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Path Integral Quantization in the Temporal Gauge

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Abstract

The quantization of non-Abelian gauge theories in the temporal gauge is studied within Feynman's path integral approach. The standard asymptotic boundary conditions are only imposed on the transverse gauge fields. The fictituous longitudinal gauge quanta are eliminated asymptotically by modified boundary conditions. This abolishes the residual time-independent gauge transformations and leads to a unique fixing of the temporal gauge. The resulting path integral for the generating functional respects automatically Gauß's law. The correct gauge field propagator is dérived. It does not suffer from gauge singularities at $n \cdot k = 0$ present in the usual treatment of axial gauges. The standard principal value prescription does not work. As a check, the Wilson loop in temporal gauge is calculated with the new propagator. To second order (and to all orders in the Abelian case) the result agrees with the one obtained in the Feynman and Coulomb gauge.

1. The problem with the gauge singularity

The question of a "natural" gauge to be used in gauge theories is very old and occurs already in classical electrodynamics. The problem of the choice of gauge is crucial for the quantization of gauge theories.

Since the conjugate momenta of the time components ($\mu = 0$) of the gauge fields $A^a_\mu(x)$ vanish (a refers to the "color" of the field in the case of non-Abelian gauge theories [1]), a straightforward quantization of gauge field theories is impossible.

The standard solution to this problem consists in adding to the original Lagrangian a gauge fixing term. This enforces, however, in non-Abelian gauge theories the introduction of additional fields, the so-called Faddeev-Popov ghosts [1].

A very attractive gauge to work with is the temporal gauge [1-5] , where the time components of the gauge fields are set equal to zero

$$A^a_o(x) = 0, \; a = 1, \ldots, N^2-1 \text{ for } SU(N). \tag{1.1}$$

In this gauge the Faddeev-Popov ghosts decouple, and the Lagrangian for the pure gauge theory without fermions is given by

$$\begin{aligned} \mathcal{L}[A^a_i, \partial_\mu A^a_i] &= -\frac{1}{4}\left(F^a_{\mu\nu} F^{a,\mu\nu}\right)_{A^a_0=0} \\ &= \mathcal{L}_0 + g\mathcal{L}_3 + \frac{g^2}{4}\mathcal{L}_4 \end{aligned} \tag{1.2}$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{1.3}$$

and

$$\mathcal{L}_0 = \frac{1}{2}\left[(\partial_0 A_i^a)(\partial_0 A_i^a) - (\partial_i A_j^a)(\partial_i A_j^a) + (\partial_i A_j^a)(\partial_j A_i^a)\right]$$

$$\mathcal{L}_3 = - f^{abc}(\partial_i A_j^a) A_i^b A_j^c \tag{1.4}$$

$$\mathcal{L}_4 = - f^{abc} f^{ade} A_i^b A_j^c A_i^d A_j^e$$

(g is the unrenormalized coupling constant, f^{abc} are the structure constants of the non-Abelian group).

The simplest derivation of the free Feynman propagator [6] starts with the Euler-Lagrange equations following from $\mathcal{L}_o$ (eq. (1.4))

$$\left(\Box\, \delta_{ij} + \partial_i \partial_j\right) A_j^a(x) = 0 \quad . \tag{1.5}$$

The translationally invariant Feynman propagator is defined as the solution of the inhomogeneous wave equation with Feynman boundary conditions

$$\left(\Box\, \delta_{ij} + \partial_i \partial_j\right) D_F^{ab}(x-x')_{j\ell} = -\delta^{ab}\, \delta_{i\ell}\, \delta^4(x-x') \quad . \tag{1.6}$$

From (1.6) one obtains ($k^\mu = (k_o, \vec{k})$, $k^2 = k_o^2 - \vec{k}^2$)

$$D_F^{ab}(x-x')_{ij} = \delta^{ab} \int \frac{d^4k}{(2\pi)^4} \, \frac{\delta_{ij} - \frac{k_i k_j}{k_o^2}}{k^2 + i\varepsilon} \, e^{ik(x-x')} \tag{1.7}$$

which can be decomposed into a transverse and longitudinal Feynman propagator

$$D_F^{ab}(x-x')_{ij} = D_F^{ab,T}(x-x')_{ij} + D_F^{ab,L}(x-x')_{ij} \tag{1.8}$$

with

$$D_F^{ab,T}(x-x')_{ij} = \delta^{ab} \int \frac{d^4k}{(2\pi)^4} \, \frac{\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}}{k^2 + i\varepsilon} \, e^{ik(x-x')} \tag{1.9}$$

$$D_F^{ab,L}(x-x')_{ij} = \delta^{ab}\, D(t-t')\, d_{ij}(\vec{x}-\vec{x}') \tag{1.10}$$

$$d_{ij}(\vec{x}-\vec{x}') = \partial_i \partial_j \Delta^{-1} \delta^3(\vec{x}-\vec{x}') \tag{1.11}$$

$$D(t-t') = \int_{-\infty}^{\infty} \frac{dk_o}{2\pi} \, \frac{1}{k_o^2} \, e^{ik_o(t-t')} \quad . \tag{1.12}$$

While the transverse propagator, eq. (1.9), is well defined and identical to the familiar propagator in Coulomb gauge, the longitudinal propagator, eq. (1.10), is undefined due to the $1/k_o^2$ singularity in eq. (1.12). This singularity is a special case of the more general $1/(n \cdot k)$ and $1/(n \cdot k)^2$ singularities, which are present in all axial gauges defined by the gauge condition $n^\mu A_\mu^a(x) = 0$ with n^μ an arbitrary constant four-vector [7].

The unphysical gauge singularities at $n \cdot k = 0$ are a consequence of the breaking of Lorentz invariance by fixing the direction n^μ. It has been argued by several authors [8,9], that a principal value prescription leads to a consistent regularization of the gauge singularities. In temporal gauge this implies that D(t-t') in eqs. (1.10, 1.12) has to be replaced by the "principal value function"

$$D_p(t-t') \equiv \int_{-\infty}^{\infty} \frac{dk_o}{2\pi} \, \mathcal{P}\frac{1}{k_o^2} \, e^{ik_o(t-t')} = -\frac{1}{2}\,|t-t'| \tag{1.13}$$

where the principal value is defined by

$$\mathcal{P}\frac{1}{k_0^2} = \frac{1}{2}\left[\frac{1}{(k_0+i\varepsilon)^2} + \frac{1}{(k_0-i\varepsilon)^2}\right] . \tag{1.14}$$

This principal value prescription has been critized [3-5, 10] *), and different regularizations and propagators have been proposed.

In this paper we shall show that a consistent quantization in temporal gauge can be carried out within Feynman's path integral approach. As a result we shall obtain a well defined propagator.

The temporal gauge condition (1.1) does not fix the gauge completely, but allows time-independent gauge transformations

$$A_i(t,\vec{x}) \longrightarrow A_i'(t,\vec{x}) = \omega(\vec{x})\,A_i(t,\vec{x})\,\omega^{-1}(\vec{x}) + \frac{i}{g}\,\omega(\vec{x})\,\partial_i\,\omega^{-1}(\vec{x})$$
$$\omega(\vec{x}) = e^{-ig\frac{\lambda^a}{2}\Theta^a(\vec{x})} . \tag{1.15}$$

(Here $A_i(x) = \frac{\lambda^a}{2}A_i^a(x)$, and $\frac{\lambda^a}{2}$ are the generators of the Lie algebra of the non-Abelian group. $\Theta^a(\vec{x})$ are arbitrary real gauge functions.) The arbitrariness in the definition of the $1/k_o^2$ singularity in eq. (1.12) is intimately related to the residual gauge transformations (1.15). In sect. 3 we shall show that the propagator is well defined if the correct boundary conditions are imposed on the free longitudinal gauge fields. Only these additional boundary conditions fix the gauge uniquely, and therefore remove the possibility of the residual gauge transformations (1.15).

---

*) Similar problems with the principal value prescription occur in the light cone gauge and have been discussed by T.T. Wu [11].

2. Canonical quantization, Gauß's law and the dressed vacuum

In this section we shall review a recent attempt [5] towards a canonical quantization in the temporal gauge. From the Lagrangian $\mathcal{L}$, eqs. (1.2 - 1.4), one obtains the following simple form of the Hamiltonian H in the temporal gauge

$$H[A_i^a, E_i^a] = \frac{1}{2}\int d^3x\left(E_i^a E_i^a + B_i^a B_i^a\right) \tag{2.1}$$

where the chromoelectric and chromomagnetic fields are given by

$$E_i^a = -\partial_0 A_i^a$$
$$B_i^a = \frac{1}{2}\varepsilon_{ijk}F_{jk}^a . \tag{2.2}$$

It is easy to see that the pairs (A_i^a, E_i^a) are canonical variables and obey the laws of Hamiltonian dynamics [1]. It turns out, however, that Gauß's law

$$G^a(x) \equiv \left(\delta^{ab}\partial_i - g f^{abc}A_i^c(x)\right)E_i^b(x) = 0 \tag{2.3}$$

is not among the equations of motion, but rather represents a constraint on the canonical variables. Actually, eq. (2.3) cannot hold as an operator equation, because it is inconsistent with the canonical equal time commutator

$$\left[E_i^a(t,\vec{x}), A_j^b(t,\vec{x}')\right] = i\,\delta^{ab}\delta_{ij}\,\delta^3(\vec{x}-\vec{x}') . \tag{2.4}$$

Since the operators $G^a(x)$ close under commutation and commute with the Hamiltonian H, one follows Dirac [12] and tries to fulfil Gauß's law in a weak form as a condition on the physical state vectors of the theory

$$G^a(x)\,|\,\text{phys}\rangle = 0 \quad . \tag{2.5}$$

In perturbation theory one then starts with eq. (2.5) at g = 0 as a condition on the unperturbed ground state $|\Omega\rangle$ [1]

$$\partial_i E_i^a(t,\vec{x})\,|\,\Omega\rangle = 0 \quad . \tag{2.6}$$

If the gauge fields are decomposed into (divergenceless) transverse fields $A_i^{aT}(x)$ and (irrotational) longitudinal fields $A_i^{aL}(x)$

$$A_i^a(x) = A_i^{aT}(x) + A_i^{aL}(x) \tag{2.7}$$

it is seen from eq. (2.2) that Gauß's law (2.6) is a condition on the unphysical longitudinal fields only. With

$$|\,\Omega\rangle = |\,0\rangle_T \otimes |\,\Omega\rangle_L \tag{2.8}$$

Gauß's law takes the form

$$\partial_i E_i^{aL}(t,\vec{x})\,|\,\Omega\rangle_L = 0 \quad . \tag{2.9}$$

In (2.8) $|\,0\rangle_T$ denotes the Fock-vacuum of the physical, transverse gauge quanta, and $|\Omega\rangle_L$ is the ground state of the longitudinal gauge quanta. $|\Omega\rangle_L$ cannot be identified, however, with the Fock-vacuum $|\,0\rangle_L$, but rather has to be the ground state of a different, unitarily inequivalent representation of the canonical commutation relations, i.e. ${}_L\langle 0\,|\,\Omega\rangle_L = 0$.

Indeed, if one tries to construct $|\Omega\rangle_L$ by means of a Bogoliubov-transformation from the Fock-vacuum, $|\Omega\rangle_L = \mathcal{U}\,|\,0\rangle_L$, one finds that the (formally unitary) operator U does not exist.

A solution to this problem has been proposed recently in [5], where $|\Omega\rangle_L$ has been defined by a limiting process *)

$$|\Omega\rangle_L = \lim_{\delta\to 0+} |\,\Omega_\delta\rangle_L \equiv \lim_{\delta\to 0+} \mathcal{U}_\delta\,|\,0\rangle_L \tag{2.10}$$

with the "dressing" operator

$$\mathcal{U}_\delta = \exp\left\{\frac{1}{4}\ln\delta\int d^3x\left[a_L^{c+}(\vec{x})\,a_L^{c+}(\vec{x}) - a_L^c(\vec{x})\,a_L^c(\vec{x})\right]\right\} \quad . \tag{2.11}$$

(Here $a_L^c(\vec{x})$, $a_L^{c+}(\vec{x})$ denote the annihilation, creation operators of the longitudinal gauge quanta with color c.)

The "dressed" vacuum $|\Omega_\delta\rangle_L$ can be interpreted as an infinite sea of pairs of longitudinal gauge quanta, where each pair has zero total momentum.

It has been shown [5] that Gauß's law is satisfied for the dressed vacuum in the limit $\delta \to 0$

$$\lim_{\delta\to 0+} \partial_i E_i^{aL}(t,\vec{x})\,|\,\Omega_\delta\rangle_L = 0 \quad . \tag{2.12}$$

The free longitudinal gauge field propagator is well defined for $\delta > 0$, and is given by [5]

---

*) In ref. [5] a parameter λ $(\lambda \to 1-)$ was used. One has $\delta = \frac{1-\lambda}{1+\lambda}$.

$$ {}_L\langle\Omega_\delta | T(A_i^{aL}(x) A_j^{bL}(x')) | \Omega_\delta\rangle_L = i\delta^{ab} D_\delta(t,t')\, d_{ij}(\vec{x}-\vec{x}') \tag{2.13} $$

with

$$ D_\delta(t,t') = -\frac{1}{2}\left\{ |t-t'| - \delta\, tt'\, \Delta^{1/2} + \frac{1}{\delta}\Delta^{-1/2} \right\} . \tag{2.14} $$

The first term in (2.14) is time-translation invariant and independent of δ, and corresponds to the usual principal value regularization, eq. (1.13). The second term in (2.14) violates time-translation invariance, but vanishes for $\delta \to 0$. The last term in (2.14) *) is time-independent, diverges for $\delta \to 0$, but does not contribute to gauge invariant quantities. Nevertheless these terms are relevant in higher orders of perturbation theory, since the product of the second and third term in eq. (2.14) can lead to δ-independent, but time-dependent contributions.

The construction of the dressed ground state $|\Omega\rangle_L$ by the limiting process (2.10) leads to a consistent canonical quantization of gauge theories in the temporal gauge, and obeys the constraint condition of Gauß's law. Although it is an interesting possibility to quantize gauge theories and to implement Gauß's law by working in a Hilbert space based on a dressed vacuum state, we think it is desirable to seek for a simpler solution.

*) This term has also been found in QED by Creutz [10], who studied the longitudinal propagator at t = t' = 0. - We are indebted to P. Carelos for drawing our attention to this paper.

Since the longitudinal gauge quanta merely play the rôle of fictituous particles, one may consider the construction of their dressed vacuum as an unnecessary luxury. One therefore has to look for an alternative quantization scheme, which does not require the detailed construction of the delicate gauge field vacuum.

In the next section we shall show that a consistent quantization in the temporal gauge can be carried out within Feynman's path integral approach.

3. Path integral quantization and determination of the Feynman propagator

The generating functional of the Green's functions of the pure gauge theory is given in the temporal gauge ($n^\mu = (1, \vec{0})$) by the following Feynman path integral

$$
\begin{aligned}
Z[J] &= \int \mathcal{D}A^a_\mu(x)\, \delta(n^\mu A^a_\mu(x)) \exp\left\{-i\int d^4x \left[\tfrac{1}{4} F^a_{\mu\nu}(x) F^{a,\mu\nu}(x) + J^{a,\mu}(x)\, A^a_\mu(x)\right]\right\} \\
&= \int \mathcal{D}A^a_i(x)\; e^{\,i\int d^4x \left\{ \mathcal{L}[A^a_i, \partial_\mu A^a_i] + \vec{J}^a(x)\cdot \vec{A}^a(x) \right\}}
\end{aligned}
\tag{3.1}
$$

where $\mathcal{L}$ is given in eqs. (1.2 - 1.4), and $J^{a,\mu}(x)$ is a conserved c-number current. For the calculation of the free gauge field propagator we only need to consider the generating functional of the free theory

$$
Z_0[J] \equiv Z[J]_{g=0} = \int \mathcal{D}A^a_i \; e^{\,i\int d^4x \left[\mathcal{L}_0 + \vec{J}^a \cdot \vec{A}^a \right]} \quad . \tag{3.2}
$$

It is well known that a complete definition of path integrals of the form (3.2) requires the specification of boundary conditions for the fields $A^a_i(x)$. If we impose on all components of A^a_i the usual asymptotic Feynman conditions

$$
A^a_i(t,\vec{x}) \xrightarrow[t\to\pm\infty]{} A^a_i(t,\vec{x})_{\substack{out\\ in}} \tag{3.3}
$$

(where A^a_i ${}^{out}_{in}$ are solutions of the free field equations (1.5)), we obtain exactly the Feynman propagator (1.8), which is still undefined due to the gauge singularity $1/k_0^2$ in the longitudinal part, eqs. (1.10, 1.12).

We mentioned already in sect. 1 that the origin of the gauge singularity lies in the residual gauge transformations (1.15), which in the weak coupling limit ($g \to 0$) - appropriate for our perturbation theoretical discussion - have the simple form

$$
A^a_i(t,\vec{x}) \longrightarrow A'^a_i(t,\vec{x}) = A^a_i(t,\vec{x}) - \partial_i \Theta^a(\vec{x}) + O(g) \quad . \tag{3.4}
$$

This implies for the transverse and longitudinal free fields the following changes under the residual gauge transformations

$$
\begin{aligned}
\delta A^{a\,T}_i(t,\vec{x}) &= 0 \\
\delta A^{a\,L}_i(t,\vec{x}) &= -\partial_i \Theta^a(\vec{x}) \quad .
\end{aligned}
\tag{3.5}
$$

Thus the physical transverse field is not affected by the residual gauge transformations, and one can impose consistently the standard asymptotic Feynman conditions

$$
A^{aT}_i(t,\vec{x}) \xrightarrow[t\to\pm\infty]{} A^{aT}_i(t,\vec{x})_{\substack{out\\ in}} \tag{3.6}
$$

where $A^{aT}_i(x)$ ${}^{out}_{in}$ are solutions of the free field equation of motion

$$
\Box A^{aT}_i(x)_{\substack{out\\ in}} = 0 \quad . \tag{3.7}
$$

On the other hand, eq. (3.5) shows that the free longitudinal field is not yet fixed uniquely by the temporal gauge condition (1.1), but rather can be changed by an arbitrary time-independent function of $\vec{x}$.

In order to obtain a complete gauge fixing, we have to impose suitable asymptotic conditions on the longitudinal free fields, which, however, must be different from the standard asymptotic conditions (3.3).

Since the longitudinal gauge quanta describe unphysical degrees of freedom, they have to disappear in asymptotic physical field configurations, i.e.

$$A_i^{aL}(t,\vec{x}) \xrightarrow[t\to\pm\infty]{} 0 \quad . \tag{3.8}$$

Condition (3.8) must hold for all gauge field copies, and therefore restricts the residual gauge transformations (3.5) to the class of gauge functions $\Theta^a(\vec{x})$ which satisfy the condition

$$\partial_i \Theta^a(\vec{x}) = 0 \quad , \text{ i.e. } \Theta^a = \text{const.} \tag{3.9}$$

We conclude that the asymptotic condition (3.8) breaks the residual gauge invariance, and therefore leads to a unique gauge fixing.

Incorporating the asymptotic conditions (3.6) and (3.8) in the path integral (3.2) should lead to a well defined generating functional and thus to the correct gauge field propagator.

To incorporate the asymptotic conditions (3.6) and (3.8) in the path integral (3.2), we put our system in a large "time box" with length $\tau = \tau_2 - \tau_1$, $\tau_1 \leq t \leq \tau_2$, and take the limit $\tau_1 \to -\infty, \tau_2 \to \infty$ after the path integration has been carried out. The conditions (3.6) and (3.8) are then replaced by

$$A_i^{aT}(\tau_{1/2},\vec{x}) = A_i^{aT}(\tau_{1/2},\vec{x})_{\substack{out\\in}} \tag{3.10a}$$

$$A_i^{aL}(\tau_{1/2},\vec{x}) = 0 \quad . \tag{3.10b}$$

The generating functional Z_o is then given by

$$Z_o[J] = \lim_{\substack{\tau_2\to\infty\\ \tau_1\to-\infty}} \hat{Z}_o[J] \equiv \tau\text{-}\lim \hat{Z}_o[J] \tag{3.11}$$

with

$$\hat{Z}_o[J] = \int\limits_{A_i^{aT}(\tau_1,\vec{x})=A_i^{aT}(\tau_1,\vec{x})_{in}}^{A_i^{aT}(\tau_2,\vec{x})=A_i^{aT}(\tau_2,\vec{x})_{out}} \mathcal{D}A_i^{aT}(t,\vec{x}) \int\limits_{A_i^{aL}(\tau_1,\vec{x})=0}^{A_i^{aL}(\tau_2,\vec{x})=0} \mathcal{D}A_i^{aL}(t,\vec{x})\, e^{i\int_{\tau_1}^{\tau_2} dt \int d^3x [\mathcal{L}_o + \vec{J}^a\cdot\vec{A}^a]}$$

$$\equiv \int_1^2 \mathcal{D}A_i^a \; e^{i\int_1^2 d^4x\, [\mathcal{L}_o + \vec{J}^a\cdot\vec{A}^a]} \quad . \tag{3.12}$$

Inserting $\mathcal{L}_o$, eq. (1.4), into (3.12) one obtains

$$\hat{Z}_o[J] = \int_1^2 \mathcal{D}A_i^a \exp\left\{-\frac{i}{2}\int_1^2 d^4x \int_1^2 d^4x'\, A_i^a(x)\hat{K}_{ij}^{ab}(x-x')A_j^b(x') + i\int_1^2 d^4x\, J_i^a(x)A_i^a(x)\right\}$$

$$\hat{K}_{ij}^{ab}(x-x') = \delta^{ab}\left(\Box\,\delta_{ij} + \partial_i\partial_j\right)\delta^4(x-x') \quad . \tag{3.13}$$

The path integration can be carried out and leads to the final result

$$\hat{Z}_0[J] = \hat{Z}_0[0]\; e^{-\frac{i}{2}\int_1^2 d^4x \int_1^2 d^4x'\, J_i^a(x)\, \hat{D}_F^{ab}(x,x')_{ij}\, J_j^b(x')} \; . \tag{3.14}$$

Here the finite time Feynman propagator $\hat{D}_F$ is defined as the inverse of the kernel $\hat{K}$ in the space of functions $A_i^a(x)$ defined by the boundary conditions (3.10a,b). $\hat{D}_F$ has the following decomposition into a transverse and longitudinal part

$$\hat{D}_F^{ab}(x,x')_{ij} = D_F^{ab,T}(x-x')_{ij} + \hat{D}_F^{ab,L}(x,x')_{ij} \; . \tag{3.15}$$

Here and in eq. (3.14) the limit $\tau_1 \to -\infty, \tau_2 \to \infty$ is carried out already in the transverse part. Therefore D_F^T is identical to the standard transverse Feynman propagator, eq. (1.9).

The longitudinal propagator $\hat{D}_F^L$ is given by

$$\hat{D}_F^{ab,L}(x,x')_{ij} = \delta^{ab}\, \hat{D}(t,t')\, d_{ij}(\vec{x}-\vec{x}') \tag{3.16}$$

where $d_{ij}(\vec{x}-\vec{x}')$ is defined in eq. (1.11), and $\hat{D}$ is the solution of the following homogeneous Dirichlet boundary value problem

$$\begin{aligned} &\frac{\partial^2}{\partial t^2}\hat{D}(t,t') = -\delta(t-t') \\ &\hat{D}(t,t') = \hat{D}(t',t) \\ &\hat{D}(\tau_{1/2},t') = 0 \qquad \forall\, t' \in [\tau_1,\tau_2] \; . \end{aligned} \tag{3.17}$$

Since the principal value function D_P, eq. (1.13), is a solution of the inhomogeneous differential equation (3.17), it remains to determine the homogeneous solution satisfying the above boundary conditions. One finds

$$\hat{D}(t,t') = -\frac{1}{2}\left\{ |t-t'| + 2\frac{tt'}{\tau} + \alpha_1(t+t') + \alpha_2 \right\} \tag{3.18}$$

where the coefficients α_i are given by

$$\begin{aligned} &\alpha_1 = -\frac{\sigma}{\tau} \; , \quad \alpha_2 = \frac{\sigma^2-\tau^2}{2\tau} \\ &\sigma = \tau_2 + \tau_1 \; , \quad \tau = \tau_2 - \tau_1 \; . \end{aligned} \tag{3.19}$$

Of particular interest are the following cases:

i) the symmetrical case $\tau_2 = \frac{\tau}{2} = -\tau_1$:

$$\begin{aligned} &\hat{D}_{sym}(t,t') = -\frac{1}{2}|t-t'| - \frac{tt'}{\tau} + \frac{\tau}{4} \\ &\forall\, t,t' \in \left[-\frac{\tau}{2}, \frac{\tau}{2}\right] \end{aligned} \tag{3.20}$$

ii) the case, where the time box is placed on the positive or negative time axis, i.e. $(\tau_1,\tau_2) = (0,\tau)$ or $(-\tau,0)$:

$$\begin{aligned} &\hat{D}_\pm(t,t') = -\frac{1}{2}|t-t'| - \frac{tt'}{\tau} \pm \frac{1}{2}(t+t') \\ &\forall\, t,t' \in \begin{cases} [0,\tau] & \text{for } (+) \\ [-\tau,0] & \text{for } (-) \end{cases} \; . \end{aligned} \tag{3.21}$$

In case i), which is appropriate for the calculation of S-matrix elements, one obtains the same time-dependence as in $D_{\mathfrak{g}}$, eq. (2.14), which was derived in the canonical quantization scheme using a dressed vacuum state [5].

The result (3.21) is almost identical (in the limit $\tau \to \infty$) to the modified propagators proposed in [3,4], apart from a constant term which was added by these authors. The correct factor $\pm\frac{1}{2}$ in front of the (t+t') term in (3.21) could not be determined by these authors a priori. It was obtained from a calculation of the Wilson loop in order g^4 and comparing the result with the known result in the Feynman and Coulomb gauge.

Obviously, $\hat{D}(t,t')$ of eq. (3.18) violates time-translation invariance. This violation of time-translations is a direct consequence of the condition (3.10b), which we had to impose in order to eliminate the unphysical longitudinal degrees of freedom. Explicit calculations show, however, that time-translation invariance is restored in gauge invariant results, see sect. 6 and refs. [3,4].

As expected, the complete fixing of the temporal gauge by the condition (3.9) (as a consequence of (3.8)) has led to a regularization of the $1/k_o^2$ singularity. Another way to see this directly is the following expansion of the symmetrical propagator function (3.20) in a Fourier series:

$$\hat{D}_{sym}(t,t') = f(t-t') - g(t) - g(t') + \frac{\tau}{12} \tag{3.22}$$

with

$$f(t-t') = \frac{1}{\tau}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} \frac{e^{i\omega_n(t-t')}}{\omega_n^2}$$

$$g(t) = \frac{1}{\tau}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} \frac{(-1)^n e^{i\omega_n t}}{\omega_n^2} \tag{3.23}$$

$$\omega_n = \frac{2\pi}{\tau}\, n \quad .$$

4. <u>Consistency of the path integral with Gauß's law</u>

In sect. 2 we discussed the problems connected with Gauß's law in the temporal gauge. Since in the derivation of the path integral (3.13) we never refered to Gauß's law, it arises the important question, whether Gauß's law is fulfilled.

In this section we shall show that our path integral implements Gauß's law automatically.

The expected value of the divergence of the chromoelectric field in the presence of a conserved c-number current $J^{a,\mu}(x) = (\rho^a(x), \vec{J}^a(x))$ is given by

$$\langle \partial_i E_i^a(x)\rangle^J \equiv \tau\text{-}lim \frac{\int_1^2 \mathcal{D}A_k^b\, (\partial_i E_i^a(x))\, e^{i\int_1^2 d^4x' [\mathcal{L}_0 + J_k^b A_k^b]}}{\int_1^2 \mathcal{D}A_k^b\, e^{i\int_1^2 d^4x' [\mathcal{L}_0 + J_k^b A_k^b]}} \tag{4.1}$$

$$= \tau\text{-}lim \left\{ \frac{1}{\hat{Z}_0[J]}\, i\partial_0\partial_i \frac{\delta}{\delta J_i^a(x)} \hat{Z}_0[J] \right\}$$

(see eqs. (3.11, 3.12) and (2.2)). With the explicit expression (3.14) for the generating functional Z_o we obtain

$$\langle \partial_i E_i^a(x)\rangle^J = \tau\text{-}lim \int_1^2 d^4x'\, \partial_0\partial_i\, \hat{D}_F^{ab,L}(x,x')_{ij}\, J_j^b(x') \tag{4.2}$$

$$= \tau\text{-}lim \int_{\tau_1}^{\tau_2} dt'\, \frac{\partial}{\partial t}\, \hat{D}(t,t')\, \partial_i'\, J_i^a(x') \quad . \tag{4.3}$$

In the last step eq. (3.16) was inserted. Using current conservation we get after an integration by parts

$$\langle \partial_i E_i^a(t,\vec{x})\rangle^J = \tau\text{-}\lim \int_{\tau_1}^{\tau_2} dt' \frac{\partial^2}{\partial t \partial t'} \hat{D}(t,t')\, \rho^a(t',\vec{x}) \tag{4.4}$$

since the surface terms vanish because of the relation

$$\frac{\partial}{\partial t} \hat{D}(t,\tau_{1/2}) = 0 \qquad \forall t \in (\tau_1,\tau_2) \,. \tag{4.5}$$

Property (4.5) and the following relation

$$\frac{\partial^2}{\partial t \partial t'} \hat{D}(t,t') = \delta(t-t') - \frac{1}{\tau} \tag{4.6}$$

are simple consequences of eq. (3.18). Making use of (4.6) we obtain from (4.4) our final expression for Gauß's law

$$\langle \partial_i E_i^a(t,\vec{x})\rangle^J = \rho^a(t,\vec{x}) - \bar{\rho}^a(\vec{x}) \tag{4.7}$$

where $\bar{\rho}^a$ denotes the time-averaged charge density

$$\bar{\rho}^a(\vec{x}) = \tau\text{-}\lim \frac{1}{\tau} \int_{\tau_1}^{\tau_2} dt'\, \rho^a(t',\vec{x}) \,. \tag{4.8}$$

In the free case the current J^μ vanishes identically, and Gauß's law is fulfilled in the form

$$\langle \partial_i E_i^a(t,\vec{x})\rangle^{J=0} = 0 \,. \tag{4.9}$$

The version (4.7) of Gauß's law has, however, a more general physical interpretation, which we want to discuss now.

Consider the following Gedankenexperiment: two pointlike infinitely massive test charges with opposite (c-number) charge g^a are at rest at $\vec{x} = 0$ and at $\vec{x} = \vec{R} = (R,0,0)$:

$$\rho^a(\vec{x}) = g^a \left[\delta^3(\vec{x}-\vec{R}) - \delta^3(\vec{x}) \right] \,. \tag{4.10}$$

Imagine that the charges are suddenly switched on at time t_1 and suddenly switched off at a later time t_2, where the time interval $[t_1,t_2]$ lies inside our time box $[\tau_1,\tau_2]$, i.e.

$$\tau_1 < t_1 < t_2 < \tau_2 \quad , \quad T \equiv t_2 - t_1 < \tau \,. \tag{4.11}$$

Obviously, this arrangement of charges realizes a rectangular Wilson loop in the (t,x_1)-plane (see sect. 6) to which belongs the following conserved c-number current

$$\begin{cases} \rho^a(t,\vec{x}) = g^a \left[\delta^3(\vec{x}-\vec{R}) - \delta^3(\vec{x})\right]\left[\Theta(t-t_1) - \Theta(t-t_2)\right] \\ J_i^a(t,\vec{x}) = g^a \delta_{i1}\, \Theta(R-x_1)\Theta(x_1)\delta(x_2)\delta(x_3)\left[\delta(t-t_1) - \delta(t-t_2)\right] \,. \end{cases} \tag{4.12}$$

For the charge distribution (4.12) the time-average $\bar{\rho}^a$ vanishes (if T is kept fixed)

$$\bar{\rho}^a(\vec{x}) = \rho^a(\vec{x}) \lim_{\tau\to\infty} \frac{T}{\tau} = 0 \tag{4.13}$$

and we obtain Gauß's law in the classical form

$$\langle \partial_i E_i^a(t,\vec{x})\rangle^J = \rho^a(t,\vec{x}) \quad . \tag{4.14}$$

This example illustrates how the limit " τ-lim" in our path integral formula has to be understood. If one wants to calculate a Green's function with time arguments $t_1, \ldots, t_n$ from the path integral formula analogous to eq. (4.1), one has to perform the limits $\tau_1 \to -\infty$, $\tau_2 \to \infty$, while keeping all t_i's fixed with $t_i \in (\tau_1, \tau_2)$.

As a further illustration of the consistency of our path integral formula we shall calculate the following time-ordered product

$$\langle T(\partial_i E_i^a(x) A_j^b(x'))\rangle^J = \tau\text{-}\lim \left\{ \frac{1}{\hat{Z}_0[J]} \partial_0 \partial_i \frac{\delta^2}{\delta J_i^a(x)\,\delta J_j^b(x')} \hat{Z}_0[J] \right\} \tag{4.15}$$

$$= \langle \partial_i E_i^a(x)\rangle^J \langle A_j^b(x')\rangle^J - i\delta^{ab}\,\tau\text{-}\lim \frac{\partial}{\partial t} \hat{D}(t,t')\,\partial_j \delta^3(\vec{x}-\vec{x}') \quad .$$

Here the effective gauge field (in the presence of the source J) is defined by

$$\langle A_j^b(x')\rangle^J = \tau\text{-}\lim \left\{ \frac{1}{\hat{Z}_0[J]} \frac{1}{i} \frac{\delta}{\delta J_j^b(x')} \hat{Z}_0[J] \right\}$$

$$= -\,\tau\text{-}\lim \int_1^2 d^4x''\, \hat{D}_F^{bc}(x',x'')_{jk} J_k^c(x'') \, . \tag{4.16}$$

In the symmetrical case $\left(\tau_2 = \frac{\tau}{2} = -\tau_1\right)$ we infer from eq. (3.20)

$$\tau\text{-}\lim \frac{\partial}{\partial t} \hat{D}_{sym}(t,t') = -\frac{1}{2}\varepsilon(t-t') - \lim_{\tau\to\infty} \frac{t'}{\tau} \tag{4.17}$$

$$= -\frac{1}{2}\varepsilon(t-t') \quad .$$

This leads for the connected part of the time-ordered product (4.15) to the following result

$$\langle T(\partial_i E_i^a(x) A_j^b(x'))\rangle_C^J = \frac{i}{2}\delta^{ab}\varepsilon(t-t')\,\partial_j \delta^3(\vec{x}-\vec{x}') \tag{4.18}$$

which is independent of the c-number current J. Eq. (4.18) is consistent with the canonical equal time commutation relation (2.4). (It is another illustration of the fact that Gauß's law cannot hold as an operator equation, as discussed already in sect. 2.)

5. The chromoelectric energy in the presence of quasi-static color sources

According to eq. (2.1) the chromoelectric part of the Hamiltonian is given by

$$H_E(t) = \frac{1}{2}\int d^3x \; E_i^a(t,\vec{x})\, E_i^a(t,\vec{x}) \quad . \tag{5.1}$$

In the presence of a conserved c-number current J we have for the expected value of the chromoelectric energy

$$\begin{aligned}\langle H_E(t)\rangle^J &= \tau\text{-}\lim \lim_{\delta t\to 0+} \frac{1}{2}\int d^3x \left\langle T\left(E_i^a(t+\delta t,\vec{x})\, E_i^a(t,\vec{x})\right)\right\rangle^J \\ &= \tau\text{-}\lim \lim_{\delta t\to 0+} \frac{1}{2}\int d^3x \left\{\frac{1}{\hat{Z}_0[J]} \frac{\partial^2}{\partial y_0 \partial t} \frac{1}{i^2} \frac{\delta^2 \hat{Z}_0[J]}{\delta J_i^a(y)\, \delta J_i^a(x)}\right\}_{\substack{y_0 = t+\delta t \\ \vec{y} = \vec{x}}}\end{aligned} \tag{5.2}$$

$$\begin{aligned} &= \langle H_E(t)\rangle^{J=0} + \\ &+ \tau\text{-}\lim \frac{1}{2}\int d^3x \int_1^2 d^4x' \int_1^2 d^4x'' \frac{\partial}{\partial t}\hat{D}_F^{ab}(x,x')_{;ij} \frac{\partial}{\partial t}\hat{D}_F^{ac}(x,x'')_{;ik} \, J_j^b(x')\, J_k^c(x'')\end{aligned} \tag{5.3}$$

with

$$\langle H_E(t)\rangle^{J=0} = \tau\text{-}\lim \lim_{\delta t\to 0+} \frac{i}{2}\int d^3x \sum_{i,a} \left(\frac{\partial^2}{\partial y_0 \partial t} \hat{D}_F^{aa}(y,x)_{;ii}\right)_{\substack{y_0 = t+\delta t \\ \vec{y} = \vec{x}}} \tag{5.4}$$

If the longitudinal contribution to the change in the chromoelectric energy is denoted by $\mathcal{E}_L^J$, i.e.

$$\mathcal{E}_L^J(t) = \langle H_E(t)\rangle_L^J - \langle H_E(t)\rangle_L^{J=0} \tag{5.5}$$

we obtain the correct classical expression for the instantaneous Coulomb interaction

$$\mathcal{E}_L^J(t) = \frac{1}{8\pi}\int d^3x' \int d^3x'' \frac{\rho^a(t,\vec{x}')\,\rho^a(t,\vec{x}'')}{|\vec{x}' - \vec{x}''|} \quad . \tag{5.6}$$

(We assumed that ρ^a has the property (4.13).) This result proves once more that our path integral quantization yields the correct expectation values.

Although the action in temporal gauge does not depend on the charge density ρ^a explicitly (since the term $\sim \rho^a A_0^a$ is absent), the derivation of eqs. (4.14) and (5.6) clearly demonstrates, how ρ^a finally reappears in physical observables.

6. Calculation of the Wilson loop

Recently it has been found [4] in a calculation of the Wilson loop [13] in quantumchromodynamics (QCD) that the principal value prescription (2.13) for the gluon propagator in temporal gauge does not reproduce the results obtained in Feynman and Coulomb gauge. Therefore the Wilson loop offers a crucial test for the validity of the gluon propagator (3.16, 3.18), which we derived from a path integral quantization.

In this section we shall calculate the Wilson loop in quantumelectrodynamics (QED). We shall obtain the correct expression to all orders in the electric charge e. This proves the correctness of our propagator in QED and QCD, for the latter case, of course, only in order g^2. The calculation of the Wilson loop in QCD up to order g^4 is presently carried out and will be published elsewhere.

The Wilson loop in QED is defined by [13]

$$W(\mathcal{C}) = \left\langle P\, e^{-ie\oint_{\mathcal{C}} dx_\mu A^\mu(x)} \right\rangle^{J=0} . \qquad (6.1)$$

Here P denotes a path-ordering; the closed, rectangular curve $\mathcal{C}$ is chosen to lie in the (x_1,t)-plane

$$\mathcal{C}:\ (t_1,0,0,0) \longrightarrow (t_1,R,0,0) \longrightarrow (t_2,R,0,0) \longrightarrow \\ \longrightarrow (t_2,0,0,0) \longrightarrow (t_1,0,0,0) . \qquad (6.2)$$

From our path integral formulae (3.11, 3.12) we obtain

$$W(\mathcal{C}) = \tau\text{-}\lim \frac{1}{\hat{Z}_0[0]} \int_1^2 \mathcal{D}A_i\; e^{ie\oint_{\mathcal{C}} dx_i A_i}\; e^{i\int_1^2 d^4x\, \mathcal{L}_0}$$

$$= \tau\text{-}\lim \frac{\hat{Z}_0[J(\mathcal{C})]}{\hat{Z}_0[0]} . \qquad (6.3)$$

Here we have introduced a current $J(\mathcal{C})$ via the relation

$$e\oint_{\mathcal{C}} dx_i A_i = \int_1^2 d^4x\, J_i(\mathcal{C}) A_i . \qquad (6.4)$$

It is obvious from the geometry of the curve $\mathcal{C}$ that the current $J^\mu(\mathcal{C}) \equiv (\rho(x), \vec{J}(x))$ is identical to the current (4.12), if we replace g^a by e and drop the color index a in (4.12). In particular, we have the time-ordering as in eq. (4.11), which guarantees that the curve $\mathcal{C}$ lies entirely inside the time box. This is crucial in order not to mix up the Wilson loop with unphysical boundary contributions.

Since $Z_0[J]$ has been calculated already in sect. 3 we get immediately from eqs. (3.14, 3.15)

$$W(\mathcal{C}) = W^T(\mathcal{C})\, W^L(\mathcal{C}) \qquad (6.5)$$

$$W^T(\mathcal{C}) = e^{-i\frac{e^2}{2}\oint_{\mathcal{C}} dx_i \oint_{\mathcal{C}} dx'_j\, D_F^T(x-x')_{ij}} \qquad (6.6)$$

$$W^L(\mathcal{C}) = \tau\text{-}\lim\, e^{-i\frac{e^2}{2}\oint_{\mathcal{C}} dx_i \oint_{\mathcal{C}} dx'_j\, \hat{D}_F^L(x,x')_{ij}} . \qquad (6.7)$$

It turns out that the dominant contribution to the Wilson loop in the limit $T \equiv t_2 - t_1 \gg R$ comes from the longitudinal part W^L. (This is the reason why we consider the Wilson loop as a crucial test for the correct regularization of the longitudinal gauge field propagator.)

A straightforward calculation gives for W^L the result

$$W^L(\mathcal{C}) = \tau\text{-}\lim \; e^{-iF(t_1,t_2)\, I(R)} \tag{6.8}$$

where the functions F and I are defined as follows

$$F(t_1,t_2) = \hat{D}(t_1,t_1) + \hat{D}(t_2,t_2) - 2\hat{D}(t_1,t_2) \tag{6.9}$$

$$I(R) = \frac{e^2}{2} \int_0^R dx_1 \int_0^R dx_1' \; d_{11}(x_1 - x_1', 0, 0) \quad . \tag{6.10}$$

With $\hat{D}$ from eq. (3.18) we get (for T fixed)

$$F(t_1,t_2) = T - \frac{T^2}{\tau} \xrightarrow[\tau\text{-}\lim]{} T \quad . \tag{6.11}$$

Inserting the expression (1.11) into (6.10) yields

$$I(R) = V(R) - V(0) \tag{6.12}$$

where V(R) is the Coulomb potential

$$V(R) \equiv -e^2 \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{R}}}{\vec{k}^2} = -\frac{\alpha}{R} \quad , \alpha = \frac{e^2}{4\pi} \quad . \tag{6.13}$$

Thus we obtain the result (exact for all T, R $\geq$ 0)

$$W^L(\mathcal{C}) = e^{-i[V(R) - V(0)]T} \quad . \tag{6.14}$$

From eq. (6.6) we derive

$$W^T(\mathcal{C}) = e^{f(T,R) - f(0,R)} \tag{6.15}$$

with

$$f(T,R) = i e^2 \int_0^R dx_1 \int_0^R dx_1' \; D_F^T(T, x_1 - x_1', 0, 0)_{11} \quad . \tag{6.16}$$

For $T > 0$ f(T,R) is only a function of the dimensionless ratio R/T. For $\frac{R}{T} < 1$ one has

$$\begin{aligned} f(T,R) &= \frac{\alpha}{\pi} \int_{-1}^{1} du \, \frac{1-u^2}{u^2} \ln\left(1 - \frac{R}{T}u\right) \\ &= -\frac{2}{3}\frac{\alpha}{\pi}\left(\frac{R}{T}\right)^2 + O\left(\left(\frac{R}{T}\right)^4\right) \end{aligned} \tag{6.17}$$

i.e. f(T,R) vanishes for $T \to \infty$, R fixed. f(0,R) is divergent and must be regularized by one of the usual methods. However, since it is independent of T, it does not contribute to the static potential derived from the Wilson loop formula (see below). Of course, the expression V(o) in eq. (6.14) is also divergent. It is due to the infinite self-energy of the static charges. It can be removed from the Wilson loop by a multiplicative mass renormalization *).

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*) The renormalization of the Wilson loop is discussed in [14-16] .

We then obtain the correct Coulomb potential

$$\lim_{T\to\infty} \frac{i}{T} \ln W(\mathcal{C})_{\text{ren}} = V(R) = -\frac{\alpha}{R} \quad . \tag{6.18}$$

The purpose of this section was to show that our modified propagator (3.18) yields the correct result for the Wilson loop. Since the longitudinal propagator entered the result only in the combination (6.9), the terms proportional to α_1 and α_2 in (3.18) were cancelled, whereas the term proportional to tt' in (3.18) went to zero in the limit, eq. (6.11). Thus the only contribution to the final expression came from the principal value term in eq. (3.18). One may guess that the additional terms in (3.18) can be omitted in Abelian gauge theories from the very beginning. As far as we know, there exists, however, no general proof of this statement. In the case of non-Abelian gauge theories it is expected that these additional terms play a crucial rôle. This has already been shown [3,4] in the special case of the propagator (3.21). These calculations [3,4] are therefore a strong support for our general expectation.

In a recent article [17] *) Friedman et al. also discussed the quantization of Yang-Mills theories in the temporal gauge. These authors found that the set of Feynman rules has to be enlarged by new vertices. Our gauge field propagator also yields additional vertices in momentum space due to the time-translation non-invariant terms in the longitudinal propagator.

It is clear to us, that further work is needed to study the question of renormalization and to give a general proof of the time-translation invariance of gauge invariant quantities.

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