment Benefit Program Though it would be cheap to activate it, we rule out the possibility for agents to be unemployed, with a view to simplifying the model structure, easing the computational burden and isolating the factors which underlie the model equilibrium outcome. Hence, we will not focus on the potential role of social security as an insurance device to protect against idiosyncratic employment risk.

Therefore, the parameter ϕ is set to zero and the matrix of transition probabilities is selected so that the probability of employment is equal to unity.

5.2 Decision Rules

This section adapts the algorithm used to compute the decision rules and the distributions of agents in the stationary setup presented in İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995 and 1999) expanding it to allow for the

²¹National Statistics Institute (Instituto Nacional de Estadística, 1991).

²²We have also tried a spacing of grid points which increases with asset levels, following Hugett and Ventura (1999).

computation of transition dynamics between steady states with a changing population structure. 23

An agent in this economy faces a finite-horizon, finite-state dynamic programming. The value functions and the decision rules for each age j can be computed working backward from the last period of life, J. The procedure to solve this finite-state, finite-horizon dynamic programming is examined in more detail in Appendix I (page 36).

Let $D = \{d_1, d_2, ..., d_m\}$ be a discrete grid of points on which asset holdings fall, and $S = \{e, u\}$ be the employment opportunities space. Let $V_{t',j}(\tilde{x}_{t',j})$ be the optimal value function of an age-j individual born in t, in period t', with beginning of period asset holdings $\tilde{x}_{t',j} = (a_{t'-1,j-1}, s) \in D \times S$, if agents are of age $j \leq j_R$, and hence face idiosyncratic employment risk. Let $V_{t',j}(x_{t',j})$ be the value function of an age-j individual born in t, in period t', with beginning of period asset holdings $x_{t',j} = a_{t'-1,j-1}$ if agents are retired $(j > j_R)$.

The agent's Bellman equations, for j = 1, 2, ..., J, are:

$$V_{t',j}\left(a_{t'-1,j-1},s\right) = \max_{c_{t',j},a_{t',j}} \left\{ u\left(c_{t',j}\right) + \beta \,\psi_{t',j+1} E_{s'} V_{t'+1,j+1}\left(a_{t',j},s\right) \right\} \tag{25}$$

subject to the constraints (2)-(5). E stands for the expectation over the distribution of s' (where the prime denotes the following age). If we substitute the budget constraint (2) into (25), the problem reduces to choosing the decision variable: $a_{t',j}$.

In each period t', for agents who are retired, i.e. $j \in (j_R, J]$, the state space, as defined by the pair $x_{t',j}$, is an $m \times 1$ vector, as the employment state does not play any role. For agents whose age is $j \in [1, j_R]$ the state space is an $m \times 2$ matrix, containing both the asset holding level and the realization of the employment state. The control space for all agents is the $m \times 1$ vector D. Therefore, for any retired agent, the dimension of the decision rules is $m \times 1$ while it is $m \times 2$ if the agents are young. The whole set of decision rules and value functions will consist of 2T j_R matrices of dimension $m \times 2$ and 2T $(J - j_R)$ vectors of dimension $m \times 1$.

²³Hugett and Ventura (1999) use a similar algorithm to that of İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995 and 1999) and focus also on a stationary setup, introducing a labour-leisure choice. Both Huang, İmrohoroğlu and Sargent (1996) and De Nardi, İmrohoroğlu and Sargent (1999) examine transitions across steady states using a preference specification that yields a linear time-and-age dependent function for the vector of decisions made by an agent, following the approach of Hansen and Sargent (1995).

5.3 Distribution of Agents

In order to compute the distribution of agents, $\lambda_{t,j}(x_{t,j})$ we have to depart from a given initial wealth distribution. Let us assume that, for that purpose, agents are born with zero wealth (zero asset holdings), so that λ_1 is an $m \times 2$ matrix consisting of zeros everywhere, except for the first row, containing the expected employment and unemployment rates. The law of motion for the distribution of agents is as follows:

$$\lambda_{t,j}(x_{t',j}) = \sum_{s} \sum_{a:a'=A_j(a,s)} \Pi(s',s) \lambda_{t-1,j-1}(a,s)$$
 (26)

Depending on the realization of the employment state, agents will be employed or unemployed, and will make asset holdings decisions correspondingly. At moment t', and at the beginning of age j they will move to different points in the state-space matrix, $\lambda_{t',j}$. Each row in the state space matrix defines the share of age-j agents which have the specified combination of asset holdings and employment status²⁴. The state space matrix is of dimension $m \times 1$ when the agents are retired, as then they are not subject to employment risk.

The age profiles for consumption, asset holdings and disposable income can then be constructed using these distributions, together with the decision rules. The aggregate values of these variables can then be easily computed.

5.4 Computation of the Equilibrium

The structure of the computational strategy is based on the Auerbach and Kotlikoff (1987) procedure, with the additional feature that there is another dimension to the time-varying transition policies, i.e. a time-varying demographic structure, as in De Nardi, İmrohoroğlu and Sargent (1999) and the model presented in Arjona (2000a).

The algorithm amounts to:

1. Computing the *initial and final steady state equilibriums* of the economy.

In order to do so, we use a backward recursion to calculate both the decision rules and value functions of the agents, given the government policy, the lumpsum transfer (accidental bequests) and the factor prices. In order to find the

²⁴Notice that $\lambda_{t',j} \geq 0$ and $\sum_{j} \lambda_{t',j} = 1$.

initial and final steady state equilibria of the model (see section 5.5 for a detailed description of the method involved), we must solve a complicated set of non-linear equations that specify the optimization behaviours of individual agents, firms and government.

For that purpose, we use a Gauss-Seidel iterative method. The algorithm starts with guesses about the endogenous variables of the model. When the solution for the initially guessed endogenous variables equals the guess itself, a true solution to the system is found. Otherwise a new guess which is obtained as a combination of the two sets of values from the previous iteration must be tried. The procedure is repeated until the true solution is found.

2. Computing the equilibrium transition path between these steady states.

This amounts to compute the transition dynamics by solving backwards the sequence of value functions and decision rules, for a given sequence of factor prices, lump-sum transfer, government policy parameters and demographic evolution.

The transition path is solved using a similar method to the one used to calculate the initial and final steady states (see section 5.6). However, as the situation in the economy changes from period to period, it is necessary to solve explicitly for the behaviour of the agent in each of the periods. Furthermore, the fact that agents take into account the future stream of prices implies that it is necessary to solve simultaneously for equilibrium in all the transition years. In order to do so we provide the economy with 150 years to adjust, i.e. to move from the initial to the final steady state²⁵.

Here we also use a Gauss-Seidel iterative method but the dimension of the problem, and hence the computational load, is increased by a factor equal to the number of transition periods (i.e. 150). Moreover, it should be noted that individuals born before the transition starts behave up to the time the policy changes as if the initial steady state would continue forever. At the time the transition starts, these individuals behave as if they were members of a new generation.

²⁵The economy would converge to the steady state asymptotically because of the endogeneity of factor prices. 150 periods provide the economy with enough time to settle down before it is forced to converge after the last period has passed, as in Auerbach and Kotlikoff (1987).

5.5 Stationary Equilibrium

For a given efficiency units profile, $\{\varepsilon_j\}_{j\in J}$, demographic parameters $\{n_t, \psi_{t,j}\}_{j\in J}$, government policy arrangements $\{\rho, \phi, x, \tau^l, \tau^u\}$, lump-sum transfer (accidental bequests), ξ , and factor prices $\{r, w\}$, the steps involved are the following:

- 1. Initialize the aggregate capital stock of the economy, K_0 and the lumpsum transfers, ξ_0 . Compute the aggregate labour input, L_0 using (16).
- 2. Given K_0 and L_0 , solve the problem of the firm and, hence, obtain the factor prices, r_0 and w_0 , as (7) and (8).
- 3. Given $\{r_0, w_0\}$, and the policy parameters $\{\rho, \phi, x, \tau^l, \tau^u\}$ compute the decision rules for each age cohort using a backward recursion from period J using the procedure specified in Appendix I.
- 4. Compute the distribution of agent types, λ_j , by completing a forward recursion.
- 5. Compute the new aggregate capital stock, K_1 and the new lump-sum transfer, ξ_1 .
- 6. Specify a convergence criterion such that if the new aggregate capital stock and lump-sum transfers are close enough to the old ones the algorithm stops. Otherwise, a new initial condition is supplied and the procedure is repeated until convergence is achieved. This new initial condition is calculated as a weighted average of the new and old capital stock (lump-sum transfers) using a given step size (damping factor) as weight.

5.6 Transition Path

For a given efficiency units profile, $\{\varepsilon_j\}_{j\in J}$, and time path for the demographic parameters $\{n_t, \psi_{t,j}\}_{j\in J}^{t\in T}$, factor prices, $\{r_t, w_t\}^{t\in T}$, government policy arrangements $\{\rho, \phi, x, \tau_t^l, \tau_t^u\}^{t\in T}$, and lump-sum transfer (accidental bequests), $\{\xi_t\}^{t\in T}$, we must solve backwards the Bellman equation specified by (25).

5.7 Welfare Measures

When examining steady state welfare, we measure it as the expected discounted utility that a newborn agent gets from the consumption policy functions, $\{C_j(a,s)\}_{j\in J}$, under a given set of social security parameters. For this set of policy options $\Omega = \{\rho, \phi, x, \tau^l, \tau^u\}$, it is calculated as:

$$W(\Omega) = \sum_{j=1}^{J} \sum_{a} \sum_{s} \beta^{j-1} \lambda_{j}(a, s) \psi^{j} U(C_{t', j}(a, s))$$
 (27)

In order to compare different social security systems, we define a baseline scenario: the equilibrium under a PAYG retirement pension system, i.e. $W_0 = W(\Omega_0)$. As in İmrohoroğlu, İmrohoroğlu and Joines (1995), we compute the lump-sum compensation (also known as the consumption supplement, consumption equivalent variation or compensating variation) required to make a newborn individual indifferent between the baseline scenario and the alternative one in each period of life²⁶, κ , i.e. we compute κ such that $W(\Omega_0 + \kappa) = W(\Omega_1)$.

During the transition, we compute the welfare measure as:

$$W_{t'}(\Omega) = \sum_{j=1}^{J} \sum_{a} \sum_{s} \beta^{j-1} \lambda_{t',j}(a,s) \psi_{t'}^{j} U(C_{t',j}(a,s))$$
 (28)

and the lump-sum compensation, $\kappa_{t'}$, is computed such that $W(\Omega_{0,t'} + \kappa_{t'}) = W(\Omega_{1,t'})$.

6 Results

In this section we present the simulation results for various gradual capitalization strategies, differing on the underlying percentages of capitalization (GCPs). All simulations start from the same initial steady state (see section 5.1) and reach a different final steady state under each of the selected policy options.

 $^{^{26}}$ We compute this measure relative to Y_0 ,. i.e. the output under the PAYG retirement pension system.

First, we will discuss the quantitative features of the steady states, focusing on the dynamic general equilibrium effects (the underlying forces driving individual agents' life-cycle choices) and comparing the outcomes obtained under each of the explored policy options. Second, we will analyze the properties of the transition paths and the implications that the implementation of the different capitalization strategies has on the intergenerational distribution of welfare.

The second aspect complements the first one. Comparing steady states only brings about "the positive aspects of taxing (or reducing pensions) and increasing savings" (as pointed out by De Nardi, Imrohoroglu and Sargent (1999)). Meanwhile an analysis of the transition dynamics takes into account the fact that different gradual capitalization policies (interacting with the demographic variations) will affect members of various generations in a different manner.

6.1 Steady-State Comparisons

We have chosen different values for the GCPs. The initial steady state (ISS) is common to all simulations, with a value of 100 percent. The final steady state has been computed: (1) under the assumption that the current PAYG system stays in place unchanged into the future (our baseline scenario), and (2) for values of the GCP of 90, 85, and 80 percent. We have also computed the final steady state for the GCP which makes individuals in the final steady state indifferent with respect to the initial steady state.

Table 1 summarizes the main quantitative features of the initial and final steady states, and compares outcomes across steady states for the different values of the GCP.

Table 1: Comparison of Steady-State Results

% Income	au	r	\overline{w}	$\frac{K}{Y}$	C	U
73.15	0.2580	0.764	0.0821	2.8295	0.1501	-83.3270
80.00	0.2572	0.790	0.0811	2.7784	0.1498	-84.7431
85.00	0.2566	0.808	0.0804	2.7426	0.1495	-85.8100
90.00	0.2560	0.827	0.797	2.7078	0.1493	-86.9060
95.00	0.2554	0.846	0.791	2.6741	0.1490	-88.0320
100.00	0.2548	0.864	0.784	2.6413	0.1487	-89.1889
100.00 (ISS)	0.1802	0.0923	0.765	2.5426	0.1434	-83.3270

Several conclusions can be extracted from the analysis of Table 1:

First, comparing the initial and final steady state results for the PAYG system, it can be observed that the path of demographic change has a major impact on the outcome of the model. The evolution of the projected old-dependency ratio brings about an increase of more than 41 percent in the social security payroll tax (in line with the results obtained in Arjona (2000a) and Arjona (2000b)). The capital-output ratio increases by ca. 4 percent and so does average consumption. The interest rates fall from 9.23 to 8.64 percent, while the wage rate increases by 2.5 percentage points. As a result of this, the measure of average utility displays a considerable decrease, implying that agents are far worse off in the final steady state in comparison to the initial one.

Second, the implementation of the different gradual capitalization strategies brings about a similar level of social security payroll tax, as expected. Both the pension benefits and the taxed income level are lowered by the same percentage in the long-run, and hence the tax levels remain practically unaltered.

Third, the smaller the GCP the greater the capital-output ratio and the average consumption is, while real interest rates become smaller and wage rates rise. In fact, the capital-output ratio increases by over 5 percent, as the GCP decreases to a level of 80 percent.

Fourth, when comparing the initial and final steady states under the PAYG scheme, we can observe that the pure effects of the demographic transition show in a lower age-asset profile between the initial and the final steady state (see Figure A3 in Appendix III). These profiles present the standard features in life-cycle models, mainly progressive asset accumulation until retirement and

depletion thereafter. When we focus on the outcome under gradual capitalization, we can observe that the lower the GCP, the higher the age-asset profiles for all ages with respect to the final PAYG steady state, and the smoother the pattern of individual asset accumulation. The demographic transition also lowers the age-consumption profiles of individual agents. When comparing the final steady state outcomes, it is worth pointing out that a higher GCP induces more consumption when old and less when young (see Figure A4). This behaviour is consistent with the lower interest rates to be found in the final steady states combined with the increase in savings incentives brought about by the improvement in life-expectancy.

Finally, we have computed the GCP threshold value which makes individual agents indifferent between being in the initial steady state and the final one (after the demographic transition has taken place and the old-dependency ratio has varied accordingly). The optimal threshold is found to be 73.15 percent, meaning that gradual capitalization strategies in which the GCP is smaller than the computed threshold will lead to a welfare level greater than the initial one, after the transition has taken place. All values above that figure (see Table 1) give a lower average utility.

6.2 Transition Dynamics

In our model, we can distinguish two main underlying mechanisms driving the evolution of the transition and determining the intergenerational distribution of welfare, along with the dynamic general equilibrium effects: (1) the demographic variations, and (2) the policy changes, i.e. the implementation of the selected gradual capitalization strategies.²⁷

Table 2 below summarizes the main results of the simulations for the baseline PAYG case and the different gradual capitalization strategies adopted (characterized by the underlying selected level of GCP). Results have been presented for selected years (with a fourty years interval between them) and are complemented with the information displayed in tables A1-A4 (in Appendix II) and Figures A5-A8 (in Appendix III).

²⁷Some other effects could also be in place under different model specifications. If we activated idiosyncratic risk, for instance, social security would act as a partial insurance device against that risk and hence substitute for the missing annuities market, playing an active role during the transition dynamics. Besides, in a model with endogenous labour-leisure choice, as in Conesa and Krueger (1999), the elimination of tax distortions would also play a fundamental role.

Table 2: Transition Paths of PAYG vs. 90% and 73% GC Strategies

	Period:	ISS	2040	2080	2120	2150
\overline{PAYG}	au	0.1802	0.2620	0.2523	0.2540	0.2548
	r	0.0923	0.0860	0.0866	0.0865	0.0864
	w	0.0765	0.0786	0.0785	0.0784	0.0784
	$\frac{K}{Y}$	2.5426	2.6494	2.6382	2.6403	2.6413
	C	0.1434	0.1490	0.1485	0.1486	0.1487
90% GC	au	0.1802	0.2754	0.2535	0.2552	0.2560
	r	0.0923	0.0847	0.0839	0.0828	0.0827
	w	0.0765	0.0790	0.0798	0.0797	0.0797
	$\frac{K}{Y}$	2.5426	2.6720	2.7040	2.7066	2.7078
	\dot{C}	0.1434	0.1502	0.1491	0.1492	0.1493
	k	0.0013	0.0113	0.0196	0.0202	0.0205
73% GC	au	0.1802	0.3014	0.2555	0.2572	0.2580
	r	0.0923	0.0823	0.0766	0.0765	0.0764
	w	0.0765	0.0799	0.0820	0.0821	0.0821
	$\frac{K}{Y}$	2.5426	2.6504	2.7159	2.8278	2.8295
	$\overset{1}{C}$	0.1434	0.1525	0.1501	0.1502	0.1502
	k	0.0047	0.0332	0.0532	0.0551	0.0557

The results presented in Table 2 can be summarized in the following manner:

First, although in the long-run the implementation of the different gradual capitalization strategies brings about a similar level of social security payroll tax (by construction) during the transition the tax level increases considerably with respect to the PAYG baseline. This is due to the fact that simultaneously with the increase in the fiscal burden generated by the ageing population structure, the social security system will have to deal with the decreasing revenue generated by the working age population, as they gradually move into the partially funded scheme. Meanwhile, the old-age population will still receive pensions equal to 100 percent of their average earnings. As it can be observed from Figure A5, the lower the GCP the higher the difference between the two curves (the continuous line plotting the PAYG baseline and the dashed lines plotting the gradual capitalization strategies) during the period 1995 – 2065. However, it is worth noticing that the required tax increases are modest, due mainly to the dynamic general equilibrium effects at work. The smaller the GCP, the larger the dynamic general equilibrium effects at work, having a

beneficial impact on the required increase in the tax rates during the transition. The implementation of the different gradual capitalization strategies brings about a similar level of social security payroll tax in the steady state, as expected.

Second, during the transition, the smaller the GCP the greater the capital-output ratio and wage rates, while real interest rates become smaller. It is worth noticing the evolution path of the interest rates under the various GCPs, as presented in Figure A6. The real interest rate decreases progressively and, in comparison with the PAYG baseline, the gap between the curves increases from years 2035-2040 onwards, stabilizing thereafter. The capital stock evolution (Figure A8) for the various gradual capitalization strategies in place displays an increasing difference with the PAYG baseline, especially from year 2050 onwards, as the model approaches its final steady state.

Third, average consumption for the gradual capitalization strategies increases considerably with respect to the PAYG baseline around the hump-shaped curve generated by the demographic variations (see Figure A7). Differences narrow and stabilize thereafter, leading to a greater average consumption in the long-run steady states relative to the PAYG baseline.

Table 3: Compensating Variations of GC Strategies vs. PAYG

Generation born in year:	$k_{73\%}$	$k_{80\%}$	$k_{85\%}$	$k_{90\%}$
1930	0.0000	0.0000	0.0000	0.0000
1950	-0.0054	-0.0053	-0.0053	-0.0052
1970	-0.0118	-0.0116	-0.115	-0.0114
1990	-0.0056	-0.0060	-0.0064	-0.0066
2010	0.0134	0.0099	0.0074	0.0049
2030	0.0272	0.0198	0.0145	0.0095
2050	0.0449	0.0325	0.0240	0.0157
2070	0.0589	0.0436	0.0326	0.0216
2090	0.0560	0.0415	0.0310	0.0205
2110	0.0547	0.0406	0.0303	0.0201
2130	0.0557	0.0412	0.0308	0.0205
2150	0.0557	0.0412	0.0308	0.0205

Table 3 reports our results on the intergenerational distribution of welfare. We use the measures of average utility defined in section 5.7 to examine the transition paths for the different gradual capitalization strategies (differing on their underlying GCPs) with respect to the PAYG baseline.²⁸

The main conclusion to be extracted from that table is that all future generations are better off under each of the proposed capitalization strategies relative to the PAYG baseline. In fact, any values reported in the table for generations born after 1995 display a positive compensating variation, implying that these generations are better off than they would have been if they had remained under the PAYG system.

The lower the GCP is, the higher the compensation must be to make individuals indifferent. This is due to the fact that, parallel to the arguments we put forward in section 6.1, reforming the social security system brings higher welfare gains supported by the dynamic general equilibrium effects. In fact, for a gradual capitalization of 80 percent, the long-run compensating variation is twice that required for a 90 percent GCP. During the transition the differences between these rates are even larger.

However, despite the fact that the different gradual capitalization strategies have similar long-run and overall positive welfare effects, as pointed out in section 6.1, the existing generations are however negatively affected by the introduction of these schemes. In fact, as shown in Table 3, the existing generations (the "old" of the model at the moment of introducing the capitalization strategies) suffer a welfare loss with respect to the PAYG transition path, irrespectively of the given GCP. These generations have negative welfare gains (positive losses) for all three proposed capitalization strategies.

7 Conclusions

The aim of this paper is to analyze the magnitude of the transition and long-run costs of changing the current Spanish Pay-As-You-Go system into a partially funded system using a gradual capitalization strategy. We perform this analysis with a view to assessing its impact on the intergenerational distribution of welfare. The study is conducted in light of the expected demographic changes in Spain.

²⁸As explained in detail in section 5.7, the compensating variations express the lump-sum consumption compensation to be given to agents born in each of the generations reported in the first column to make them indifferent between participating in the gradual capitalization strategies with respect to the PAYG baseline.

For that purpose, we build a stochastic dynamic general equilibrium model with overlapping generations of long-lived agents facing mortality and idiosyncratic employment risk and expand the computational procedure of İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995) to examine transition dynamics between steady states with a changing population structure, in light of the forecasted demographic changes that will occur in Spain in the medium to the long run. The model consists of an overlapping generations model of long-lived agents facing mortality risk and borrowing constraints. We focus on the contribution-related pension benefits of the General Regime and use a demographic forecast of the Spanish Institute of Demography (SID).

We examine both the quantitative features of the steady states and the properties of the transition paths, with special emphasis on their implications for the intergenerational distribution of welfare and focusing on the role of dynamic general equilibrium effects.

Our main findings can be summarized as follows. First, demographic variations have a large impact on the outcome of the model, increasing the social security payroll tax to more than 41 percent in the final steady state. Second, the gradual capitalization percentage threshold which makes individual agents indifferent between the initial steady state and the final one is found to be 73 percent, meaning that selected gradual capitalization strategies in which the GCP is smaller than the threshold will deliver a greater welfare level than the one agents enjoyed in the initial steady state. Third, in the long-run the implementation of the different gradual capitalization strategies brings about, by construction, a similar level of social security payroll taxes. However, during the transition the tax level increases considerably relative to the PAYG baseline, as the social security system deals with the decreasing revenue generated by the working age population while the existing "old" generations still receive 100 percent of their pension benefits. Fourth, the smaller the GCP, the larger the dynamic general equilibrium effects at work, generating a positive impact on the required increase in the tax rates during the transition. Fifth, all future generations are better off under each of the proposed capitalization strategies relative to the PAYG baseline. However, those paying the costs are the older generations, which suffer a welfare loss with respect to the PAYG transition path for all examined gradual capitalization strategies.

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Appendix I: Dynamic Programming

In order to solve the finite-horizon, finite-state dynamic program, we use a single recursion starting from the last period of life. The last period problem is as follows:

$$V_{t',J}(x_{t',J}) = \max_{c_{t',J},a_{t',J}} u(c_{t',J})$$
s.t.
$$c_{t',J} = (1 + r_{t'}) a_{t'-1,J-1} + b_{t',J} + \zeta_{t'}$$
(29)

For an agent of age J the value function at t'+1 is zero. Therefore, the solution to the problem (29) is a decision rule $A_{t',J}$ which is an $m \times 1$ vector of zeros²⁹. The value function, $V_{t',J}$, is calculated by evaluating the objective function at the budget constraint, with $a_{t'-1,J-1} = D$.

The recursion is then worked backwards until age j_R , by solving the problem $(j_R < j < J)$:

$$V_{t',j}(x_{t',j}) = \max_{c_{t,j}, a_{t',j}} \left\{ u(c_{t',j}) + \beta \psi_{t',j+1} E_{s'} V_{t'+1,j+1}(x_{t'+1,j+1}) \right\}$$
(30)
s.t.
$$c_{t',j} + a_{t',j} = (1 + r_{t'}) a_{t'-1,j-1} + y_{t',j} + \zeta$$

$$c_{t',j} + a_{t',j} = (1 + r_{t'}) \ a_{t'-1,j-1} + y_{t',j} + \zeta$$

$$c_{t',j} \ge 0, \ a_{t',j} \ge 0$$
(31)

In order to find the decision rule, we let $a_{t'-1,j-1}$ take each of the values of the elements in D and for each of those we obtain the value of $a_{t',j} \in D$ that solves problem (30) subject to (31) by evaluating the objective function at each point on this grid, D. The value obtained is stacked on the decision rule $m \times 1$ vector, $A_{t',j}$.

The problem is different from age j_R onwards, as then the disposable income of agents is dependent on the idiosyncratic employment risk, and hence the decision rule becomes an $m \times 2$ matrix, i.e. will be different depending on the

²⁹This is due to the fact that: (1) there are no bequests for individuals of age J, and (2) death is certain after age J.

realization of the employment state, s. The value function is also a matrix of dimension $m \times 2$.

The problem to be solved by the agents in period j_R is:

$$V_{t',j}\left(\tilde{x}_{t',j}\right) = \max_{c_{t,j}, a_{t'+1,j+1}} \left\{ u\left(c_{t',j}\right) + \beta \,\psi_{t',j+1} E_{s'} \,V_{t'+1,j+1}\left(x_{t'+1,j+1}\right) \right\}$$
s.t. (31)

For periods $j \leq j_R$ the problem is:

$$V_{t',j}(\widetilde{x}_{t',j}) = \max_{c_{t,j}, a_{t'+1,j+1}} \left\{ u(c_{t',j}) + \beta \psi_{t',j+1} \sum_{s'} \Pi(s',s) \ V_{t'+1,j+1}(\widetilde{x}_{t'+1,j+1}) \right\}$$
s.t. (31)

The procedure to find the decision rules and the value functions is similar to the one described above, but, due to the employment risk, we must solve the problem for the two different states. The decision rules $A_{t',j}$ and the value functions, $V_{t',j}$, will be of dimension $m \times 2$.

The whole set of decision rules and value functions for each period t' will consist of 2 j_R matrices of dimension $m \times 2$ and 2 $(J - j_R)$ vectors of dimension $m \times 1$.

Appendix II: Tables

Table A1: Evolution of (Demographic) Transition with a PAYG System

	Pay-As-You-Go							
Period	au	r	w	$\frac{K}{Y}$	C			
ISS	0.1802	0.0923	0.0765	2.5426	0.1434			
2020	0.2038	0.0902	0.0771	2.5765	0.1454			
2040	0.2620	0.0860	0.0786	2.6494	0.1490			
2060	0.2758	0.0850	0.0789	2.6671	0.1495			
2080	0.2523	0.0866	0.0785	2.6382	0.1485			
2100	0.2532	0.0866	0.0784	2.6393	0.1486			
2120	0.2540	0.0865	0.0784	2.6403	0.1486			
2140	0.2548	0.0864	0.784	2.6413	0.1487			
2150	0.2548	0.0864	0.784	2.6413	0.1487			

Table A2: Evolution of Transition under a 90% Gradual Capitalization Strategy

:	90% Gradual Capitalization							
Period	$\overline{\tau}$	r	\overline{w}	$\frac{K}{Y}$	C	k		
ISS	0.1802	0.0923	0.0765	2.5426	0.1434	0.0049		
2020	0.2111	0.0887	0.0776	2.6023	0.1459	0.0077		
2040	0.2754	0.0847	0.0790	2.6720	0.1502	0.0113		
2060	0.2792	0.0815	0.0802	2.7296	0.1502	0.0206		
2080	0.2535	0.0829	0.0798	2.7040	0.1491	0.0196		
2100	0.2543	0.0829	0.0797	2.7053	0.1492	0.0203		
2120	0.2552	0.0828	0.0797	2.7066	0.1492	0.0202		
2140	0.2560	0.0827	0.0797	2.7078	0.1493	0.0205		
2150	0.2560	0.0827	0.0797	2.7078	0.1493	0.0205		

Table A3: Evolution of Transition under an 85% Gradual Capitalization Strategy

	85% Gradual Capitalization							
Period	$\overline{\tau}$	r	\overline{w}	$\frac{K}{V}$	C	k		
ISS	0.1802	0.0923	0.0765	2.5426	0.1434	0.0013		
2020	0.2149	0.0879	0.0779	2.6159	0.1462	0.0118		
2040	0.2827	0.0840	0.0793	2.6842	0.1508	0.0174		
2060	0.2810	0.0798	0.0808	2.7620	0.1506	0.0286		
2080	0.2541	0.0811	0.0803	2.7384	0.1494	0.0295		
2100	0.2549	0.0810	0.0804	2.7398	0.1495	0.0307		
2120	2.2549	0.0809	0.0804	2.7412	0.1495	0.0305		
2140	0.2558	0.0808	0.0804	2.7426	0.1495	0.0308		
2150	0.2566	0.0808	0.0804	2.7426	0.1495	0.0308		

Table A4: Evolution of Transition under a 73% Gradual Capitalization Strategy

	73% Gradual Capitalization							
Period	au	r	w	$\frac{K}{Y}$	C	k		
ISS	0.1802	0.0923	0.0765	2.5426	0.1434	0.0047		
2020	0.2245	0.0859	0.0786	2.6504	0.1469	0.0216		
2040	0.3014	0.0823	0.0799	2.7159	0.1525	0.0332		
2060	0.2861	0.0757	0.0823	2.8421	0.1514	0.0565		
2080	0.2555	0.0766	0.0820	2.8244	0.1501	0.0532		
2100	0.2563	0.0765	0.0820	2.8261	0.1502	0.0553		
2120	0.2572	0.0765	0.0821	2.8278	0.1502	0.0551		
2140	0.2580	0.0764	0.0821	2.8295	0.1502	0.0557		
2150	0.2580	0.0764	0.0821	2.8295	0.1502	0.0557		

Appendix III: Figures

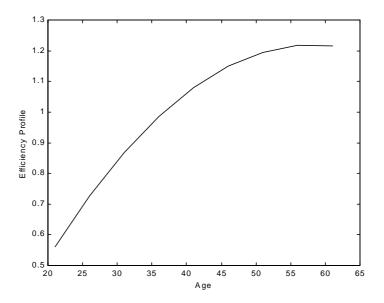


Figure A1: Smoothed Efficiency Profile

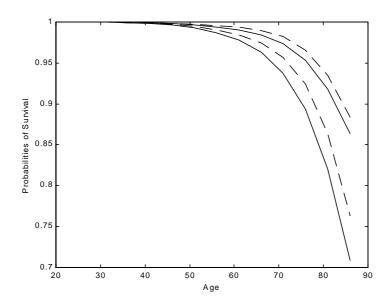


Figure A2: Simple and Unconditional Probabilities of Survival in years 1995 and 2051. Continuous: Initial. Dashed: Final. Lower curves: Conditional. Upper curves: Unconditional.

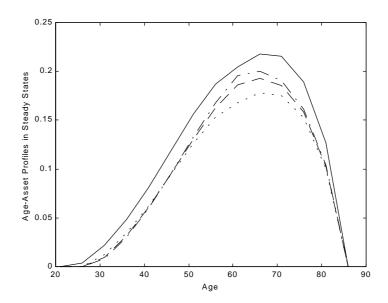


Figure A3: Age-Asset Profiles in Steady State. Upper: ISS. Lower: FSS-PAYG. Dashed: FSS-90%. Dash-dotted: FSS-85%.

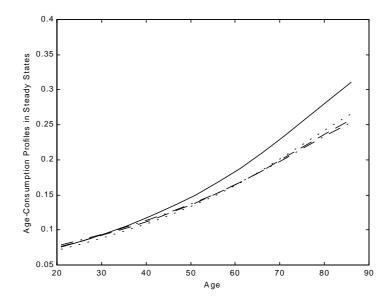


Figure A4: Age-Consumption Profiles in Steady States. Upper: ISS. Dotted: FSS-PAYG. Dashed: FSS-90%. Dash-dotted: FSS-85%.

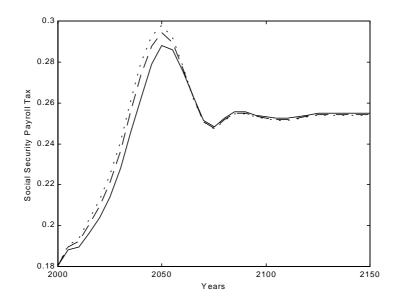


Figure A5: Social Security Payroll Tax during Transition. Continuous: PAYG. Dashed: 90%. Dotted: 85%.

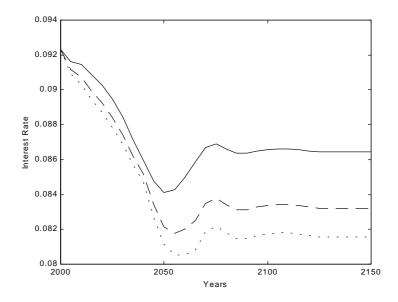


Figure A6: Interest Rate during Transition. Continuous: PAYG. Dashed: 90%. Dotted: 85%.

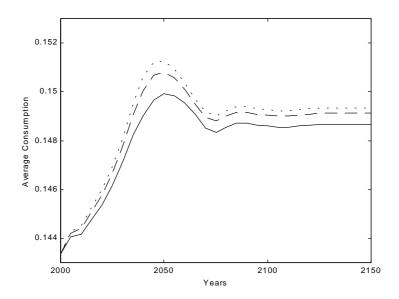


Figure A7: Average Consumption during Transition. Continuous: PAYG. Dashed: 90%. Dotted: 85%.

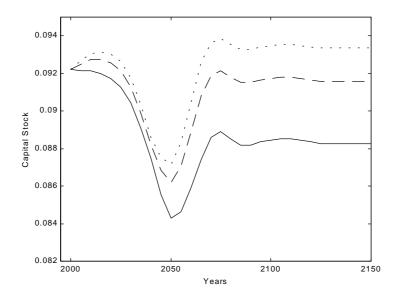


Figure A8: Capital Stock during Transition. Continuous: PAYG. Dashed: 90%. Dotted: 85%.