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Axial-vector Anomaly for Dirac-Kähler Fermions on the Lattice

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Abstract

The axial-vector current of Dirac-Kähler fermions on the lattice is studied. We consider a U(1) gauge theory in two dimensions as well as an SU(N) gauge theory in four dimensions. Using a short-distance expansion of the fermion propagator in an external gauge field, we show that the correct anomaly is reproduced in the continuum limit.

1. Introduction

The incorporation of fermions into the scheme of lattice gauge theories encounters the well-known degeneracy problem: The naively discretized Dirac action describes more than one species of fermions in the continuum limit. Several methods have been proposed to cope with this unwanted "doubling", e.g. by Wilson /1/, Susskind /2/ and Drell et al. /3/. A somewhat different approach due to Becher and Joos /4/ (see also /5/) starts from the Dirac-Kähler equation. It has the advantage that there exists a straightforward correspondence between the continuum and the lattice description and that the degeneracy is no lattice artifact /6/.

An important question concerning fermions on the lattice is the following: Does the lattice description reproduce the correct anomaly of the axial-vector current /7,8/ in the continuum limit? Karsten and Smit /9/ have shown that the anomaly vanishes in the continuum limit, if one uses the naive action and the most obvious definition of the current on the lattice. This result originates from the fact that the contributions of the different species to the would-be anomaly cancel. On the other hand, applying Wilson's degeneracy regularization they found the correct anomaly in weak coupling perturbation theory at the one-loop level. That Wilson's action reproduces the continuum anomaly was also proved by Kerler /10/ in a nonperturbative analysis. Defining an axial current different from that used by Karsten and Smit, Sharatchandra et al. /11/ derived the correct anomaly for Susskind fermions as well as in the naive formulation.

In this paper we study the axial anomaly in the Dirac-Kähler framework. The close connection between lattice and continuum, which exists in this approach because of its geometrical interpretation, leads immediately to the definition of an axial-vector current on the lattice. This current is conserved in the free case. We want to show that the corresponding gauge invariant current of the interacting theory has the correct anomaly in the continuum limit. The plan of the paper is as follows: In Sect. 2 we set up our notation and give the necessary preliminaries. Sect. 3 contains the calculation of the anomaly.

2. Preliminaries

In the Dirac-Kähler formalism the Dirac field is described by a general differential form

$$\Phi = {}^{0}\varphi(x) + \sum_{\mu=1}^{n} \varphi_{\mu}(x)\, dx^{\mu} + \sum_{\substack{\mu,\nu=1 \\ \mu<\nu}}^{n} \varphi_{\mu\nu}(x)\, dx^{\mu} \wedge dx^{\nu} + \cdots \tag{2.1}$$

(For details about this formalism see /4/.) We take Φ to be complex and work in n = 2 or 4 Euclidean dimensions. Using a multi-index notation we write

$$\Phi = \sum_{H} \varphi(x,H)\, dx^{H} , \tag{2.2}$$

where the sum extends over all ordered sets of indices $H = \{\mu_1, \mu_2, \cdots, \mu_h\}$ with $1 \le \mu_1 < \mu_2 < \cdots < \mu_h \le n$ and

$$dx^{H} = dx^{\mu_1} \wedge dx^{\mu_2} \wedge \cdots \wedge dx^{\mu_h} . \tag{2.3}$$

The action for a free Dirac-Kähler field reads

$$\begin{aligned} S_c &= \frac{1}{n} \int d^n x \sum_{H} \bar{\varphi}(x,H) \Big((d - \delta + m)\Phi \Big)(x,H) \\ &= \int d^n x \sum_{b=1}^{n} \bar{\Psi}^{(b)}(x) \Big(\sum_{\mu=1}^{n} \gamma_\mu \partial_\mu + m \Big) \Psi^{(b)}(x) . \end{aligned} \tag{2.4}$$

Here d is the exterior derivative, δ is the coderivative, and the connection between the components $\varphi(x,H)$, $\bar{\varphi}(x,H)$ and the Dirac spinors $\Psi, \bar{\Psi}$ is given by

$$\begin{aligned} \varphi(x,H) &= \sum_{a,b=1}^{n} \Psi_a^{(b)}(x) \left(\gamma_H^{+} \right)_{ba} , \\ \bar{\varphi}(x,H) &= \sum_{a,b=1}^{n} \bar{\Psi}_a^{(b)}(x) \left(\gamma_H^{T} \right)_{ba} . \end{aligned} \tag{2.5}$$

For the Euclidean γ-matrices we use the following conventions:

$$\{ \gamma_\mu, \gamma_\nu \} = 2\, \delta_{\mu\nu} , \tag{2.6}$$

$$\gamma_H = \gamma_{\mu_1} \gamma_{\mu_2} \cdots \gamma_{\mu_h} \quad \text{for} \quad H = \{\mu_1, \mu_2, \cdots, \mu_h\} ,\ \mu_1 < \mu_2 < \cdots < \mu_h ,\ \gamma_{\emptyset} = 1 . \tag{2.7}$$

Eq. (2.4) shows that the Dirac-Kähler field Φ represents n "flavours" of Dirac fermions in n = 2 or 4 dimensions.

We can define the 1-form

$$\sum_{\mu=1}^{n} j_{\mu}^{A}(x)\, dx^{\mu} \tag{2.8}$$

of an axial-vector current:

$$\begin{aligned} j^A &= \frac{(-1)^{n/2}}{n}\, e^{2\pi i/n} \star^{-1} (\bar{\Phi}, \varepsilon \vee \Phi)_1 \\ &= \frac{(-1)^{n/2}}{n}\, e^{-2\pi i/n} \star^{-1} (\varepsilon \vee \bar{\Phi}, \Phi)_1 \\ &= \sum_{\mu} \Big\{ i \sum_{b=1}^{n} \bar{\Psi}^{(b)}(x)\, \gamma_\mu \gamma_5\, \Psi^{(b)}(x) \Big\}\, dx^{\mu} , \end{aligned} \tag{2.9}$$

where

$$\varepsilon = dx^1 \wedge dx^2 \wedge \dots \wedge dx^n , \qquad (2.10)$$

$$\gamma_5 = -i\,\gamma_1 \gamma_2 \qquad \text{for } n = 2, \qquad (2.11)$$

$$\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \qquad \text{for } n = 4. \qquad (2.12)$$

The definitions of the Hodge star operator $\star$, the Clifford product $\vee$ and the expressions $(\ \ ,\ \ )_p$ (p = 0,1,...,n) may be found in /4/. We have chosen the phase factors in (2.9) such that the conventional axial-vector current emerges when we rewrite j^A in terms of Dirac spinors by using (2.5). If ϕ and $\bar{\phi}$ are solutions of the Dirac-Kähler equation and its adjoint, respectively,

$$(d - \delta + m)\,\phi = 0 , \qquad (2.13)$$

$$(d - \delta - m)\,\bar{\phi} = 0 , \qquad (2.14)$$

we have

$$\delta j^A = \frac{2}{n}\, e^{-2\pi i/n}\, m \star^{-1} (\bar{\phi}, \varepsilon \vee \phi)_0 , \qquad (2.15)$$

i.e.,

$$\sum_\mu \partial_\mu j^A_\mu(x) = 0 \qquad \text{for } m = 0. \qquad (2.16)$$

Now we introduce a hypercubic lattice with lattice spacing a. Let e_μ be a vector of length a in μ-direction. Then the cells of our lattice are:

lattice points $\quad x \equiv (x, \emptyset)$,

links $\quad (x, x + e_\mu) \equiv (x, \{\mu\})$,

plaquettes $\quad (x, x + e_\mu, x + e_\nu) \equiv (x, \{\mu, \nu\})$, $\mu < \nu$,

etc. A general cochain ϕ (the lattice analogue of a differential form) is

written as

$$\phi = \sum_{x,H} \varphi(x,H)\, d^{x,H} , \qquad (2.17)$$

where H has the same meaning as above and the elementary cochains $d^{x,H}$ act on the cells (x',H') according to

$$d^{x,H}((x',H')) = a^h\, \delta_{x,x'}\, \delta_{H,H'} . \qquad (2.18)$$

Here and in the following h is equal to the number of elements in H. Using the prescriptions of /4/, we get for the free action on the lattice:

$$S_0 = \frac{a^n}{n} \sum_{x,H} \bar{\varphi}(x,H) \Big((d_L - \delta_L + m)\,\phi \Big)(x,H) , \qquad (2.19)$$

where d_L and δ_L are the lattice versions of d and δ , respectively.

In order to define an axial-vector current on the lattice we write down the lattice analogues of (2.9) according to the rules in /4/. We have now

$$\varepsilon = \sum_x d^{x,\{1,2,\dots,n\}} , \qquad (2.20)$$

and one realizes that the two expressions

$$\frac{(-1)^{n/2}}{n}\, e^{2\pi i/n} \star^{-1} (\bar{\phi}, \varepsilon \vee \phi)_1 \qquad (2.21)$$

and

$$\frac{(-1)^{n/2}}{n}\, e^{-2\pi i/n} \star^{-1} (\varepsilon \vee \bar{\phi}, \phi)_1 , \qquad (2.22)$$

which are equal in the continuum, do not coincide on the lattice. Nevertheless, each of them represents a conserved current (for m = 0) in the sense that

$$\delta_L\left(\frac{(-1)^{n/2}}{n}\, e^{2\pi i/n}\, \star^{-1}(\bar\Phi, \varepsilon\vee\Phi)_1\right) \tag{2.23}$$

$$= \delta_L\left(\frac{(-1)^{n/2}}{n}\, e^{-2\pi i/n}\, \star^{-1}(\varepsilon\vee\bar\Phi, \Phi)_1\right) = 0 \quad ,$$

if Φ and $\bar\Phi$ are solutions of the free lattice Dirac-Kähler equation and its adjoint, respectively. So it seems reasonable to take

$$\frac{(-1)^{n/2}}{2n}\left(e^{2\pi i/n}\, \star^{-1}(\bar\Phi, \varepsilon\vee\Phi)_1 + e^{-2\pi i/n}\, \star^{-1}(\varepsilon\vee\bar\Phi, \Phi)_1\right)$$

$$= \frac{1}{2n}\sum_{x,\mu}\ \sum_{\substack{H\\ \mu\in H}} (-1)^{\binom{h}{2}+h+1}\, \rho_{H\setminus\{\mu\}, CH} \tag{2.24}$$

$$\cdot\Big\{e^{2\pi i/n}\Big[(-1)^{\mu}\, \bar\varphi(x+e_\mu, H\setminus\{\mu\})\, \varphi(x+e_H, CH) + \bar\varphi(x,H)\, \varphi(x+e_H, CH\cup\{\mu\})\Big]$$

$$+ e^{-2\pi i/n}\Big[(-1)^{\mu}\, \bar\varphi(x+e_H, CH)\, \varphi(x+e_\mu, H\setminus\{\mu\}) + \bar\varphi(x+e_H, CH\cup\{\mu\})\, \varphi(x,H)\Big]\Big\}\, d^{x,\{\mu\}}$$

as the axial-vector current on the lattice. Here $\sum_{\substack{H\\ \mu\in H}}$ means: sum over all ordered index sets H which contain μ. The complement CH of an ordered index set H, the difference $H\setminus\{\mu\}$ etc. are again taken to be naturally ordered. The sign function $\rho_{H,K}$ is defined for index sets H,K with $H\cap K = \emptyset$. It is equal to $(-1)^{\nu}$, where ν is the number of pairs (i,j) such that $i\in H$, $j\in K$ and $i>j$. Furthermore, $\rho_{\emptyset,H} = \rho_{H,\emptyset} = 1$. Note the particular point splitting in (2.24) and that the term with coefficient $e^{-2\pi i/n}$ looks like a "hermitian conjugate" of the term with coefficient $e^{2\pi i/n}$.

A lattice gauge field is introduced as usual. On each link $(x,\{\mu\})$ we

have a variable $U(x,\mu)$ which takes its values in the gauge group $\mathcal{G} = U(1)$ or SU(N). The Dirac-Kähler field carries an additional colour index (for $\mathcal{G} = SU(N)$) and transforms according to some unitary representation of $\mathcal{G}$. The local gauge transformations are given by

$$\begin{aligned} \varphi(x,H) &\mapsto g(x)\, \varphi(x,H) \\ \bar\varphi(x,H) &\mapsto \bar\varphi(x,H)\, g(x)^{-1} \\ U(x,\mu) &\mapsto g(x+e_\mu)\, U(x,\mu)\, g(x)^{-1} \end{aligned} \tag{2.25}$$

with $g(x)\in\mathcal{G}$. For $H = \{\mu_1,\mu_2,\dots,\mu_h\}$ $(\mu_1<\mu_2<\dots<\mu_h)$ we define

$$e_H = \sum_{\mu\in H} e_\mu \tag{2.26}$$

and

$$U(x,H) = U(x+e_{\mu_1}+e_{\mu_2}+\dots+e_{\mu_{h-1}}, \mu_h)\cdots U(x+e_{\mu_1},\mu_2)\, U(x,\mu_1)\ . \tag{2.27}$$

Then products of the form

$$\begin{aligned} &\bar\varphi(x,K)\, U^{+}(x,H)\, \varphi(x+e_H, K')\ , \\ &\bar\varphi(x+e_H, K)\, U(x,H)\, \varphi(x,K') \end{aligned} \tag{2.28}$$

are gauge invariant. When taking the continuum limit we shall write

$$U(x,\mu) = e^{i g a A_\mu(x)}\ . \tag{2.29}$$

Here g denotes the gauge coupling and $A_\mu(x)$ is a smooth continuum gauge field.

The gauge invariant action reads

$$S = S_g[U] - \sum_{\substack{x,H\\ y,K}} \bar\varphi(x,H)\, G^{-1}(x,H|y,K)\, \varphi(y,K)\ , \tag{2.30}$$

where $S_g[U]$ is the action of the pure gauge field and

$$G^{-1}(x,H|y,K) = \frac{a^n}{n}\Big\{\sum_\mu C_\mu(H,K)\frac{1}{a}\left(U(x-e_\mu,\mu)\,\delta_{y,x-e_\mu} - \delta_{y,x}\right)$$

$$+\sum_\mu C^T_\mu(H,K)\frac{1}{a}\left(\delta_{y,x} - U^+(x,\mu)\,\delta_{y,x+e_\mu}\right) - m\,\delta_{x,y}\;\delta_{H,K}\Big\}. \tag{2.31}$$

In this formula colour indices have been suppressed, and the $2^n \times 2^n$-matrices $C_\mu(H,K)$ (H = row index, K = column index) are given by

$$C_\mu(H,K) = \begin{cases} \rho_{\{\mu\},H}\;\delta_{K,H\cup\{\mu\}} & \text{if } \mu \notin H, \\ 0 & \text{otherwise}, \end{cases} \tag{2.32}$$

$\mu = 1,2,\dots,n.$

The matrices C_μ, C^T_ν satisfy the anticommutation relations

$$\{C_\mu, C_\nu\} = \{C^T_\mu, C^T_\nu\} = 0\;, \quad \{C_\mu, C^T_\nu\} = \delta_{\mu\nu} \tag{2.33}$$

and are analogous to the matrices A^+_i, A^-_j of /12/. As gauge invariant version of the axial-vector current (2.24) we take

$$j^A = \sum_{x,\mu} j^A(x,\mu)\, d^{x,\{\mu\}} \tag{2.34}$$

with

$$j^A(x,\mu) = \frac{1}{2n}\sum_{\substack{H\\ \mu\in H}} (-1)^{\binom{h}{2}+h+1}\, \rho_{H\setminus\{\mu\},CH}$$

$$\cdot\Big\{e^{2\pi i/n}\Big[(-1)^\mu\, \bar\varphi(x+e_\mu,H\setminus\{\mu\})\,U^+(x+e_\mu,H\setminus\{\mu\})\,\varphi(x+e_H,CH) \tag{2.35}$$

$$+\bar\varphi(x,H)\,U^+(x,H)\,\varphi(x+e_H,CH\cup\{\mu\})\Big]$$

$$+e^{-2\pi i/n}\Big[(-1)^\mu\,\bar\varphi(x+e_H,CH)\,U(x+e_\mu,H\setminus\{\mu\})\,\varphi(x+e_\mu,H\setminus\{\mu\})$$

$$+\bar\varphi(x+e_H,CH\cup\{\mu\})\,U(x,H)\,\varphi(x,H)\Big]\Big\}.$$

In the following we shall need the propagator G of the Dirac-Kähler field in an arbitrary external gauge field. We define

$$Z_\phi = \int D\phi\, D\bar\phi\; e^{-S}\;, \tag{2.36}$$

where $\int D\phi\, D\bar\phi$ is the usual integration over the Grassmann variables $\varphi(x,H)$, $\bar\varphi(x,H)$, and get

$$G(x,H|y,K)_{ij} = Z_\phi^{-1}\int D\phi\, D\bar\phi\; e^{-S}\;\bar\varphi_j(y,K)\,\varphi_i(x,H)\;, \tag{2.37}$$

$$i,j = \text{colour indices.}$$

In order to find a series expansion for G we split the Dirac-Kähler operator G^{-1} into a free part and an interaction part:

$$G^{-1}(x,H|y,K)_{ij} = -a^n\left[G_0^{-1}(x,H|y,K)\,\delta_{ij} + V(x,H|y,K)_{ij}\right] \tag{2.38}$$

with

$$G_0^{-1}(x,H|y,K) = \frac{1}{na}\sum_\mu\Big[C_\mu(H,K)\left(\delta_{y,x} - \delta_{y,x-e_\mu}\right)$$

$$+C^T_\mu(H,K)\left(\delta_{y,x+e_\mu} - \delta_{y,x}\right)\Big] + \frac{m}{n}\,\delta_{x,y}\;\delta_{H,K}\;, \tag{2.39}$$

$$V(x,H|y,K)_{ij} = \frac{1}{na}\sum_\mu\Big[C_\mu(H,K)\,\delta_{y,x-e_\mu}\left(\delta_{ij} - U_{ij}(y,\mu)\right) \tag{2.40}$$

$$+C^T_\mu(H,K)\,\delta_{y,x+e_\mu}\left(U^+_{ij}(x,\mu) - \delta_{ij}\right)\Big]$$

and consider $G^{-1}(x,H|y,K)_{ij}$, $V(x,H|y,K)_{ij}$ etc. as matrices with row index (i, x,H) and column index (j,y,K). The free propagator $-a^{-n}\hat G_0$ has the usual representation as a Fourier integral:

$$\hat{G}_0(x,H|y,K)_{ij} = G_0(x,H|y,K)\,\delta_{ij} \,, \tag{2.41}$$

$$G_0 = aA + a^2 M \,, \tag{2.42}$$

$$A(x,H|y,K) = \sum_\mu \left(C_\mu(H,K) f_\mu(x-y) - C_\mu^T(H,K) f_\mu(y-x) \right), \tag{2.43}$$

$$M(x,H|y,K) = m\,\delta_{H,K}\, f(x-y) \,, \tag{2.44}$$

$$f_\mu(x) = \frac{n}{(2\pi)^n} \int_{-\pi}^{\pi} d^n q \; g_0(q)\, e^{iq\cdot x/a} \left(e^{-iq_\mu} - 1\right), \tag{2.45}$$

$$f(x) = \frac{n}{(2\pi)^n} \int_{-\pi}^{\pi} d^n q \; g_0(q)\, e^{iq\cdot x/a} \,, \tag{2.46}$$

$$g_0(q) = \left(4 \sum_\mu \sin^2\left(\tfrac{1}{2} q_\mu\right) + a^2 m^2 \right)^{-1} . \tag{2.47}$$

Now we can expand G in a series:

$$G = -a^{-n}\, \hat{G}_0 \sum_{r=0}^{\infty} (-1)^r \left(V \hat{G}_0\right)^r . \tag{2.48}$$

Applying (2.29) we have for $a \to 0$:

$$\hat{G}_0 \left(V \hat{G}_0\right)^r = \begin{cases} O(a^{r+1}) & \text{if } r < n-1, \\ O(a^n \ln a) & \text{if } r = n-1, \\ O(a^n) & \text{if } r > n-1. \end{cases} \tag{2.49}$$

Eq. (2.48) is essentially a short-distance expansion of G. Since the axial anomaly is a quantum effect induced by the singularities of the propagator, only the first few terms of (2.48) will contribute to the final result.

Later on it will be useful to write

$$V = V^0 + aV^1 + O(a^2) \tag{2.50}$$

with

$$V^0(x,H|y,K) = -\frac{ig}{n} \sum_\mu \Big[C_\mu(H,K)\, \delta_{y,x-e_\mu} A_\mu(y) + C_\mu^T(H,K)\, \delta_{y,x+e_\mu} A_\mu(x) \Big] , \tag{2.51}$$

$$V^1(x,H|y,K) = \frac{g^2}{2n} \sum_\mu \Big[C_\mu(H,K)\, \delta_{y,x-e_\mu} A_\mu(y)^2 - C_\mu^T(H,K)\, \delta_{y,x+e_\mu} A_\mu(x)^2 \Big] . \tag{2.52}$$

Note the following symmetry properties:

$$\begin{aligned} A(x,H|y,K) &= -A(y,K|x,H) \,, \\ M(x,H|y,K) &= M(y,K|x,H) \,, \\ V^0(x,H|y,K)_{ij} &= V^0(y,K|x,H)_{ij} \,, \\ V^1(x,H|y,K)_{ij} &= -V^1(y,K|x,H)_{ij} \,. \end{aligned} \tag{2.53}$$

3. Calculation of the anomaly of the axial-vector current

We start from the equations of motion on the lattice, which follow from the action (2.30):

$$\sum_{\mu\in H} \rho_{\{\mu\},H\setminus\{\mu\}} \frac{1}{a}\Big(U^+(x,\mu)\,\varphi(x+e_\mu,H\setminus\{\mu\}) - \varphi(x,H\setminus\{\mu\})\Big)$$

$$+\sum_{\mu\in CH} \rho_{\{\mu\},H} \frac{1}{a}\Big(\varphi(x,H\cup\{\mu\}) - U(x-e_\mu,\mu)\,\varphi(x-e_\mu,H\cup\{\mu\})\Big) + m\,\varphi(x,H) = 0 \,, \tag{3.1}$$

$$\sum_{\mu\in H} \rho_{\{\mu\},H\setminus\{\mu\}} \frac{1}{a}\Big(\bar\varphi(x+e_\mu,H\setminus\{\mu\})\,U(x,\mu) - \bar\varphi(x,H\setminus\{\mu\})\Big)$$

$$+\sum_{\mu\in CH} \rho_{\{\mu\},H} \frac{1}{a}\Big(\bar\varphi(x,H\cup\{\mu\}) - \bar\varphi(x-e_\mu,H\cup\{\mu\})\,U^+(x-e_\mu,\mu)\Big) - m\,\bar\varphi(x,H) = 0 \,. \tag{3.2}$$

Using (3.1) and (3.2) one gets for the lattice divergence of the axial current:

$$\sum_\mu \frac{1}{a}\big(j^A(x,\mu) - j^A(x-e_\mu,\mu)\big)$$

$$= m\,\frac{(-1)^{n/2}}{n} \sum_H (-1)^{\binom{h}{2}} \rho_{CH,H}\Big[e^{2\pi i/n}\,\bar\varphi(x+e_H,CH)\,U(x,H)\,\varphi(x,H) \tag{3.3}$$

$$-e^{-2\pi i/n}\,\bar\varphi(x,H)\,U^+(x,H)\,\varphi(x+e_H,CH)\Big] + B(x) \,.$$

We shall show that the vacuum expectation value of B(x) will give the anomaly in the continuum limit. The explicit expression for this B-term reads

$$B(x) = \frac{(-1)^{n/2}}{2na} \sum_\mu \sum_{\substack{H\\ \mu\in H}} (-1)^{\binom{h}{2}+h} \rho_{H\setminus\{\mu\},CH}$$

$$\cdot\Big\{e^{-2\pi i/n}\Big[(-1)^\mu\,\bar\varphi(x+e_\mu,H\setminus\{\mu\})\,W_1(x,\mu,H)\,\varphi(x+e_H,CH)$$

$$+(-1)^\mu\,\bar\varphi(x,H\setminus\{\mu\})\,W_2(x,\mu,H)\,\varphi(x+e_H,CH)$$

$$-\bar\varphi(x+e_{CH},H)\,W_3(x,\mu,H)\,\varphi(x,CH\cup\{\mu\}) \tag{3.4}$$

$$-\bar\varphi(x+e_{CH},H)\,W_4(x,\mu,H)\,\varphi(x-e_\mu,CH\cup\{\mu\})\Big]$$

$$+e^{2\pi i/n}\Big[(-1)^\mu\,\bar\varphi(x+e_H,CH)\,W_1^+(x,\mu,H)\,\varphi(x+e_\mu,H\setminus\{\mu\})$$

$$+(-1)^\mu\,\bar\varphi(x+e_H,CH)\,W_2^+(x,\mu,H)\,\varphi(x,H\setminus\{\mu\})$$

$$-\bar\varphi(x,CH\cup\{\mu\})\,W_3^+(x,\mu,H)\,\varphi(x+e_{CH},H)$$

$$-\bar\varphi(x-e_\mu,CH\cup\{\mu\})\,W_4^+(x,\mu,H)\,\varphi(x+e_{CH},H)\Big]\Big\}$$

with

$$W_1(x,\mu,H) = U^+(x+e_\mu,H\setminus\{\mu\}) - U(x,\mu)\,U^+(x,H) \,,$$

$$W_2(x,\mu,H) = U^+(x,H) - U^+(x,H\setminus\{\mu\})\,U^+(x+e_{H\setminus\{\mu\}},\mu) \,, \tag{3.5}$$

$$W_3(x,\mu,H) = U(x,CH) - U^+(x+e_{CH},\mu)\,U(x,CH\cup\{\mu\}) \,,$$

$$W_4(x,\mu,H) = U(x-e_\mu,CH\cup\{\mu\}) - U(x,CH)\,U(x-e_\mu,\mu) \,.$$

Three points should be mentioned at this place. First, because

$$\delta_L\Big(\sum_{x,\mu} j(x,\mu)\,d^{x,\{\mu\}}\Big) = -\frac{1}{a}\sum_{x,\mu}\big(j(x,\mu) - j(x-e_\mu,\mu)\big)\,d^{x,\emptyset} \,, \tag{3.6}$$

the expression

$$\frac{1}{a}\sum_\mu \big(j(x,\mu) - j(x-e_\mu,\mu)\big) \tag{3.7}$$

is the natural (from the geometrical point of view) lattice analogue of the divergence of a current in the continuum. Secondly, in the classical continuum limit

$$m \frac{(-1)^{n/2}}{n} \sum_{H} (-1)^{\binom{h}{2}} \varrho_{CH,H} \Big[e^{2\pi i/n} \bar{\varphi}(x+e_H, CH) U(x,H) \varphi(x,H) - e^{-2\pi i/n} \bar{\varphi}(x,H) U^+(x,H) \varphi(x+e_H, CH) \Big] \tag{3.8}$$

$$\rightarrow 2mi \sum_{b=1}^{n} \bar{\Psi}^{(b)}(x) \gamma_5 \Psi^{(b)}(x) .$$

Thirdly, in the free theory we have $U(x,\mu) = 1$ **and consequently** B(x) = 0. **If in addition** m = 0, **Eq.** (3.3) **shows that our axial-vector current is conserved on the lattice.**

If one replaces the products of fermion fields in the B-term by the propagator in the external gauge field, the resulting expression should tend to the anomaly as a → 0. **Therefore we study**

$$Z_\phi^{-1} \int D\phi D\bar{\phi}\, e^{-S} B(x) = \frac{(-1)^{n/2}}{2na} \sum_{\mu} \sum_{\substack{H \\ \mu \in H}} (-1)^{\binom{h}{2}+h} \varrho_{H\setminus\{\mu\}, CH}$$
$$\cdot \Big\{ (-1)^{\mu} tr_c \Big[e^{-2\pi i/n} W_1(x,\mu,H) G(x+e_H, CH | x+e_\mu, H\setminus\{\mu\}) + e^{2\pi i/n} W_1^+(x,\mu,H) G(x+e_\mu, H\setminus\{\mu\} | x+e_H, CH) \Big]$$
$$+ (-1)^{\mu} tr_c \Big[e^{-2\pi i/n} W_2(x,\mu,H) G(x+e_H, CH | x, H\setminus\{\mu\}) + e^{2\pi i/n} W_2^+(x,\mu,H) G(x, H\setminus\{\mu\} | x+e_H, CH) \Big] \tag{3.9}$$
$$- tr_c \Big[e^{-2\pi i/n} W_3(x,\mu,H) G(x, CH\cup\{\mu\} | x+e_{CH}, H) + e^{2\pi i/n} W_3^+(x,\mu,H) G(x+e_{CH}, H | x, CH\cup\{\mu\}) \Big]$$
$$- tr_c \Big[e^{-2\pi i/n} W_4(x,\mu,H) G(x-e_\mu, CH\cup\{\mu\} | x+e_{CH}, H) + e^{2\pi i/n} W_4^+(x,\mu,H) G(x+e_{CH}, H | x-e_\mu, CH\cup\{\mu\}) \Big] \Big\}$$

for a → 0. Here tr_c means trace over colour indices. Because

$$W_1(x,\mu,H) = -iga^2 \sum_{\substack{\lambda \in H \\ \lambda < \mu}} F_{\mu\lambda}(x) + O(a^3) ,$$
$$W_2(x,\mu,H) = -iga^2 \sum_{\substack{\lambda \in H \\ \lambda > \mu}} F_{\mu\lambda}(x) + O(a^3) ,$$
$$W_3(x,\mu,H) = -iga^2 \sum_{\substack{\lambda \in CH \\ \lambda > \mu}} F_{\mu\lambda}(x) + O(a^3) , \tag{3.10}$$
$$W_4(x,\mu,H) = -iga^2 \sum_{\substack{\lambda \in CH \\ \lambda < \mu}} F_{\mu\lambda}(x) + O(a^3)$$

with

$$F_{\mu\lambda}(x) = \partial_\mu A_\lambda(x) - \partial_\lambda A_\mu(x) - ig\left[A_\mu(x), A_\lambda(x)\right] , \tag{3.11}$$

we need G up to terms of order a^{-1}.

Consequently, in the case n = 2 with gauge group U(1) only the first term of (2.48) contributes to the anomaly. So we can replace G in (3.9) by $-a^{-1}A$. ($M(x,H|y,K) = 0$ for the combinations (H,K) occurring in (3.9).) Inserting the explicit expression (2.43) we find:

$$Z_\phi^{-1} \int D\phi D\bar{\phi}\, e^{-S} B(x) = ig\big(f_1(e_1) + f_1(e_{12})\big) F_{12}(x) + O(a \ln a) . \tag{3.12}$$

The integral $f_1(e_1) + f_1(e_{12})$ can be calculated in the limit a → 0 (see /11/):

$$\lim_{a\to 0} \big(f_1(e_1) + f_1(e_{12})\big) = \lim_{a\to 0} \frac{2}{\pi^2} \int_{-\pi}^{\pi} d^2q\, g_0(q) \sin^2(\tfrac{1}{2}q_1) \cos^2(\tfrac{1}{2}q_2)$$
$$= \frac{2}{\pi^2} \int_{-\pi/2}^{\pi/2} d^2q \frac{\sin^2 q_1 \cos^2 q_2}{\sum_\mu \sin^2 q_\mu} = \frac{2}{\pi} . \tag{3.13}$$

So we obtain the result

$$\lim_{a\to 0} Z_\Phi^{-1} \int D\Phi\, D\bar\Phi\, e^{-S}\, B(x) = \frac{ig}{\pi} \sum_{\mu,\nu} \varepsilon_{\mu\nu} F_{\mu\nu}(x) \tag{3.14}$$

($\varepsilon_{12} = 1$), which is the correct anomaly for our model with two flavours.

Evaluating the vacuum expectation value of the B-term in $n = 4$ dimensions we have to take into account the terms with $r = 0,1,2$ in the expansion (2.48) for G. A typical contribution in (3.9) is of the form

$$\frac{1}{a} \operatorname{tr}_c \Big(W\, G(x,H|y,K) - W^+ G(y,K|x,H) \Big) , \tag{3.15}$$

where W is a matrix in colour space, which is of order a^2 and antihermitian in this order. It is easy to see that

$$C_\mu(H,K) = C_\mu^T(H,K) = \delta_{H,K} = 0 \tag{3.16}$$

for all combinations (H,K) which occur in (3.9). So G_o does not contribute. Therefore one can write by means of (2.53):

$$\begin{aligned} &\frac{1}{a} \operatorname{tr}_c \Big(W\, G(x,H|y,K) - W^+ G(y,K|x,H) \Big) \\ &= a^{-3} \sum_{i,j} \Big\{ (W - W^+)_{ji}\ (A V^o_{ij} A)(x,H|y,K) \\ &\quad + a W_{ji} \sum_k \Big[(A V^o_{kj} A V^o_{ik} A)(x,H|y,K) - (A V^o_{ik} A V^o_{kj} A)(x,H|y,K) \Big] \Big\} \\ &\quad + O(a \ln a) \end{aligned} \tag{3.17}$$

for all (H,K) which appear in (3.9). With (3.17) we get the following contributions to the a $\to$ 0 limit of the vacuum expectation value of the B-term:

$$Z_\Phi^{-1} \int D\Phi\, D\bar\Phi\, e^{-S}\, B(x) = B_1(x) + B_2(x) + O(a \ln a) \quad . \tag{3.18}$$

Here

$$\begin{aligned} B_1(x) = \frac{i}{8a^3} \sum_\mu \sum_{\substack{H \\ \mu\in H}} (-1)^{\binom{k}{2}+k}\, \varrho_{H\setminus\{\mu\},CH} \\ \cdot \sum_{i,j} \Big\{ &(-1)^{\mu+1}\, \hat W_1(x,\mu,H)_{ji}\ (A V^o_{ij} A)(x+e_H, CH|x+e_\mu, H\setminus\{\mu\}) \\ &+ (-1)^{\mu+1}\, \hat W_2(x,\mu,H)_{ji}\ (A V^o_{ij} A)(x+e_H, CH|x, H\setminus\{\mu\}) \\ &+ \hat W_3(x,\mu,H)_{ji}\ (A V^o_{ij} A)(x+e_{CH}, H|x, CH\cup\{\mu\}) \\ &+ \hat W_4(x,\mu,H)_{ji}\ (A V^o_{ij} A)(x+e_{CH}, H|x-e_\mu, CH\cup\{\mu\}) \Big\} , \end{aligned} \tag{3.19}$$

$$\begin{aligned} B_2(x) = \frac{i}{8a^2} \sum_\mu \sum_{\substack{H \\ \mu\in H}} (-1)^{\binom{k}{2}+k}\, \varrho_{H\setminus\{\mu\},CH} \\ \cdot \sum_{i,j,k} \Big\{ &(-1)^{\mu+1} W_1(x,\mu,H)_{ji} \big(A V^o_{kj} A V^o_{ik} A - A V^o_{ik} A V^o_{kj} A\big)(x+e_H, CH|x+e_\mu, H\setminus\{\mu\}) \\ &+ (-1)^{\mu+1} W_2(x,\mu,H)_{ji} \big(A V^o_{kj} A V^o_{ik} A - A V^o_{ik} A V^o_{kj} A\big)(x+e_H, CH|x, H\setminus\{\mu\}) \\ &+ W_3(x,\mu,H)_{ji} \big(A V^o_{kj} A V^o_{ik} A - A V^o_{ik} A V^o_{kj} A\big)(x+e_{CH}, H|x, CH\cup\{\mu\}) \\ &+ W_4(x,\mu,H)_{ji} \big(A V^o_{kj} A V^o_{ik} A - A V^o_{ik} A V^o_{kj} A\big)(x+e_{CH}, H|x-e_\mu, CH\cup\{\mu\}) \Big\} \end{aligned} \tag{3.20}$$

and

$$\hat W_\alpha(x,\mu,H) = W_\alpha(x,\mu,H) - W_\alpha^+(x,\mu,H) , \quad \alpha = 1,2,3,4 . \tag{3.21}$$

So we have to calculate products of A and V^o. We find:

$$(AV^oA)(x,H|y,K) = \sum_{\substack{z,z'\\L,L'}} A(x,H|z,L)\,V^o(z,L|z',L')\,A(z',L'|y,K)$$

$$= \frac{ig}{2^6\pi^8}\int_{-\pi/a}^{\pi/a} d^4p\; e^{ip\cdot(x+y)/2} \sum_{\sigma,\tau,\nu} \tilde{A}_\nu(p)$$

$$\cdot\Big\{ e^{-ia(p_\sigma+p_\nu)/2} K_{\sigma\tau}(p, e_\sigma+e_\nu-x+y)\,(C_\sigma C_\nu C_\tau)(H,K)$$

$$+ e^{-ia(p_\sigma+p_\tau+p_\nu)/2} K_{\sigma\tau}(p, e_\sigma+e_\nu-e_\tau-x+y)\,(C_\sigma C_\nu C_\tau^T)(H,K)$$

$$+ e^{-ia(p_\sigma+p_\nu)/2} K_{\sigma\tau}(p, e_\sigma-e_\nu-x+y)\,(C_\sigma C_\nu^T C_\tau)(H,K)$$

$$+ e^{-ia(p_\sigma+p_\tau+p_\nu)/2} K_{\sigma\tau}(p, e_\sigma-e_\nu-e_\tau-x+y)\,(C_\sigma C_\nu^T C_\tau^T)(H,K) \tag{3.22}$$

$$+ e^{-iap_\nu/2} K_{\sigma\tau}(p, e_\nu-x+y)\,(C_\sigma^T C_\nu C_\tau)(H,K)$$

$$+ e^{-ia(p_\tau+p_\nu)/2} K_{\sigma\tau}(p, e_\nu-e_\tau-x+y)\,(C_\sigma^T C_\nu C_\tau^T)(H,K)$$

$$+ e^{-iap_\nu/2} K_{\sigma\tau}(p, -e_\nu-x+y)\,(C_\sigma^T C_\nu^T C_\tau)(H,K)$$

$$+ e^{-ia(p_\tau+p_\nu)/2} K_{\sigma\tau}(p, -e_\nu-e_\tau-x+y)\,(C_\sigma^T C_\nu^T C_\tau^T)(H,K)\Big\}.$$

Here we have introduced the Fourier transform of $A_\nu(x)$:

$$\tilde{A}_\nu(p) = a^4 \sum_x e^{-ip\cdot x} A_\nu(x) \quad . \tag{3.23}$$

Furthermore, we have used:

$$\sum_z A_\nu(z) f_\sigma(z+x) f_\tau(z+y) = \frac{1}{16\pi^4}\int_{-\pi/a}^{\pi/a} d^4p\; \tilde{A}_\nu(p)\, e^{-ip\cdot(x+y)/2}\, K_{\sigma\tau}(p, x-y) \tag{3.24}$$

with

$$K_{\sigma\tau}(p,x)$$

$$= \int_{-\pi}^{\pi} d^4q\; g_o(q-\tfrac{a}{2}p)\, g_o(q+\tfrac{a}{2}p)\, e^{iq\cdot x/a} \left(1-e^{-iq_\sigma+iap_\sigma/2}\right)\left(1-e^{iq_\tau+iap_\tau/2}\right). \tag{3.25}$$

By means of (2.32) the products of the matrices C_μ and C_ν^T are evaluated, and the resulting expression for AV^oA is inserted in (3.19). Next we expand the exponentials and the functions $K_{\sigma\tau}$ up to first order in a, keeping p fixed and taking into account that $|e_\mu| = a$. All terms that might give contributions of order a^{-1} in B_1 cancel, and we see that $\hat{W}_\alpha(x,\mu,H)$ is needed only in lowest order. So we use (3.10) and get finally ($\varepsilon_{1234} = 1$):

$$B_1(x)$$

$$= -i\frac{g^2}{2\pi^4}(I_1-3I_2+3I_3-I_4)\sum_{\mu,\nu,\sigma,\tau} \varepsilon_{\mu\nu\sigma\tau}\, tr_c\big(\partial_\tau A_\nu(x) - \partial_\nu A_\tau(x)\big) F_{\mu\sigma}(x)$$

$$-i\frac{g^2}{\pi^4}(I_1-2I_2+I_3)\sum_{\mu,\nu,\sigma,\tau} \varepsilon_{\mu\nu\sigma\tau}\, tr_c\big(\partial_\tau A_\nu(x)\big)\big(F_{\mu\tau}(x) + F_{\mu\nu}(x)\big) \tag{3.26}$$

$$+ O(a \ln a)$$

with

$$I_j = \int_{-\pi/2}^{\pi/2} d^4q\; \frac{\sin^2 q_1 \sin^2 q_2 \cdots \sin^2 q_j}{\left(\sum_\mu \sin^2 q_\mu\right)^2} \quad , \quad j = 1,2,3,4 \; . \tag{3.27}$$

The evaluation of B_2 proceeds along similar lines. With the help of the anticommutation relations (2.33) and Eq. (3.16) we find for combinations (H,K) actually occurring in B_2:

$$\sum_k \left(AV^\circ_{kj} A V^\circ_{ik} A - AV^\circ_{ik} A V^\circ_{kj} A\right)(x,H|y,K)$$

$$= \frac{g^2}{4\pi^4} \sum_{\mu,\lambda,\tau} \left[A_\mu(x), A_\lambda(y)\right]_{ij}$$

$$\times\Big\{\mathcal{N}_\tau(x-y-e_\mu-e_\lambda)(C_\tau C_\mu C_\lambda)(H,K) + \mathcal{N}_\tau(x-y-e_\mu+e_\lambda)(C_\tau C_\mu C^T_\lambda)(H,K) \tag{3.28}$$

$$+\mathcal{N}_\tau(x-y+e_\mu-e_\lambda)(C_\lambda C_\tau C^T_\mu)(H,K) + \mathcal{N}_\tau(x-y+e_\mu+e_\lambda)(C_\tau C^T_\mu C^T_\lambda)(H,K)$$

$$+\mathcal{N}_\tau(x-y+e_\tau-e_\mu-e_\lambda)(C_\mu C_\lambda C^T_\tau)(H,K) + \mathcal{N}_\tau(x-y+e_\tau-e_\mu+e_\lambda)(C_\mu C^T_\lambda C^T_\tau)(H,K)$$

$$+\mathcal{N}_\tau(x-y+e_\tau+e_\mu-e_\lambda)(C_\lambda C^T_\tau C^T_\mu)(H,K) + \mathcal{N}_\tau(x-y+e_\tau+e_\mu+e_\lambda)(C^T_\mu C^T_\lambda C^T_\tau)(H,K)\Big\}$$

$$+ O(a \ln a),$$

where

$$\mathcal{N}_\tau(x) = \int_{-\pi}^{\pi} d^4q \left(4\sum_\mu \sin^2(\tfrac{1}{2}q_\mu)\right)^{-2} e^{iq\cdot x/a}\left(e^{-iq_\tau}-1\right) . \tag{3.29}$$

Using again (3.10) we obtain

$$B_2(x) = \frac{g^3}{2\pi^4} \sum_{\mu,\nu,\sigma,\tau} \varepsilon_{\mu\nu\sigma\tau}\, tr_c \left[A_\nu(x), A_\tau(x)\right]$$

$$\times\left\{(I_1-3I_2+3I_3-I_4)\, F_{\mu\sigma}(x) + (I_1-2I_2+I_3)\left(F_{\mu\tau}(x)+F_{\mu\nu}(x)\right)\right\} \tag{3.30}$$

$$+ O(a \ln a) .$$

Combining (3.26) and (3.30) one gets

$$Z_\Phi^{-1} \int D\Phi\, D\bar\Phi\, e^{-S}\, B(x)$$

$$= -i\frac{g^2}{2\pi^4}(I_1-3I_2+3I_3-I_4)\sum_{\mu,\nu,\sigma,\tau} \varepsilon_{\mu\nu\sigma\tau}\, tr_c\, F_{\mu\nu}(x)\, F_{\sigma\tau}(x) \tag{3.31}$$

$$+ O(a \ln a).$$

Because

$$(2\pi)^{-4}(I_1-3I_2+3I_3-I_4) = (2\pi)^{-4}\int_{-\pi/2}^{\pi/2} d^4q\, \frac{\sin^2 q_1 \cos^2 q_2 \cos^2 q_3 \cos^2 q_4}{\left(\sum_\mu \sin^2 q_\mu\right)^2} \tag{3.32}$$

is equal to the integral C_4, which was shown by Sharatchandra et al. /11/ to have the value $(32\,\pi^2)^{-1}$, we arrive at the final result for the vacuum expectation value of the B-term in the continuum limit:

$$\lim_{a\to 0} Z_\Phi^{-1}\int D\Phi\, D\bar\Phi\, e^{-S}\, B(x) = -i\frac{g^2}{4\pi^2}\sum_{\mu,\nu,\sigma,\tau}\varepsilon_{\mu\nu\sigma\tau}\, tr_c\, F_{\mu\nu}(x)\, F_{\sigma\tau}(x) . \tag{3.33}$$

This is the correct anomaly of the axial-vector current for four flavours.

We want to make some remarks. First, we have checked that the anomaly in the continuum limit does not depend on the ordering of the indices $\mu_i \in H$ in (2.27). Instead of the natural order, which we have chosen in (2.27) for all H, one could have taken an arbitrary order for each index set H separately without affecting the result. Secondly, the series (2.48) might, of course, diverge. But actually we need only the first few terms explicitly, and with respect to the rest it suffices to know certain regularity properties. It should be possible to establish these without relying on an expansion like (2.48).

We close with remarking that there exists an alternative method for the calculation of the axial anomaly due to Sharatchandra et al. /11/ (see also /10/). Applying this procedure one performs an infinitesimal local chiral transformation of the fermion field in the integral

$$\int D\phi D\bar{\phi}\, e^{-S}\, \Theta \ , \tag{3.34}$$

where Θ is an arbitrary observable. In this way one obtains a kind of anomalous Ward-Takahashi identity on the lattice. As a → 0, one should recover from this identity the divergence of the axial-vector current in the continuum including the anomaly. So one has to find a suitable chiral transformation. In the continuum,

$$-e^{-2\pi i/n}\, \varepsilon \vee \phi = -e^{-2\pi i/n} \sum_{H} (-1)^{\binom{h}{2}+n/2} \rho_{CH,H}\, \varphi(x,CH)\, dx^{H} \tag{3.35}$$

corresponds to $i\gamma_5 \psi^{(W)}(x)$, and $e^{2\pi i/n}\, \varepsilon \vee \bar{\phi}$ is equivalent to $i\bar{\psi}^{(W)}(x)\gamma_5$, if one uses the correspondence (2.5). Since on the lattice

$$\varepsilon \vee \phi = \sum_{x,H} (-1)^{\binom{h}{2}+n/2} \rho_{CH,H}\, \varphi(x+e_H, CH)\, d^{x,H} \ , \tag{3.36}$$

one takes as an infinitesimal local chiral transformation that preserves gauge invariance:

$$\begin{aligned} \delta\phi &= -e^{-2\pi i/n} \sum_{x,H} (-1)^{\binom{h}{2}+n/2} \rho_{CH,H}\, U^{+}(x,H)\, \varphi(x+e_H, CH)\, d^{x,H}\, \delta\alpha(x) \ , \\ \delta\bar{\phi} &= e^{2\pi i/n} \sum_{x,H} (-1)^{\binom{h}{2}+n/2} \rho_{CH,H}\, \bar{\varphi}(x+e_H, CH)\, U(x,H)\, d^{x,H}\, \delta\alpha(x) \ . \end{aligned} \tag{3.37}$$

One should notice the characteristic point splitting in (3.37). Now it is easy to derive the above mentioned Ward-Takahashi identity. But it turns out that it

requires much more work to evaluate the anomaly by taking the continuum limit of this identity than to perform the calculations outlined in the main part of this section. Therefore we have used this second method only for the U(1) theory in two dimensions. In that case we found the correct continuum anomaly as above.

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References

/1/ K.G. Wilson, in "New phenomena in subnuclear physics", ed. A. Zichichi (Plenum, New York, 1977) (Erice, 1975)

/2/ L. Susskind, Phys. Rev. D16 (1977) 3031

/3/ S.D. Drell, M. Weinstein and S. Yankielowicz, Phys. Rev. D14 (1976) 487, 1627

/4/ P. Becher and H. Joos, Z. Phys. C - Particles and Fields 15 (1982) 343

/5/ J.M. Rabin, Nucl. Phys. B201 (1982) 315

/6/ M. Göckeler, Dirac-Kähler fields and the lattice shape dependence of fermion flavour, DESY preprint 82-082 (December 1982)

/7/ S.L. Adler, Phys. Rev. 177 (1969) 2426

/8/ J.S. Bell and R. Jackiw, Nuovo Cim. 60A (1969) 47

/9/ L.H. Karsten and J. Smit, Nucl. Phys. B183 (1981) 103

/10/ W. Kerler, Phys. Rev. D23 (1981) 2384

/11/ H.S. Sharatchandra, H.J. Thun and P. Weisz, Nucl. Phys. B192 (1981) 205

/12/ P. Becher, Phys. Lett. 104B (1981) 221