DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 83-008
February 1983

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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

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QCD Self-Energy Contribution to the Mass-Shifts in the ^3P_J-States

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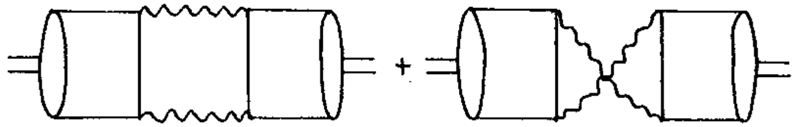
Abstract

We calculate the self-energy contribution to the mass shift of the ^3P_J-states via the two gluon intermediate state. The effect consists of an overall shift for all states, which depends on the potential energy, and individual displacements. The corrections are typically of the order of a few MeV.

Nonrelativistic potential models together with QCD describe in a reasonable way the masses and various decay modes of Quarkonium states /1/. The hyperfine $^3S_1-{}^1S_o$ splitting and the splitting of the $^3P_{J=0,1,2}$ states arise from spin-spin, spin-orbit and tensor forces of the Breit-Fermi-Hamiltonian. They are of the order of 50 - 100 MeV.

In the next step contributions of order α_s^2 have to be included. Among them one particular contribution arises from the self-interaction of the various C = + states via a two gluon exchange. We calculate this contribution for the ^3P_J, $J^{PC} = 0^{++}$, 1^{++}, 2^{++} states which, in the case of the P_c/χ states of Charmonium is of the order of 5-10 MeV for the experimentally measured decay-width of 16.3 $\pm$ 3.6 MeV for χ_o /2/.

The self-energy contribution is given by the two following diagrams

 (1)

The coupling of two gluons to the various ^3P_J states can be obtained by the bound state formalism as described e.g. in ref. /3/

$$A^J_{\mu_1\mu_2}\,\varepsilon_1^{\mu_1}\varepsilon_2^{\mu_2} = 8\sqrt{2}\sqrt{\frac{3}{4\pi M^3}}\;R_P'(0)\left(\frac{1}{k_1\cdot k_2 - i\varepsilon}\right)^2 \; a_J$$

$$a_o = \frac{1}{\sqrt{6}}\Big\{\left[\varepsilon_1\cdot\varepsilon_2\, k_1\cdot k_2 - \varepsilon_1\cdot k_2\,\varepsilon_2\cdot k_1\right]\left[M^2 + k_1\cdot k_2\right]$$
$$+\varepsilon_1\cdot k_2\,\varepsilon_2\cdot k_2\,k_1^2 + \varepsilon_1\cdot k_1\,\varepsilon_2\cdot k_1\,k_2^2 - \varepsilon_1\cdot\varepsilon_2\,k_1^2k_2^2 - \varepsilon_1\cdot k_1\,\varepsilon_2\cdot k_2\,k_1\cdot k_2\Big\} \quad (2)$$

$$a_1 = \frac{M}{2}\Big\{\left[k_1^2\,\epsilon(e^*,\varepsilon_1,\varepsilon_2,k_2) + \varepsilon_1\cdot k_1\,\epsilon(e^*,\varepsilon_2,k_1,k_2)\right] + \left[1\leftrightarrow 2\right]\Big\}$$

$$a_2 = \frac{M^2}{\sqrt{2}}\Big\{k_1\cdot k_2\,\varepsilon_1^a\varepsilon_2^b + k_2^a k_1^b\,\varepsilon_1\cdot\varepsilon_2 - k_1^a\varepsilon_2^b\,\varepsilon_1\cdot k_2 - k_2^a\varepsilon_1^b\,\varepsilon_2\cdot k_1\Big\}\,e^{*ab}$$

Here $k_{1,2}$, $\varepsilon_{1,2}$ denote the momenta and polarization vectors of the two gluons. $e_a(e_{ab})$ stands for the polarization vector (tensor) of the spin 1 (2) bound state of momentum $P = k_1 + k_2$ and mass M and $R_p'(o)$ is the derivative of the radial P-wave function at the origin. In this formula we neglected the binding energy in the term $(k_1k_2 - i\varepsilon)^{-2}$. This approximation has to be removed at a later step in the calculation.

Combining (1) and (2) the complex mass-shift is given by:

$$\delta M_J^2 = 2M_J\left(\Delta M_J - \frac{i}{2}\Gamma_J\right)$$
$$= \frac{1}{2J+1}\left(\frac{2}{3}\right)(4\pi\alpha_s)^2\left(8\sqrt{2}\sqrt{\frac{3}{4\pi M^3}}\,R_p'(0)\right)^2\frac{1}{2}i\cdot \qquad (3)$$
$$\cdot\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k_1^2+i\varepsilon)(k_2^2+i\varepsilon)(k_1k_2-i\varepsilon)^4}\sum_{polarisations}(a_J^*a_J)$$

$$k_{1,2} = \frac{1}{2}P \pm k$$

When evaluating (3) it proved convenient to use the Cauchy integral formula for the k_o integration. Since the dependence on the angular variables is trivial in this case, one is then left with just one integration over $|\vec{k}| \equiv k$:

$$\int dk_o\,k^2dk\,d\Omega\,\frac{F(k_o,k)}{(k_1^2+i\varepsilon)(k_2^2+i\varepsilon)(k_1k_2-i\varepsilon)^4}$$
$$= 8\pi^2 i\int k^2dk\left\{-\frac{F(k_o=-\frac{1}{2}M-k,k)}{4M^5k^5(k+\frac{1}{2}M)}+\frac{F(k_o=\frac{1}{2}M-k,k)}{4M^5k^5}\left[P\left(\frac{1}{k-\frac{1}{2}M}\right)+i\pi\delta(k-\tfrac{1}{2}M)\right]\right. \qquad (4)$$
$$\left.+\frac{1}{3!}\frac{d^3}{dk_o^3}\left[\frac{F(k_o,k)}{[(k_o^2-k^2+\frac{1}{4}M^2)^2-M^2k_o^2][k_o-\sqrt{k^2+\frac{1}{4}M^2}]^4}\right]_{k_o=-\sqrt{k^2+\frac{1}{4}M^2}}\right\}$$

In our case the integrals coming from the first two terms diverge logarithmically for $k \to 0$. We introduce a cut-off to make them finite. Physically this

cut-off is determined by the binding energy of $Q\bar{Q}$, which was neglected so far. The term $k_1k_2 - i\varepsilon$ actually reads $k_1k_2 + (m_Q^2 - \frac{1}{4}M^2 - \langle q^2\rangle) - i\varepsilon$ with m_Q being the effective quark mass and 2q the relative momentum between Q and $\bar{Q}$. In a nonrelativistic approximation one has

$$M = 2m_Q + E_{kin} + E_{pot}$$
$$\langle q^2\rangle = -\langle\vec{q}^{\,2}\rangle = -m_Q E_{kin} \qquad (5)$$

The term neglected in $k_1k_2 - i\varepsilon$ is thus given by

$$m_Q^2 - \frac{1}{4}M^2 - \langle q^2\rangle \simeq -\frac{1}{2}M E_{pot} \qquad (6)$$

which leads to a cut-off of order $|E_{pot}|$.

Our result for the complex mass shift is

$$\delta M_J = \left(\Delta M - \frac{i}{2}\Gamma\right)_J = \frac{8}{3}\frac{\alpha_s^2}{\pi}(R_p'(0))^2\frac{1}{M^4}\cdot$$
$$\cdot\begin{Bmatrix} 8\ln\left(\frac{M^2}{4E_{pot}^2}\right) + 36\ln(2) + \frac{58}{3} - 18i\pi \\ 8\ln\left(\frac{M^2}{4E_{pot}^2}\right) + \frac{52}{3} - 0 \\ 8\ln\left(\frac{M^2}{4E_{pot}^2}\right) + \frac{48}{5}\ln(2) + \frac{128}{15} - \frac{24}{5}i\pi \end{Bmatrix} \text{ for } \begin{Bmatrix} J=0 \\ J=1 \\ J=2 \end{Bmatrix} \qquad (7)$$

The missing imaginary part for J = 1 is a direct consequence of "Yang's"-theorem. The logarithmic part of the mass shift, which is common for all states can be determined experimentally from the decay width of $^3P_1$ /4/:

$$\Gamma(^3P_1 \to gq\bar{q}) = \left(\frac{n}{3}\alpha_s\right)\frac{64}{3}\frac{\alpha_s^2}{\pi}(R_p'(0))^2\frac{1}{M^4}\ln\left(\frac{M^2}{4(E_{kin}+E_{pot})^2}\right) \qquad (8)$$

The different arguments in the logarithms of (7) and (8) come from neglecting

$\langle q^2 \rangle = -m_Q E_{kin}$ in deriving (8).

Given an experimental width around 1 MeV /2/, $n \approx 3$ for the number of flavors and an $\alpha_s = 0.2$ (0.5) we get for the common mass shift a value of 5 (2) MeV. The remaining individual mass shifts can be obtained from the annihilation width of the ^3P_o state, $\Gamma^{o^{++}} = (16.3 \pm 3.6)$ MeV /2/. The total mass shifts are thus given by

$$\Delta M_J = 5\ (2)\ \text{MeV} + \begin{Bmatrix} 6.3 \\ 2.5 \\ 2.1 \end{Bmatrix} \text{MeV for} \begin{Bmatrix} J=0 \\ J=1 \\ J=2 \end{Bmatrix} \qquad (9)$$

These corrections are too small and in addition do not have the right signs to correct the experimentally wrong level-splitting as obtained from the Breit-Fermi-Hamiltonian

$$R = \frac{M_2^{++} - M_1^{++}}{M_1^{++} - M_o^{++}} = \begin{Bmatrix} >0.8 \text{ for Breit Fermi} \\ 0.48 \text{ experiment} \\ 0.47 \text{ experiment corrected} \end{Bmatrix} \qquad (10)$$

The last entry comes from subtracting our theoretical mass-shifts from the experimental values.

Finally we should mention that our corrections lead to a shift of the center of gravity of the P_c/χ states with respect to the $^1P_1$ state of 5(2) + 2.7 MeV.

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