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COMPLETE SNAKE AND ROTATOR SCHEMES FOR SPIN POLARIZATION IN PROTON RINGS AND LARGE ELECTRON RINGS

by

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Principal remarks on snake schemes

Why snake schemes are needed in ring accelerators and how they function was explained in some detail in the introduction of a previous DESY report[5].

For describing the spin motion we introduce, as usual, a rectangular, right-handed coordinate system $\{\vec{z}, \vec{x}, \vec{s}\}$ which follows the beam, where $\vec{x}$ is the transverse horizontal direction, $\vec{s}$ the beam direction (usually also horizontal) and $\vec{z}$ the (usually vertical) direction orthogonal to the $\vec{x}$, $\vec{s}$-plane.

Then, in generalizing previous usage, we call a "snake" any sequence of magnets that rotates the spin by 180° about an arbitrary axis lying in the $\vec{x}$, $\vec{s}$-plane, i.e. about an arbitrary horizontal axis in the usual orientation.

The overall spin rotation of a snake can then be described by a vertical rotation of 180° about the horizontal $\vec{x}$-axis, followed by a (horizontal) rotation by an angle α about the (vertical) $\vec{z}$-axis, where α may have any value between -180° and +180°. The value of this spin precession angle characterizes the snake, and for $\alpha = n \cdot 90°$, n integer, special names have been previously introduced:

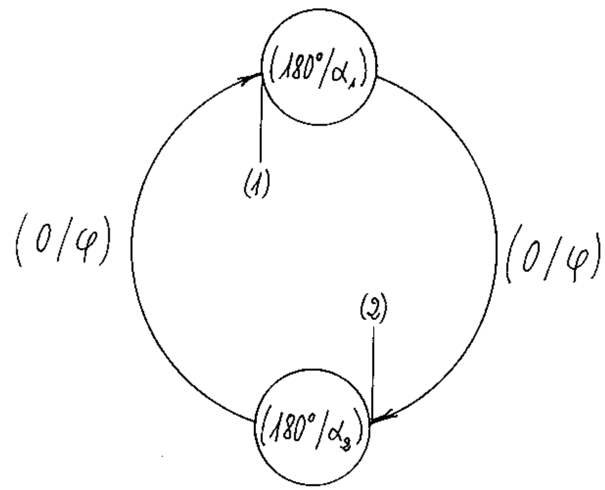
spin precession angle α	overall rotation axis of the snake	snake character	abbreviated
180°	$\vec{s}$-axis	snake of 1st kind	1st
90°	$(\vec{s}-\vec{x})$-axis	left pointed snake	L
0°	$\vec{x}$-axis	snake of 2nd kind	2nd
-90°	$(\vec{s}+\vec{x})$-axis	right pointed snake	R

After their proposal by Derbenev and Kondratenko[1,2] in 1977/78, the snakes of the 1st and 2nd kind were for a long time the only ones discussed for application in accelerator rings. Recently, however, more general snake schemes were found to be applicable, and practicable specimens were suggested[5]. Together with other new samples of snakes, they will be reviewed here in a broader context.

With the spin precession angle α, the overall spin rotation of a snake may be denoted by the symbol

$$(180°/\alpha)$$

where 180° is the vertical spin rotation and α the subsequent horizontal spin rotation. We can then quite simply write down and evaluate the overall spin rotation in a ring with two snakes of precessions α_1 and α_2 in opposite symmetry points:



starting at (1): $(0/\varphi)(180°/\alpha_2)(0/\varphi)(180°/\alpha_1) = (0/\alpha_2 - \alpha_1)$

starting at (2): $(0/\varphi)(180°/\alpha_1)(0/\varphi)(180°/\alpha_2) = (0/\alpha_1 - \alpha_2)$

The equal spin precession φ in the two arcs obviously drops out, and the overall spin precession per turn is $(\alpha_2 - \alpha_1)$ in half ring (1) and $-(\alpha_2 - \alpha_1)$ in half ring (2), yielding the spin tune

$$\nu = \begin{cases} +\dfrac{\alpha_2 - \alpha_1}{360°} & \text{in half ring (1)} \\ -\dfrac{\alpha_2 - \alpha_1}{360°} & \text{in half ring (2)} \end{cases}$$

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Complete snake and rotator schemes for spin polarization in proton rings and large electron rings

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Abstract

In order to maintain spin polarization in proton rings and large electron rings, some generalized Siberian Snake[1,2] scheme may be required to make the spin tune almost independent of energy and thus avoid depolarizing resonances. The practical problem of finding such schemes that, at reasonable technical effort, can be made to work over large energy ranges has been addressed before[3,4,5] and is here revisited in a broadened view and with added new suggestions. As a result, possibly optimum schemes for electron rings (LEP) and proton rings are described. In the proposed LEP scheme, spin rotation is devised such that, at the interaction points, the spin direction is longitudinal as required for experiments.

If particles of different energy see the same spin rotation in the snake and if the snake performance is also maintained when changing the energy of the ring, only the spin precession φ in the half rings will change and the spin tune will stay the same independent of energy. This is the purpose of the snake pair and we see that, in principle, any combination of snakes will do the job, provided that ν is not near a spin resonance which would depolarize the beam. For $\alpha_1 = \alpha_2$, i.e. equal snakes, we would, of course, be on the integer resonance; α_1 and α_2 must therefore be chosen to be sufficiently different. Generally, one would design for a spin tune near $\nu = 0.5$, which is, for example, the case when using a 1st and a 2nd kind snake or a left and a right pointed snake.

In accelerator rings with more than two symmetry points there are advantages in using more than two snakes[6,7]. Then, with n snakes of precession angles α_i, the overall spin precession per turn will be

$$(0 \;/\; \pm \sum_{i=1}^{n} (-1)^i \alpha_i)$$

If, for example, a single snake pair were designed to give a spin tune $\nu = \pm 0.17$, then the insertion of 3 such snake pairs in the ring would give a spin tune near $\nu = 0.5$ again.

Snake arrays

Turning now to practical snake design, we will consider here only alternating sequences of vertical and horizontal bending magnets, with each magnet being characterized by its vertical or horizontal spin rotation.

If, in this sequence, all vertical spin rotations are given as certain multiples of a rotation angle ψ_V, and all horizontal spin rotations as multiples of another rotation angle ψ_H, the sequence is characterized by the two parameters ψ_V, ψ_H. Then, for an arbitrary pair of values ψ_V, ψ_H, i.e. for an arbitrary point in the (ψ_V, ψ_H)-plane, the sequence will, in general, not act as a snake. But there may, in this plane, exist a continuous locus of points where it does, and in that case we call this continuous set of snakes a snake array.

A snake array thus is a continuous set of snakes obtained, for a given sequence of vertical spin rotations proportional to ψ_V interlaced with horizontal spin rotations proportional to ψ_H, by varying ψ_V, ψ_H along the locus of snake points in the (ψ_V, ψ_H)-plane. The subsequent examples of snake arrays will illustrate this.

Four different snake arrays were found to be of particular practical interest and are therefore here investigated. They are denoted by (A), (B), (C) and (D) and are shown schematically in figures 1 through 4. In all cases, ψ_V and ψ_H are taken to be one half of the maximum vertical or horizontal spin rotation angle occuring in the snake. Writing ψ_V = V and ψ_H = H, the snake arrays are described by the sequences

(A): (-V, -H, +2V, +2H, -V, -H, $\underbrace{[-\tfrac{4}{3}H] \| [+\tfrac{4}{3}H]}_{\text{Alternative (A1)}}$, +H, -V, -2H, +2V, +H, -V)

(B): (+V, +2H, -2V + 90°, +2H, +V - 90° ‖ +V - 90°, +2H, -2V + 90°, +2H, +V)

(C): (-V, -2H, +2V, +2H, -V ‖ -V, -2H, +2V, +2H, -V)

(D): (-H, -V, +2H, +V ‖ +V, -2H, -V, +H)

Alternative (D1): (-V, +2H, +V ‖ +V, -2H, -V)

For each of these snake arrays, the locus of snake points in the (ψ_V, ψ_H)-plane is shown in Figs. 5 and 6. Surprisingly, the locus is outside and quite close to a circle in all four cases, and it is therefore possible to denote a certain snake in the array by using an obvious clock notation. For example, the 7.30 h-version of array (A) with $\psi_V = \psi_H = 45°$ is the old snake of 1st kind suggested in ref. 3). In a practical snake design, ψ_V and ψ_H should be chosen as small as possible, which means that the quadrant between 6.00 h and 9.00 h ist most eligible. Also indicated in the figures is the snake precession angle α which may or may not vary along the array.

Numerically, the four snake arrays are given in tables 1, 2, and 3. For individual snake points distributed about the almost circular locus, the tables give rather accurate values of vertical and horizontal spin rotation ψ_V, ψ_H, the spin precession angle α and the corresponding contribution $\Delta\nu$ of the snake to the spin tune. At certain points, the character of the snake is indicated in previous notation (1st/2nd; L/R).

The snakes of all four arrays have a certain symmetry about their centerline (℄) where, in a storage ring, on would place the interaction point. Since collision experiments will generally require a well defined helicity, a longitudinal spin orientation at the center of the snake is of great interest. Therefore, the tables also list the angle ψ_s that the spin forms with the beam direction $\vec{s}$ at the snake centerline (the spin is always in the $(\vec{x},\vec{s})$-plane at the ℄). The longitudinal spin component n_s at this point is also given.

Regarding variation of beam energy, a snake may be operated in two different modes. In the first mode, all spin rotations in the snake are kept constant, and thus also the magnetic fields (for protons approximately), which means that the horizontal and vertical geometry varies with energy and requires large magnet apertures in both directions. In this mode, the snake is chosen as one point of the array and stays there invariably.

In the second mode, the snake geometry is kept constant in one coordinate, say the horizontal, during energy variation, i.e. the horizontal magnets are ramped with energy, while the vertical magnets are adjusted such that snake action is maintained. In this case, the energy corresponding to the 6.00 h-point may be called the design energy, and for negative energy deviation the snake will migrate along the array toward the 9.00 h-point and for positive energy variation toward the 3.00 h-point. The maximum relative energy deviation δ_{max} then depends on the "radius" of the array and is 66 % for array (A) and 55 % for array (B), for example. In the opposite case, when keeping the vertical snake geometry constant and varying the horizontal, the design energy corresponds to the 9.00 h-point, and the snake moves along the array from the 12.00 h-point through the 9.00 h-point to the 6.00 h-point while varying the energy deviation from $-\delta_{max}$ to $+\delta_{max}$. The behaviour of the snakes in this second mode (except for array (D)) is shown in figures 7, 8 in terms of the relative energy deviation δ. Fig. 7 also gives the relative vertical beam excursion as defined in figs. 1-4 as a function of δ.

It is the advantage of this mode that, for energy variation, the magnet apertures need to be large only in one direction, but it only allows to raise the energy by a factor of the order of 3. If larger energy factors are required, the snake must be operated in the first mode.

Some comments regarding each snake array will subsequently be given.

Snake array (A)

was found starting from the old snake of 1st kind described in ref. 3), which turned out to be the 7.30 h-point that is closest to the origin and therefore most interesting for practical application (see also ref. 12)). The precession angle α is 180° along the whole array, i.e. the snakes are of the 1st kind everywhere. The variation of snake parameters upon energy variation in the second mode, i.e. with constant horizontal geometry, is shown at the top of fig. 7. The constant spin tune is then a fundamental advantage, but the variation of spin orientation at the centerline may be a disadvantage. Therefore, an alternative with added horizontal s-bend as shown in fig. 3 is designed to bring the longitudinal spin component at the centerline into the vicinity of 100 % over most of the energy range.

As a curiosity, we note that the symmetry point of the array also represents a snake of the 1st kind.

Snake array (B)

While all other snake arrays described here are straight, this array is curved with an overall horizontal deflecting angle corresponding to 720° spin rotation at the design energy. Therefore, when incorporated in a ring accelerator, this snake must be operated at constant horizontal geometry, i.e. in the second mode, for which the parameter variation is shown in the middle of fig. 7. Since the 4 horizontal bends can replace part of the normal arc bending, the installation of this snake in principle only requires to add a few vertical bends to the ring and is therefore quite economic. Another advantage is that the spin orientation at the centerline is longitudinal throughout the array. The spin precession α is zero everywhere, which means that the snake is of the 2nd kind throughout. Near the 8.00 h-point at $\psi_V = \psi_H = 45°$, the second vertical bend is zero and each half snake resembles the old LEP/HERA rotator[8]. The correction proposed for this rotator[9] may, in retrospect, be viewed as a first step along the array. The symmetry point of the array also represents a snake of the 2nd kind.

Looking at vertical beam geometry, the beam lines in fig. 2 include a pair of energy-dependent beam translations which keep the vertical beam position constant at the centerline, independent of energy. Although not needed in principle, such translations will be mandatory in case of beam-beam collision at the centerline.

Snake array (C)

In contrast to the previous two arrays, the spin precession α, i.e. the snake character varies when going around this array. It is, therefore, not very interesting for operation in the second mode with constant horizontal geometry. The sign of α can be inverted by inverting the sign of the horizontal bends, shown as a dashed line in fig. 3. The figure shows the snake preceded and followed by a horizontal translation which does not affect the overall spin motion and serves only to restore the straight beam line. The 6.00 h-point of this array was once proposed for LEP[10] as a rotator and - optionally - as a snake of 2nd kind.

Snake arrays (D) and (D1)

As operated in the second mode with constant geometry in one coordinate, these arrays are already described in ref. 5). They are both shown at the top of fig. 6, with the values for (D) outside and the values for (D1) inside the circle. The 6.00 h-point of arrays (D) and (D1) denotes the pair of "standard-type" snakes described in figs. 4, 5 of that report, where energy variation about this point is at fixed horizontal geometry. The 9.00 h-point of array (D) denotes the pair of "novel-type" snakes described in figs. 6, 7 of that report, where energy variation about this point is at fixed vertical geometry and the sign of α is inverted by inverting the sign of the horizontal bends.

Alternative (D1) is obtained from (D) by omitting the first and last horizontal bending magnet. Then, as shown in fig. 4, the snake is preceded and followed by a horizontal translation for restoring the beam line.

Design applications

We shall now try to evaluate the given variety of snakes and rotators (half snakes) for practical application and make design suggestions for electron and proton rings. As an example of a large electron ring we discuss the LEP storage ring. For proton rings, we consider the snake geometry at injection, which determines the required magnet apertures.

Complete snake and rotator scheme for LEP

Such a scheme was already proposed two years ago[11] and is here improved by what has been learnt since. Thereby, the following aspects are considered:

1) We assume for the LEP ring that the polarization is generated at some energy in the working energy range between, say, 40 GeV and 90 GeV, for instance by the Sokolov-Ternov effect in kink magnets or wigglers[11] or by some external polarizing mechanism. With polarization, the machine energy, then, varies by a factor of the order of 2.5 only, and this makes it possible to use snakes and rotators which operate in the second mode at a fixed horizontal geometry.

2) When varying the working energy with polarization, the spin tune should remain constant.

3) Although in principle one snake pair would be sufficient to make the spin tune almost independent of particle energy, there should be as many snakes in the machine as interaction straight sections in order to reduce the influece of field errors on the spin tune[6].

4) At the interaction point at the centerline of each snake, the spin should be longitudinal at all operating energies. On the other hand, the snakes are not required to procure an inversion of helicity at this point since this would likewise affect electrons and positrons such that the relative helicity stays unchanged. In a conceivable future ep addition, the snake must also not necessarly be able to invert the helicity since we have assumed a polarizing mechanism that allows to produce both signs of polarization such that at least a simultaneous inversion of helicity in all interaction points is possible.

5) The rotator-snakes should be as cheap as possible, i.e. the required amount of extra bending and the required magnet apertures should be as small as possible.

Taking this all together, it appears that one snake of the 1st kind of array (A1) plus 3 (or 7) snakes of the 2nd kind of array (B) as shown in fig. 9 are the best solution. The spin tune is $\nu = 0.5$ throughout. The snakes (B) need a minimum of extra bending since the horizontal bends replace part of the arc bending. The spin is always longitudinal at the centerline of snakes (B) and close to longitudinal at the centerline of snake (A1).

When choosing the 6.00 h-point of all snakes to correspond to a design energy of 58 GeV, for example, then the energy can comfortably be varied between 40 GeV and 90 GeV with a polarized beam. Some snake parameters for this energy range are composed in the following small table, including the relative beam excursion h/h_0:

energy		E	37.8 GeV	46.5 GeV	E_0=58.2 GeV	90.2 GeV
rel. energy deviation		δ	- 35 %	- 20 %	0	+ 55 %
in (A1)	longit. spin comp. at ℄	n_s	.93	.99	1	.99
in (A1)	rel. vert. excursion	h/h_0	1.84	1.32	1	.97
in (B)	longit. spin comp. at ℄	n_s	1	1	1	1
in (B)	rel. vert. excursion in 1st vert. bend	h/h_0	2.06	1.37	1	1.73
in (B)	rel. vert. excursion in 2nd vert. bend	h/h_0	1.54	1.25	1	0.65

Installation of the curved snakes (B) implies a slight modification of ring geometry which will be easy to accomodate and should be quantitatively studied in a sample layout.

Snake schemes for proton rings

In contrast to the electron ring where we assumed the polarization to be generated in the working energy range, the protons will be injected as a polarized beam and must then be accelerated to the top energy. Their energy will be raised by a factor of 20 or more, and this completely rules out the use of any snakes operated in the second mode at constant geometry in one coordinate. For accelerating polarized proton beams, one must use snakes operated in the first mode at variable geometry throughout.

Thus we may select particular snakes samples from any of the straight snake arrays and maintain them at fixed spin rotation, i.e. at nearly constant field through the acceleration cycle. The beam excursions in a given snake have their maximum at the lowest energy, at injection, and become small at high energies. Since the magnet apertures must enclose the beam position at injection as well as the straight line that the beam approaches at very high energies, the spin rotation angles should be as small as possible, which makes the 7.30 h-vicinity of arrays (A), (C), and (D) most eligible for snake selection.

Also for protons, it is advisable to use more than one snake pair[6,7]. Once knowing how many snakes to install, they can be chosen such that their precession angles add up to give an overall spin tune shift near $\nu = 0.5$. The smallest of all beam excursions has the 7.30 h-version of array (A), the old snake of 1st kind of ref. 3), but I have not found a similarly favourable snake of the 2nd kind to pair it. The one described in ref. 4) could be used, but it seems preferable instead to use pairs of snakes from the vicinity of the 7.30 h-point of array (D), with alternating signs of α, i.e. with alternating polarity of the horizontal bending.

When employing 2 pairs in the ring, the snake shown in fig. 10 can be used which has a spin precession angle of ± 135°. The horizontal and vertical excursions are shown to relative scale. At an injection energy of E_{kin} = 2 GeV, for example, the horizontal deflecting angle per magnet unit is 162 mrad and, with a unit length of 1 m and a field of 15 KG, the horizontal beam excursion is 32.4 cm, which might be considered the limit of what is practically feasible. For various energies, the horizontal beam excursion h is given in the following table, and also the magnetic field which, for given spin rotation, must be varied as $\frac{pc}{E}$.

kinetic energy	E_{kin}	GeV	0.5	1	2	5	10	50	100
total energy	E	GeV	1.438	1.938	2.938	5.938	10.938	50.938	100.938
momentum	pc	GeV/c	1.090	1.696	2.784	5.864	10.898	50.930	100.934
ratio	$\frac{pc}{E}$		.758	.875	.948	.987	.9963	.9998	.99996
integr. field for ψ_H = 52.13°	$B\ell$	T m	1.203	1.389	1.505	1.567	1.581	1.587	1.587
hor. bending angle		mrad	331.1	245.7	162.1	80.2	43.5	9.35	4.72
hor. excursion (with ℓ_{mag} = 1 m)	h	cm	66.2	49.1	32.4	16.0	8.7	1.9	0.9

The ring with its 4 snakes is shown schematically at the bottom of fig. 11. The spin tune is $\nu = \frac{4 \cdot 135°}{360°} = 0.5$. The longitudinal spin component at the centerline is $n_s \approx 0.5$ ($\psi_s = -60.24°$). If a longitudinal spin orientation is required at that point, the snake version at the bottom of fig. 10 can in principle be used, which is an alternative obtained by inserting one horizontal and two vertical beam translations near the centerline. But the space for experiments may be too restricted. In that case, one can split the snake in two half snakes, move the halves away from each other and insert between them a horizontal S-bend of fixed geometry such that, at a certain energy, the spin is longitudinal at the centerline and, when varying the energy, will rotate in the horizontal plane. With a certain energy increment, the spin will then repeatedly assume the orientations $\psi_s = 0°$ and $\psi_s = 180°$, and in the vicinity of these distinct energies, experiments with positive or negative helicity can be done.

A second ring example with 3 pairs of (D)-type snakes is shown at the top of fig. 11. Here, the snakes correspond to the 7.30 h-point of array (D) exactly, with a spin precession of $\alpha = \pm 145.7°$, yielding a spin tune of $\nu = \frac{6 \cdot 145.7°}{360°} = \pm 0.43$. Following these examples, it is straightforward to find, for a wanted number of snake pairs, a snake sample with small beam excursion in the vicinity of the 7.30 h-point of array (D).

There might also exist similarly good snake pairs in the 8.00 h-region of array (C), but these have not been quantitatively compared yet with those of array (D).

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Tab. 1

Snake array (A)

vert. spin rotation angle Ψ_V	150°	147.235°	135°	106.875°	90°	73.125°	45°	32.765°	30°	32.765°	45°	73.125°	90°	106.875°	135°	147.235°	150°
hor. spin rotation angle Ψ_H	90°	67.5°	45°	31.5°	30°	31.5°	45°	67.5°	90°	112.5°	135°	148.5°	150°	148.5°	135°	112.5°	90°
rel. energy offset δ					-.6667	-.65	-.5	-.25	0	.25	.5	.65	.6667				
snake precession α	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°
snake tune shift $\Delta\nu$	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
snake character	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st	1st
(A) spin forward angle at ℄ Ψ_s	-120°	-60.21°	0°	45.7°	60°	71.3°	90°	105.21°	120°	140.33°	180°	-105.05°	-60°	-11.95°	90°	174.68°	120°
(A) longit. spin comp. at ℄ n_s	-.5	.497	1	.698	.5	.321	0	-.262	-.5	-.770	-1	.260	.5	.978	0	-.996	-5
(A1) spin forward angle at ℄ Ψ_s	120°	-150.21°	-60°	3.7°	20°	29.3°	30°	15.21°	0°	-9.68°	0°	56.95°	100°	150.05°	-90°	122.18°	120°
(A1) longit. spin comp. at ℄ n_s	-.5	-.868	.5	.998	.940	.872	.866	.965	1	.986	1	.545	-.174	-.866	0	-.533	-.5

Figure captions

Fig. 1: Snake geometry of array (A), schematic, with alternative (A1)

Fig. 2: Snake geometry of array (B), schematic

Fig. 3: Snake geometry of array (C), schematic

Fig. 4: Snake geometry of arrays (D) and (D1), schematic

Fig. 5: Snake arrays (A) and (B): Locus of snake points in the plane of horizontal spin rotation angle ψ_H and vertical spin rotation angle ψ_V

Fig. 6: Snake arrays (D), (D1), and (C): Locus of snake points in the plane of horizontal spin rotation angle ψ_H and vertical spin rotation angle ψ_V. At each snake point, the spin precession angle α is given

Fig. 7: Parameters of snakes (A), (B), and (C) as a function of relative energy deviation δ, when operated in the second mode at constant horizontal geometry. Displayed are the vertical spin rotation angle ψ_V, the relative beam excursion h/h_o in the second vertical magnet, the angle ψ_s that the spin forms with the beam direction at the snake centerline, and the longitudinal spin component n_s at that point.

Fig. 8: Addendum to fig. 7, showing ψ_s and n_s at the snake centerline as a function of δ for the snake alternative (A1)

Fig. 9: Snake scheme proposed for LEP

Fig. 10: Snake geometry of array (D) as proposed for practical application in a proton ring

Fig. 11: Examples of snake schemes proposed for proton rings

Tab.2

Snake array (B)

vert. spin rotation angle Ψ_V	112.5°	110.359°	101.542°	90°	67.5°	45°	33.458°	24.641°	22.5°	24.641°	33.458°	45°	67.5°	90°	101.542°	110.359°	112.5°
hor. spin rotation angle Ψ_H	90°	72°	54°	45°	40°	45°	54°	72°	90°	108°	126°	135°	140°	135°	126°	108°	90°
rel. energy offset δ						-.5	-.4	-.2	0	.2	.4	.5	.556				
snake precession α	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°
snake tune shift $\Delta\nu$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
snake character	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd	2nd
spin forward angle at ℄ Ψ_s	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°	180°
longit. spin comp. at ℄ n_s	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Tab.3

Snake array (C)

Ψ_V	157.5°	156.273°	150°	139.4°	122.56°	90°	57.44°	40.6°	30°	23.727°	22.5°	23.727°	30°	40.6°	57.44°	90°	122.56°	139.4°	150°	156.273°	157.5°
Ψ_H	90°	72°	49.94°	36"	27°	22.5°	27°	36°	49.94°	72°	90°	108°	130.06°	144°	153°	157.5°	153°	144°	130.06°	108°	90°
δ						-.75	-.7	-.6	-.445	-.2	0	.2	.445	.6	.7	.75					
α	0	39.08°	88.31°	122.23°	149.3°	-180°	-149.3°	-122.23°	-88.31°	-39.08°	0°	39.08°	88.31°	122.23°	149.3°	180°	-149.3°	-122.23°	-88.31°	-39.08°	0
$\Delta\nu$	0	.109	.245	.34	.415	-.5	-.415	-.34	-.245	-.109	0	.109	.245	.34	.415	.5	-.415	-.34	-.245	-.109	0
char.	2nd		≈ L			1st			≈ R		2nd		≈ L			1st			≈ R		2nd
Ψ_s	0°	19.54	44.16°	61.12°	74.67°	90°	105.33°	118.88°	135.84°	160.46°	180°	-160.46°	-135.84°	-118.88°	-105.33°	-90°	-74.67°	-61.12°	-44.16°	-19.54°	0°
n_s	1	.942	.717	.483	.264	0	-.264	-.483	-.717	-.942	-1	-.942	-.717	-.483	-.264	0	.264	.483	.717	.942	1

Snake array (D)

Ψ_V	135°	131.97°	120°	110°	90°	70°	57.235°	54.736°	48.03°	45°	48.03°	54.736°	57.235°	70°	90°	110°	120°	131.97°	135°
Ψ_H	90°	72°	54.736°	43.808°	45°	48.808°	57.235°	60°	72°	90°	108°	120°	122.765°	131.192°	135°	131.192°	125.264°	108°	90°
δ					-.5	-.458	-.364	-.333	-.2	0	.2	.333	.364	.458	.5				
(D) α	180°	92.17°	0	-39.69°	-90°	-125.07°	-145.65°	-150°	-164.17°	180°	164.17°	150°	145.65°	125.07°	90°	39.69°	0	-92.17°	180°
(D) $\Delta\nu$	.5	.256	0	-.110	-.25	-.347	-.405	-.417	-.456	.5	.456	.417	.405	.347	.25	.110	0	-.256	.5
(D) char.	1st	≈ L	2nd		R					1st					L		2nd	≈ R	1st
(D1) α	0	-51.83°	-109.47°	-137.31°	180°	137.31°	99.88°	90°	51.83°	0°	-51.83°	-90°	-99.88°	-137.31°	180°	137.31°	109.47°	51.83°	0
(D1) $\Delta\nu$	0	-.144	-.304	-.381	.5	.381	.277	.25	.144	0	-.144	-.25	-.277	-.381	-.5	.381	.304	.144	0
(D1) char.	2nd		R		1st			L		2nd		R			1st			L	2nd
Ψ_s at ℄	180°	-154.09°	-125.26°	-111.35°	-90°	-68.65°	-49.94°	-45°	-25.91°	0	25.91°	45°	49.94°	68.65°	90°	111.35°	125.26°	154.09°	180°
n_s at ℄	-1	-.899	-.577	-.364	0	.364	.644	.707	.899	1	.899	.707	.644	.364	0	-.364	-.577	-.899	-1

Fig. 1: (A) Snake of 1st kind

Spin rotations $\begin{cases} V = \psi_V(\delta) & [\psi_{V,min} = 30^\circ \text{ at } \delta = 0] \\ H = \psi_H = (1+\delta)\cdot 90^\circ & [90^\circ \text{ at } \delta = 0] \end{cases}$

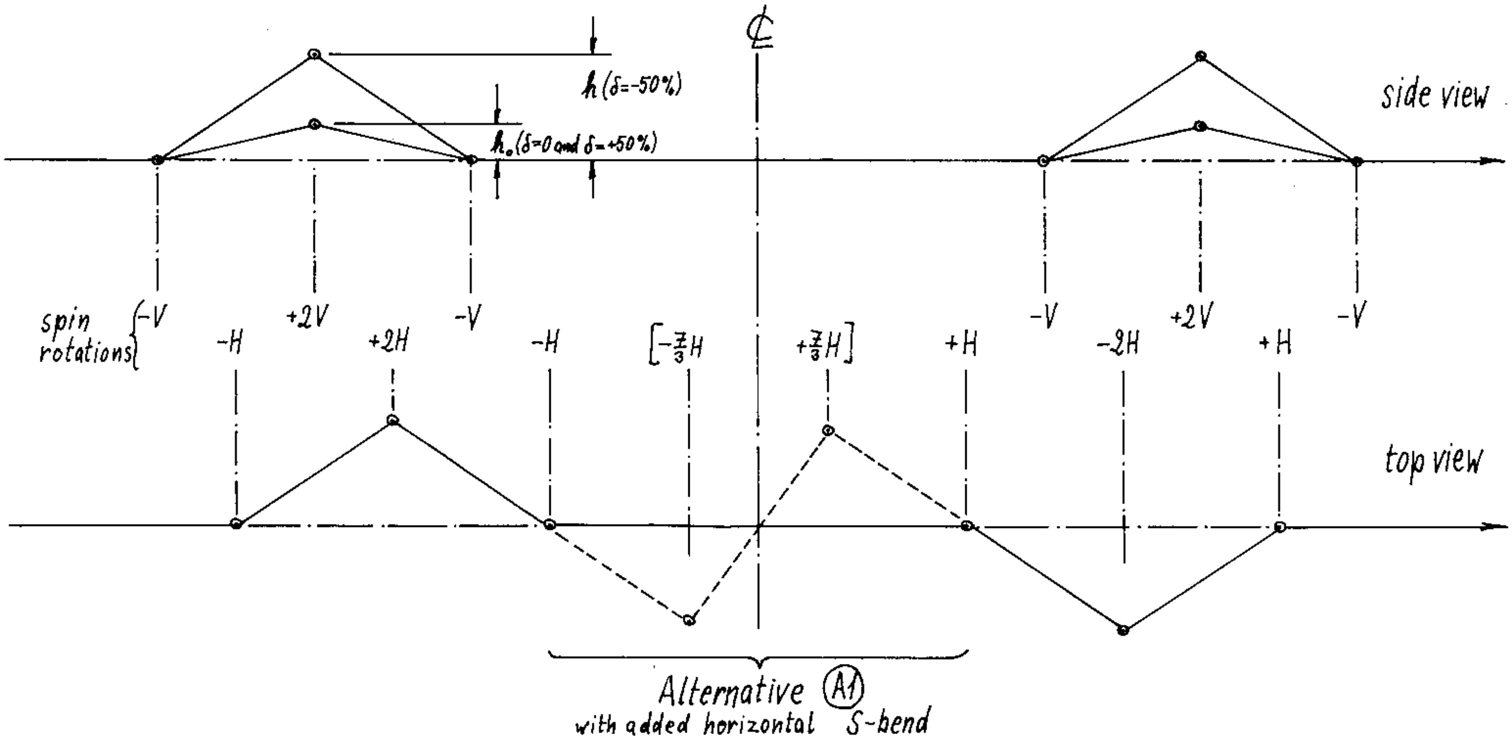


Fig. 2: (B) Snake of 2nd kind, curved

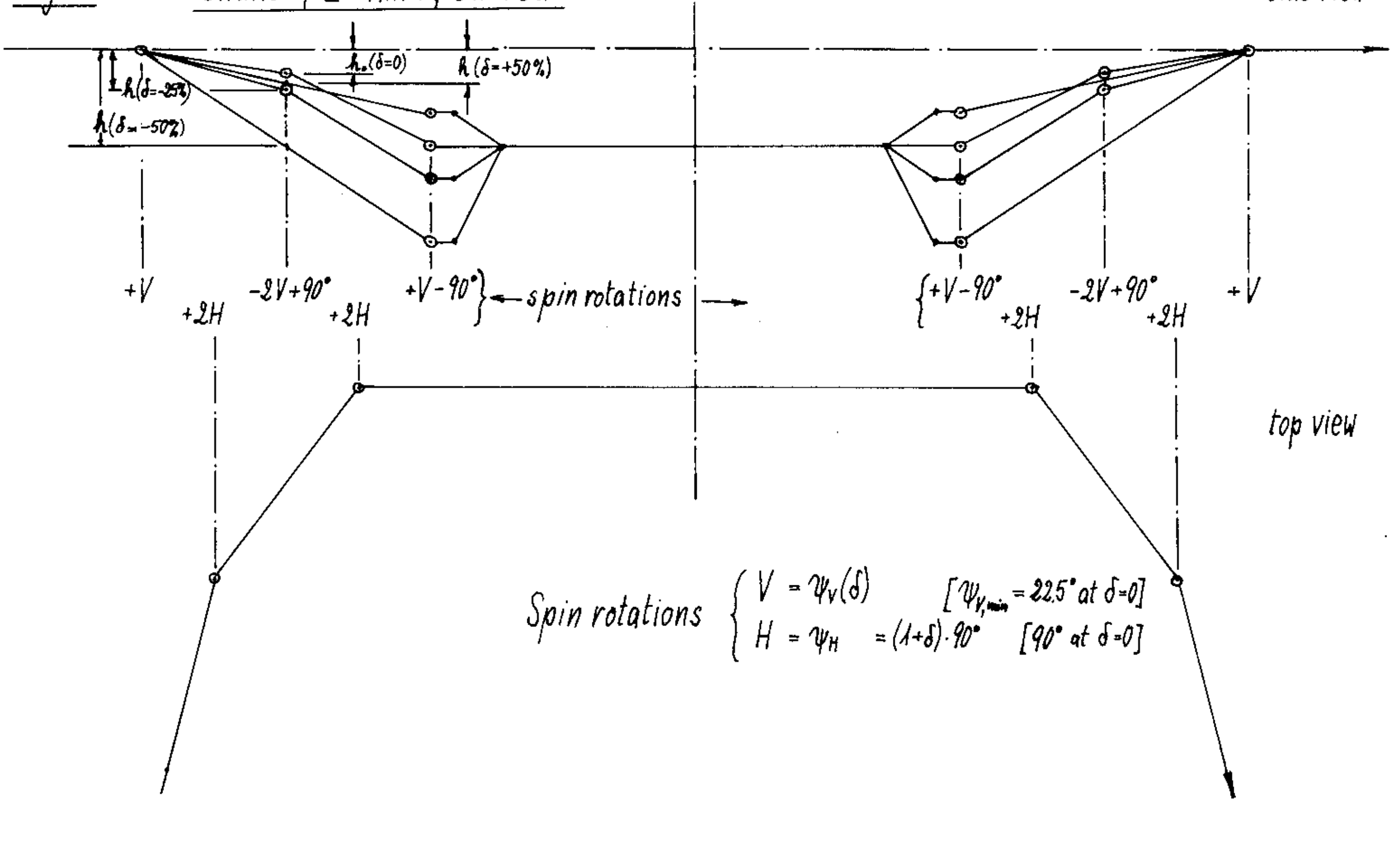


Fig.3: Ⓒ Snake of 2nd kind at $\psi_{V,min} = 22.5°$

Spin rotations $\begin{cases} V = \psi_V(\delta) & [\psi_{V,min} = 22.5° \text{ at } \delta = 0] \\ H = \psi_H = (1+\delta)\cdot 90° & [90° \text{ at } \delta = 0] \end{cases}$

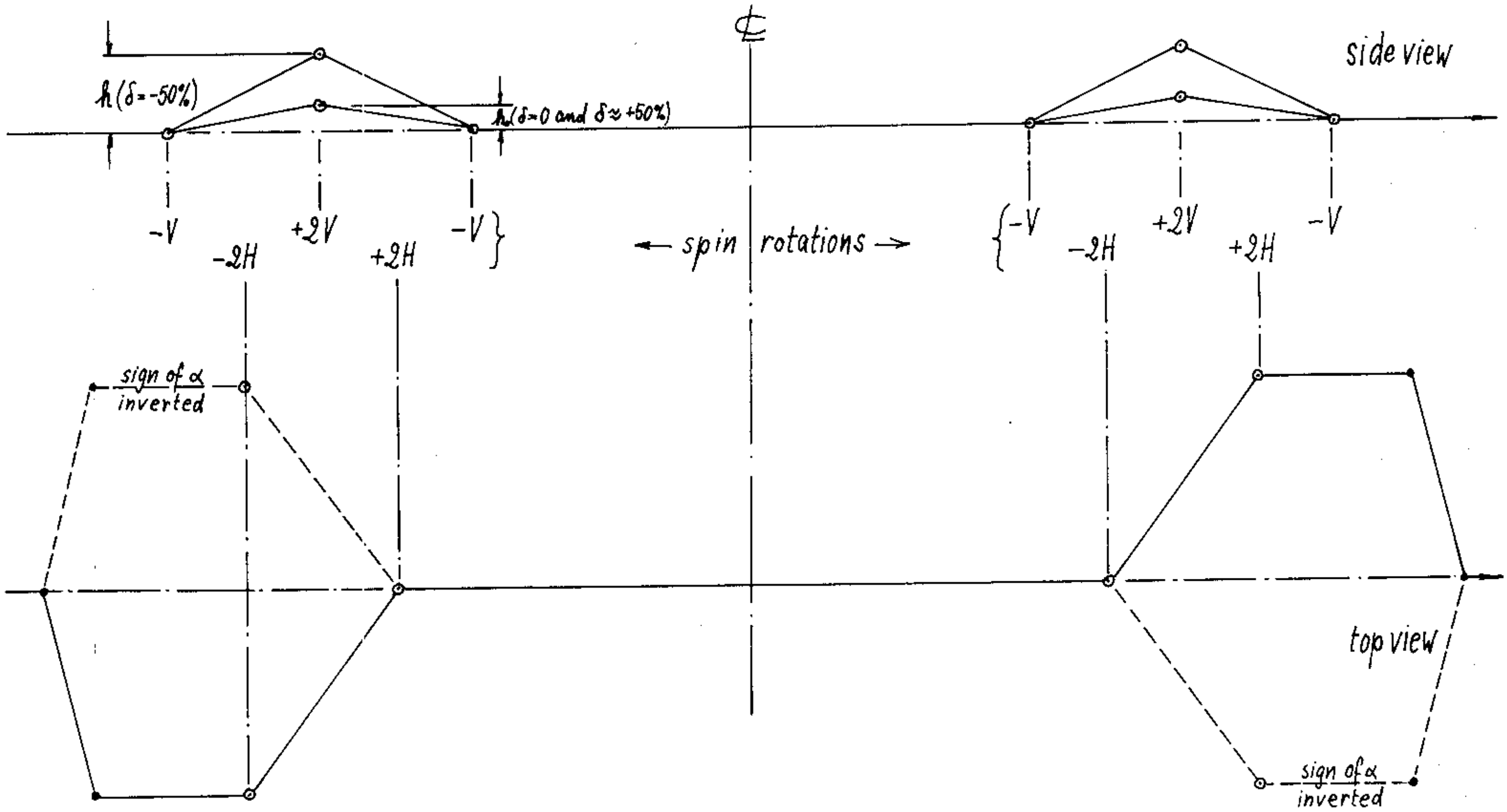


Fig.4: Ⓓ Snake of 1st kind at $\psi_{V,min} = 45°$

Spin rotations $\begin{cases} V = \psi_V(\delta) & [\psi_{V,min} = 45° \text{ at } \delta = 0] \\ H = \psi_H = (1+\delta)\cdot 90° & [90° \text{ at } \delta = 0] \end{cases}$

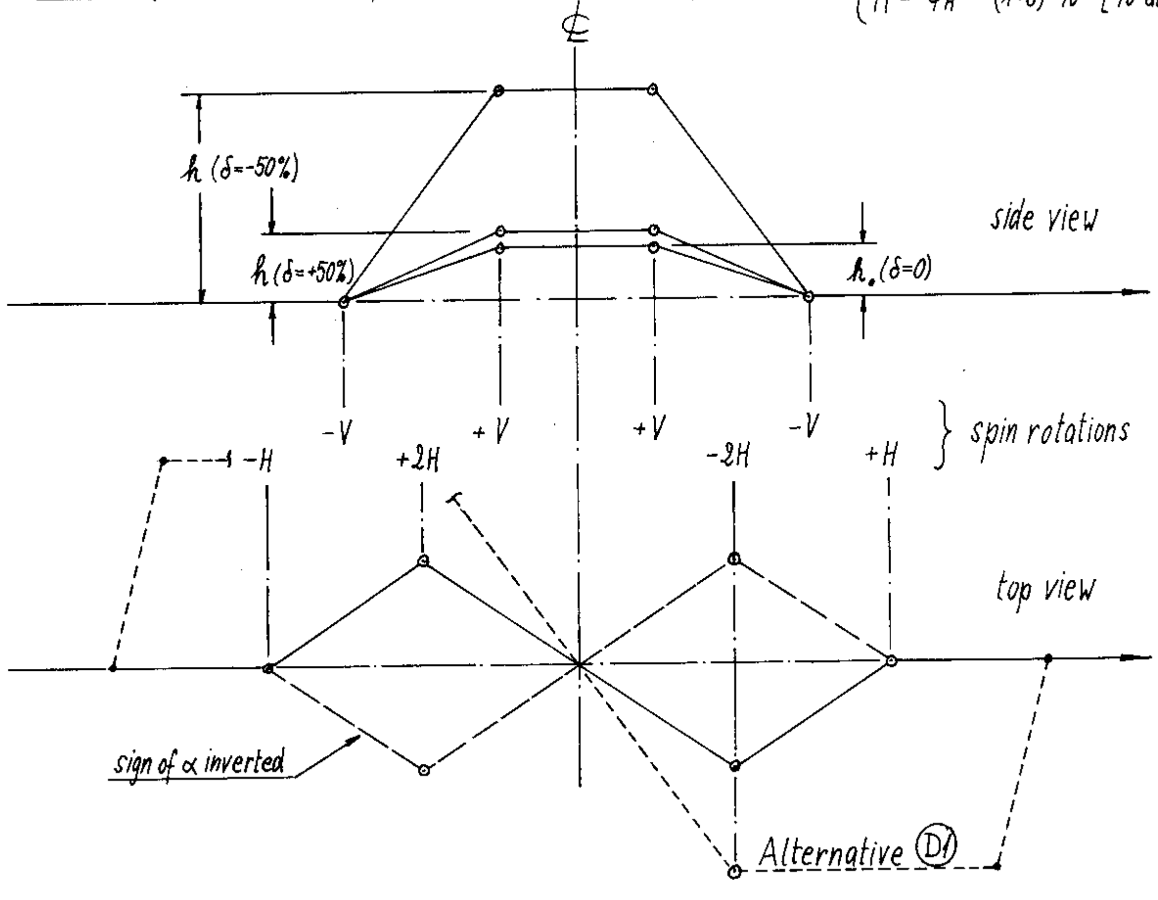


Fig. 5

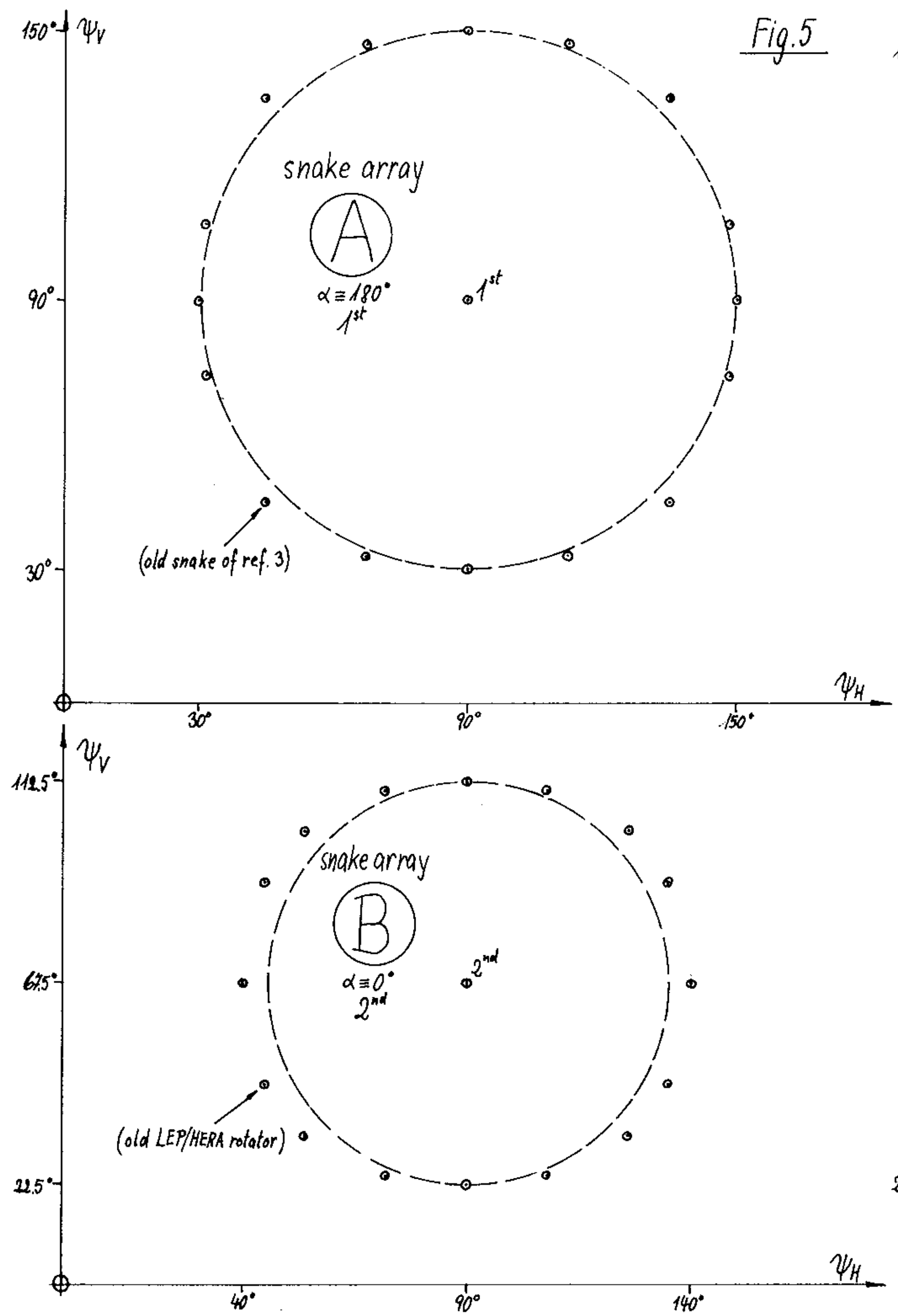

snake array
A
α ≡ 180°
1st
(old snake of ref. 3)
snake array
B
α ≡ 0°
2nd
(old LEP/HERA rotator)
ψV
ψH
150°
90°
30°
112.5°
67.5°
22.5°
40°
140°

Fig. 6

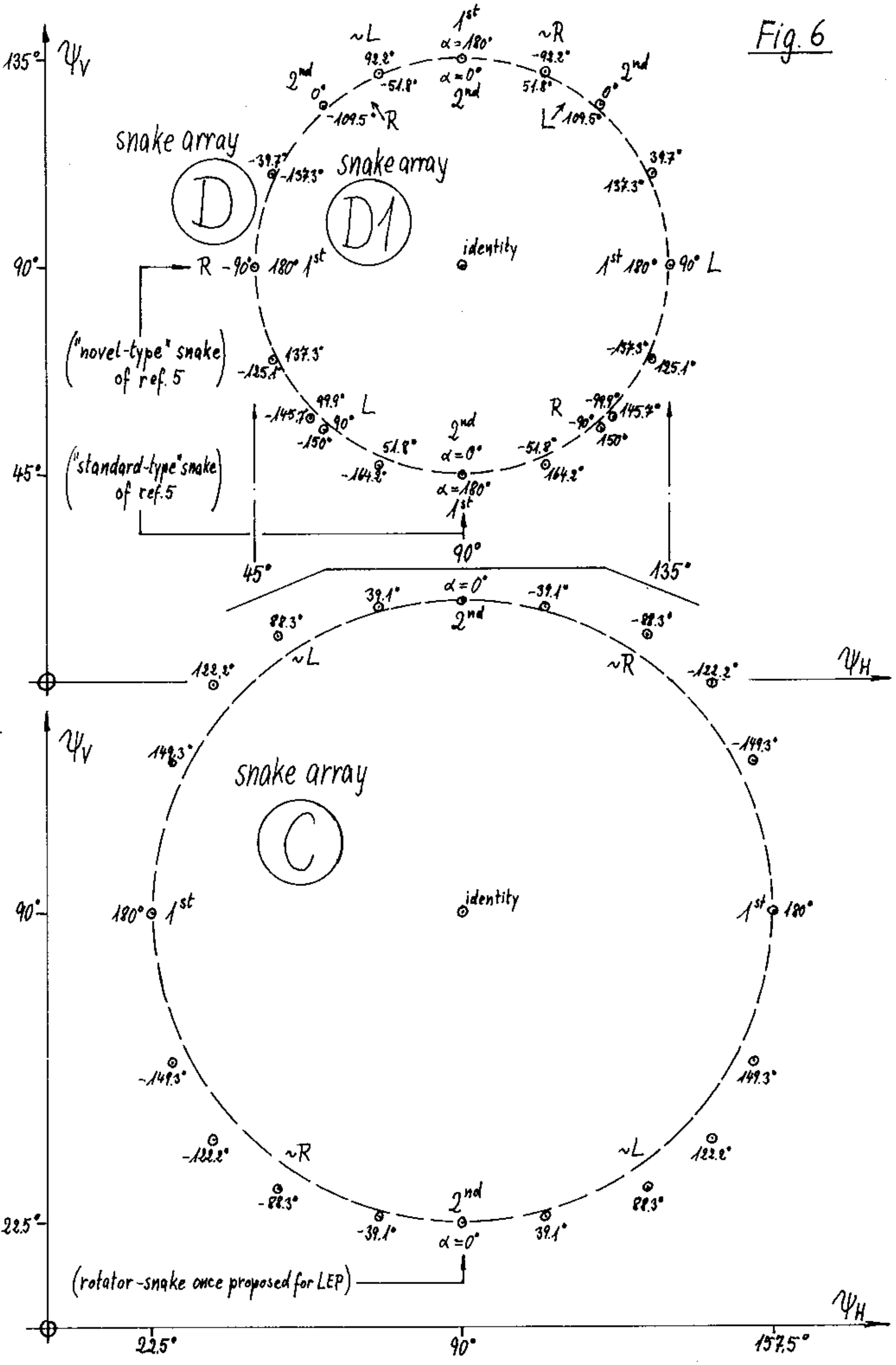
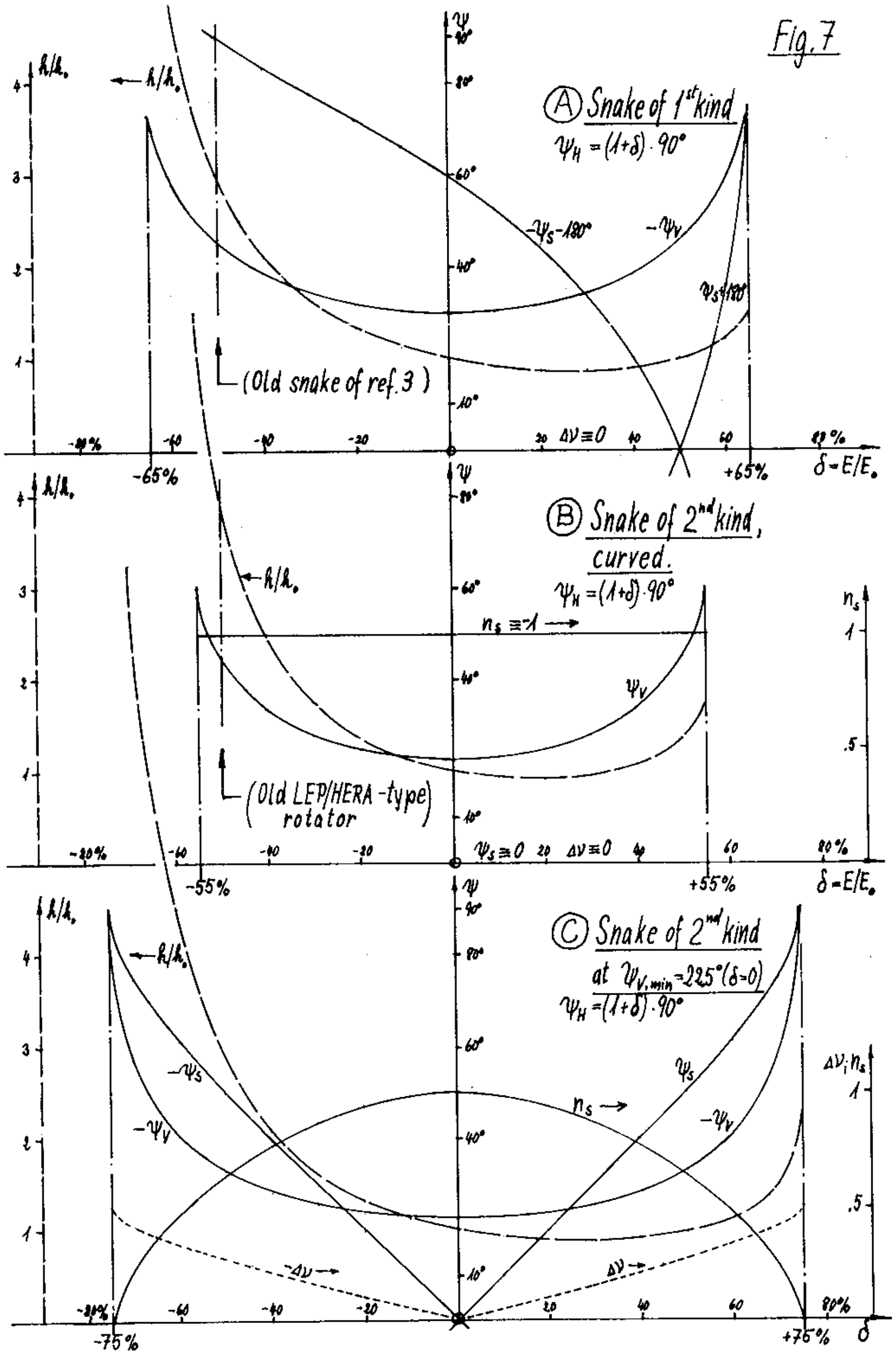
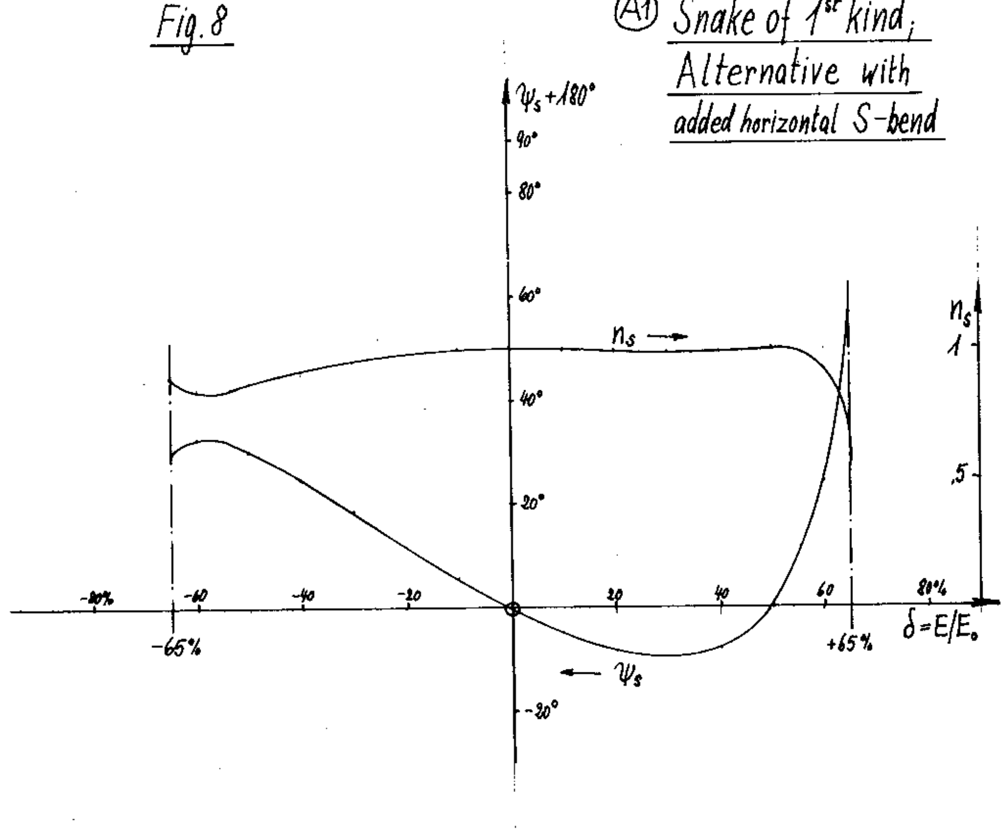

snake array
D
snake array
D1
identity
("novel-type" snake of ref. 5)
("standard-type" snake of ref. 5)
snake array
C
identity
(rotator-snake once proposed for LEP)
ψV
ψH
135°
90°
45°
22.5°
157.5°

Fig. 7



Ⓐ Snake of 1st kind
ψH = (1+δ)·90°
−ψs−180°
−ψv
ψs+180°
(Old snake of ref. 3)
ΔV ≡ 0
−65%
+65%
δ = E/E₀
h/h₀
Ⓑ Snake of 2nd kind, curved.
ψH = (1+δ)·90°
ns ≡ 1
ψv
(Old LEP/HERA-type rotator)
ψs ≡ 0
ΔV ≡ 0
−55%
+55%
Ⓒ Snake of 2nd kind at ψV,min = 22.5° (δ=0)
ψH = (1+δ)·90°
ψs
−ψs
−ψv
ns
−Δν
Δν
Δν; ns
−75%
+76%
δ

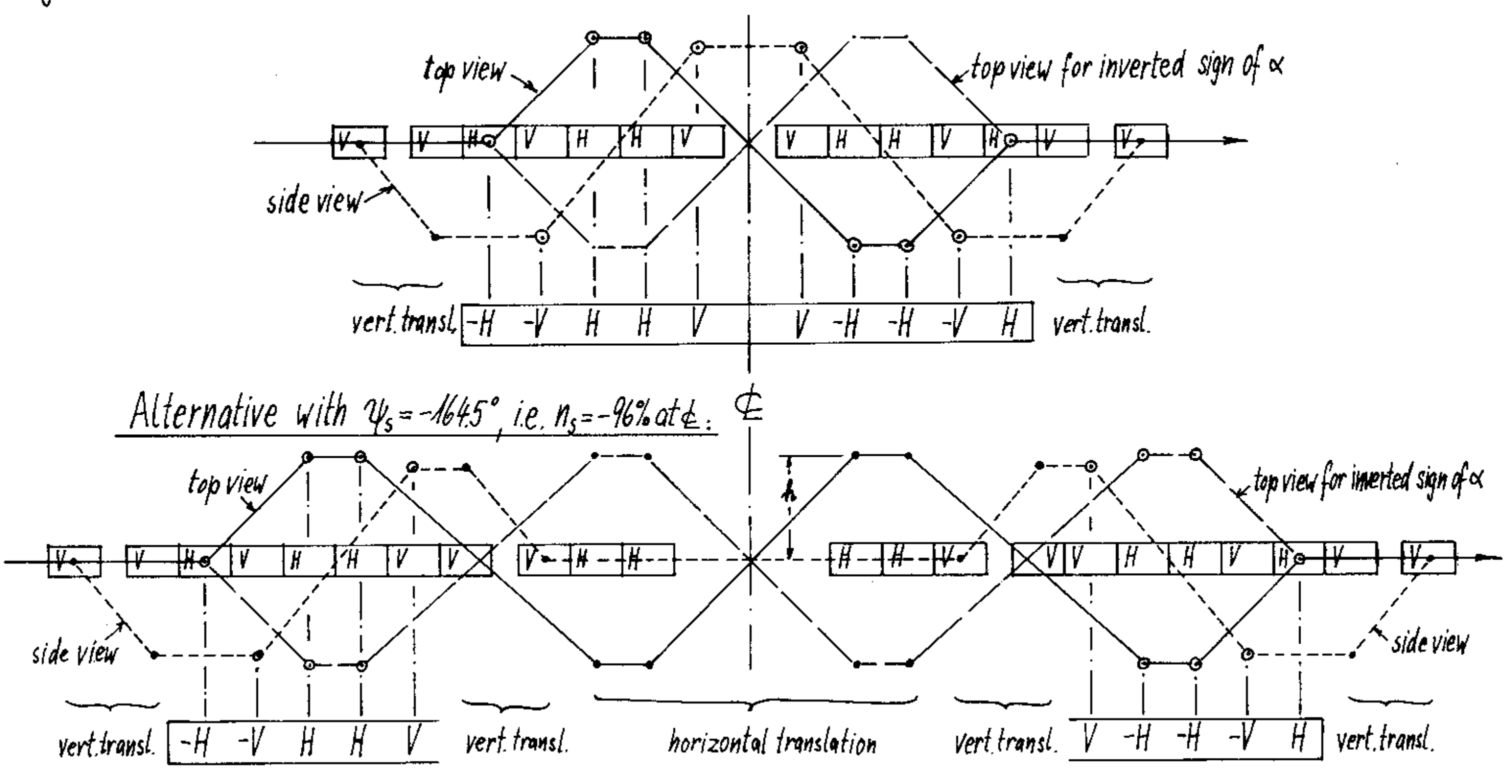
Fig. 8



Ⓐ1 Snake of 1st kind; Alternative with added horizontal S-bend
ψs + 180°
ns
ψs
−65%
+65%
δ = E/E₀

Fig. 10: Snake of array (D) with $V=\psi_V=63.6°$; $H=\psi_H=52.133°$; $\alpha=135.26°$ (i.e. $2\cdot\Delta\nu=0.75$); $\psi_s=-60.24°$ at ℄:

Alternative with $\psi_s=-164.5°$, i.e. $n_s=-96\%$ at ℄:



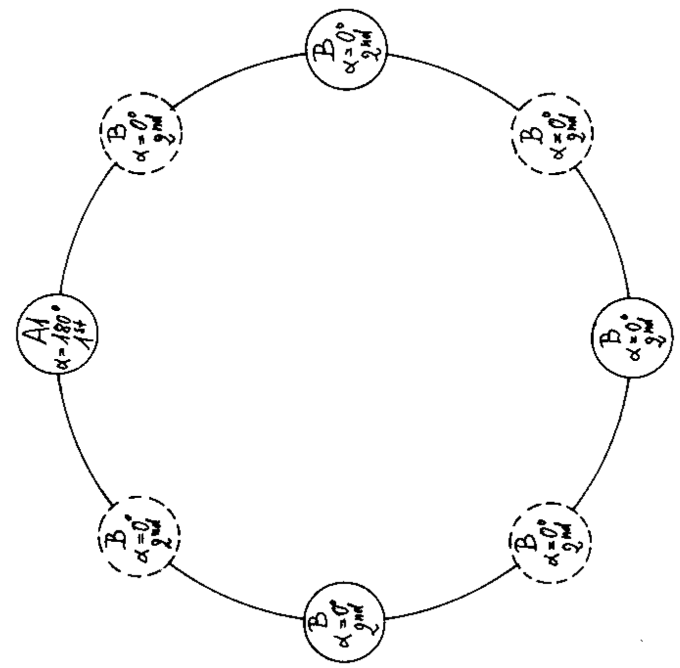
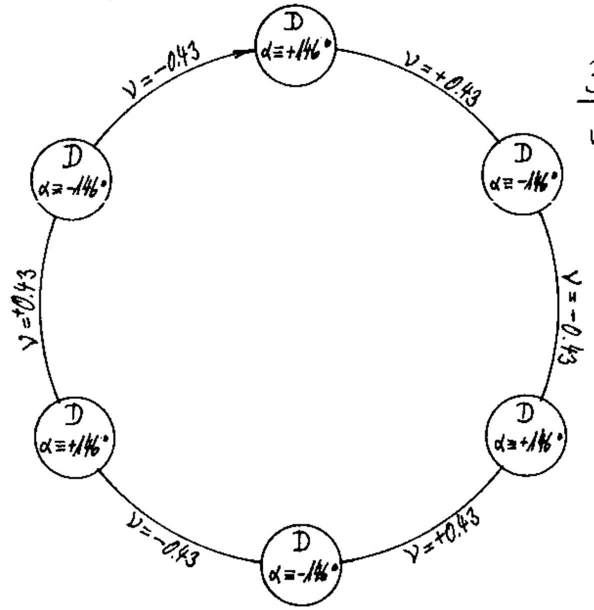


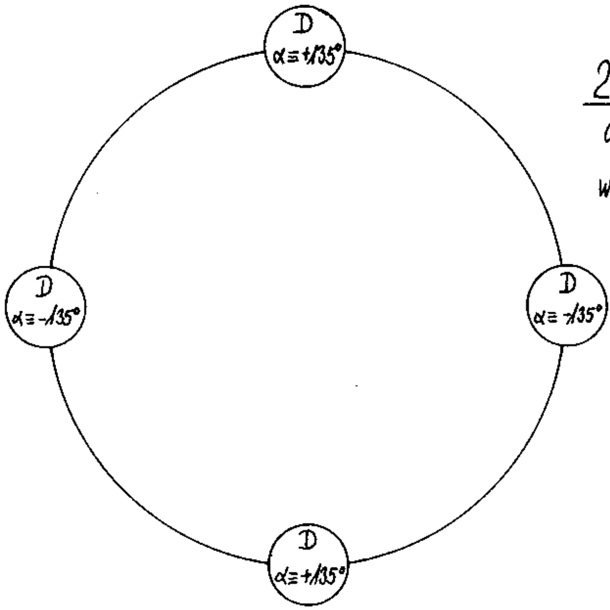
Fig. 9.: LEP ring with one A1-type snake (1st kind) and 3 (or 7) B-type snakes (2nd kind) of fixed horizontal geometry and variable vertical geometry, with variable spin rotations. Spin tune ν = 0.5.

Fig. 11: Examples of p-rings with 3 or 2 D-type snake pairs of fixed spin rotations, i.e. variable geometry.



3 Snake pairs

with $\psi_V = \psi_H = 57.235°$;
$\alpha = 145.7°$ and
spin tune $\nu = \pm 0.43$



2 Snake pairs

as in Fig. 10

with $\psi_V = 63.6°$; $\psi_H = 52.1°$;
$\alpha = 135°$ and
spin tune $\nu = 0.5$