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OBSERVATION OF FOUR-JET STRUCTURE IN e^+e^- - ANNIHILATION AT $\sqrt{s}$ = 33 GeV

by

JADE Collaboration

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Observation of Four-Jet Structure
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JADE Collaboration

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ABSTRACT

Topological distributions of hadrons from the reaction $e^+e^- \rightarrow$ hadrons are studied at center of mass energies of about 33 GeV. The experimental distributions in the parameters acoplanarity and tripodity, both sensitive to events with a four-jet structure, show significant deviations from the expectations for two- and three-jet events. They can be described well by the inclusion of four-jet events. The relative magnitude of the observed effect indicates second order QCD as its probable origin.

Investigations of e^+e^- - annihilations into hadrons at PETRA energies have shown that, in addition to events with a dominant two-jet structure, there is evidence for a significant percentage of planar three-jet events 1). These events are generally interpreted as being due to hard gluon bremsstrahlung by quarks, as predicted by QCD 2). Within this framework, however, higher order diagrams give rise to events with four partons in the final state, leading to four-jet events, at a rate proportional to α_s^2. In this letter we report on a search for such four-jet events.

In the following analysis, two event shape parameters are used. The first is the acoplanarity 4)

$$A = 4 \min \left\{ \frac{\sum_i |\vec{p}_i^{\,\perp}|}{\sum_i |\vec{p}_i|} \right\}^2$$

where the $\vec{p}_i$ are the particle or parton momenta and the $\vec{p}_i^{\,\perp}$ are their components perpendicular to a plane which is oriented such that the quantity in brackets is minimized. Whereas, before hadronisation, two- and three-jet events have zero acoplanarity, four-jet events in general are nonplanar and give nonzero values for this parameter.

The second jet variable used is the tripodity D_3, recently introduced by

Nachtmann and Reiter 5), and defined as

$$D_3 = 2 \max \left\{ \frac{\sum_i |\vec{p}_i{}^T| \cos^3 \sphericalangle(\vec{n}, \vec{p}_i{}^T)}{\sum_i |\vec{p}_i|} \right\}$$

where the $\vec{p}_i{}^T$ are the particle or parton momenta projected onto the plane perpendicular to the event thrust axis, and $\vec{n}$ is a unit vector in this plane, oriented such that the quantity in brackets is maximized.

D_3 measures the symmetry of the momentum distribution in the plane normal to the thrust axis. If this distribution is symmetric with respect to the interaction point, $D_3 = 0$. The allowed range of D_3 is most easily understood for events without fragmentation of quarks and gluons. For two- and three-jet events, D_3 vanishes due to a symmetric distribution. Four-jet events fall into two separate classes (see insert in Fig. 1) : events in class I have two parton momenta on each side of the plane normal to the thrust axis, which leads to a symmetric distribution and vanishing D_3. Events in class II have one high momentum in one hemisphere and the three remaining ones in the opposite one, leading to $D_3 \geqslant 0$ with a maximum value of 0.324 5). The distribution of D_3, expected from QCD and other models, is given in Ref. 5.

The aim of this investigation is not to isolate possible four-jet events but instead to study the total sample of multihadron events, in order to keep the analysis as free as possible from the influence of cuts. The analysis was carried out on multihadron events obtained with the JADE detector 3) at the PETRA e^+e^- storage ring at center of mass energies between 30 and 35 GeV, yielding 33 GeV on average. To obtain a clean sample of multihadron events the following cuts were made : the total visible energy was required to be greater than 12 GeV; charged particles originating from the vertex as well as neutrals were used. In order to minimize the number of events with significant particle loss it was required that the angle between the event thrust axis and the beam axis be larger than 37 degrees. Events with a high momentum imbalance were eliminated by the requirement $|\sum_i \vec{p}_i| < 10$ GeV/c, where the sum extends over all particle momenta $\vec{p}_i$, charged and neutral. The remaining events were momentum balanced through the introduction of a hypothetical particle with the missing momentum.

The variables acoplanarity A and tripodity D_3 were evaluated in two different ways, both from reconstructed parton axes and directly from the particle momenta. For the following discussion primarily the first approach was used. For each event four jet axes were determined, using a jet reconstruction algorithm 6). The algorithm has been tested with artificially generated events and was found to reconstruct the original parton direction typically within 10 degrees, for four-jet events 6). In order to define meaningful jets, at least three particles were required per axis. With this additional requirement 2560 multihadron events remain in the sample. The experimental distribution A versus D_3 obtained from the reconstructed axes is given in Fig. 1. The data lie in two well defined bands as expected from the kinematics of four momentum-balanced axes.

Since four axes have been fitted for all events, the A and D_3 distribution for two- and three-jet events had to be studied as well as for the four-jet events predicted by second order QCD. For that purpose two- and three-jet events were generated using the Lund model 7) with fragmentation along the colour strings. As described elsewhere 8), the Lund hadronisation appears to describe the two- and three-jet data better than models with fragmentation along the parton axes 9,10). Four-jet events were generated according to the generator of Ali et al. 10); for fragmentation again the Lund scheme was used 11). The influence of details of this fragmentation model will be discussed later. The fraction δ of four-jet events contained in the total number of events depends on the strong coupling constant α_s, and the cut in acoplanarity made to avoid infrared divergences. The value of α_s obtained in leading order from the analysis of the three-jet events is $\alpha_s = 0.17$ 1). With this value and for a cut in acoplanarity of $A > 0.05$, which was used for the four-jet generation in the present analysis, Ali et al. obtain a fraction $\delta \simeq 5\%$ 10). All events were generated taking into account radiative corrections. The experimental resolution and acceptance functions of the JADE detector were applied to these artificially created events. Furthermore, the events underwent the identical chain of analysis programs as the real data.

In Fig. 2a) the experimental D_3 distribution is shown and compared with the theoretical expectations. Here and in the following figures the theoretical curves are normalized to the total number of events. The expectation for two- and three-jet events alone (curve L23 in Fig. 2a) extends to large values of D_3. This is the effect expected from fragmentation, initial state radiation and resolution as well as the fact that four axes were fitted to events with two and three jet structures. The data, however, fall off markedly slower with D_3 and are incompatible with two and three jets alone. Inclusion of four-jet events (curve L234 in Fig. 2a) changes the shape of the theoretical expectation and gives a significantly better fit to the data.

As can be seen from the scatter-plot A versus D_3 (Fig. 1), A and D_3 are independent variables for $D_3 < 0.01$ and the acoplanarity distribution of the events in that bin, 73% of all events, has not been used in the D_3-histogram. For this reason the events with $D_3 < 0.01$ are projected onto the acoplanarity axis. The resulting A-distribution is given in Fig. 2b, together with the theoretical expectations. As in the case of D_3, it is seen that QCD with two and three jets alone cannot explain the data which are, however, well described by the inclusion of four-jet events. This is reflected by the χ^2 values obtained by comparing the data with the curves L23 and L234 (see Table 1). If both the A- and D_3-distributions are considered together, a fraction of 5% four-jet events results in a χ^2 of 11.0 for 12 degrees of freedom, whereas the corresponding number for two and three jets alone is 63.9. It is not possible to fit the data without four-jet events by adjusting only the fraction of three-jet events. The data would then require 100% three-jet events and even then the χ^2 is 25.1 for 12 degrees of freedom.

In the above discussion the proportion δ of four-jet events relative to the total sample was fixed at 0% for L23 and at 5% for L234. We note again that δ is the fraction of four-jet events generated. To see which value of δ the data favour, its value was varied as a free parameter in the fit of L234 to both variables, tripodity for events with $D_3 \geq 0.01$ and acoplanarity for events with $D_3 < 0.01$ (Table 1). The resulting χ^2 distribution as a function of δ is given in Fig. 3. The best fit is obtained for $\delta = (7.2 \pm 1.2)\%$. This value should be compared with the QCD expectation of 5%.

In the determination of α_s, rigorously both three-jet events and four-jet events, and possibly even higher orders, should be considered simultaneously. The close agreement between the value of δ predicted (using a value of α_s based on the experimental observation of three-jet events) and the value of δ obtained in the fit indicates that such a more refined determination of α_s will not change the currently accepted value very much.

The data discussed above are for center of mass energies $\sqrt{s} > 30$ GeV. At lower energies the existence of four-jet events is also predicted by QCD. Their shape is, however, expected to be indistinguishable from that of two- and three-jet events. In figures 2c,d the tripodity- and acoplanarity-distributions are shown for data obtained at $\sqrt{s} = 22$ GeV. Both distributions are well described by the model with two and three-jet events alone, with a χ^2 of 9.1 for 12 degrees of freedom. Only at energies above 30 GeV do the data require the inclusion of four-jet events. This indicates that the effect is not caused by instrumental resolution or uncertainties in the fragmentation model.

In the following, the systematic effects of the method, of different models, and of variations of the model parameters used are discussed in detail.

In the above analysis the variables acoplanarity and tripodity have been calculated from reconstructed jet axes, which were obtained by fitting four axes to each event. In order to see to what extent the above conclusions depend on this method of fitting four axes, the analysis was repeated by determining the variables A and D_3 directly from the observed particle momenta. The resulting distributions are given in Fig. 4. Due to effects of hadronisation, the shapes of the distributions are distorted relative to those obtained from the jet axes. When compared with the model expectations, however, the same conclusion as before is reached about the existence of four-jet events. Considering both distributions together the data require a four-jet event fraction of $(8.2 \pm 1.2)\%$ which results in a χ^2 of 15.7 for 14 degrees of freedom, compared to $\chi^2 = 117.1$ without the inclusion of four-jet events.

It is interesting to note that an analysis of the momentum tensor Q 12) leads to similar conclusions. The experimental ratio of events in the spherical region ($Q_1 \geq 0.06$ and $(Q_3-Q_2)/\sqrt{3} \leq 0.35$) to the total number of events is $(4.6 \pm 0.4)\%$. The expectation with the inclusion of 5% four-jet events is $(4.7 \pm 0.3)\%$ compared to $(2.7 \pm 0.2)\%$ for two and three jets alone.

In order to check the influence of different fragmentation schemes the data were also compared with events generated with the fragmentation along the parton axes 9,10). The same conclusion results. The shape of the two- and three-jet expectation does not fit the data; these are again described much better if four-jet events are included. The best results were obtained with a ratio $\delta = (6.5 \pm 1.2)\%$.

In the model calculations used above, a mean transverse momentum of σ_q = 350 MeV has been used. To check the sensitivity of the analysis to this variable, it was investigated how well the data can be described by the two- and three-jet hypothesis with a variation of σ_q between 320 and 390 MeV. As expected, large transverse momenta during fragmentation make the events more spherical and hence the A and D_3 distributions are described better with larger σ_q - values. Nevertheless, even at the highest value of σ_q studied (390 MeV) the quality of fit without four-jet events is poor and (5.5 ± 1.5)% four-jet events are needed. Such high values of σ_q are not compatible with other experimental distributions like the mean transverse momentum of the particles out of the event plane $<p_\perp^{out}>$. In Table II experimental values of $<p_\perp^{out}>$ are compared with the values expected from the two- and three-jet hypothesis for the above range of σ_q. This is done for all events as well as for those with a spherical configuration ($Q_1 > 0.06$). A value of σ_q = 390 MeV/c is not sufficiently high for spherical events and too high for describing $<p_\perp^{out}>$ for all events. Thus a consistent description of the data is not obtained.

The kinematic region in which four-jet events mainly contribute (large A or large D_3) is enriched by events with the production and decay of the bottom quark. Since the b-quark might fragment with higher transverse momenta than the other quarks, and thus might simulate four-jet events, this question was studied separately. The model was hence modified such that all quarks except the b fragmented with σ_q = 350 MeV, and the b-quark fragmented with σ_q = 390 MeV. With a $b\bar{b}$ production rate of 9% corresponding to its charge this model was unable to describe the A and D_3 distribution. The χ^2 of the fit was 43.6 for 10 degrees of freedom. That b-quark fragmentation is treated correctly in the original model can also be seen from the fact that the A- and D_3-distributions are well described by two and three jets alone at the center of mass energy of 22 GeV - well above the bottom threshold.

The question arises of whether the observed shape of the D_3 and A distribution might be mainly due to two-, three-jet events with a high missing momentum. The analysis was repeated with the cut in missing momentum reduced from 10 to 4 GeV/c. Although statistics are now reduced the conclusion is the same. It was also checked that the distributions in missing momentum for the data and the model events have the same shape. Any effects of the missing momentum would hence also appear in the model predictions.

Finally, the acoplanarity was computed for the data directly from the particle momenta with no correction for the missing momentum at all. Also in this case the data are incompatible with the distributions for two and three jets alone and favour the four-jet interpretation.

In conclusion, the experimental distributions of the jet parameters acoplanarity and tripodity show significant deviations from the expectations for two- and three-jet events alone. These effects which are observed only at center of mass energies above 30 GeV can be explained by the inclusion of events with a four-jet configuration and are well described by different fragmentation models. Both the shape of the distributions and the relative proportion of the excess events indicate second order QCD effects as their probable origin.

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   The following parameters were used : A fragmentation function $f(z) = (1+\beta)\cdot(1-z)^{\beta}$ with $\beta = 0.4$, a production ratio of secondary u, d and s quarks of 3:3:1 and a fraction of pseudoscalar mesons compared to all mesons produced of 50%. The mean transverse momentum relative to the fragmentation axis was $\sigma_q = 0.350$ GeV. For the generation of three-jet events the value of $\alpha_s = 0.17$ was used.

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11. The four-momenta of quarks and gluons were generated according to the generator by Ali et al. 10). Hadrons were then produced according to the Lund model 7) along three colour strings between $\text{quark}_1 - \text{gluon}_1$, $\text{quark}_2 - \text{gluon}_2$, and $\text{gluon}_1 - \text{gluon}_2$, respectively. There is an ambiguity as to the choice of the quark-gluon pairs. Within the statistics of the experiment, both choices resulted in the same distributions. The contribution of four-quark final states to four-jet events is expected to be less than 10% 5) and has been neglected.

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TABLE I

χ^2 - values of the simultaneous fit to the D_3- and A-distributions at $\sqrt{s}$ = 33 GeV (Fig. 2a,b) for different four-jet fractions δ

	4-jet fraction	χ^2/NDF
L23	0 %	63.9 / 12
L234	5 %	11.0 / 12
Best fit L234	(7.2 ± 1.2) %	6.4 / 12

TABLE II

Comparison of experimental mean transverse momentum out of the event plane $<p_\perp^{out}>$ with that obtained for different values of the fragmentation parameter σ_q in MeV/c for two- and three-jet events.

	$<p_\perp^{out}>$ in MeV/c	
	all events	$Q_1 > 0.06$
Data $\sqrt{s}$ = 33 GeV	159 ± 0.5	224 ± 2.7
L23 σ_q = 320	155 ± 0.4	203 ± 2.6
L23 σ_q = 350	161 ± 0.4	212 ± 2.6
L23 σ_q = 390	166 ± 0.4	216 ± 2.4
Data $\sqrt{s}$ = 22 GeV	148 ± 0.8	188 ± 2.8
L23 σ_q = 320	145 ± 0.4	184 ± 1.3
L23 σ_q = 350	151 ± 0.4	183 ± 1.2

FIGURE CAPTIONS

Fig. 1 Experimental scatter-plot of acoplanarity A versus tripodity D_3, as defined in the text, for multi-hadron events.
The data lie in two well defined bands as expected from the kinematics of four momentum balanced axes. Also illustrated are the configurations possible for four axes in space.

Fig. 2 Experimental D_3-distribution for all events and A-distribution for the events with $D_3 < 0.01$. The variables D_3 and A were calculated from four reconstructed axes.
Figs. a) and b) are for $\sqrt{s}$ = 33 GeV, Figs. c) and d) for $\sqrt{s}$ = 22 GeV.
L23 is the model expectation for two- and three-jet events alone. L234 includes in addition four-jet events (5% at generation).
Both L23 and L234 are normalized to the total number of events.

Fig. 3 The χ^2- distribution (12 degrees of freedom) of the simultaneous fit to the experimental A- and D_3-distributions of Fig. 2a,b, as a function of the four jet fraction δ.

Fig. 4 Experimental D_3-distribution (Fig. a) and A-distribution (Fig. b) for all events. In contrast to Fig. 2, here the variables D_3 and A were calculated from the particle momenta. The histograms are model expectations without (L23) and with (L234) the inclusion of four-jet events (for δ = 8% as obtained from best fit).

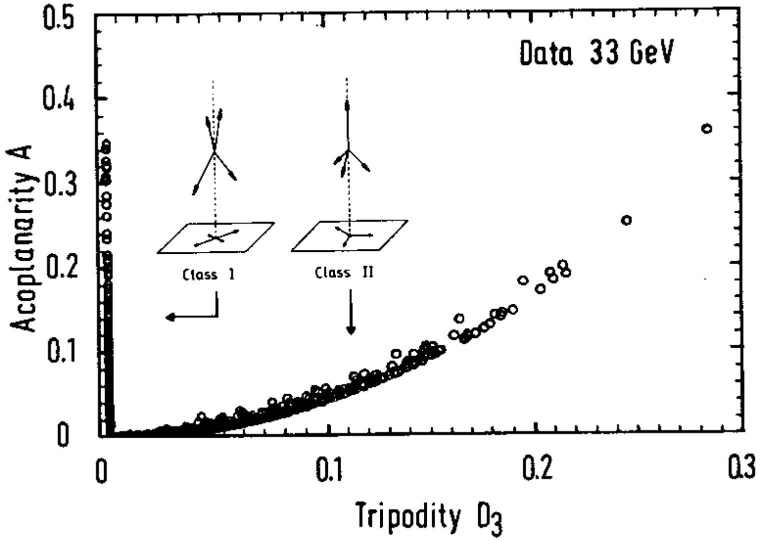
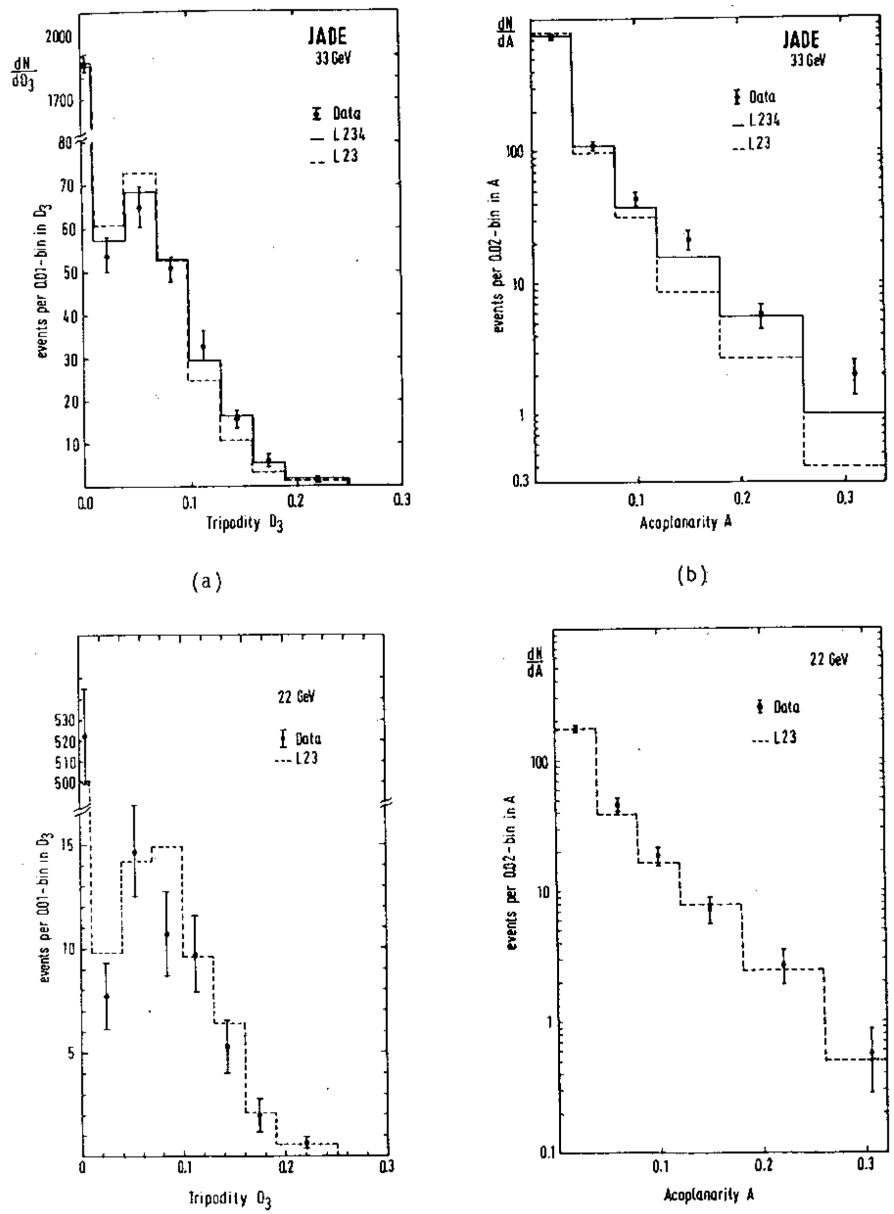


Fig. 1



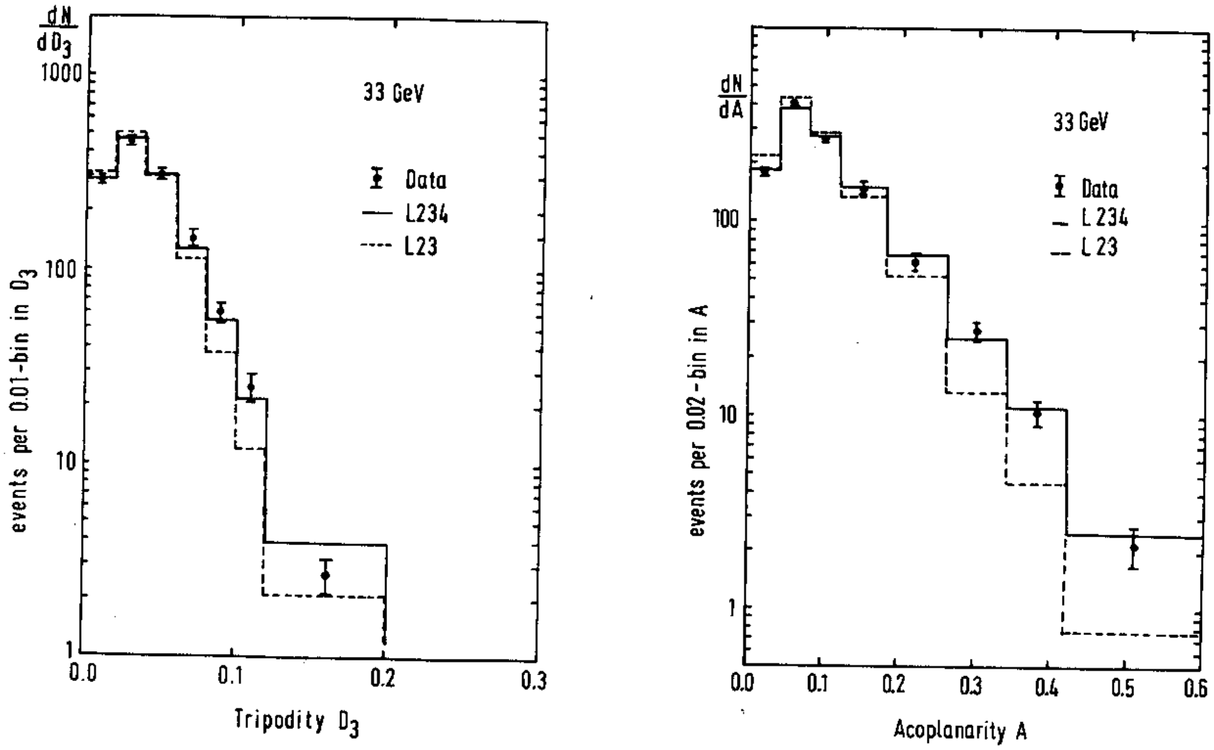
(a)

(b)

(c)

(d)

Fig. 2


dN/dD3
1000
100
10
1
33 GeV
Data
L234
L23
events per 0.01-bin in D3
0.0
0.1
0.2
0.3
Tripodity D3
dN/dA
100
10
1
33 GeV
Data
L 234
L 23
events per 0.02-bin in A
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Acoplanarity A

(a)

(b)

Fig. 4

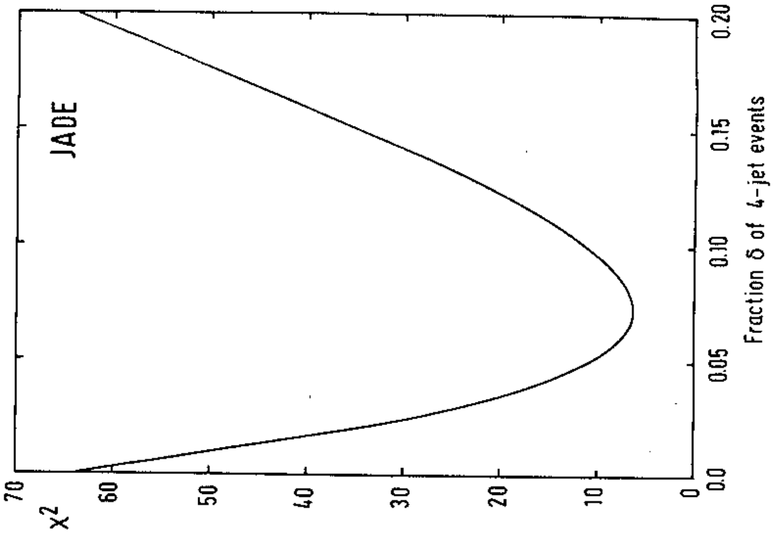

JADE
χ²
70
60
50
40
30
20
10
0
0.0
0.05
0.10
0.15
0.20
Fraction δ of 4-jet events

Fig. 3