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Abstract

The continuum limit of a Z_4 gauge plus matter lattice theory is identified with massless scalar and vector fields with quartic self-interactions ϕ^4 and $(A_\mu A_\mu)^2$, respectively. The analysis is based on the mean field approximation after gauge fixing.

1. Introduction.

In a recent paper[1] a method based on the mean field approximation combined with a loop expansion around it, has been suggested by Brézin and Drouffe as a general approach to investigate the field theoretic content of lattice theories in the neighbourhood of a critical point.[2] They applied the method to a Z_2 gauge model coupled to matter. They showed that the scaling limit of this model is a continuum one component massless scalar field with quartic self-interaction.

In this note we apply the same approach to a Z_4 lattice gauge theory coupled to bosonic matter. We are motivated by the fact that a richer structure of the gauge group will be reflected in a more complex structure of the continuum limit. After gauge fixing, the mean field approximation leads us to the detection of a second order transition point (critical point), end of a first order line. The choice of a particular gauge is necessary in order to avoid the restriction imposed by Elitzur's theorem[3], namely the vanishing of the expectation value of any non-gauge invariant quantity. We then study the correlation functions which are relevant to the long distance limit of the model at this critical point, and obtain the vertices and propagators of the associated field theory. These correspond to massless scalar and vector fields, with quartic self-interactions of the type ϕ^4 and $(A_\mu A_\mu)^2$, respectively, and uncoupled among themselves. An important remark is that in connection with the vector field, euclidean invariance is not completely restored. A cubic (but not euclidean) symmetric piece appears in the lattice four point correlation function. Nevertheless, we can extract from it its tensorial component, which, when combined with the euclidean piece of the correlation function, leads to the full euclidean covariant vertex for the four vectors interaction.

We do not include the loop expansion around the mean field approximation[4]. This would lead to a quantum field theory with vertices and propagators as given by the tree approximation considered here.

2. Critical point.

Let us begin by recalling the definition of the model. With each site i of a d-dimensional hypercubic lattice and each oriented link i, μ (μ is a unit vector along the positive axis μ) we associate, respectively, the variables $h_i = e^{i\phi_i}$ and $U_{i,\mu} = e^{i\phi_{i,\mu}}$, with $\phi_i = \pi n_i/2$, $\phi_{i,\mu} = \pi n_{i,\mu}/2$, n_i, $n_{i,\mu} = 0,1,2,3$. The Z_4 locally invariant Action is defined by

$$S = S_G + S_{GM}, \qquad (1)$$

where

$$S_G = \beta_p \sum_{i,\mu\nu} \mathrm{Re}\,(U_{i,\mu}\, U_{i+\mu,\nu}\, U^*_{i+\nu,\mu}\, U^*_{i,\nu}) \qquad (2)$$

and

$$S_{GM} = \beta_l \sum_{i,\mu} \mathrm{Re}\, h_i^*\, U_{i,\mu}\, h_{i+\mu} \quad . \qquad (3)$$

β_p and β_l are non-negative real parameters (plaquette and link coupling constants, respectively). For later convenience, we shall express the pure gauge part of the Action, S_G, in terms of two-dimensional real unit vectors.[5] Defining

$$\hat{k}_{i,\mu} \equiv (\cos\phi_{i,\mu}\ ,\ \sin\phi_{i,\mu}) \equiv k^{\alpha}_{i,\mu} \quad , \ \alpha = 1,2$$

it is easy to obtain

$$S_G = \beta_p \sum_{i,\mu\nu} \Big((\hat{k}_{i,\mu}\cdot\hat{k}_{i+\nu,\mu})(\hat{k}_{i+\mu,\nu}\cdot\hat{k}_{i,\nu}) - (\hat{k}_{i,\mu}\cdot\hat{k}_{i+\mu,\nu})(\hat{k}_{i+\nu,\mu}\cdot\hat{k}_{i,\nu})$$
$$+ (\hat{k}_{i,\mu}\cdot\hat{k}_{i,\nu})(\hat{k}_{i+\nu,\mu}\cdot\hat{k}_{i+\mu,\nu})\Big) \quad . \qquad (2a)$$

As we mentioned in the Introduction, we are interested in the detection of a second order transition point, end of a first order line. In a straightforward application of the mean field approximation this point disappears, therefore making it necessary to first fix the gauge. We shall work in the unitary gauge, defined by the condition that the Action depends only upon the gauge variables. After gauge fixing, the gauge-matter piece of the Action becomes

$$S_{GM} = \beta_l\, \hat{1}\cdot \sum_{i,\mu} \hat{k}_{i,\mu} \quad , \qquad (3a)$$

where $\hat{1}$ is a unit vector along the positive direction 1 in internal space.

In the mean field approximation, the interaction among the dynamical variables is replaced by an external field $\vec{K}_{i,\mu} = |\vec{K}_{i,\mu}|(\cos \pi n_{i,\mu}/2, \sin \pi n_{i,\mu}/2)$, $n_{i,\mu} = 0,1,2,3$, coupled to the gauge variables through the independent link Action

$$S_o = \sum_{i,\mu} \vec{K}_{i,\mu} \cdot \hat{k}_{i,\mu} \quad . \tag{4}$$

By using standard techniques, one obtains for the generating functional of vertex functions the expression

$$\Gamma(\{\vec{x}_{i,\mu}\}) = -\beta_p \sum_{i,\mu\nu} ((\vec{x}_{i,\mu} \cdot \vec{x}_{i+\nu,\mu})(\vec{x}_{i+\mu,\nu} \cdot \vec{x}_{i,\nu}) - (\vec{x}_{i,\mu} \cdot \vec{x}_{i+\mu,\nu})(\vec{x}_{i+\nu,\mu} \cdot \vec{x}_{i,\nu})$$

$$+ (\vec{x}_{i,\mu} \cdot \vec{x}_{i,\nu})(\vec{x}_{i+\nu,\mu} \cdot \vec{x}_{i+\mu,\nu})) - \beta_l \sum_{i,\mu} x_{i,\mu} \cos \pi n_{i,\mu}/2 \tag{5}$$

$$+ \sum_{i,\mu} ((1+x_{i,\mu}) \ln (1+x_{i,\mu}) + (1-x_{i,\mu}) \ln (1-x_{i,\mu})) \quad ,$$

where $\vec{x}_{i,\mu} = \langle \hat{k}_{i,\mu} \rangle_o$ and $x_{i,\mu} = |\vec{x}_{i,\mu}|$. $\langle \ \rangle_o$ denotes the expectation value with e^{iS_o} as weight factor.

The vertex functions are obtained by taking derivatives of Γ with respect to the averaged link variables $\vec{x}_{i,\mu}$, and evaluating them at the configuration $\{\vec{x}^*_{i,\mu}\}$ which minimizes Γ. This configuration is easily obtained if we consider the translationally invariant situation $\vec{x}_{i,\mu} = \vec{x} = x(\cos \pi n/2, \sin \pi n/2)$. In this case, (5) reduces to

$$\frac{\Gamma(\vec{x})}{Nd} = -\frac{1}{4} \beta x^4 - \beta_l x \cos \pi n/2 + (1+x) \ln(1+x) + (1-x) \ln (1-x) \quad , \tag{6}$$

where N is the number of lattice sites and $\beta = 2\beta_p$ (d-1). The minima of $\Gamma(\vec{x})$ are obtained by setting n = o and by solving the equation

$$\beta_l = -\beta x^{*3} + \ln\left(\frac{1+x^*}{1-x^*}\right) \quad . \tag{7}$$

In Fig. 1 we plot the r.h.s. of (7) for different values of β. For $\beta > 8/3$ there exists a first order transition line in the (β,β_l) - plane defined by the condition $\Gamma(x_1^*) = \Gamma(x_2^*)$. This line ends at the critical point

$$C \equiv (\beta_c, \beta_l^c) = \left(\frac{8}{3}, 2\left(-\frac{\sqrt{2}}{3} + \ln(1+\sqrt{2})\right)\right) , \quad \vec{x}_c^* = \frac{1}{\sqrt{2}} \hat{1} \quad , \tag{8}$$

where the transition is of second order. In fact, $\Gamma''(1/\sqrt{2}) = 0$ for $\beta = \beta_c$. In the neighbourhood of the critical point the equation for the first order line is approximately given by

$$\frac{\beta - \beta_c}{2\sqrt{2}} + (\beta_l - \beta_l^c) = 0 \quad . \tag{9}$$

(See Fig. 2.)

3. Continuum limit.

In this Section we shall consider the vertex functions which are relevant to the long distance behaviour of the model, namely the two-, three- and four-point functions. Their study will lead us to the identification of the continuum field theories associated with the discrete theory at its critical point.

Let $\vec{x}^* = x^* \hat{1}$ be a minimum of $\Gamma(\{\vec{x}_{i,\mu}\})$. For the two-link inverse two-point function a straightforward calculation leads to the result

$$\Gamma^{(2)\,\alpha\beta}_{i_1\mu_1;\,i_2\mu_2} \equiv \left.\frac{\partial^2 \Gamma(\{\vec{x}_{i,\mu}\})}{\partial x^\alpha_{i_1,\mu_1} \partial x^\beta_{i_2,\mu_2}}\right|_{\vec{x}_{i,\mu} = \vec{x}^*} \tag{10}$$

$$= \begin{bmatrix} -\beta x^{*2} P_{i_1\mu_1;\,i_2\mu_2} + \frac{2}{1-x^{*2}} \delta_{i_1\mu_1;\,i_2\mu_2} & 0 \\ 0 & \beta_p x^{*2}(\mp P_{i_1\mu_1;\,i_2\mu_2} + 2(d-1)\,\delta_{i_1\mu_1;\,i_2\mu_2}) \end{bmatrix} .$$

The - and + signs refer to the choice of links indicated in Figs. 3a and 3b, respectively, and

$$P_{i_1,\mu_1;i_2,\mu_2} = \begin{cases} 1, & \text{if the links } i_1,\mu_1 \text{ and } i_2,\mu_2 \text{ belong to the same plaquette} \\ 0, & \text{otherwise.} \end{cases} \qquad (11)$$

Notice that $\Gamma^{(2)\,11}_{i_1,\mu_1;i_2,\mu_2}$ is the same as the corresponding quantity in Ref. 1. The Fourier transform of (10) is given by

$$\Gamma^{(2)\,\alpha\beta}_{\mu_1\mu_2}(\vec{q}) = \sum_{\vec{r}_2} e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}\,\Gamma^{(2)\,\alpha\beta}_{i_1\mu_1;i_2\mu_2}$$

$$= \begin{bmatrix} -2\beta_p x^{*2}\Big(\delta_{\mu_1\mu_2}\sum_{\mu\neq\mu_1}\cos q_\mu + 2(1-\delta_{\mu_1\mu_2})\cos\tfrac{1}{2}q_{\mu_1}\cos\tfrac{1}{2}q_{\mu_2}\Big) + \dfrac{2\delta_{\mu_1\mu_2}}{1-x^{*2}} & 0 \\ 0 & -2\beta_p x^{*2}\Big(\delta_{\mu_1\mu_2}\sum_{\mu\neq\mu_1}\cos q_\mu + 2(1-\delta_{\mu_1\mu_2})\sin\tfrac{1}{2}q_{\mu_1}\sin\tfrac{1}{2}q_{\mu_2} - \delta_{\mu_1\mu_2}(d-1)\Big) \end{bmatrix} \qquad (12)$$

where $\vec{r}_j$ is the position vector of the center of the link i_j,μ_j.
At the critical point and for small $\vec{q}$ (long distances) this expression reduces to

$$\Gamma^{(2)\,\alpha\beta}_{\mu_1\mu_2;c}(\vec{q}) = \frac{2}{3(d-1)}\begin{bmatrix} 4d\big(\delta_{\mu_1\mu_2}-\tfrac{1}{d}\big) + \delta_{\mu_1\mu_2}\big(q^2-2q^2_{\mu_1}\big) + \tfrac{1}{2}\big(q^2_{\mu_1}+q^2_{\mu_2}\big) & 0 \\ 0 & q^2\Big(\delta_{\mu_1\mu_2} - \dfrac{q_{\mu_1}q_{\mu_2}}{q^2}\Big) \end{bmatrix} \qquad (13)$$

$$+\,O(|\vec{q}|^4).$$

A simple analysis of this matrix reveals the continuum limit that is associated with each of the two directions in internal space: i) $\Gamma^{(2)\,11}_{\mu_1,\mu_2;c}(\vec{q})$ is not an euclidean tensor. However, the diagonalization of the matrix $\Gamma^{(2)\,11}_{\mu_1\mu_2;c}(\vec{0})$

leads to one zero eigenvalue in the direction $(\mu_1,\mu_2,\ldots,\mu_d) = (1,1,\ldots,1)$ and to d-1 non-zero eigenvalues in the transverse directions. Therefore, there is only one mode relevant to the long distance limit of the discrete model, namely a massless scalar field with an inverse propagator given by

$$\Gamma^{(2)\,11}_{c}(\vec{q}) = \frac{1}{d^2}\sum_{\mu_1,\mu_2}\Gamma^{(2)\,11}_{\mu_1,\mu_2;c}(\vec{q}) = \frac{4}{3d^2}\,q^2 + O(|\vec{q}|^4) \quad . \qquad (14)$$

ii) $\Gamma^{(2)\,22}_{\mu_1,\mu_2;c}(\vec{q})$ is a tensor, which can be identified with the inverse propagator of a massless vector field in the unitary gauge.

The nature of the interactions among these fields is obtained from the three- and four-point functions evaluated at zero momenta and projected along the direction (1,1, ..., 1) when they involve the massless scalar field.

For the three point function, after a long but straightforward calculation we obtain

$$\Gamma^{(3)\,\alpha\beta\gamma}_{i_1,\mu_1;i_2,\mu_2;i_3,\mu_3} = \frac{\partial^3\Gamma(\{\vec{x}_{i,\mu}\})}{\partial x^\alpha_{i_1,\mu_1}\,\partial x^\beta_{i_2,\mu_2}\,\partial x^\gamma_{i_3,\mu_3}}\Bigg|_{\vec{x}_{i,\mu}=\vec{x}^*}$$

$$= -\beta_p\,T^{\alpha\beta\gamma}P_{i_1,\mu_1;i_2,\mu_2;i_3,\mu_3} + \delta_{i_1,\mu_1;i_2,\mu_2}\,\delta_{i_2,\mu_2;i_3,\mu_3}\,U^{\alpha\beta\gamma} \quad , \qquad (15)$$

where

$$T^{\alpha\beta\gamma} = x^\beta\delta^{\gamma\alpha} \mp x^\alpha\delta^{\beta\gamma} \pm x^\gamma\delta^{\alpha\beta}\Big|_{\vec{x}^*} \quad , \qquad (16)$$

(-+ and +- correspond to the choice of links indicated in Figs. 4a and 4b, respectively),

$$P_{i_1,\mu_1;\, i_2,\mu_2;\, i_3,\mu_3} = \begin{cases} 1, & \text{if the links } i_1,\mu_1 \,;\, i_2,\mu_2 \text{ and } i_3,\mu_3 \\ & \text{belong to the same plaquette} \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

and

$$U^{111} = \frac{4x^*}{(1-x^{*2})^2} \quad ,$$

$$U^{122} = U^{212} = U^{221} = \frac{1}{x^{*2}}\left(\beta_l - \ln\left(\frac{1+x^*}{1-x^*}\right) + \frac{2x^*}{1-x^{*2}}\right) \quad , \tag{18}$$

$$U^{222} = U^{112} = U^{121} = U^{211} = 0 \quad .$$

By summing over the positions of the centers of the links, we obtain the three-point function evaluated at zero momenta:

$$\Gamma^{(3)\,\alpha\beta\gamma}_{\mu_1\mu_2\mu_3}(\vec{q}_1,\vec{q}_2,\vec{q}_3)\Big|_{\vec{q}_i=\vec{0}} = \sum_{\vec{r}_1,\vec{r}_2,\vec{r}_3} \Gamma^{(3)\,\alpha\beta\gamma}_{i_1,\mu_1;\, i_2,\mu_2;\, i_3,\mu_3}$$

$$= N\Big(-\beta_p\big(\delta_{\mu_1\mu_2}(1-\delta_{\mu_2\mu_3})\, A^{\alpha\beta\gamma} + (1-\delta_{\mu_1\mu_2})(\delta_{\mu_3\mu_1} B^{\alpha\beta\gamma} + \delta_{\mu_3\mu_2} C^{\alpha\beta\gamma})\big) \tag{19}$$

$$+ \delta_{\mu_1\mu_2}\delta_{\mu_2\mu_3} U^{\alpha\beta\gamma}\Big) \quad ,$$

with

$$A^{112} = B^{121} = C^{211} = A^{221} = B^{212} = C^{122} = 4x^* \tag{20}$$

and the rest of components equal to zero. Explicitely,

$$\Gamma^{(3)\,111}_{\mu_1\mu_2\mu_3} = N\Big(-\beta_p\big(\delta_{\mu_1\mu_2}(1-\delta_{\mu_2\mu_3})4x^* + (1-\delta_{\mu_1\mu_2})(\delta_{\mu_3\mu_1}4x^* + \delta_{\mu_3\mu_2}4x^*)\big) \tag{21a}$$

$$+ \delta_{\mu_1\mu_2}\delta_{\mu_2\mu_3}\frac{4x^*}{(1-x^{*2})^2}\Big) \quad ,$$

$$\Gamma^{(3)\,222}_{\mu_1\mu_2\mu_3} = 0 \quad , \tag{21b}$$

$$\Gamma^{(3)\,221}_{\mu_1\mu_2\mu_3} = N\Big(-\beta_p\delta_{\mu_1\mu_2}(1-\delta_{\mu_2\mu_3})4x^* + \delta_{\mu_1\mu_2}\delta_{\mu_2\mu_3}\frac{1}{x^{*2}}\Big(\beta_l - \ln\Big(\frac{1+x^*}{1-x^*}\Big) + \frac{2x^*}{(1-x^{*2})^2}\Big)\Big) \,, \tag{21c}$$

$$\Gamma^{(3)\,212}_{\mu_1\mu_2\mu_3} = N\Big(-\beta_p\delta_{\mu_3\mu_1}(1-\delta_{\mu_1\mu_2})4x^* + \delta_{\mu_1\mu_2}\delta_{\mu_2\mu_3}\frac{1}{x^{*2}}\Big(\beta_l - \ln\Big(\frac{1+x^*}{1-x^*}\Big) + \frac{2x^*}{(1-x^{*2})^2}\Big)\Big) \,, \tag{21d}$$

$$\Gamma^{(3)\,122}_{\mu_1\mu_2\mu_3} = N\Big(-\beta_p\delta_{\mu_2\mu_3}(1-\delta_{\mu_3\mu_1})4x^* + \delta_{\mu_1\mu_2}\delta_{\mu_2\mu_3}\frac{1}{x^{*2}}\Big(\beta_l - \ln\Big(\frac{1+x^*}{1-x^*}\Big) + \frac{2x^*}{(1-x^{*2})^2}\Big)\Big) \tag{21e}$$

and

$$\Gamma^{(3)\,112}_{\mu_1\mu_2\mu_3} = \Gamma^{(3)\,121}_{\mu_1\mu_2\mu_3} = \Gamma^{(3)\,211}_{\mu_1\mu_2\mu_3} = 0 \quad . \tag{21f}$$

Eqs. (21b) and (21f) imply the vanishing of the vector-vector-vector and vector-scalar-scalar couplings, respectively. In particular this result holds at the critical point (8). The scalar-scalar-scalar and scalar-vector-vector couplings vanish when they are evaluated at this point. In fact, as can be easily verified,

$$\sum_{\mu_1\mu_2\mu_3}\Gamma^{(3)\,111}_{\mu_1\mu_2\mu_3;c} = \sum_{\mu_1}\Gamma^{(3)\,122}_{\mu_1\mu_1\mu_3;c} = \sum_{\mu_2}\Gamma^{(3)\,212}_{\mu_1\mu_2\mu_3;c} = \sum_{\mu_3}\Gamma^{(3)\,221}_{\mu_1\mu_2\mu_3;c} = 0 \quad . \tag{22}$$

For the four-point function, at zero momenta and at the critical point, a similar calculation leads to the result

$$\Gamma^{(4)\,\alpha\beta\gamma\delta}_{\mu_1\mu_2\mu_3\mu_4;c}(\vec{q}_1,\vec{q}_2,\vec{q}_3,\vec{q}_4)\Big|_{\vec{q}_i=\vec{0}}$$

$$= N\Big(-\frac{4}{3(d-1)}\big(\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}(1-\delta_{\mu_2\mu_3})A^{\alpha\beta\gamma\delta} + (1-\delta_{\mu_1\mu_2})(\delta_{\mu_3\mu_1}\delta_{\mu_4\mu_2}B^{\alpha\beta\gamma\delta}$$

$$+ \delta_{\mu_3\mu_2}\delta_{\mu_4\mu_1}C^{\alpha\beta\gamma\delta})\big) + \delta_{\mu_1\mu_2}\delta_{\mu_2\mu_3}\delta_{\mu_3\mu_4}\big(64x_c^{*\alpha}x_c^{*\beta}x_c^{*\gamma}x_c^{*\delta}/x_c^{*4}$$

$$+\frac{16}{3}\left(\delta^{\alpha\beta}\delta^{\gamma\delta}+\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma}\right)\Big)\Big) \quad , \tag{23}$$

where

$$A^{1111}=A^{2222}=A^{2211}=A^{1122}=B^{1111}=B^{2222}=B^{1212}=B^{2121}=C^{1111}=C^{2222}$$

$$=C^{1221}=C^{2112}=4 \quad , \tag{24}$$

and the rest of the components being zero. From the last two equations,

$$\sum_{\mu_1\mu_2\mu_3\mu_4}\Gamma^{(4)\,1111}_{\mu_1\mu_2\mu_3\mu_4;c}=64Nd \quad , \tag{25a}$$

$$\Gamma^{(4)1112}_{\mu_1\mu_2\mu_3\mu_4;c}=\Gamma^{(4)1121}_{\mu_1\mu_2\mu_3\mu_4;c}=\Gamma^{(4)1211}_{\mu_1\mu_2\mu_3\mu_4;c}=\Gamma^{(4)2111}_{\mu_1\mu_2\mu_3\mu_4;c}$$

$$=\Gamma^{(4)2221}_{\mu_1\mu_2\mu_3\mu_4;c}=\Gamma^{(4)2212}_{\mu_1\mu_2\mu_3\mu_4;c}=\Gamma^{(4)2122}_{\mu_1\mu_2\mu_3\mu_4}=\Gamma^{(4)1222}_{\mu_1\mu_2\mu_3\mu_4;c}=0 \quad , \tag{25b}$$

$$\sum_{\mu_3\mu_4}\Gamma^{(4)2211}_{\mu_1\mu_2\mu_3\mu_4;c}=\sum_{\mu_2\mu_4}\Gamma^{(4)2121}_{\mu_1\mu_2\mu_3\mu_4;c}=\sum_{\mu_1\mu_4}\Gamma^{(4)1221}_{\mu_1\mu_2\mu_3\mu_4;c}$$

$$=\sum_{\mu_1\mu_2}\Gamma^{(4)1122}_{\mu_1\mu_2\mu_3\mu_4;c}=\sum_{\mu_1\mu_3}\Gamma^{(4)1212}_{\mu_1\mu_2\mu_3\mu_4;c}=\sum_{\mu_2\mu_3}\Gamma^{(4)2112}_{\mu_1\mu_2\mu_3\mu_4;c}=0 \tag{25c}$$

and

$$\Gamma^{(4)\,2222}_{\mu_1\mu_2\mu_3\mu_4;c}=-\frac{16}{3(d-1)}\left(\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}+\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}+\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3}\right)$$

$$+16\frac{d}{d-1}\sum_{\mu}\delta_{\mu_1\mu}\delta_{\mu_2\mu}\delta_{\mu_3\mu}\delta_{\mu_4\mu} \quad . \tag{25d}$$

Eqs. (25b) and (25c) show the vanishing of the scalar-scalar-scalar-vector, vector-vector-vector-scalar and scalar-scalar-vector-vector couplings, while (25a) shows that the scalar field has a quartic self-interaction. Thus, to the order considered, the correlation functions for this field are the same as those corresponding to a single component ϕ^4 theory. Finally, let us consider (25d). Except for the last term in its r.h.s. this would correspond to a quartic self-interaction of the form $(A\mu A\mu)^2$ for the massless vector field. This interaction is clearly not gauge invariant, what is consistent with the fact that we are working in a fixed gauge. As for the last term, we notice that it is not a tensor. However, by performing its average over the rotation group we extract from it the euclidean tensor

$$16\frac{d}{(d-1)(d+2)}\left(\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}+\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}+\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3}\right) \tag{26}$$

which is just of the same form as the first term. (The non tensorial component averages to zero.) Thus, the full euclidean covariant piece of the lattice four-point function to be identified with the field vertex is given by

$$\left(\Gamma^{(4)\,2222}_{\mu_1\mu_2\mu_3\mu_4;c}\right)_{eucl.}=\frac{32}{3(d+2)}\left(\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}+\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}+\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3}\right). \tag{27}$$

In summary, by applying to a Z_4 lattice gauge model with bosonic matter the mean field approximation in the unitary gauge, we have detected a critical point, end of a first order line, where a second order phase transition occurs. At this point, the long distance limit of the relevant correlation functions of the model has the same structure as that of the correlation functions associated with continuum massless one-component scalar and vector fields with quartic self-interactions of the type φ^4 and $(A_\mu A_\mu)^2$, respectively, and uncoupled among themselves.

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Figure captions

Fig. 1 $f(x) = -\beta x^3 + \ln\left(\frac{1+x}{1-x}\right)$ for different values of β.

Fig. 2 First order transition line in the neighbourhood of the critical point.

Fig. 3 Choice of links in the calculation of the two-point function.

Fig. 4 Choice of links in the calculation of the three-point function.

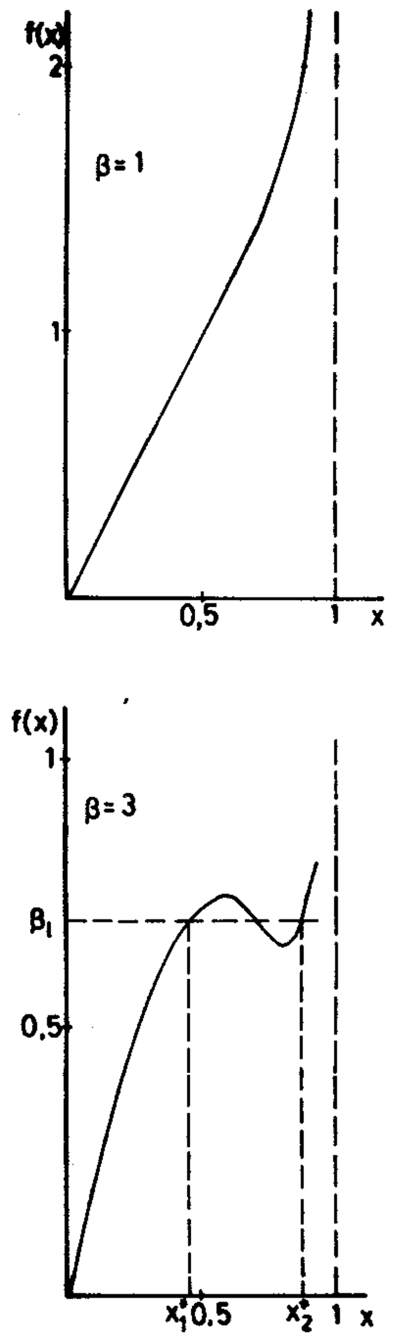

f(x)
2
1
β= 1
0,5
1
x
f(x)
1
0,5
β= 8/3
0,5
1/√2
1
x
f(x)
1
β= 3
βl
0,5
x1*
0,5
x2*
1
x

Fig. 1

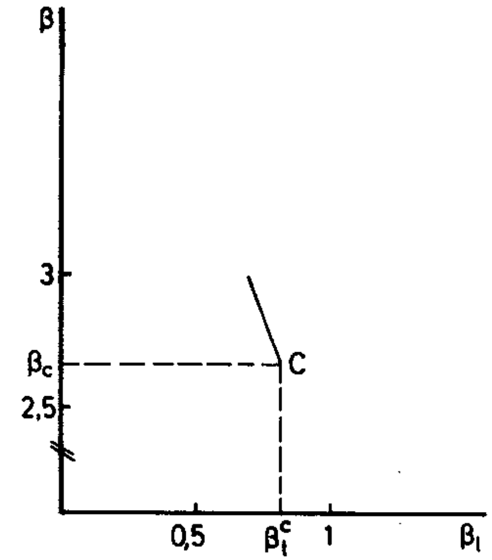
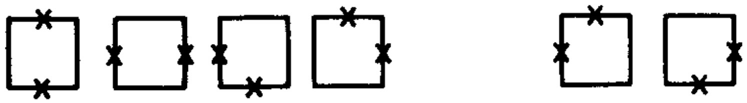
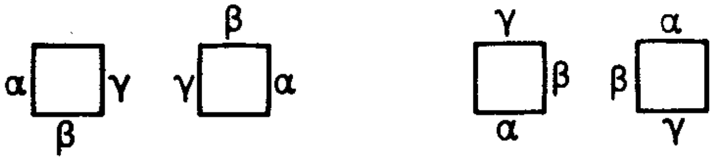

β
3
βc
2,5
C
0,5
βlc
1
βl

Fig. 2



(a) (b)

Fig. 3



(a) (b)

Fig. 4