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## PROPERTIES OF GLUON JETS

by

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ABSTRACT

We compute the expected properties of gluon jets in a model based on the KUV jet calculus and recombination. Emphasis is placed on a) the production of baryons, and b) the question of whether hadrons produced by the decays of Zweig rule stable quarkonia (e.g. the  $\psi$ ) in  $e^+e^-$  have markedly different energy spectra from those produced by the adjacent quark-antiquark continuum.

PROPERTIES OF GLUON JETS

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## I. INTRODUCTION

One of the important questions in current jet phenomenology is whether jets produced by gluons are distinguishable in the laboratory from those produced by quarks. Although jet calculus indicates that the multiplicity of elementary partons in a gluon jet should be roughly twice that in a quark jet/ $1/2$ , to date there have been few indications that this relation for the multiplicities carries over to the hadrons produced when these partons become confined.

By observing the hadrons "on" and "off" resonances which are believed to decay via a three gluon intermediate state, one might hope to isolate the gluon jets. At times, such observations have led to speculations that the gluons do indeed have different properties; in particular it has been proposed that they may be particularly effective in producing baryons/ $2/3$  (see, however, the later results of Ref 3), or that their relative effectiveness in producing baryons may have a different  $x$  dependence than expected/ $4/5$ . Other methods have also been used to isolate gluon jets/ $5/6$ .

Various calculations have appeared in the literature which address these questions in different models. An excellent review of baryon production in  $e^+e^-$  annihilation can be found in the talk by Gutbrod/ $6/7$ . In addition to numerous papers using cascade-type integral equations/ $7/8$ , there are various more detailed models. The Lund group/ $8/9$  have a model in which diquark-antidiquark pairs are created by a tunnelling process. They find that the probability to have a  $p$  or  $\bar{p}$  as fastest charged particle is slightly greater in a gluon jet than in a quark jet, and that a higher baryon fraction should be produced on the "onia" resonances than off them (but this difference should be less pronounced at higher energies). Meyer/ $9/10$  uses a more phenomenological but rather similar approach in which quark-antiquark pairs are created and align with other quarks to make baryons.

The work of Hoffmann/ $10/11$  uses a weighted phase space to generate the distribution of quarks within the jets, and the recombination model to produce baryons and mesons from them. He finds that the fraction of baryons is augmented by 30-45% in  $T$  decays as compared to  $q\bar{q}$  jets, and that the distribution in  $x$  of both protons and mesons is steeper on the  $T$  than off it.

Recently we have been exploring a model for jet fragmentation which utilizes the Konishi-Ukawa-Veneziano (KUV)/ $1/2$  jet calculus to produce the partons in the jet) and the recombination model (to stick them together into mesons or baryons). Results of the procedure for the  $e^+e^-$  continuum annihilation into quark-antiquark pairs are contained in our previous

papers/ $11,12/$  along with discussion and rationalizations. In this paper we apply the same method to the gluon jets, and we compute the spectrum for baryons and mesons "on" and "off" the upsilon and toponium resonances. Our results turn out to be very similar to those of Hoffmann, despite the difference in method for producing the parton distribution in the jets.

A brief summary of our basic formulae is given in Section II. In Section III we display the basic properties of the fragmentation functions for gluons into baryons, pions and kaons and compare with quark jet fragmentation functions. In Section IV we compare the expected energy spectra (per event) of baryons and mesons "on" and "off" the presently available upsilon resonance and the expected top quark resonance (which we have assumed for purposes of calculation to be at 42 GeV). Finally Section V contains conclusions and summary.

In three Appendices we provide additional discussion and comparison with data. Appendix A shows the relative sizes of the baryon and meson predictions; Appendix B compares our parameterization with available meson data; and Appendix C discusses some technical details, including the relation of our work to a previous calculation by Eilam and Zahir/ $25/26$  of the quark and gluon fragmentation into baryons.



## QUARKONIUM DECAY SPECTRA

To obtain the baryon energy spectrum per event at the upsilon and toponium resonances, we have taken the probability for decay into three gluon jets /15/

$$(2.5) \quad \frac{1}{\Gamma_{3g}} \frac{d\Gamma}{dx_1 dx_2 dx_3} = \frac{1}{\pi^2 - 9} \cdot \int (x_1 + x_2 + x_3 - 2) \cdot \left\{ \frac{x_1^2 (1-x_1)^2 + x_2^2 (1-x_2)^2 + x_3^2 (1-x_3)^2}{x_1^2 x_2^2 x_3^2} \right\}$$

multiplied by the probability for a gluon at  $x_1$  to produce a baryon at  $x$

$$(2.6) \quad \int d\bar{x}_1 D_g^{b+\bar{b}}(\bar{x}_1, Q^2) \delta(x - \bar{x}_1, x_1)$$

multiplied by 3 to allow for each of the jets to produce a baryon, and integrated over the allowed phase space:

$$(2.7) \quad \frac{1}{\sigma} \frac{d\sigma}{dx} = 3 \int d\bar{x}_1 \int dx_1 dx_2 dx_3 \frac{1}{\Gamma_{3g}} \frac{d\Gamma}{dx_1 dx_2 dx_3} D_g^{b+\bar{b}}(\bar{x}_1, Q^2) \delta(x - \bar{x}_1, x_1)$$

In Section IV these spectra are compared with the per event spectra calculated at the resonance energy using the quark-antiquark jet program of refs. 11 and 12.

We should mention that in these applications the  $Q^2$  of all jets (i.e. the three gluon jets in the case of the quarkonia and the two quark jets in the case of the continuum) has been taken in all calculations to be equal to the center of mass energy squared of the entering  $e^+e^-$  system. While

there is some justification for this in the case of the  $e^+e^- \rightarrow q \bar{q}$  reactions (it has been shown that there is a gauge in which there is only one jet, with off-shellness  $Q^2/16$ ), there is no clear-cut choice for the three-gluon decays. Some idea of the dependence of the answers on a different choice of  $Q^2$  can be obtained from Fig 2.

### III. RAW JET FRAGMENTATION FUNCTIONS

#### BARYON PRODUCTION

From the formulae in Section II we obtain the gluon jet fragmentation functions into proton+ antiproton shown in Figs 2-4. The dependence on the  $Q^2$  of the jet is shown in Fig 2a; for comparison we show in Fig 2b the  $Q^2$  dependence of the fragmentation function for the up quark into the same final state, and in Fig 2c the relative sizes of fragmentation functions of various partons into baryons. Notice that the gluon jet reaches its "asymptotic" Altarelli-Parisi behavior earlier in  $Q^2$  than the up quark jet. As we explain in our previous papers/11,12/ this dependence is strongly dependent on the parameters  $Q_0^2$  and  $\Lambda$  used in the evaluation of the jet calculus.

In Fig 3 we show the dependence of the gluon fragmentation function on the parameters  $\Lambda$  and  $Q_0$  at  $Q^2=89.5\text{GeV}^2$ . This is similar to the behavior of the quark fragmentation functions shown in reference 12. As discussed there, we choose to limit our considerations to small values of  $\Lambda$  near 100 MeV because of recent fits to the scale breaking behavior of the deep inelastic structure functions. Also we limit our considerations for  $Q_0^2$  to relatively small values larger than the mass of the proton (to allow for the possibility of the "intrinsic" fragmentation term discussed in Ref.12.) In the rest of this paper we have used the values  $\Lambda=.2\text{ GeV}$  and  $Q_0^2=.2\text{ GeV}^2$  for all baryon calculations.

Figure 4 contains the decomposition into the various contributions: three gluon (with all gluons converted into quark antiquark pairs according to the recipe of Eq.2.2), two gluon, etc. We note that the three gluon contribution dominates at all  $x$ . Although at first this may appear different from the behavior shown in Ref.12, it is in fact similar: at large  $x$  the contribution is dominated by the original gluon plus two radiated gluons, whereas at small  $x$  the radiated gluons dominate. Both these possibilities are contained in the three gluon contribution. However, the gluon jet curve is steeper than the quark jet curve (see Fig. 2c) because the quark coming from the "leading" gluon does not get all its momentum; hence the produced baryons tend to come out at smaller  $x$ .

At very large  $Q^2$ , the two gluon and one gluon terms dominate the three gluon term at large  $x$ . Asymptotically, therefore, the jet can produce enough quarks at large  $x$  to overwhelm those coming from the gluon splitting.

In all these curves and those to follow, we have set the normalization parameter  $R_B$  of Eq.2.3 equal to  $27/4$  for the baryon spectrum analysis.

As we discussed in Ref.12 the maximum value allowed by probability arguments/17/ would be  $27$  (the corresponding value for the mesons would be  $4$ ), and the expected value would be this divided by some number between  $2$  and  $4$  (to allow for the possibility of resonance and meson formation). These arguments were shown to be consistent with the observed size of  $J/\psi$  decay into  $p+\bar{p}/18/$ . In the present paper the factor  $4$  is suggested by the following arguments:

- If all recombination probabilities are divided by  $4$ , this allows roughly for equal chances to form baryons and mesons, and equal chances to form the ground states and the higher excited states. If we had nothing else to go on, this would at least be a reasonable guess.
- For the mesons, dividing by a factor  $4$  in fact works fairly well. This is shown in Appendix B, where we display calculations of the pion and kaon production in  $e^+e^-$  scattering compared with the data.
- Dividing by a factor  $4$  gives baryon cross sections in  $e^+e^-$  scattering of approximately the right size relative to the mesons. This is shown in Appendix A, where we compare our calculations of pion, kaon and proton production in  $e^+e^-$  scattering. This is to be compared with the data of Ref. 19 which shows  $\pi^+$ ,  $\pi^0$ ,  $K^0$ , and  $\Lambda$  production. Data from protons at large  $x$  is not available. When it becomes available the exact value of  $R_B$  can be adjusted slightly.
- The production of lambdas in gluon jets is exactly proportional to the production of protons, unlike the case of quark jets shown in Ref 12. If we use exactly the same arguments presented there, ignoring SU(3) breaking, there would be twice as many lambdas as first generation protons in the jet. Allowing for SU(3) breaking, we expect this factor to be reduced by a factor of  $2$  to  $3$ . This would give roughly equal numbers of lambdas and "elementary" protons produced; this is more or less what is seen at the upsilin  $/3/$ .

#### MESON PRODUCTION

For the calculation of gluon fragmentation functions into mesons we use  $R_M=1$ ,  $\Lambda=.2\text{ GeV}$  (this should be the same everywhere) and  $Q_0^2=.2\text{ GeV}^2$ . As shown in Appendix B these fit the presently available data reasonably well. The small value of  $Q_0$  is chosen to reproduce the apparent scale invariance of the data in the presently available energy range. It is still large enough to allow for an "intrinsic" fragmentation of the partons at the end of the chain into pions, although it is a bit small for kaons.

Because of this parametrization, the gluon jets are also more "asymptotic" in this case, as evidenced by the behavior in Figures 5a and b. The assumption of SU(3) invariance for the quark flavors leads to the fact that Fig. 5a depicts both the neutral kaon and the charged pion fragmentation functions. As usual, there will be corrections due to the mass of the strange quark which alter this result; but the number of neutral kaons should not be much more than a factor two smaller than the number of charged pions, at each  $x$ .

#### PARTICLE RATIOS

As discussed by Schierholz and Teper /4/, the baryon/meson ratio in gluon jets may be flat or even increasing in presently available data, whereas one might expect it to be a decreasing function of  $x$  based on counting rule arguments for the fragmentation functions. In Fig 6a we display this ratio as a function of  $Q^2$  using the baryon and meson parametrizations discussed above (see Figs.1 and 4). Notice that the  $x$  dependence changes as a function of  $Q^2$ , due to the fact that at presently accessible energies our parametrization for the baryons is much less asymptotic than the one for the mesons. For those who insist on seeing the asymptotic behavior, we also present in Figs.6a and b the results at  $Q^2=10^{23}$ . At this large value, the baryon to meson ratio does decrease as  $x$  approaches 1, although it does not have exactly the  $(1-x)^2$  behavior mentioned by Schierholz and Teper.

In Fig 6b we show the same ratio for up quark jets. The qualitative change with  $Q^2$  from a possible rise near  $x=1$  to rapid decrease near this point also occurs here, but at small  $Q^2$  there is less of a rise than for the gluon jets. Some sample values for down and strange quark jets are also shown.

One possible difficulty with our approach is displayed in Fig.6c, which shows the ratio of charged to neutral pions in the gluon jets, as a function of  $x$ . This is different from the behavior in quark jets. As can be seen in Fig A1, the calculations for  $e^+e^-$  annihilation produce a neutral pion spectrum which is almost exactly half the charged pion spectrum at all  $x$  values. In the gluon jet case, the allowed vertex in which a gluon can split into a  $u\bar{u}$  or  $d\bar{d}$  quark pair produces low order graphs which can contribute to the  $\pi^0$  spectrum at large  $x$ ; and these graphs have no exact counterpart in the charged pion calculation.

We regard the behavior shown in Fig 6c to be very unlikely. It would not be present in improved jet calculus models /16,20/ where colorless clusters are produced by preconfinement and these colorless, chargeless,

clusters decay in some phenomenological way. Its appearance here is, we feel, an artifact of the jet calculus approximation neglecting transverse momentum. Nevertheless, it would be nice to have an experimental check on this point.



#### IV. QUARKONIUM DECAYS

We can now produce the  $x$  spectrum of particles produced at the decay of the  $\psi$  and toponium resonances by inserting the gluon jet spectra found in the preceding section into Eq. 2.7. To have a simple means of comparing the situation on and off the resonances, we present the expected spectra per event (dividing out the very different on-resonance and continuum total cross sections).

Our results are presented in Figures 7-10. We see from Fig 7 that the answer to the question "are there more baryons per event on or off the resonance" depends to some extent whether one is looking at fast or slow baryons. At the  $\psi$ , for instance, there will be more slow baryons on the resonance and more fast baryons in the continuum surrounding it. At the toponium, the dominance of fast baryons from the 3 gluon jets on resonance is less marked, but there still are more fast baryons in the continuum.

Unfortunately, it is very difficult for us to compare with the numbers quoted in the 1981 Bonn conference proceedings<sup>3/</sup>, which give an integrated number of baryons per event. This is because we cannot calculate down to very small values of  $x$  and have any confidence in the results. The main problem here is that the kinematic corrections due to the masses of the baryons are not present in our formalism, and we don't completely understand how they should be incorporated. If one simply guesses that the curve must turn over somehow near small  $x$ , the integral of the "on resonance" curve in Fig 7a will come to about .28, not inconsistent with the value given in Ref 3. (Needless to say, with the present set of approximations we are unable to produce different values for the different excitations of the heavy quark-antiquark bound state.)

The curves in Fig 7 may be moved up and down by changing the value of  $R_B$  in the recombination function. Such adjustments will also change the relative position of the proton curve in Fig. A1. We expect that when enough data are available to justify such an adjustment, the value of  $R_B$  will not change from our value of  $27/4$  by much more than a factor of 2.

The results for the mesons are similar, except that the dominance of the "on resonance" events at small  $x$  is not nearly so marked. This is consistent with the data reported in Refs. 2 and 27.

In Figs. 7-10 two off-resonance curves are shown, labelled  $Q\bar{Q}3$  and  $Q\bar{Q}4$ . The difference lies in the number of quarks produced at the photon vertex which are "jetting". In  $Q\bar{Q}3$ , only the up, down and strange quarks produce jets; in  $Q\bar{Q}4$  the charmed quark is also assumed to produce a jet. The QCD

evolution of all these jets uses only three flavors of quarks, i.e. the gluon can split only into  $u\bar{u}, d\bar{d}$  and  $s\bar{s}$  pairs. We regard it as quite unlikely that  $c\bar{c}$  pairs are produced in the jet evolution up through the present measured range.

At the  $\psi$  it is likely that  $Q\bar{Q}3$  is more relevant than  $Q\bar{Q}4$ , at least for the baryon production. At the toponium, it seems reasonable to have at least four flavors jetting. (The possible inclusion of the fifth flavor, as shown in Fig B1, makes very little difference).

Because the production mechanism of both baryons and mesons in this model at low  $x$  arises predominantly from the production and "conversion" of multiple gluons, the number produced per event, as shown in Figs 7-10, is not greatly affected by the number of jet flavors even though the overall cross section of course increases as more flavors are taken into account. At large  $x$ , the inclusion of flavors like charm which do not directly contribute quarks to the produced particle leaves the absolute yields unchanged (see Fig. B1); hence the number per event is reduced.

Those flavors which are produced and contributing to the hadron cross section but not "jetting" will of course decay weakly, providing additional mesons and perhaps protons. Complete comparison with the data will require that these decays be modelled and added to the calculations here.

## V.SUMMARY AND CONCLUSIONS

We have studied baryon and meson production in gluon jets, using a form of the jet calculus to generate parton distributions within the jet and the recombination model to package these into hadrons. All our parameters for both the mesons and the baryons ( $A(Q_0^2)_B$ ,  $R_B(Q_0^2)_M, R_M$ ) are selected to make predictions consistent with data coming from quark jets; hence within this framework the gluon jet fragmentation functions are completely determined.

Because our gluon fragmentation function into baryons is larger than the quark fragmentation at low  $x$ , we predict more low energy baryons on the  $T$  than in adjacent parts of the  $e^+e^-$  continuum. In this our results agree with those of the Lund group /8/ and Hoffmann/10/, who had different approaches.

Within our framework, our expression for baryon production is in a very nonasymptotic region in  $Q^2$ . It remains to be seen whether this is a reasonable approach. However, as we stress in Ref.12, we reproduce many features of the present data (leading quark effect, possible rise with  $Q^2$  of the number of baryons, etc.). See Appendix C for a detailed discussion of the  $Q^2$  dependence of this type of model. We are planning to recalculate the quantities studied here and in Refs.11 and 12 by use of an improved jet calculus /16,20/. In this other method, colorless clusters of quarks and gluons are produced and these then decay phenomenologically into the observed hadrons. While this approach potentially overcomes some of the theoretical limitations to our present calculation, it has the disadvantage that the naive but practical recombination model must be replaced by a potentially more complicated cluster decay function.

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## FIGURE CAPTIONS

Fig 1. Diagrammatic version of Eq. (2.1) for the three parton inclusive distribution

Fig.2. Jet fragmentation functions into proton plus anti-proton. All curves were computed using the parameters  $Q_0^2=2 \text{ GeV}^2$ ,  $\Lambda=2 \text{ GeV}$ , and  $R_B=27/4$ .

a) Dependence on  $Q^2$  of the gluon fragmentation function into proton plus antiproton

b) Dependence on  $Q^2$  of the up quark fragmentation function into proton plus antiproton

c) Comparison at  $Q^2=10^6$  of the various possible fragmentation functions into proton plus antiproton .

Fig 3.. Behavior of the gluon fragmentation function into proton plus antiproton at  $Q^2=89.5 \text{ GeV}^2$  (the epsilon). All curves were computed with  $R_B=27/4$ . Here we show the dependence on the parameters  $Q_0^2$  and  $\Lambda$  in the jet calculus expressions.

a)  $\Lambda=200 \text{ MeV}$ ;

b)  $\Lambda=50 \text{ MeV}$ .

Fig.4..Decomposition of the gluon fragmentation function in Fig 1a) into proton plus antiproton. We see that at this  $Q^2$  most of the contribution comes from those jet calculus terms in which three gluons are produced. ( $Q^2=89.5 \text{ GeV}^2$ ,  $Q_0^2=2. \text{ GeV}^2$ ,  $\Lambda=2 \text{ GeV}$ ,  $R_B=27/4$ ). At very large  $Q^2$ , this is no longer true.

Fig.5..Dependence on  $Q^2$  of the gluon fragmentation into mesons. All these curves are calculated with  $Q_0^2=2. \text{ GeV}^2$ ,  $\Lambda=2 \text{ GeV}$ , and  $R_M=1$ .

a) production of  $K^0+K^0$ , equal to  $\pi^+\pi^-$  in the approximations used here.

b) production of neutral pions.

Fig 6..Particle ratios in the jets using the parametrization of Figures 2 and 5.

a) protons to pions in gluon jets;

b) protons to pions in quark jets;

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c) charged to neutral pions in gluon jets.

Fig.7..Protons plus antiprotons per event, on and off the quarkonium resonances. All curves are calculated with  $Q_0^2=2 \text{ GeV}^2$ ,  $\Lambda=2 \text{ GeV}$ , and  $R_B=27/4$ . The possibilities  $Q\bar{Q}3$  and  $Q\bar{Q}4$  for the continuum are discussed in the text.

a) epsilon;

b) toponium

Fig.8..Neutral kaons per event, on and off the quarkonium resonances. All curves are calculated with  $Q_0^2=2. \text{ GeV}^2$ ,  $\Lambda=2 \text{ GeV}$ , and  $R_M=1$ .

Fig.9..Neutral pions per event, on and off the quarkonium resonances. Parameters are the same as in Fig.8

Fig.10..Charged pions per event, on and off the quarkonium resonances. Parameters are the same as in Fig.9.

Fig.A.1..Predictions at  $Q^2=1089 \text{ GeV}^2$  for the meson and proton plus antiproton production in  $e^+e^-$  annihilation using the parameters of Fig.2 for the baryons and Fig.5 for the mesons. The option  $Q\bar{Q}4$  (four flavors of quarks are making jets, but only 3 flavors are active in the evolution of the jets) is used.

Fig.B1..The various options  $Q\bar{Q}3, Q\bar{Q}4$  and  $Q\bar{Q}5$  are compared with recent Petra data using the parameters  $Q_0^2=2 \text{ GeV}^2$ ,  $\Lambda=2 \text{ GeV}$ , and  $R_M=1$ . All curves have been calculated for  $Q^2=1089 \text{ GeV}^2$

a) Charged pions; the data shown are Tasso data at  $Q^2=1156 \text{ GeV}^2$  (Ref.21)

b) Neutral pions; the data shown are Cello data at  $Q^2=1089 \text{ GeV}^2$  (Ref.22)

c) Neutral kaons; the data shown are TASSO data at  $Q^2=1089 \text{ GeV}^2$  (Ref.23)

Fig.B2..Energy dependence of charged pion production using the same parameterization as in the other meson graphs.

a) Comparison with Tasso data (Ref 21) using the option  $Q\bar{Q}4$ . (see text)

b) Comparison with DASP data (ref 24) showing the two options  $Q\bar{Q}3$  and  $Q\bar{Q}4$ . (see text)

Figure Captions

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using the Eilam-Zahir parameterization  $R_B=2$ ,  $Q_0^2=64$ ,  $\Lambda=0.65$ . Results are presented for  $Q^2=900$  GeV<sup>2</sup>. Compare Fig.C2a of this paper and Fig.3 of Ref.25.

Fig.C1..Dependence of the gluon fragmentation function into protons on the power n of the recombination function

$$R_n \left( \frac{x_1 x_2 x_3}{x^3} \right)^n \int (x_1 + x_2 + x_3 - x)$$

All curves are calculated with  $Q_0^2=2$ GeV<sup>2</sup>,  $\Lambda=2$ GeV. With our normalization prescription  $R_1=27/4$ ;  $R_2=27^2/4$ .

- a)  $Q^2=89.5$ GeV<sup>2</sup>
- b)  $Q^2=10^5$ GeV<sup>2</sup>

Fig.C.2..To prepare this figure, we computed the full jet calculus at  $Q^2=30$ GeV<sup>2</sup> with our form of the recombination function and the Eilam-Zahir parameterization  $R_B=2$ ,  $Q_0^2=64$ GeV<sup>2</sup>,  $\Lambda=0.65$ GeV. We then used the Altarelli-Parisi equations to evolve these values to larger  $Q^2$ . This should be equivalent to the Oregon group's technique of cutting off the y integrals.

- a)  $Q^2=900$ . GeV<sup>2</sup> (compare Fig 3 of Ref.25)
- b)  $Q^2=(180)^2$  GeV<sup>2</sup> (compare Fig 4 of Ref 25)
- c)  $Q^2=10^{10}$  GeV<sup>2</sup>

Fig.C.3..The jet calculus preserves "moment" at the vertices.

Fig.C.4.. $Q^2$  dependence of the moments of  $D_i^{pF}$  computed with the recombination function

$$\frac{(27)^2}{4} \left( \frac{x_1 x_2 x_3}{x^3} \right)^2 \int (x_1 + x_2 + x_3 - x)$$

( $Q_0^2=2$ GeV<sup>2</sup>,  $\Lambda=2$ GeV). Although in principle these moments behave asymptotically like solutions of the Altarelli-Parisi equations, in practice asymptopia is not reached at reasonable  $Q^2$ .

Fig.C.5.. $Q^2$  dependence of the moments of  $D_i^{pF}$  computed with recombination function

$$\frac{27}{4} \left( \frac{x_1 x_2 x_3}{x^3} \right) \int (x_1 + x_2 + x_3 - x)$$

Fig.C.6..Computation of the full jet calculus for our recombination function

$$R_B \frac{x_1 x_2 x_3}{x^3} \int (x_1 + x_2 + x_3 - x)$$

## APPENDIX A

We present in Fig.A1 our predictions for  $e^+e^- \rightarrow \pi^+, \pi^0, K^0$ , and  $p+\bar{p}$  at  $Q^2=1089\text{GeV}^2$ . These should be compared with the Tasso data shown in Fig 4 of Ref 24. Since many of these data are plotted in Appendix B, we do not reproduce the Tasso plot here. The main features of this data which interest us here are:

- The large x cross sections for pions, kaons, and lambdas are more or less parallel
- The lambda cross sections are about a factor two smaller than the kaon cross sections
- The charged pion yields are almost exactly twice the neutral pion yields

Since we expect there to be somewhat more protons than lambdas, the overall normalization of the proton curve in our calculation seems about right. As discussed in the text this can be adjusted somewhat once more data is known.

Also, we see that twice the pi zero cross section is almost exactly equal to the charged pion cross section; and that all the curves have a similar shape at large x.

## APPENDIX B

Here we compare our parameterization  $Q_0^2=.2\text{ GeV}^2, A=.2\text{ GeV}, R_M=1$  with available data for meson production in  $e^+e^-$  annihilation. The Teper probability argument/17/ applied to the recombination function of Eq.2.4 would allow a maximum value  $R_M=4$ . Older fits /11,13/ to the  $\pi^+$  inclusive spectrum found  $R_M=1$  gave a good fit, and we continue to find this reasonable. In Fig. B1, results are presented for various, different numbers of flavors of jets produced at the photon vertex; in all cases only three flavors are assumed to take part in the QCD evolution of the jet. Due to the small electric charge of the b quark, its inclusion is almost irrelevant.

Notice in Fig B1c that the fit to the neutral kaon data is not as good as the fit to the pion data in the first two figures. There are two adjustments possible to the model, both of which are in the right direction to correct this discrepancy. The first is that our  $Q_0^2$  of .2  $\text{GeV}^2$  is a little small for the kaons; we might expect a  $Q_0$  at least as big as the kaon mass. As can be seen from Fig.3 (or similar graphs in our previous papers), increasing this parameter will tend to pull the curve down at small x, thus decreasing its slope. The other likely adjustment is the inclusion of mass corrections at the end of the cascade where gluon splittings occur. These would tend to decrease the amount of strange quarks produced compared with the number of nonstrange quarks.

Another important effect which matters at small x is mass corrections due to the mass of the produced particle. This will affect the proton production as well. At present we do not know how to incorporate these effects into our calculation, so we put most emphasis on the behavior at larger x where the calculation is more accurate.

In Figure B2 we address the problem of the energy dependence of the predictions. The Tasso data is plotted in Fig B2a, along with our  $Q\bar{Q}$  predictions. With the present choice of parameters, the calculations show very little change with energy in this region; this is consistent with the data. In Fig B2b we compare with the lower energy DASP data. This data has a 15 % normalization uncertainty; however our predictions lie a bit below the data even if this is allowed for.

APPENDIX C

The purpose of this Appendix is to discuss the relation of our work to the similar calculation of Eilam and Zahir(EZ)/25/ and to discuss a number of the properties of the method, with emphasis on those which should be improved in better models.

COMPARISON WITH EILAM AND ZAHIR

The work of these two authors is superficially very similar to ours in that they use the KUV jet calculus to generate a spray of partons which are then recombined. Since those authors have much smaller values for their fragmentation functions in general, and since their graph comparing the size of the gluon and quark fragmentation functions looks quite different from our Fig.2c, we feel some discussion is in order.

There are, unfortunately, many differences in the two actual calculations.

- First, EZ use a different recombination function,

$$C.1) \quad \frac{19.9}{x} \left( \frac{x_1 x_2}{x} \right)^{1.65} \frac{x_3}{x} \int (x_1 + x_2 + x_3 - x) \quad (n=1)$$

which not only has a different power behavior in the individual  $x_i$  but also is normalized according to a different prescription.

- Second, they use different values for the parameters  $Q_0^2$  and  $\Lambda$  in the jet calculus. (They have  $Q_0^2 = .64 \text{ GeV}^2$ ,  $\Lambda = .65 \text{ GeV}$ )

- Thirdly, instead of performing the full integrals in Eq.2.1, they cut off the integrals at an upper  $Q^2 = 30 \text{ GeV}^2$ ; the intent of this was to restrict the relative transverse momentum of the quarks.

Let us deal with these differences in turn. Because we invert moments with Yndurain's method, we are not particularly well equipped to deal with noninteger powers in the recombination function so it is not easy for us to exactly duplicate their work. We can, however, compute the results for the two functions

$$C.2) \quad \frac{27}{4} \frac{x_1 x_2 x_3}{x^3} \int (x_1 + x_2 + x_3 - x) \quad (n=1)$$

and

$$C.3) \quad \frac{(27)^2}{4} \left( \frac{x_1 x_2 x_3}{x^3} \right)^2 \int (x_1 + x_2 + x_3 - x) \quad (n=2)$$

The results are shown in Fig C1. We see that the values of the answers are reduced by about a factor of 4 with our normalization prescription for the recombination function. If we had instead used the valon model prescription for normalization/26/, we would have 120 for the normalization in Eq. C.3 instead of (27)<sup>2</sup>/4. Thus the overall answers would be reduced by a factor of about 6 (i.e. Our n=1 answers with our normalization of 27/4 would be a factor 6 larger than n=2 answers with the valon model prescription). Since the EZ function is partway between n=1 and n=2, we might expect an overall factor of not more than 10 difference between our calculation and theirs from the different power and normalization prescriptions.

The effect of using the Oregon values for  $Q_0^2$  and  $\Lambda$  is to make the baryon results "asymptotic" at a much lower  $Q^2$  than is the case with our values  $Q_0^2 = 2 \text{ GeV}^2$  and  $\Lambda = 2 \text{ GeV}$ . (if no cutoff is used in the jet calculus). This is shown in Fig 12 of Ref 12. Actually, no figures are necessary since the jet calculus answer is a function of only  $Y - Y_0$ , and it is easy to compute that with

$$Y - Y_0 = \frac{1}{2\pi b_j} \ln \left[ \frac{(1 + a_{j,b} \ln Q^2/\Lambda^2)}{(1 + a_{j,b} \ln Q_0^2/\Lambda^2)} \right]$$

the value of  $Q^2$  corresponding for our calculation to their value of 900 is about  $2 \times 10^{23}$ . As a practical matter, therefore, we are always operating in the nonasymptotic region of our expression. This may be objectionable on aesthetic grounds.

Finally, the cutoff placed on the y integrals by Eilam and Zahir should have the effect of giving their results exact Altarelli-Parisi behavior, since for all  $Q^2$  above  $30 \text{ GeV}^2$  their expression can be written in the form

$$C.4) \quad D_j^p(Q^2, \alpha) = D_j^p(Y - Y(30), \alpha) D_j^p(30, \alpha)$$

Unfortunately the  $Q^2$  dependence of their results at small  $x$  is not consistent with this.

To get some idea of what their results would look like with our (Yndurain)inversion technique, (or what our results would look like with a cutoff on the y integrals) we have computed the moments at  $Q^2=30$ .  $\text{GeV}^2$  and have evolved these values with the Altarelli-Parisi equations, using our recombination function Eq.2.3 but their  $Q_0^2, \Lambda$  and the vaion normalization  $R_B=2$ . This involves no approximations. Then we inverted them with the Yndurain technique. The results are shown in Fig C.2. Comparing with Figs 3 and 4 of Ref. 25, we see that

- At large x, where they believe their method to be more accurate, and where our inversion technique is certainly most accurate, our results differ from theirs by a factor of about  $10^2$  (far greater than the factor expected from power differences)
- For the two  $Q^2$  values they plot,  $Q^2=900$  and  $Q^2=(180)^2 \text{GeV}^2$ , the gluon function is relatively larger in our calculation than in theirs
- We must go to quite large  $Q^2$  (we show  $Q^2=10^{10}$  here) to obtain a gluon function which lies below the quark function everywhere above  $x=.3$ . In their Figs. 3 and 4 the gluon function is depicted as lying below the quark function at all x.

In summary, we do not understand the size of their answers. When we reproduce their technique as closely as we can, we find much larger values. Also we find that in the energy range they covered, the gluon jets are relatively more important than they show.

#### GENERAL PROPERTIES OF THE METHOD; PLACES FOR IMPROVMENT

##### $Q^2$ dependence

If we begin by considering a recombination function for mesons of the general form

$$C.5) \quad R_M \left( \frac{x_1 x_2}{x} \right)^{MN} f(x_1 + x_2 - x)$$

where NN is some integer power, we see that the moments of the recombined fragmentation function for mesons take the form /11/

$$C.6) \quad D_i^M(N, Y) = \int_{Y_0}^Y D_{ij}^M(N, Y - y) f_j^M(y) dy$$

$$\text{with } f_i^N(\gamma) = \sum_{a_1, a_2, b_1, b_2} \sum_{m=0}^{N-2NN} \binom{N-2NN}{m} C_{a_1, a_2} \rightarrow M$$

$$C.7) \quad D_{a_1, b_1}^{N-NN-m}(\gamma - Y_0) D_{a_2, b_2}^{m+NN}(\gamma - Y_0) P_{j \rightarrow b_1, b_2}^{N-NN-m, m+NN}$$

Of course there is a similar but much more complicated expression for the baryons/12/.

If we go into a representation where the parton propagators  $D_{ij}$  are diagonal, contributions to Eq.C.6 from the diagram of Fig. C.3 will have the form

$$C.8) \quad D(n+m, Y) \sim \lambda^{( \lambda_n + \lambda_m ) (\gamma - Y_0)} - \lambda^{m+m} (\gamma - Y_0)$$

As long as n and m are greater than 1, there is always one n+m eigenvalue such that  $\lambda_{n+m} > \lambda_n + \lambda_m$ . In this case, as  $Y - Y_0$  gets large, the contributions to the n+m'th moment will asymptotically have the canonical Altarelli-Parisi behavior in  $Q^2$ .

Provided the recombination function power NN is 2 or greater, no first moments will contribute to the sums in Eq. C.7 at all and in principle one has ensured that asymptotically the  $Q^2$  dependence will be of this simple form.

However, as we show in Fig C.4, where we plot the  $Q^2$  dependence of the  $D_{i, p+\bar{p}}$  moments for the NN=2 case, asymptopia will never be attained for our parameterization. None of the moments falls nearly as fast as one would expect from the Altarelli-Parisi eigenvalues for the appropriate total moment number values. The reason for this is simple:  $Y - Y_0$  is never very big, and the actual  $Q^2$  dependence of the moments comes from the cancellation between the terms of Eq.C.8. (Doubtours should plot the function  $\exp(-3x) - \exp(-5x)$ , and recall that with our parameters  $Y - Y_0$  at  $Q^2=10^6$  is .32 for the baryon case and .51 for the meson case).

This determination of the  $Q^2$  behavior of the moments by cancellation between the terms holds also for the NN=1 case - the case used for the calculations presented in this paper. In this case, there is a potential problem of principle because the n=1 moments have a zero eigenvalue, and in the terms where this occurs one has  $\lambda_{n+m} < \lambda_n + \lambda_m$ . These contributions



therefore will not have Altarelli-Parisi behavior asymptotically as  $Y \rightarrow Y_0$  becomes large.

As shown in Fig C.5, all the moments ultimately decrease with  $Q^2$  except the  $n=3$  moment, which can have three zero eigenvalues contributing for the baryon case (or two for the meson case). Furthermore, the rate of decrease of these  $n \neq 3$  moments with  $Q^2$  is similar to that of the  $NN=2$  moments.

In the inversion routine, the  $n=3$  moment contributes only to the lowest point in  $x(.3077$  for our baryon case). Hence if this point is excluded, the contribution at each of the higher points  $(J+1)/(NMOM+2)$  is obtained from terms each of which has decreasing behavior with  $Q^2$  (see Eq. C.9 below). In Yndurain's method, the moments contribute with alternating signs to the value at each  $x$  point (except for the point at the largest  $x$ , where only one moment contributes). The increasing values with  $Q^2$  of our calculations at small  $x$  are due partly to this cancellation between the decrease of various moments and partly (at lower  $Y$ ) to the increase of terms like Eq. C.8 from zero at  $Y = Y_0$ .

Some idea of the practical difference between our full jet calculus and the cutoff version can be got by comparing Fig C2a with Fig C.6. These are computed with the EZ parameters; with our values of  $Q_0^2$  and  $A$  the difference will be much more marked because of the growth shown in Fig.2a.

#### INVERSION METHOD

The values of functions of  $x$  presented in this paper have been obtained from the basic Yndurain formula/14/.

$$C.9) \quad F\left(\frac{J+1}{NMOM+2}\right) \approx \frac{(NMOM+1)!}{J!} \sum_{\ell=0}^{NMOM-J} \frac{(-1)^\ell}{\ell!(NMOM-J-\ell)!} \mu^{\frac{J+\ell}{2}}$$

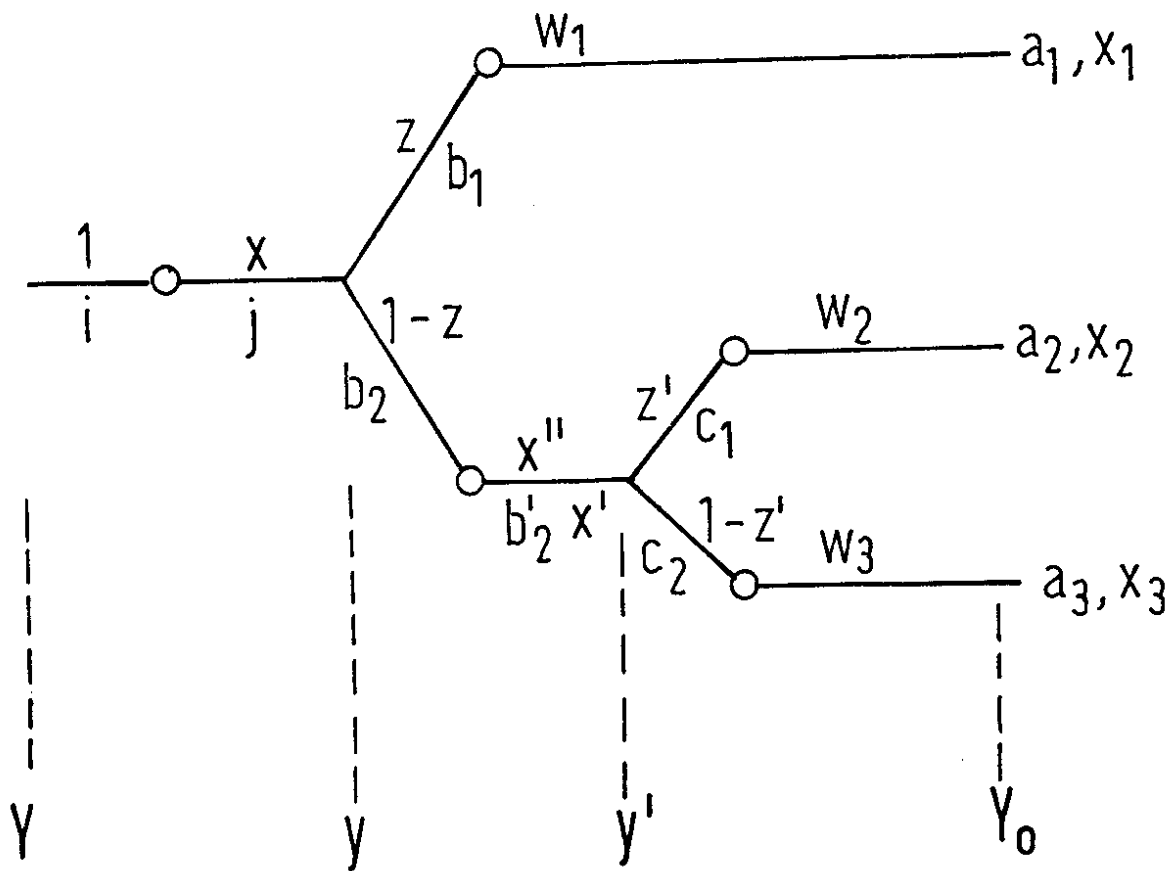
$$\text{where} \quad \mu_{J+\ell} \equiv \int_0^1 x^{J+\ell} F(x) dx$$

We do not use the "correction term", which we find frequently makes agreement with the true function worse.

The simple formula Eq.C.9 gives very good reproduction of functions at large  $x$ ; at small  $x$  on test functions with shapes similar to those of the experimental fragmentation functions, Eq.C.9 may overestimate the values at small  $x$  by as much as 20 or 30 %. However, we have never found a case in which it underestimates the function in this region. Hence we believe the relative sizes of the curves plotted are approximately correct despite potential problems with the method.

As mentioned above, the alternative method used by Eilam and Zahir, that of fitting the large  $n$  moments with moments of a function in  $x$  designed to have reasonable behavior at large  $x$ , apparently has the property that it treats the small  $x$  points incorrectly. Our method is also inaccurate in the small  $x$  region, but we believe the inaccuracies to be smaller.





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Fig.1

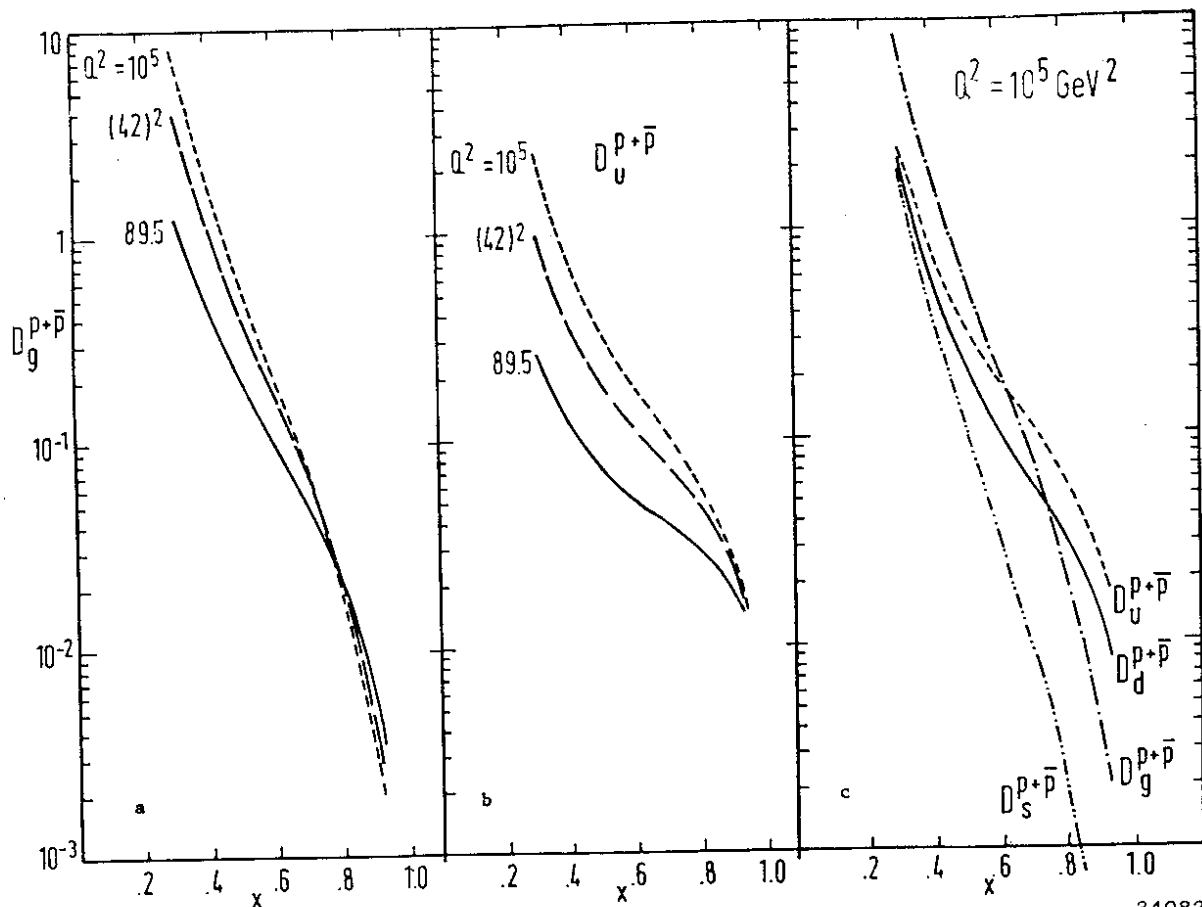
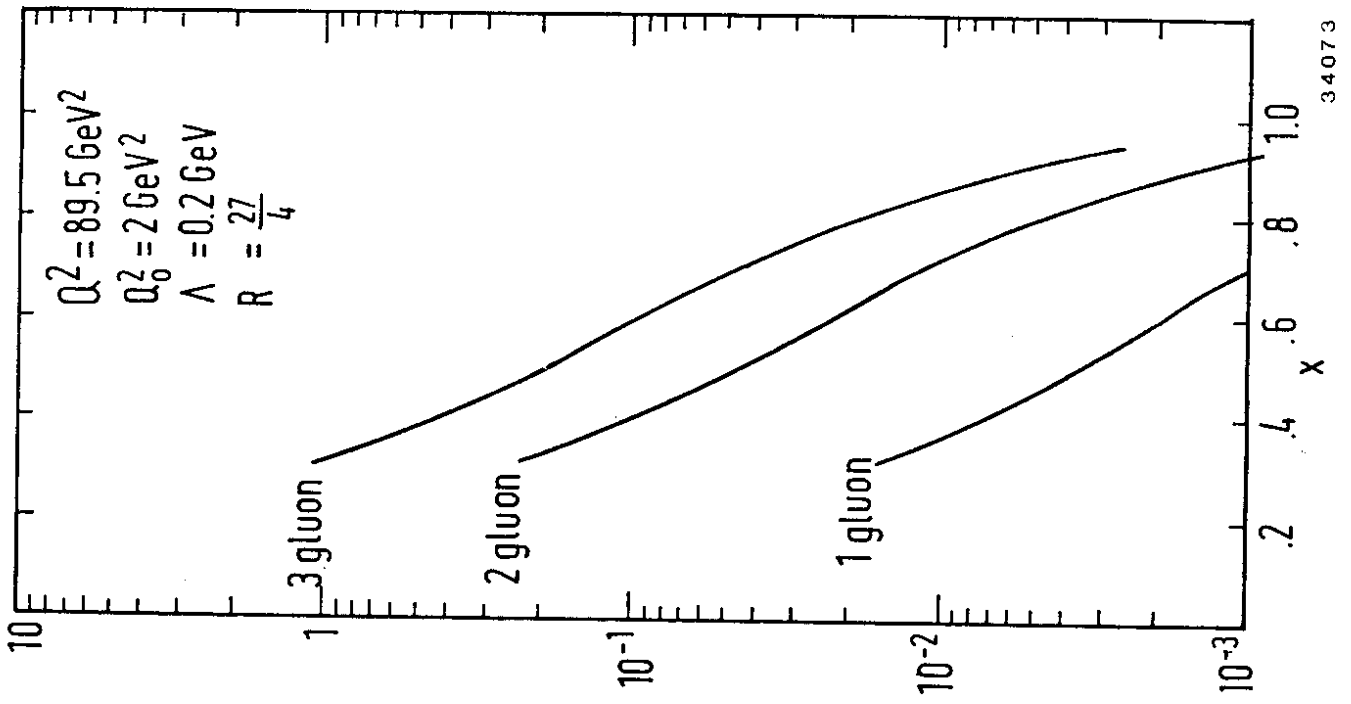


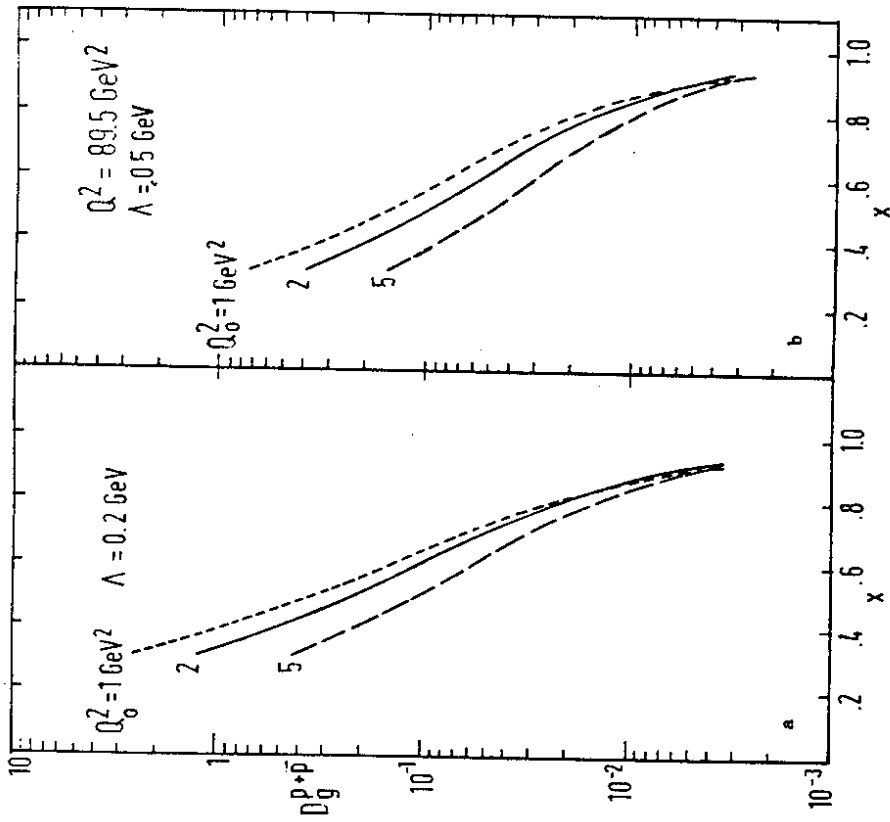
Fig.2

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FIG 4



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FIG 3

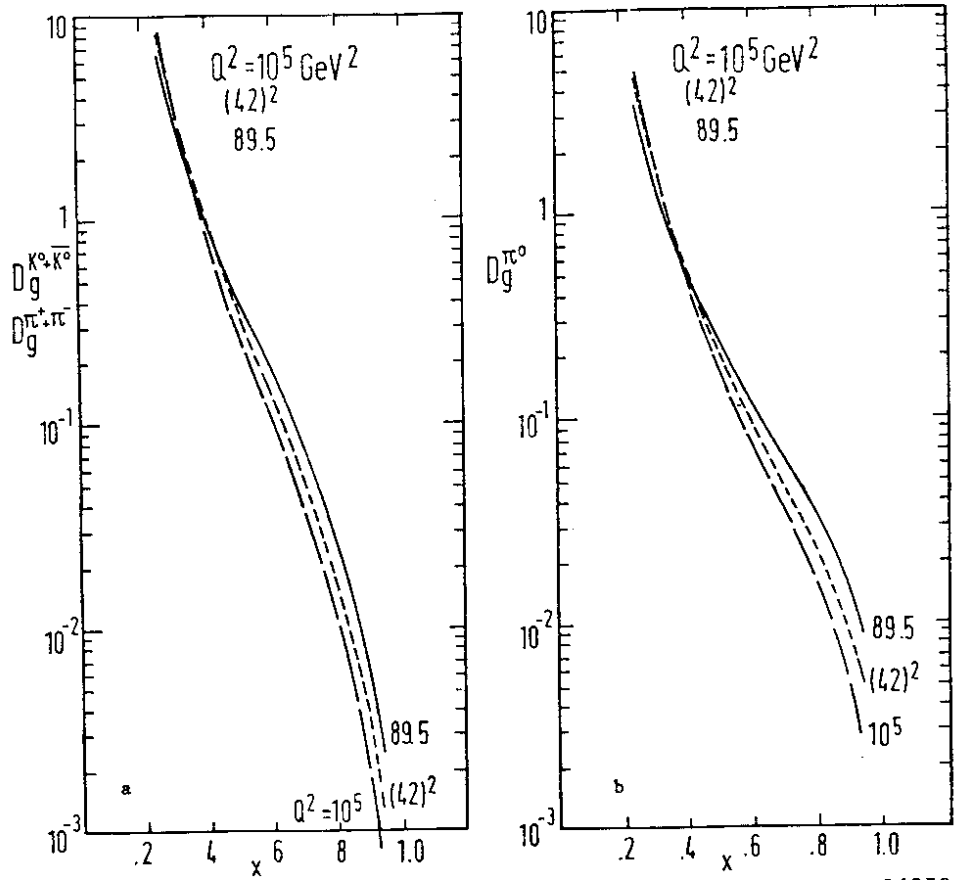


Fig 5

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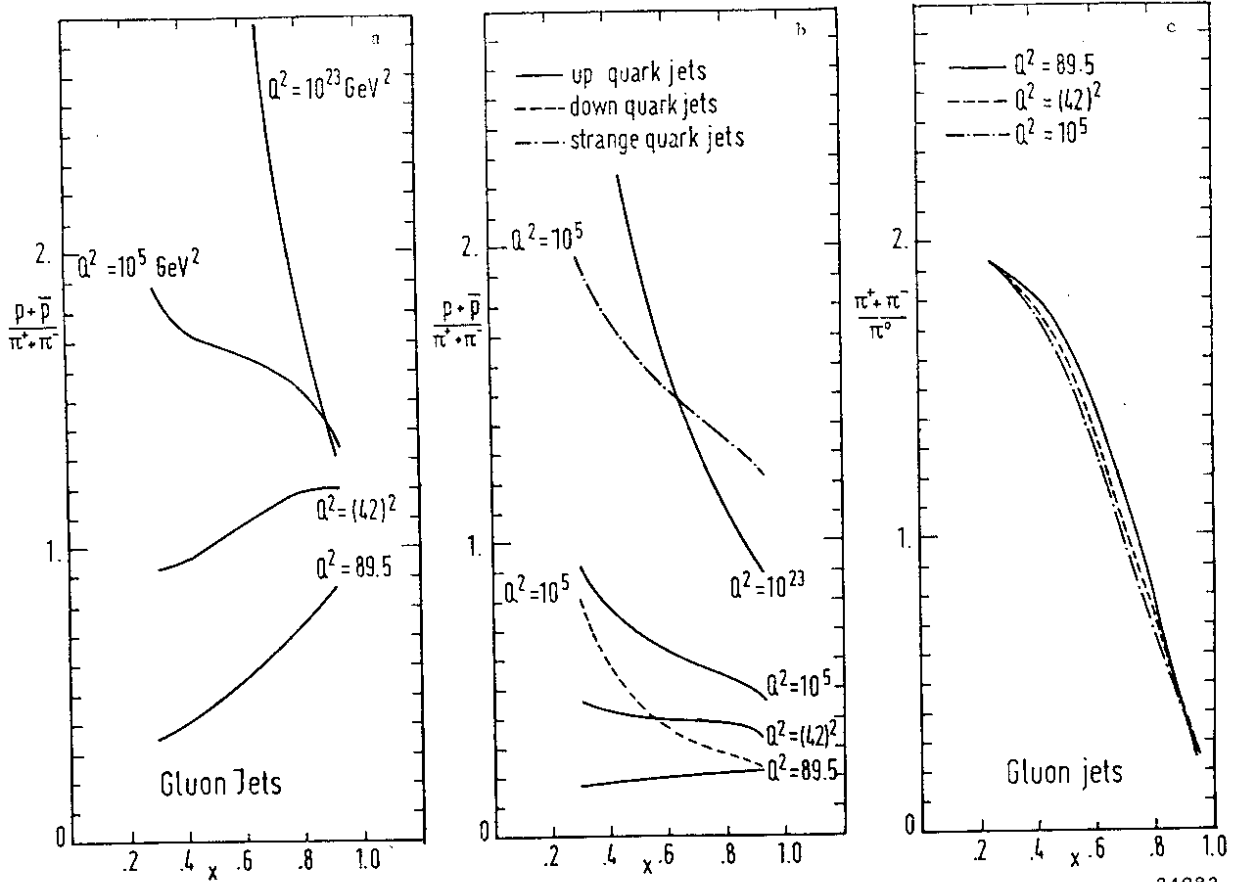


Fig 6

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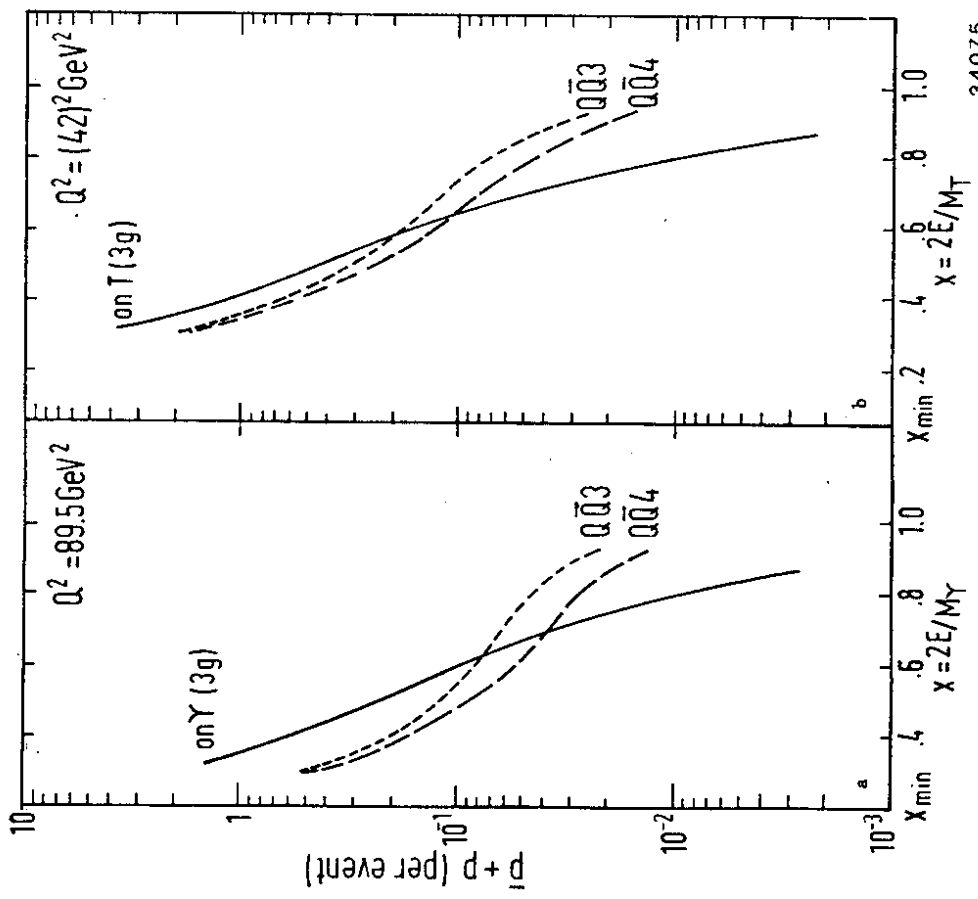


Fig 7

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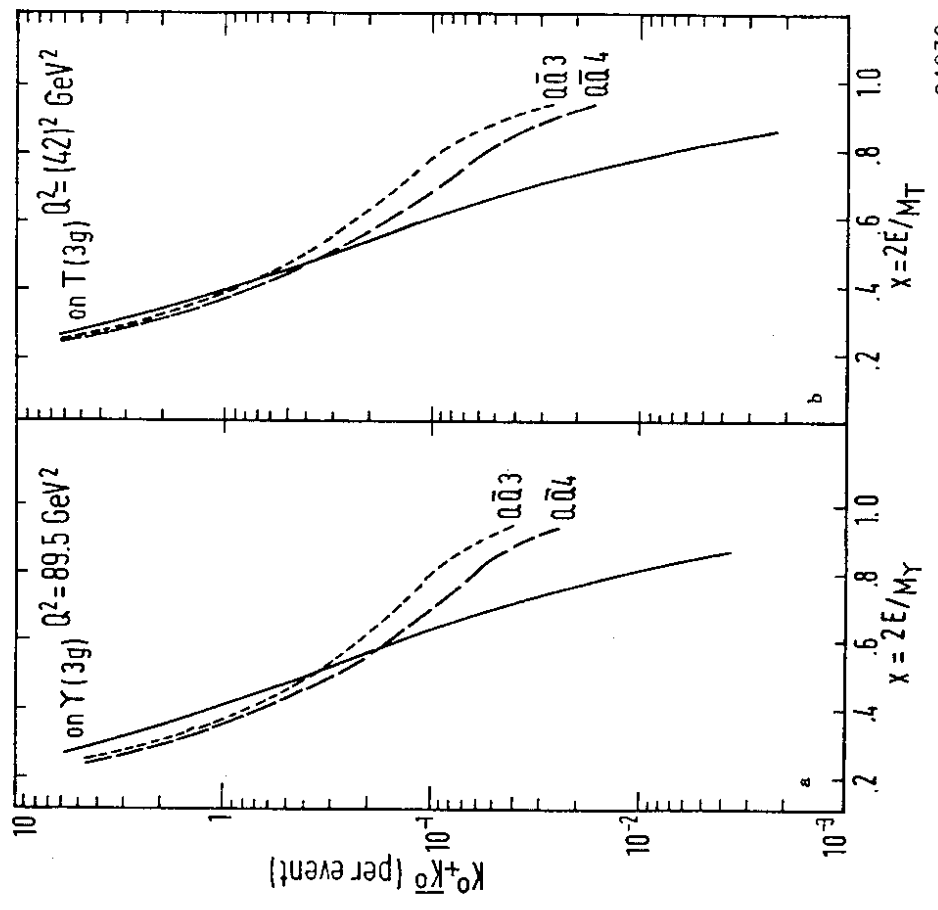


Fig 8

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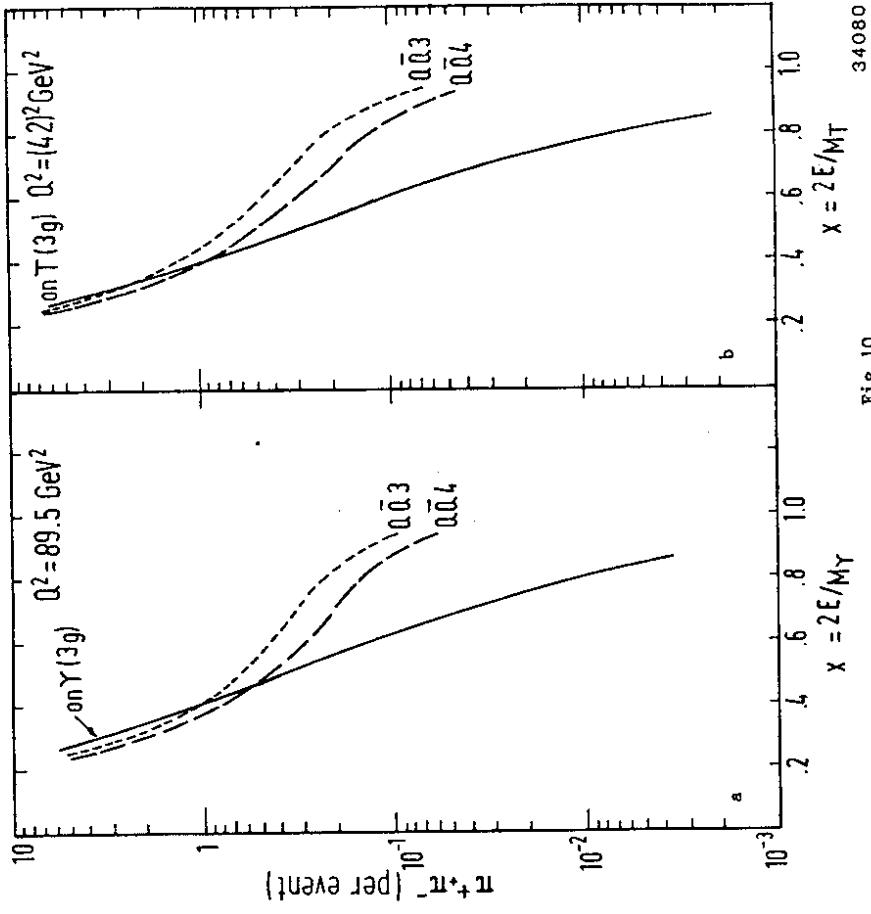


FIG 10

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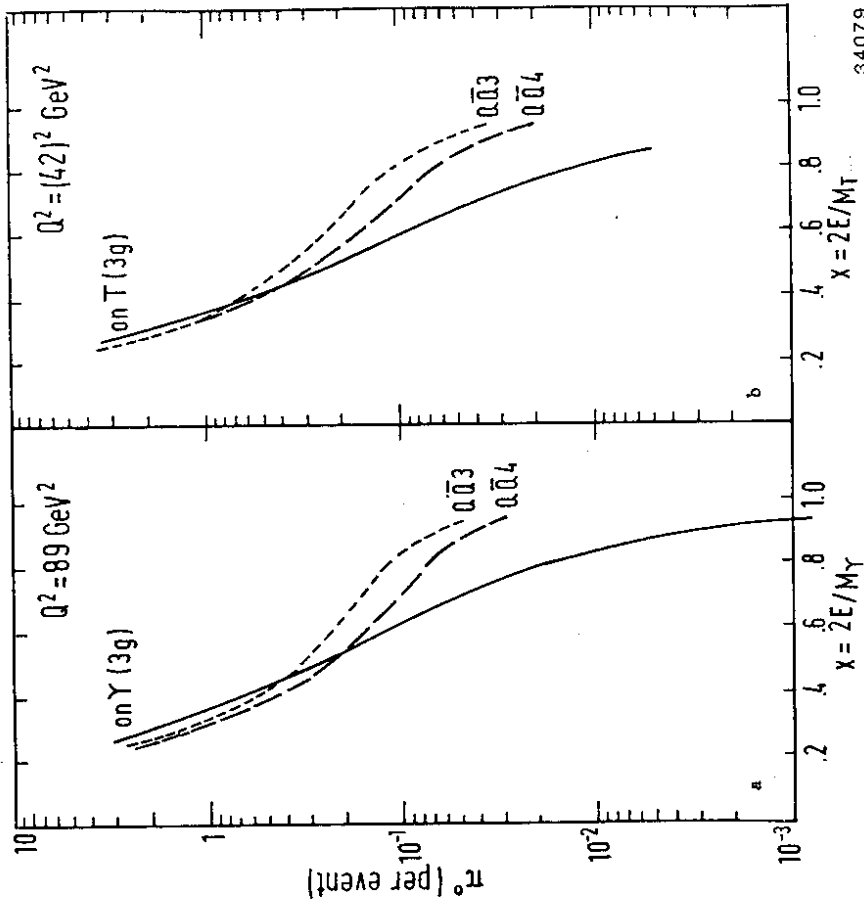


FIG 9

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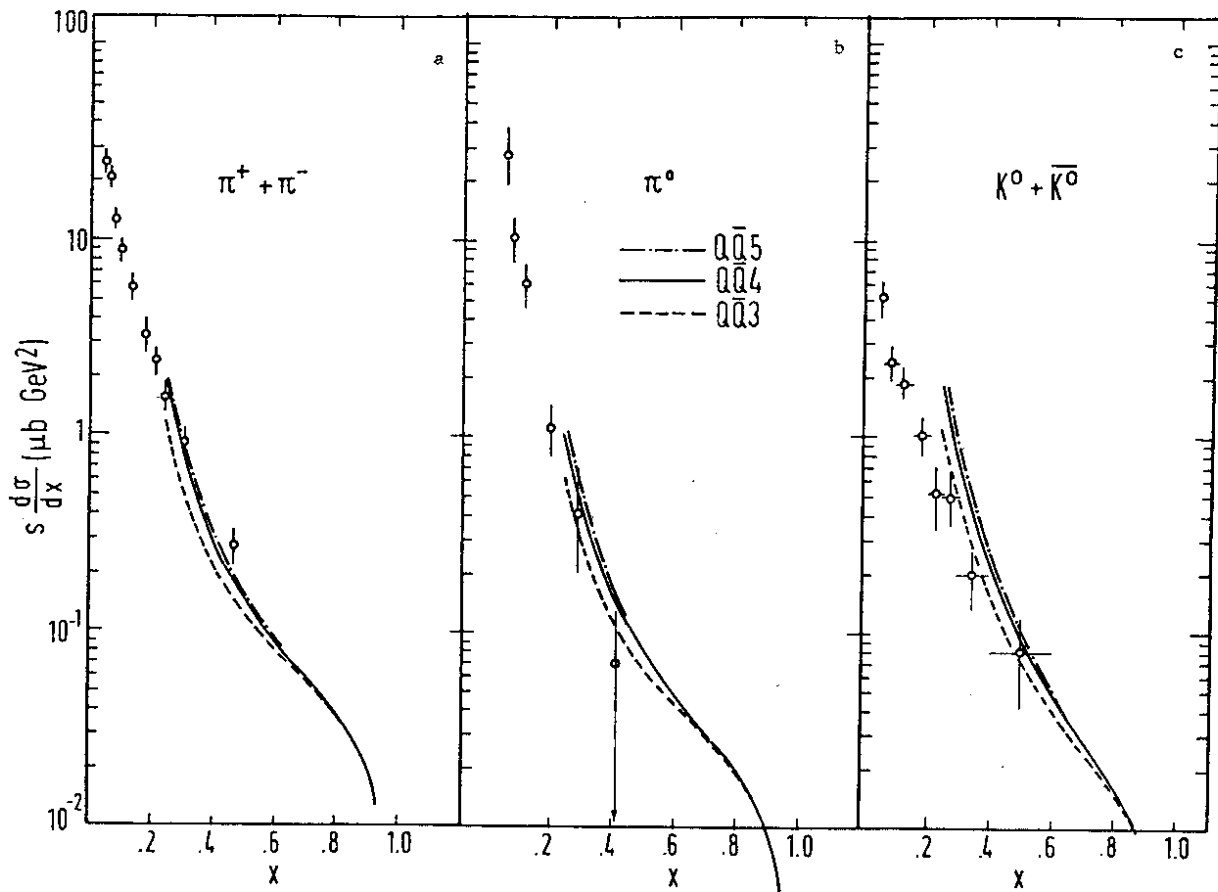


Fig B.1

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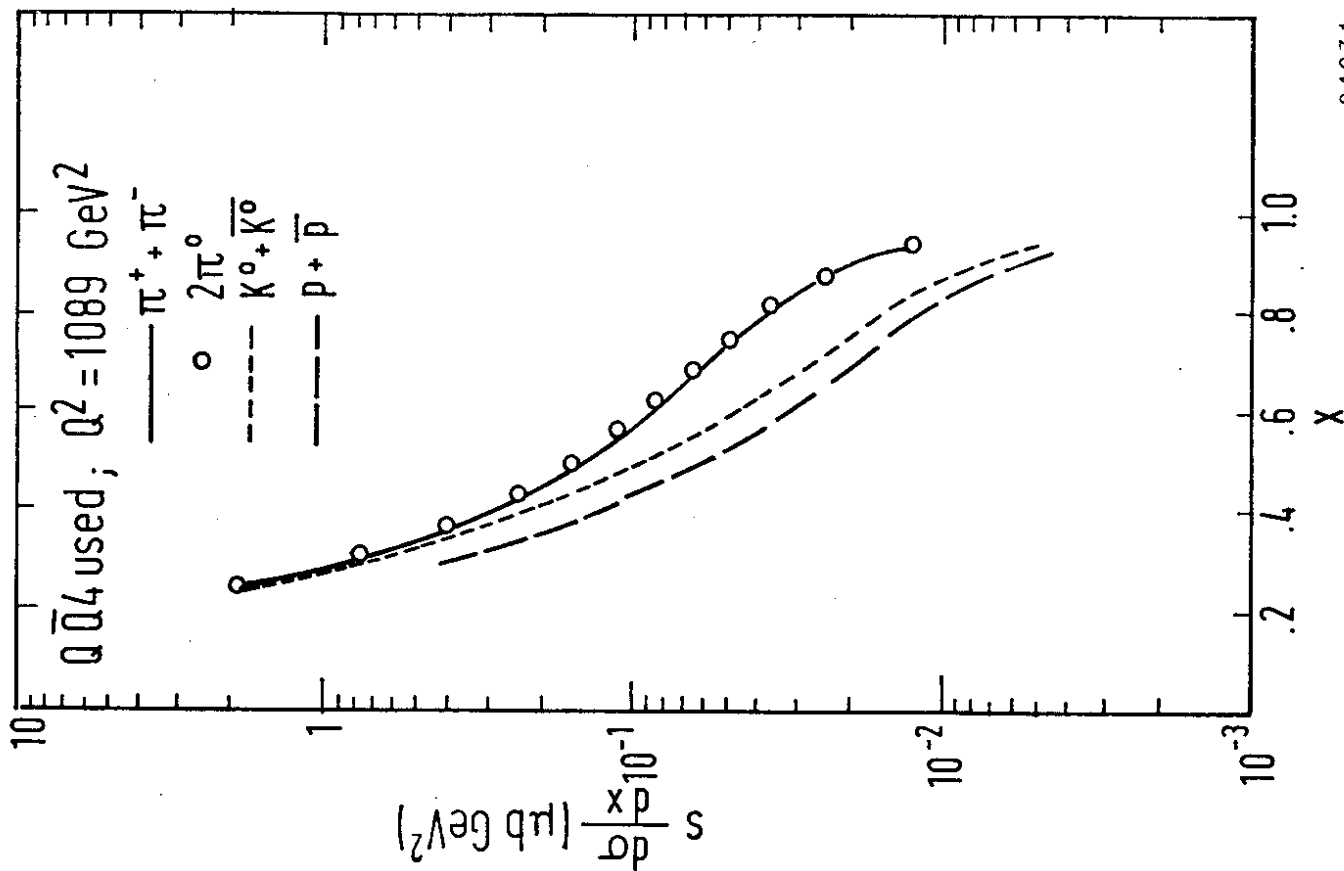
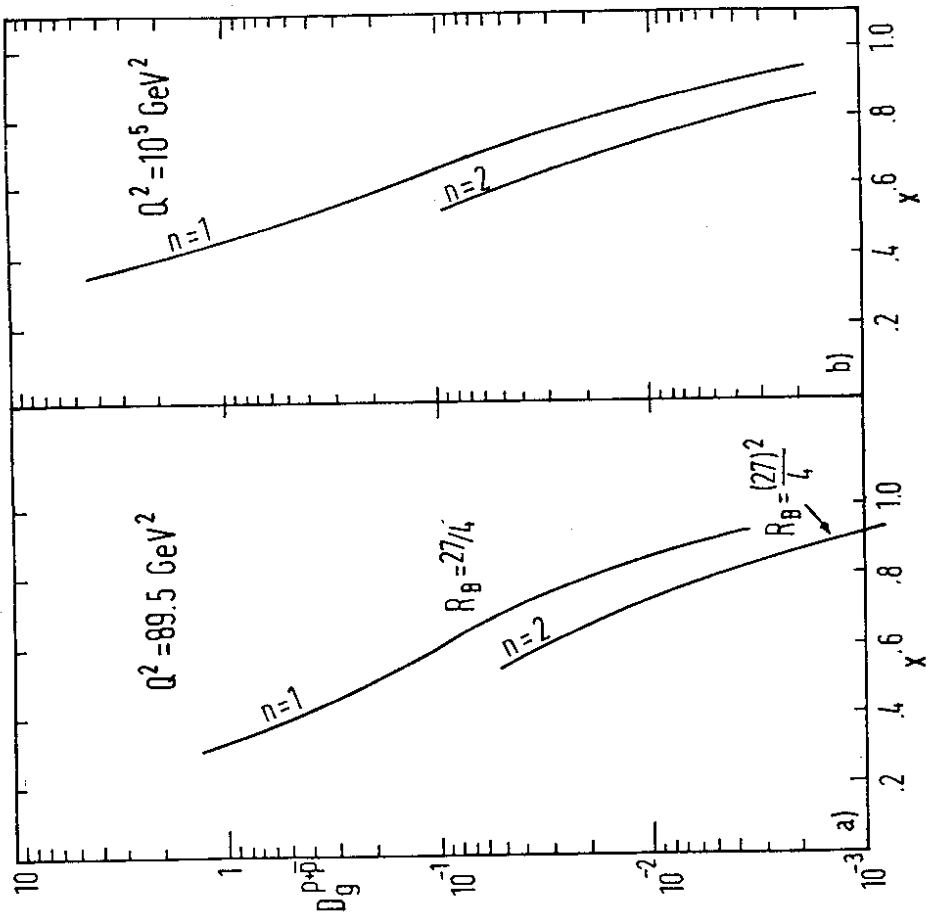


Fig A.1

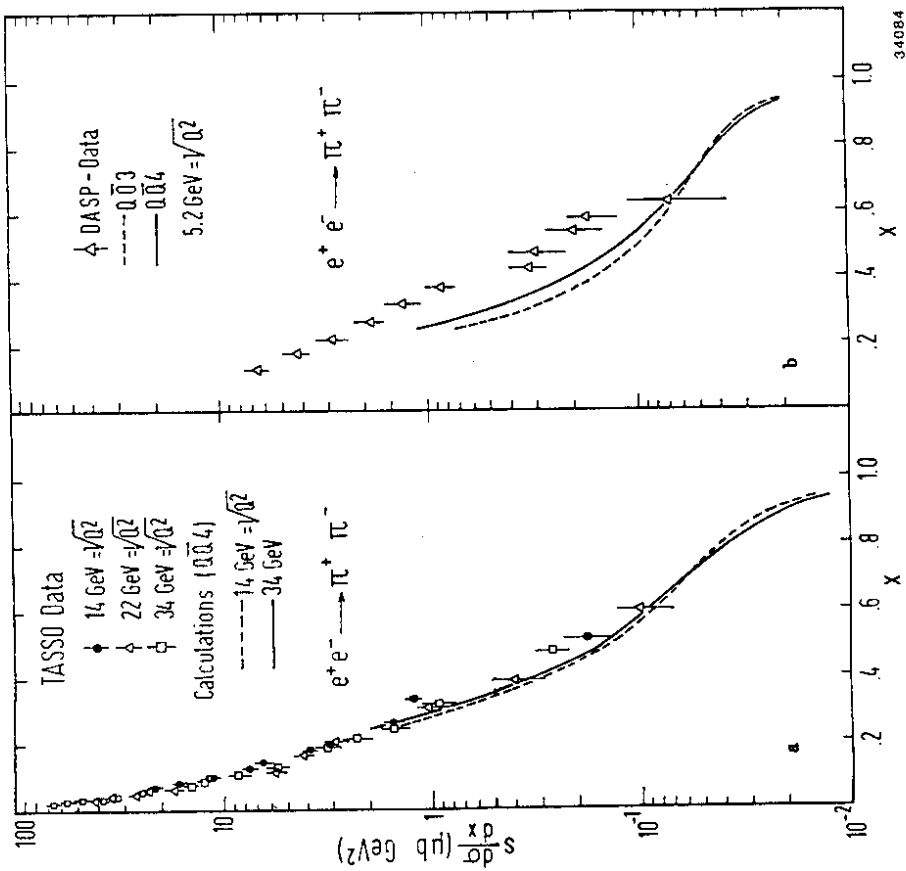
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Fig. C.1



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Fig B.2

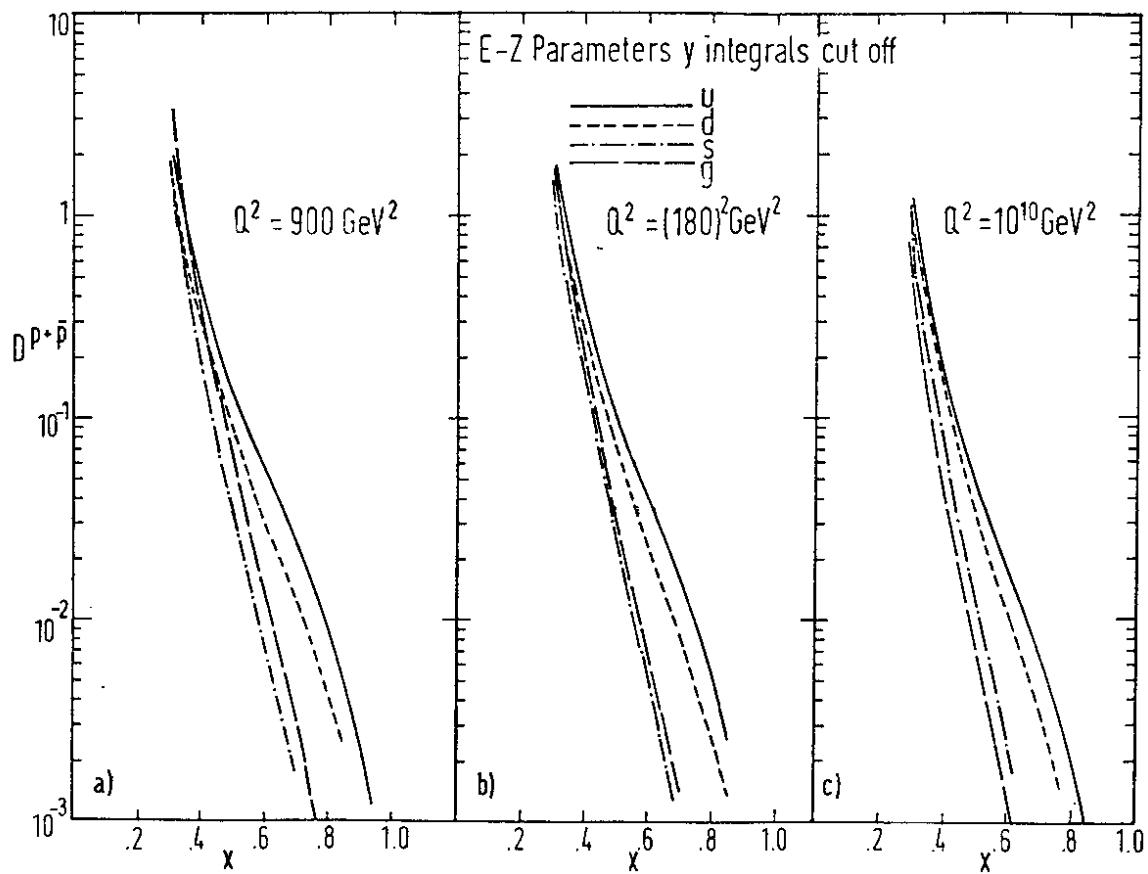
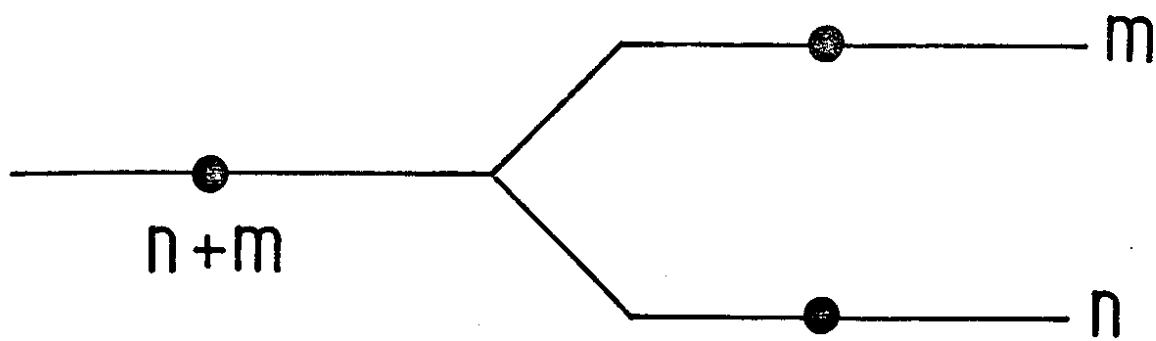


Fig. C 2

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Fig. C. 3

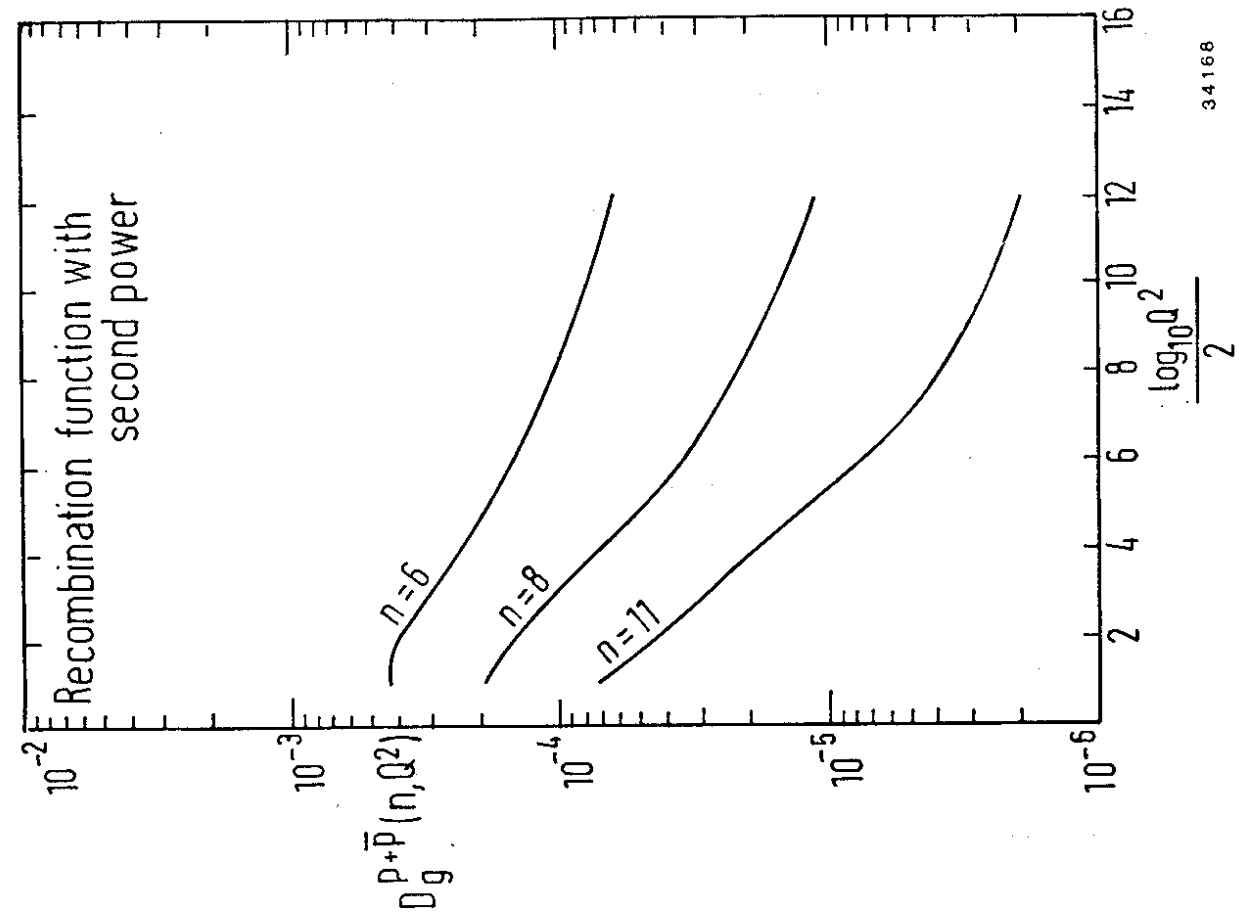


Fig. C.4

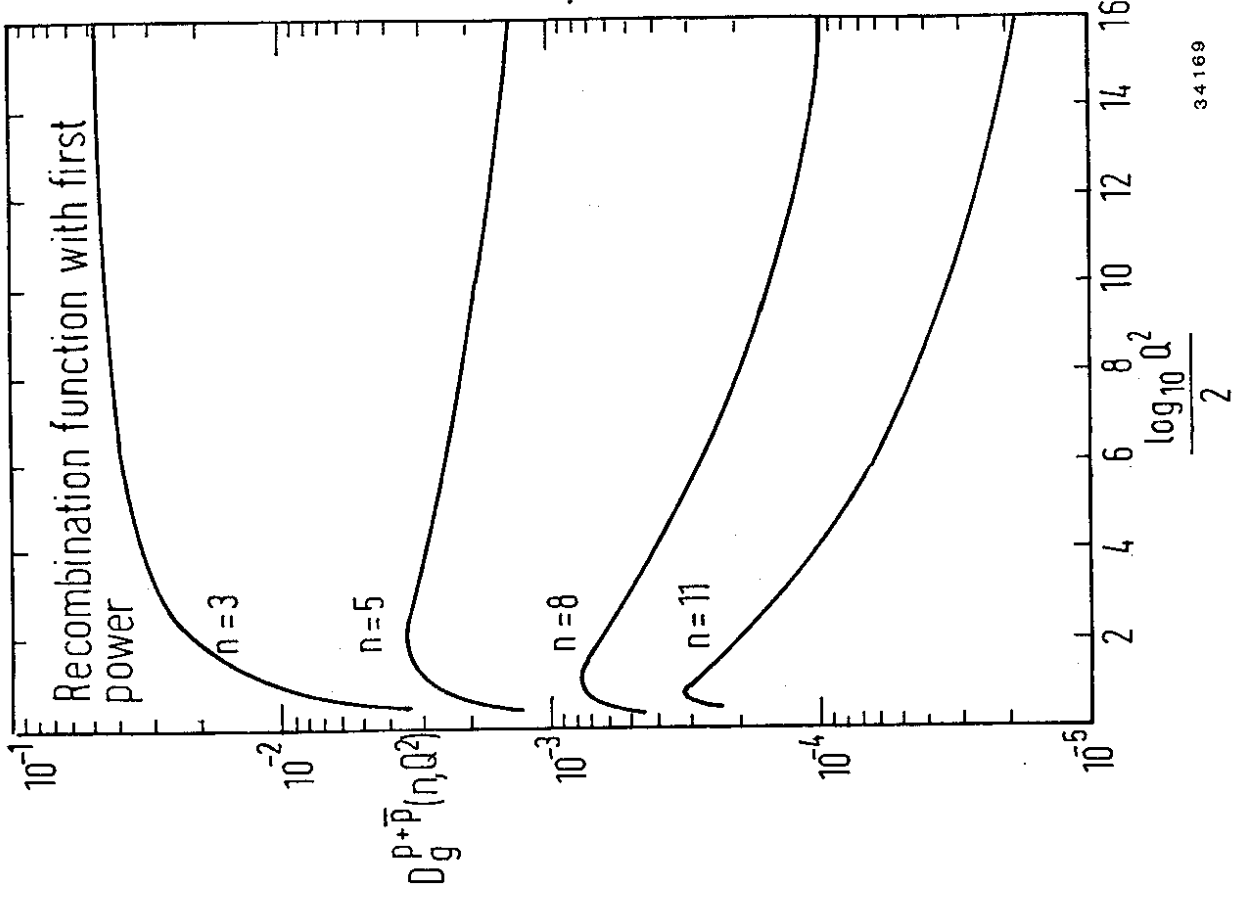


Fig. C.5

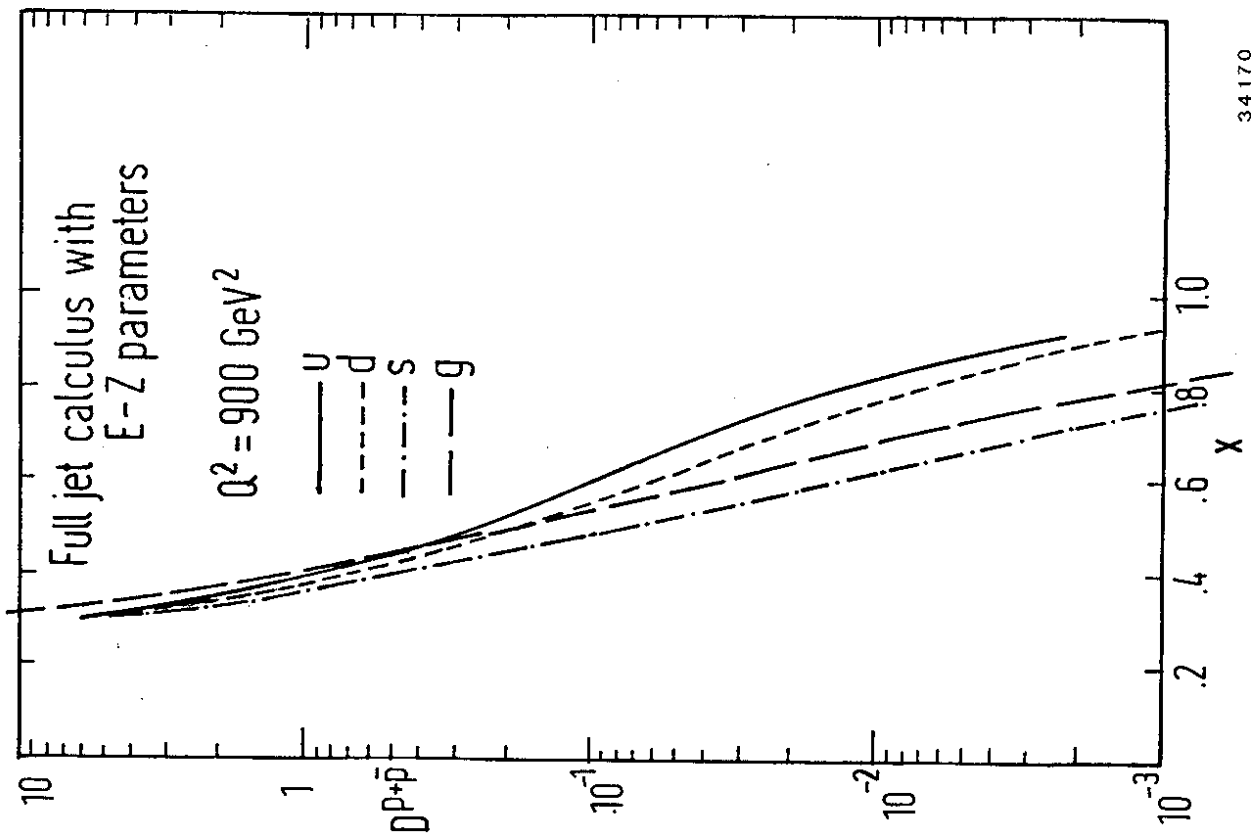


Fig. C.6