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NOTE ADDED

B. Berg and C. Panagiotakopoulos

The Photon in U(1) Lattice Gauge Theory

In the above paper we have called our 1^{+-} axial vector "photon". This is justified in the following sense: Let us consider in free field theory a 1^{--} photon vector state $|\vec{p},s\rangle$ with $\vec{p}$ momentum and s helicity. The combination

$$|\hat{\vec{p}}\rangle = \frac{1}{\sqrt{2}} \left(|\vec{p},s\rangle - |\vec{p},-s\rangle \right)$$

has parity $P = +1$. Using free fields it is easily checked that in the naive continuum limit this state has an overlap with our 1^{+-} state.

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Abstract

We carry out a MC calculation of the spectrum of 4d U(1) lattice gauge theory. In the scaling limit $\beta \to \beta_c$ massive 0^{++}, 1^{+-} and 2^{++} states are indicated. On the critical line $\beta > \beta_c$ we find striking evidence for a massless 1^{+-} photon, and no signal for other states.

In the strong coupling (SC) region (β small) abelian as well as non-abelian lattice gauge theories (LGT) are in the confinement phase. The famous confinement problem for non-abelian LGT consists in proving that this phase extends to the continuum limit: lattice constant $a \to 0, \beta \to \infty$. On the other hand we like to recover a free field theory of massless photons from the 4d abelian U(1) LGT. In a fundamental paper on LGT Wilson [1] therefore conjectured that the U(1) LGT undergoes a phase transition as the coupling constant β is varied, with a nonconfining phase at weak coupling .

Later the existence of these two phases has been rigorously proven by Guth [2] and the result has been generalized by Fröhlich and Spencer [3] . Monte Carlo calculations [4] indicate a second order phase transition at $\beta \gtrsim 1.0$. For large enough β perturbation theory becomes applicable and the existence of a zero mass state has been proven [3] . No rigorous results exist for the whole region $\beta > \beta_c$. In analogy to the 2d X-Y-model one expects a critical line of mass zero field theories. In this letter we demonstrate by a MC simulation that this picture is correct, and that the massless excitation has the quantum numbers 1^{+-} of the photon.

We consider U(1) LGT with the Wilson [1] action. At each link b of a hypercubic 4d lattice there is an element $U(b) = e^{i\theta_b} \in U(1)$, and averages are calculated with the partition function

$$Z = \int_{-\pi}^{\pi} \prod_b d\theta_b \exp\left(-\beta \sum_p U(\dot{p})\right) \tag{1}$$

For each plaquette p, $U(\dot{p})$ is the ordered product of the four link matrices surrounding the plaquette.

For our MC calculation we use as in Ref. [4] the Metropolis method and approximate U(1) by Z (1000). Most of our calculations are carried out on an $4^3 \cdot 8$ lattice with cyclic boundary conditions. 4^3 ist the spacelike box and 8 is the extension in Euclidean "time" direction. At $\beta = 1.3$ some finite size consistency checks are carried out on an 8^4 lattice.

Our results are based on (diagonal) correlations between Wilson loops up to length 6, as depicted in Fig. 1. In Ref. [5,6] the irreducible representations of the cubic group on these were constructed. We like to remind the reader that there exist five irreducible representations of the cubic group. In the standard notation for point groups A_1, A_2 are the one-dimensional representations, E is the two-dimensional representation and T_1, T_2 are the three-dimensional representations. Under certain assumptions [6] we have the following correspondence in the continuum limit of a LGT:

$$A_1^{PC} \to 0^{PC}, \quad T_1^{PC} \to 1^{PC}, \quad E^{PC} \to 2^{PC}, \quad T_2^{PC} \to 2^{PC} \tag{2}$$

and $A_2^{PC} \to 3^{PC}$ (P = parity, C = C-parity).

In the present paper we consider for some of the irreducible representations states of momentum $\vec{k} = (k_1, k_2, k_3)$, $k_i = \frac{2\pi n}{L}$, $(n = 0, \pm 1, \ldots, \pm[\frac{L}{2}])$ L is the spacelike lattice size. Problems with phases are avoided by the trick of Ref. [7] : We first construct the irreducible representation in question on a spacelike cube and then we perform the Fourier transformation for the cube operators $C_j(\vec{x},t)$:

$$\tilde{C}_j(\vec{k},t) = \sum_{\vec{x}} e^{i\vec{k}\cdot\vec{x}} C_j(\vec{x},t) \quad . \tag{3}$$

Here $\vec{x}$ is the position of the center of the cube. The index $j = (OP, R)$ labels the operators OP and representations $R = A_1^{++}, T_1^{+-}, E^{++}$, etc.. Of course a Wilson loop may contribute to several cubes. In Ref. [7] this construction was carried out for the 1-plaquette operator in the A_1^{++} representation. The generalization to other representations and operators is, however, straightforward.

We will calculate correlation functions

$$\rho_j(\vec{k},t) = \mathrm{Real} \langle 0| \overline{\tilde{C}_j(\vec{k},t)}\, \tilde{C}_j(\vec{k},0)|0\rangle \tag{4}$$

and define corresponding energies by means of

$$E_j(|\vec{k}|,t) = -\ln\left(\rho_j(\vec{k},t)/\rho_j(\vec{k},0)\right) \tag{5}$$

These energies are upper bounds for the energy $E_R(|\vec{k}|)$ of the lowest state, which couples to the irreducible representation R in question. According to equation (2) we now abbreviate these states by 0^{++}, 1^{+-}, 2^{++} etc.. If the relativistic energy-momentum dispersion is restored, we expect

$$E_R(|\vec{k}|) = \sqrt{m_R^2 + \vec{k}^2} \tag{6}$$

In the practical MC calculation statistical noise limits us to rather short distances: $t = 0, 1$, and only in some cases reasonable results are also obtained for $t = 2$. If the state $\tilde{C}_j(\vec{k},t)|0\rangle$ is a good approximation to the wave function of the lowest state in question, already $E_j(|\vec{k}|, 1)$ may be a rather close bound to the energy $E_j(|\vec{k}|)$.

We now present our results. At each considered β -value on the $4^3 \cdot 8$ lattice we have performed about 10 000 double sweeps and we did measurements after each double sweep. We have used random upgrading [6] and a sweep is defined by upgrading each link variable once in the mean. At each β -value between 1200 - 1800 sweeps without measurements were done for reaching equilibrium.

Let us first consider momentum $\vec{k} = 0$ states. In Figure 2 our distance $t = 1$ energy results for the lowest lying states 0^{++}, 1^{+-} and 2^{++} are given. For guiding the eyes MC points of the same state are connected with straight lines. The bounds $E(\vec{k} = 0, t = 1)$ on the energies (= masses) decrease by approaching the critical point from below: $\beta \to \beta_c \approx 1.0$, ($\beta < \beta_c$). The energy results from distance $t = 2$ are of course better (= lower) bounds, but as in non-abelian gauge theories [6] reliable results can hardly be obtained if $E(|\vec{k}| = 0, t = 1) \gtrsim 2$. For the states 0^{++} and 1^{+-} the distance $t = 2$ results are given in Table 1. We use always the operator, which gives the lowest result also used at distance $t = 1$. For the other considered representations distance $t = 1$ energies are higher.

From Figure 2 we note a clear difference between U(1) and non-abelian gauge theories: The relative lightness of the 1^{+-} state. To summarize: In the scaling limit $\beta \to \beta_c$ a spectrum of massive 0^{++}, 1^{+-}, 2^{++} (and evtl. other) states is indicated with

$$m(0^{++}) < m(1^{+-}) < m(2^{++}). \tag{7}$$

As in non-abelian gauge theories it would be pointless to estimate precise mass ratios with the present method. Due to bad wave functions, ratios at distance $t = 1$ are not stable and at distance $t = 2$ statistical noise is a severe problem. For small values of β ($\beta = 0.8$) our results for 0^{++} are, within statistical errors, in agreement with existing strong coupling expansion results [8] .

Above or near the critical point β_c the short distance energy definitions $E_j(0,t)$ begin to approach their spin wave ($\beta \to \infty$) limits. For $t = 1, 2$ the values are presumably high. Some leading order $(\beta \to \infty)$ calculations were carried out in Ref. [9,6]. In the present case on an $4^3\cdot 8$ lattice these results read $E_{(1,A_1^{++})}(0,1) \gtrsim 3.96$ and $E_{(1,E^{++})}(0,1) \gtrsim 3.83$.

Our final Fig. 3 represents results from the 1-plaquette operator for the 0^{++}, 1^{+-} and 2^{++} states with momentum

$$|\vec{K}| = \frac{2\pi}{4} \cdot \qquad (8)$$

For the T_1^{+-} representation, this means the photon 1^{+-}, a dramatic change (as compared with Fig. 2) is observed. For $\beta \to \beta_c$ the T_1^{+-} energy values start to undershoot the A_1^{++} and E^{++} energies, and for $\beta > \beta_c$, $\beta \to \infty$ we find:

$$E_{(1,T_1^{+-})}\left(\frac{2\pi}{4}, 1\right) \to \text{const} \approx 1.38 \qquad (9)$$

From the relativistic dispersion law (6) of a free photon we get $\sqrt{\left(\frac{2\pi}{4}\right)^2} = \frac{\pi}{2} \approx 1.57$, and the discrepancy with (9) is argued to be due to our small spacelike lattice. Indeed replacing equation (6) by $E_R(|\vec{K}|) = \sqrt{m_R^2 + \sum_i (2 - 2\cos K_i)}$ yields $\sqrt{2 - 2\cos\frac{2\pi}{4}} = \sqrt{2} \approx 1.41$ in good agreement with (9). Distance t = 2 results are similar; they are collected in Table 3. Furthermore we did a finite size check at $\beta = 1.3$ on an 8^4 lattice. We carried out 3000 double sweeps with measurement (186 sweeps for equilibrium). The results for the T_1^{+-} state and lowest momentum $|\vec{K}| > 0$ are

$$E\left(\frac{2\pi}{8}, 1\right) = 0.870 \pm 0.036,\; E\left(\frac{2\pi}{8}, 2\right) = 0.780 \pm 0.033 \qquad (10)$$

in good agreement with the relativistic dispersion $\sqrt{\left(\frac{2\pi}{8}\right)^2} = \frac{\pi}{4} \approx 0.785$.

We interpret the result as clear evidence for a massless photon state on the critical line $\beta > \beta_c$. It is amazing that the photon can be detected at short distances in a MC calculation, on a finite lattice although the correlation length is infinite. For momentum $\vec{K} = 0$ the power law behaviour of the correlation function leads at short distances to spin wave results, which prevent us from seeing massless excitations. By giving a small momentum $\vec{K}$ to our considered states we can, however, clearly project out the photon 1^{+-}. Presumably other excitations in the T_1^{+-} channel have a much higher mass or decouple from the 1-plaquette operator.

In conclusion we have recovered the massless 1^{+-} photon from 4d U(1) lattice gauge theory by means of a MC simulation. Several analytic checks *) could be carried out. An extended version of this letter is in preparation.

*) We would like to thank Martin Lüscher for pointing this out.

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Figure captions:

Fig. 1: Wilson loops up to length 6.

Fig. 2: $E(|\vec{K}| = 0, t = 1)$ for the three lowest lying states.

Fig. 3: $E(|\vec{K}| = \frac{2\pi}{4}, t = 1)$ for the three lowest lying states.

β	$E_{A_1^{++}}(0,2)$, (O,P)	$E_{T_1^{+-}}(0,2)$, (O,P)
0.90	2.4 ± 0.3, (3)	2.9 ± 0.3, (3)
0.95	1.82 ± 0.10, (3)	2.25 ± 0.10, (3)
0.975	0.94 ± 0.10, (2)	1.99 ± 0.06, (3)
1.0	0.83 ± 0.05, (2)	2.27 ± 0.15, (3)
1.025	1.58 ± 0.10, (3)	-
1.05	2.04 ± 0.11, (2)	-
1.1	2.55 ± 0.25, (4)	-

Table 1

$E(|\vec{K}| = 0,\ t = 2)$ results for 0^{++} and 1^{+-} states. Because of the limited statistics the given error bars are not always reliable. In brackets the used operator as explained in the text is indicated.

	$\beta = 0.9$	$\beta = 1.0$	$\beta = 1.1$
A_2^{++}, (OP)	$4.83^{+0.83}_{-0.45}$, (2)	$4.84^{+0.74}_{-0.43}$, (2)	3.95 ± 0.20, (2)
A_2^{+-}, (OP)	> 5.48, (4)	> 4.85, (4)	$5.68^{+0.87}_{-0.46}$, (4)
A_1^{--}, (OP)	$4.92^{+0.60}_{-0.38}$, (3)	$5.80^{+2.40}_{-0.66}$, (3)	-, (3)
E^{--}, (OP)	$5.29^{+0.67}_{-0.40}$, (3)	$5.02^{+0.58}_{-0.39}$, (3)	-, (3)
T_2^{++}, (OP)	4.58 ± 0.20, (3)	3.76 ± 0.10, (3)	3.77 ± 0.08, (4)
T_2^{+-}, (OP)	$4.98^{+0.46}_{-0.31}$, (2)	$5.38^{+0.67}_{-0.41}$, (2)	4.66 ± 0.25, (2)
T_1^{-+}, (OP)	> 5.45, (3)	> 5.25, (3)	$5.60^{+1.21}_{-0.54}$, (3)
T_2^{-+}, (OP)	$5.40^{+0.48}_{-0.32}$, (3)	4.25 ± 0.15, (3)	4.23 ± 0.18, (3)
T_2^{--}, (OP)	> 6.26, (3)	$5.66^{+0.73}_{-0.43}$, (3)	$5.36^{+0.67}_{-0.39}$, (3)

Table 2

$E(|\vec{K}| = 0,\ t = 1)$ results for A_2^{++}, A_2^{+-}, A_1^{--}, E^{--}, T_2^{++}, T_2^{+-}, T_1^{-+}, T_2^{-+} and T_2^{--} states.

β	$E_{(1,\ T_1^{+-})}\ (\frac{2\pi}{4}\ ,\ 2)$
0.90	-
0.95	2.70 ± 0.20
0.975	2.22 ± 0.09
1.0	1.63 ± 0.03
1.025	1.51 ± 0.02
1.05	1.44 ± 0.02
1.1	1.41 ± 0.02
1.3	1.37 ± 0.02
1.5	1.36 ± 0.02

Table 3

$E(|\vec{K}| = \frac{2\pi}{4}\ ,\ t = 2)$ results for the photon 1^{+-}.

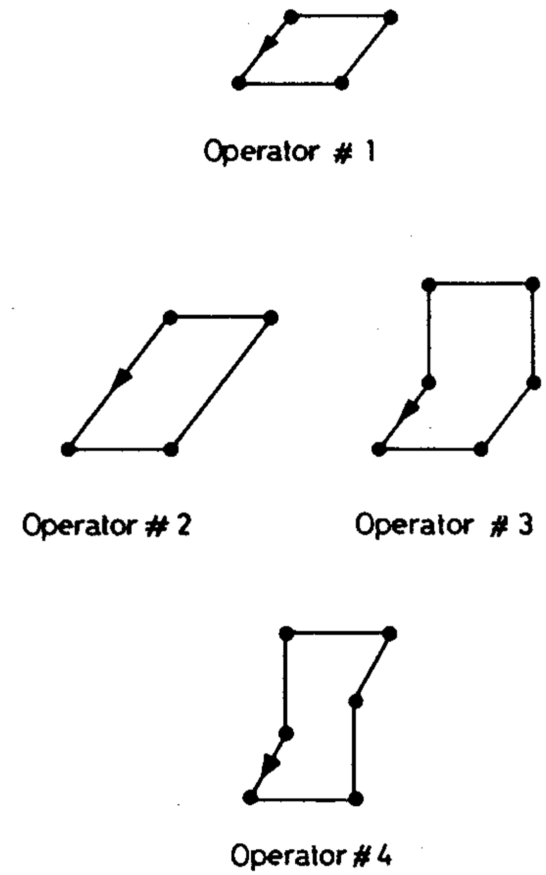
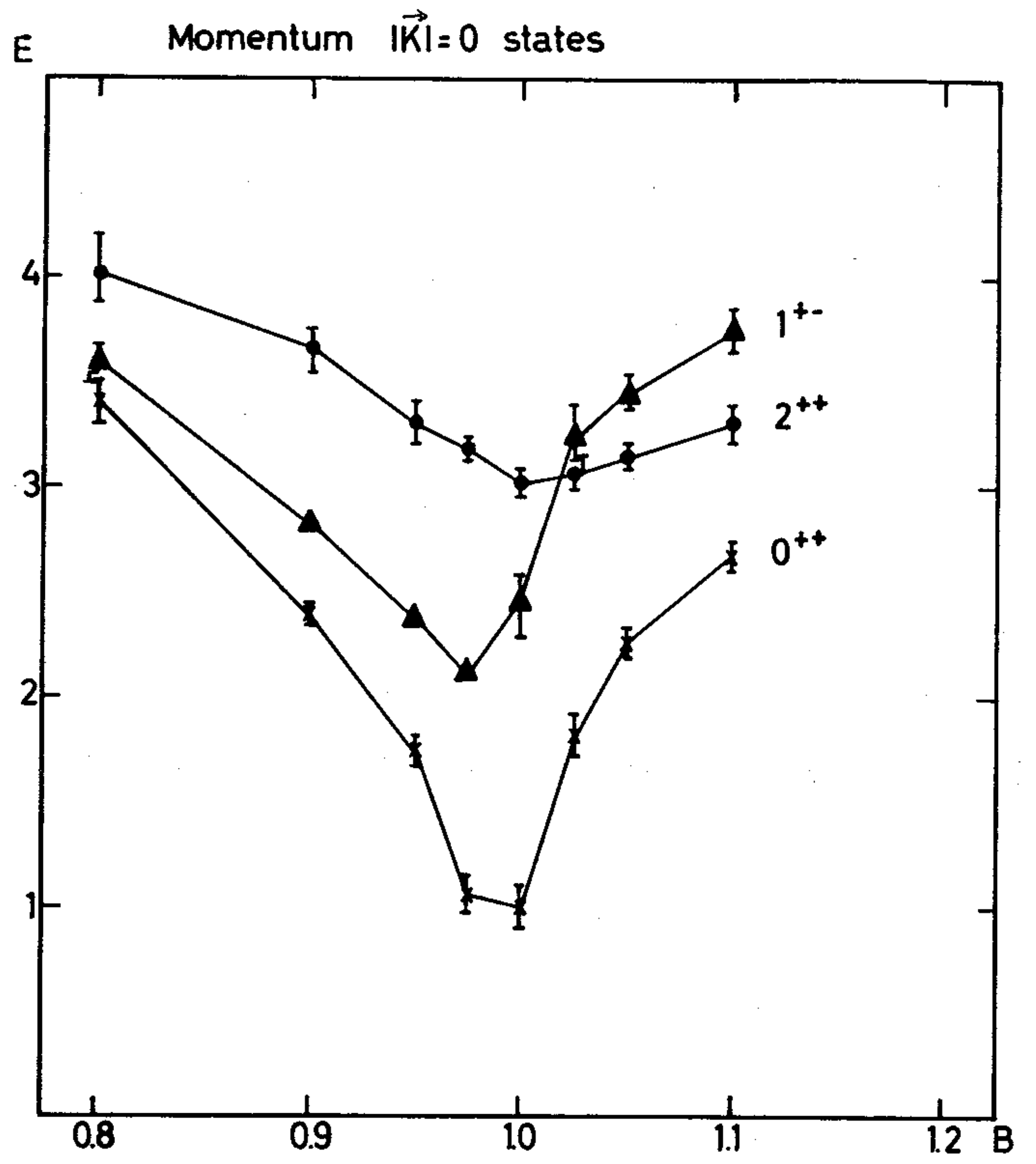
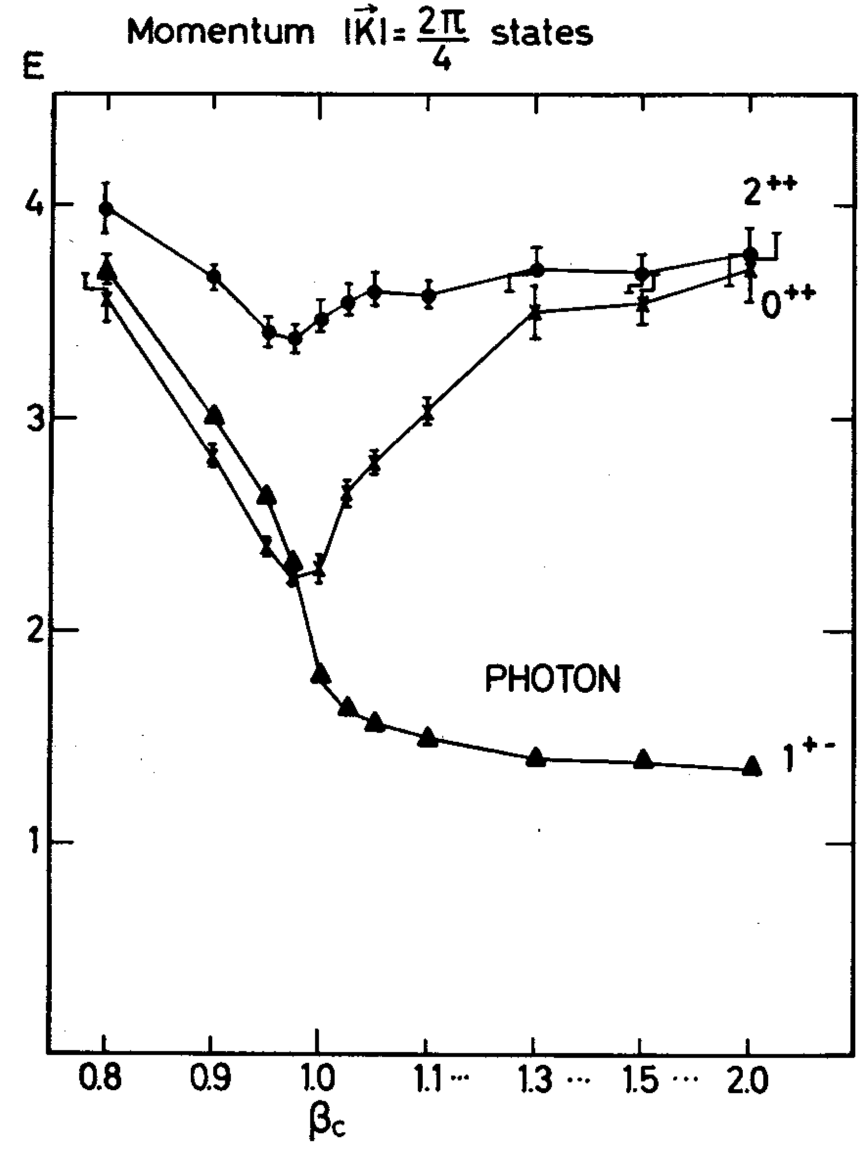


Fig. 1



E
Momentum |K| = 0 states
4
3
2
1
0.8
0.9
1.0
1.1
1.2
B
1+-
2++
0++

Fig.2



Momentum |K| = 2π/4 states
E
4
3
2
1
2++
0++
PHOTON
1+-
0.8
0.9
1.0
1.1 ···
1.3 ···
1.5 ···
2.0
βc

Fig.3