DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 83-057
July 1983

MONTE CARLO CALCULATION OF SU(2) GLUEBALL STATES WITH SYMANZIK's TREE-LEVEL IMPROVED ACTION

by

B. Berg
II. Institut für Theoretische Physik der Universität Hamburg

A. Billoire
CEN Saclay, Service de Physique Théorique

S. Meyer
Fachbereich Physik der Universität Kaiserslautern

C. Panagiotakopoulos
International Centre for Theoretical Physics
Miramare, I-34100 Trieste

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 83-057
July 1983

MONTE CARLO CALCULATION OF SU(2) GLUEBALL STATES WITH SYMANZIK's TREE-LEVEL IMPROVED ACTION

by

B. Berg [1], A. Billoire [2], S. Meyer [3]
and C. Panagiotakopoulos [4]

Abstract

With the tree-level improved 4d SU(2) lattice gauge theory we carry out a Monte Carlo calculation of the spectrum. We find a scaling window for the 0^+ state, leading to $m(0^+) \approx 53\ \Lambda_L^{TI}$. Results for the 2^+ state are somewhat inconclusive but also consistent with scaling.

1.) II.Institut für Theoretische Physik der Universität Hamburg, Luruper Chaussee 149, D-2000 Hamburg 50

2.) CEN Saclay, Service de Physique Théorique, Orme des Merisiers, F-91191 Gif-Yur-Yvette, Cedex

3.) Fachbereich Physik der Universität Kaiserslautern, Erwin-Schrödinger-Strasse, 675 Kaiserslautern; supported by Deutsche Forschungsgemeinschaft

4.) International Centre for Theoretical Physics, Miramare, I-34100 Trieste

Let us consider 4d SU(2) lattice gauge theory. In the continuum limit each physical quantity is proportional to an appropriate power of the correlation length (inverse mass gap) ξ with an universal coefficient. For non-zero lattice spacing $a \neq 0$ this "scaling" is violated by non-universal terms of order $({}^a/_\xi)^2\ \ell n\ ({}^\xi/a)$. Symanzik /1/ proposed to reduce these violations to order $({}^a/_\xi)^4\ \ell n\ ({}^\xi/a)$ by including in the lattice action irrelevant terms, which can be determined in perturbation theory, in ${}^1/N$ expansion or, in principle, also by Monte Carlo checks of scaling. For 4d lattice gauge theories the tree-level improved (TI) action is known due to work by Weisz /2/ and Curci et al./3/. The latter authors showed that Symanzik's program, to all leading logarithms, can be accomplished by adding to the Wilson Lagrangian the simplest term of dimension 6, i.e. the rectangle.

As in the Monte Carlo (MC) calculation of ref./4/ we use the choice

$$S^{TI} = -\frac{4}{g^2}\left[\frac{5}{3}\sum_{\square} \mathrm{Re}\ (\mathrm{Tr}\ \square) - \frac{1}{12}\sum_{\square\!\square} \mathrm{Re}\ (\mathrm{Tr}\,\square\!\square)\right] . \tag{1}$$

Here $\square\!\square$ represents the rectangular double plaquettes of size 1×2.

Using Wilson's /5/ action Creutz /6/ carried out a well-known MC calculation of the SU(2) and SU(3) string tension, and established a scaling window. With some reservations in mind this allows to extrapolate the continuum limit. A reasonable estimate of the SU(2) string tension (close to that of ref. /6,7/) is

$$\sqrt{K^W} = (79 \pm 12)\ \Lambda_L^W . \tag{2}$$

For the SU(2) TI action (1) a MC calculation of the string tension was carried out in ref. /4/. A scaling window is found, which leads to the estimate

$$\sqrt{K^{TI}} = (17.9 \pm 1.0)\ \Lambda_L^{TI} . \tag{3}$$

Combining equations (2) and (3) yields

$$\Lambda_L^{TI} / \Lambda_L^{W} = 4.4 \pm 0.9 \qquad (4)$$

in good agreement with the perturbative claim about $\Lambda_L^{TI} / \Lambda_L^{W}$ in ref. /3/. The very low statistics of ref. /4/ prevent, however, conclusions about improvements. Also the definition of the string tension was not taken TI, but the numerical relevance of this can argued to be minor. We like to emphasize that improvement in the sense of Symanzik /1/ is relevant for mass ratios.

For the SU(2) O^+ glueball (mass gap) a scaling window was first obtained in ref. /8/. Wilson's action was used and the final estimate reads

$$m(O^+)^W = (170 \pm 30)\ \Lambda_L^W. \qquad (5)$$

Excited SU(2) glueball states were also studied in the literature /9-12/, but the situation is unsatisfactory, because no convincing scaling windows could be established.

In this letter we report first results of a MC calculation of the SU(2) spectrum with the TI action (1). We use a variant of the MC variational method, which was pioneered in ref. /8-10/. Our lattice size is $5^3 \cdot 8$ (5^3 is the spacelike box and 8 is the extension in time direction). In view of the double plaquettes in the action (1) 5^3 seems to be the smallest feasible spacelike extent. As in ref./13/ our calculation is based on 21 different Wilson loops W_i ($i = 1, \ldots, 21$) of length ≤ 8. We perform always two MC sweeps before measuring the 21 loops. Each of our considered ß-values is based on 20 000 such double sweeps and

measurements, 1200 sweeps without measurements were always done first for reaching equilibrium.

In view of our previous experience /8,13/ we decided to restrict minimization to on-diagonal correlations

$$C_i(t) = \langle W_i(t)\, W_i(o)\rangle - \langle W_i(o)\rangle^2. \qquad (6)$$

More precisely: We consider the glueball mass definitions /8/

$$m_i(t) = -\frac{1}{t} \ln \left(\frac{C_i(t)}{C_i(o)} \right) \qquad (7.a)$$

and

$$\hat{m}_i(t) = -\ln \left(\frac{C_i(t)}{C_i(t-1)} \right). \qquad (7.b)$$

For t=1 we pick out the best = lowest values $m_i(1) = \hat{m}_i(1)$, and verify that they give compatible results for $m_i(2)$ and $\hat{m}_i(2)$.

The best results for the O^+ state (i.e. the A_1^+ representation /13/) are always obtained with the operators of Fig.1. Their $m_i(t)$ values are collected in Table 1. The values $\beta = 1.5$, 1.6 and 1.7 indicate a scaling curve, which should be improved. From Figure 2 we obtain the preliminary estimate

$$m(o^+) = (53 \pm 8)\ \Lambda_L^{TI} \qquad (8)$$

From all considered 21 operators we have in Figure 2 always used the one (as indicated in Table 1) which gives the lowest value for $m_i(1)$.

As mass ratios are TI-improved, we are particularly interested in excited glueball states. Following the classification of ref./13/ we have considered o^-, 1^+ and 2^+ states (A_1^-, T_1^+ and E^+ representations of the cubic group). For o^- and 1^+ we obtain only at distance t=1 results out off the statistical noise. As in previous investigations /12,13/ the mass values are high and there is no signal for scaling.

The situation is less clear for the 2^+ state. With the exception ß=1.9 the lowest value for $m_i^{E^+}(1)$ is always obtained from the operator of Figure 3. Some other operators give slightly higher results and the $m_i^{E^+}(2)$, $\hat{m}_i^{E^+}(2)$ results of all these operators are compatible. The 2^+ values of Fig. 4 are based on the operator of Figure 3. We recognize that the results are consistent with scaling (in the same region where o^+ scales), albeit rather inconclusive. A very tentative continuum estimate is

$$m(2^+) = (75 \pm 13)\ \Lambda_L^{TI} \quad . \tag{9}$$

An analysis with much higher statistics may confirm or contradict this estimate.

In conclusion: The TI action (1) has given a o^+ estimate (8), which is consistent with the result (5) for Wilson's action. Combining equations (5) and (8) gives

$$\Lambda_L^{TI} / \Lambda_L^{W} = 3.6 \pm 0.8 \tag{10}$$

and should be compared with equation (4). For the 2^+ state the TI action seems to allow more definite results than previous investigations with

other actions. A very high MC statistic is, however, required and the final results could still be in contradiction with scaling.

Symanzik's improvement program was first investigated within the 2d O(3) non-linear σ-model /14,15/. TI action and 1-loop improved action significantly give different results. We like to argue that 4d gauge theories are more well-behaved, but MC calculations with a 1-loop improved SU(2) action are certainly an important next step and consistency check.

Acknowledgements

One of the authors (B.B.) likes to thank Prof. K.Symanzik and Dr. P. Weisz for useful discussions. C. Panagiotakopoulos thanks the DESY theory group and the CEN Saclay theory group for travel support. The computer calculations were done on the Siemens 7561 at the University of Kaiserslautern and the CYBER 205 at the University of Karlsruhe.
We are particularly indepted to Prof. Schreiner and Dr. Fengler (Univers. Karlsruhe) for their support.

References

1.) K. Symanzik in "Mathematical problems in Theoretical Physics", edited by R. Schrader et al., Springer, Berlin 1982; preprints, DESY, 83 - 016, 83 - 026.

2.) P. Weisz, Nucl.Phys. B212 (1983) 1

3.) G. Curci, P. Menotti and G.P. Paffuti, preprint, IFUH-TH 83/5, Pisa

4.) S. Belforti, G. Curci, P. Menotti and G.P. Paffuti, preprint, IFUH-TH 83/6, Pisa

5.) K. Wilson, Phys.Rev. D10 (1974) 2445

6.) M. Creutz, Phys.Rev. D21 (1980) 2308; Phys.Rev.Lett. 45 (1980) 313

7.) B. Berg and J. Stehr, Z.Phys. C9 (1981) 333

8.) B. Berg, A. Billoire and C. Rebbi, Ann.Phys. (NY) 142 (1982) 185, addendum 146 (1983) 470

9.) M. Falcioni, E. Marinari, M.L. Paciello, G. Parisi, F. Rapuano, B. Taglienti and Zhang-Yi-Chang,Phys.Lett. 110B (1982) 295

10.) K. Ishikawa, M. Teper and G. Schierholz, Phys.Lett. 110B (1982) 399

11.) K. Ishikawa, G. Schierholz and M. Teper, preprint, DESY 83-004

12.) B. Berg, A. Billoire and K. Koller, preprint, DESY 83-45

13.) B. Berg and A. Billoire, Nucl.Phys. B221 (1983) 109; preprint,CEN-SACLAY SPH. T/42 (1983),to be published in Nucl.Phys.B

14.) B. Berg, S. Meyer, I. Montvay and K. Symanzik, preprint, DESY 83-015, to be published in Phys.Lett.B; B. Berg, S. Meyer and I. Montvay, in preparation; B. Berg, preprint, DESY 83-031, to be published in Z. Phys. C

15.) M. Falcioni, G. Martinelli, M. L. Paciello, G.Parisi and B. Taglienti, preprint, Roma (1983).

β	OP	m(1)	m(2)	$\hat{m}(2)$	Used
1.5	1	2.07 ± 0.03	2.15 ± 0.13	2.23 ± 0.28	-
1.5	2	2.07 ± 0.03	2.19 ± 0.14	2.31 ± 0.30	-
1.5	3	2.03 ± 0.03	2.12 ± 0.11	2.22 ± 0.24	3
1.6	1	1.81 ± 0.03	1.80 ± 0.08	1.79 ± 0.16	1
1.6	2	1.82 ± 0.03	1.79 ± 0.08	1.77 ± 0.16	-
1.6	3	1.83 ± 0.02	1.80 ± 0.07	1.77 ± 0.16	-
1.7	1	1.75 ± 0.03	1.59 ± 0.06	1.43 ± 0.12	-
1.7	2	1.74 ± 0.03	1.56 ± 0.06	1.38 ± 0.12	2
1.7	3	1.81 ± 0.03	1.63 ± 0.06	1.45 ± 0.12	-
1.8	1	2.03 ± 0.04	1.91 ± 0.09	1.78 ± 0.21	-
1.8	2	2.01 ± 0.03	1.89 ± 0.09	1.76 ± 0.21	2
1.8	3	2.08 ± 0.03	1.92 ± 0.09	1.77 ± 0.20	-
1.9	1	2.26 ± 0.03	2.04 ± 0.13	1.82 ± 0.30	1
1.9	2	2.23 ± 0.04	2.02 ± 0.13	1.82 ± 0.27	-
1.9	3	2.32 ± 0.04	2.11 ± 0.16	1.90 ± 0.34	-

Table 1

0^+ estimates from the operators of Figure 1

Figure Captions

Fig.1: Best trial operators for the 0^+ state

Fig.2: $m(0^+)$ estimate

Fig.3: Best trial operator for the 2^+ state

Fig.4: $m(2^+)$ estimate

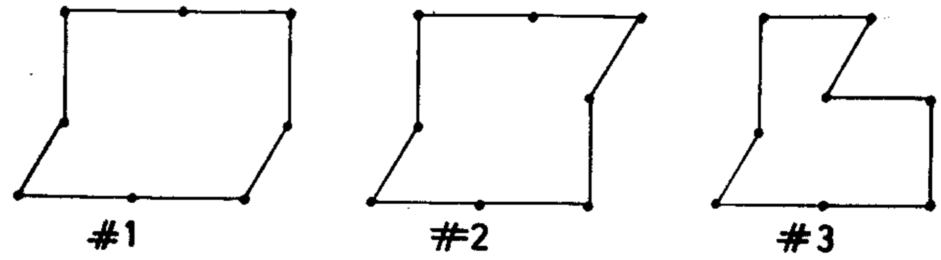

Fig.1

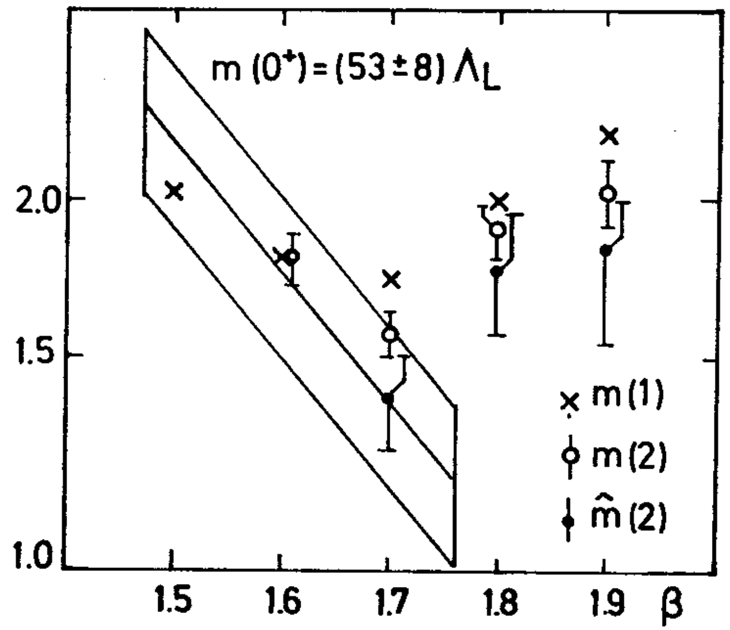

Fig.2

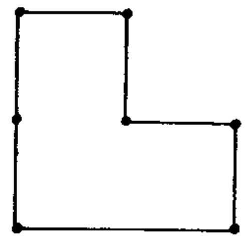

Fig.3

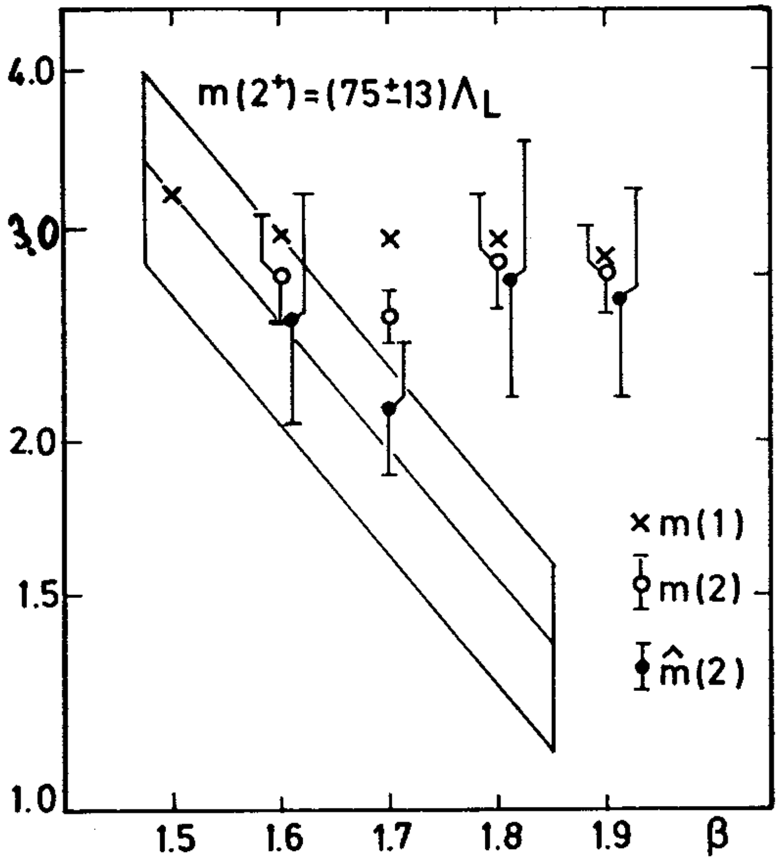

Fig.4