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THE QCD EFFECTIVE COUPLING CONSTANT IN e^+e^- ANNIHILATION

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The QCD effective coupling constant in e^+e^- annihilation*

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Abstract

The QCD effective coupling constant $\alpha_s(Q^2)$ is determined by comparing the $o(\alpha_s)^2$ jet-distributions with the high energy e^+e^- data from PETRA. We get $\alpha_s(Q^2 = 1225\ \mathrm{GeV}^2) = 0.125 \pm 0.01$, which corresponds to $\Lambda_{\overline{MS}} = 110^{+70}_{-50}$ MeV with five flavours.

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1. The observation of multijet events at PETRA(1) can be interpreted as an evidence in support of Quantum Chromodynamics, QCD. Detailed comparison of QCD-based models incorporating the $O(\alpha_s)$ and the $O(\alpha_s)^2$ Born diagrams show remarkable consistency with the data, <u>if</u> the intrinsic transverse momentum of the hadrons $\langle p_T\rangle \approx 400$ MeV is taken into account(1),(2).

2. A theoretically meaningful determination of the effective QCD coupling constant, $\alpha_s(Q^2)$, is possible only if the complete $O(\alpha_s)^2$ calculations have been performed for the quantity being analysed. The caluclations for the total hadronic cross-section σ_{tot} were performed by several groups(3) which agree with each other. The result is

$$\sigma_{tot} = \sigma_0 \left(1 + \frac{\alpha_s(s)}{\pi} + (1.98 - .116\, n_f)\left(\frac{\alpha_s(s)}{\pi}\right)^2\right) \quad (1)$$

where the coefficient is specific to a certain regularization prescription, the so-called $\overline{MS}$-scheme(4). The coupling constant $\alpha_s(Q^2)$ in (1) is defined as

$$\alpha_s(Q^2) = \frac{2\pi}{b_0 \ln(Q^2/\Lambda^2) + b_1/b_0 \ln\ln(Q^2/\Lambda^2)} \quad (2)$$

with

$$b_0 = (33 - 2n_f)/6$$
$$b_1 = (153 - 19n_f)/6 \quad (3)$$

The coefficient of $(\alpha_s/\pi)^2$ for $n_f = 5$ is 1.4, which is small and hence inspires confidence in using total cross-section measurements to determine $\alpha_s(Q^2)$. However, the demands that such a determination impose on the experimental accuracy are formidable, if not impossible. The variation of σ_{tot} with $\alpha_s(Q^2)$ at $\sqrt{s} = 35$ GeV can be seen in table 1. At low energies the analysis gets involved due to threshold effects and higher twist contributions.

3. The multijet event rates and distributions are ideally suited for the determination of $\alpha_s(Q^2)$. Since $Q^2 \simeq 1300\ \mathrm{GeV}^2$ at the present PETRA/PEP energies, it is possible to analyse a process in a kinematic region where every sub-energy and invariant mass is large. Consequently, one could apply perturbation theory with confidence. In perturbative QCD such multijet distributions receive contribution in order α_s or higher. This is what makes them preferred quantities over the inclusive measurement of σ_{tot}. The $O(\alpha_s)^2$ calculations for the multijet rates and distributions have also been performed by three independent groups. The calculations of Ellis, Ross and Terrano (ERT)[5] and those of Vermaseren, Gaemers and Oldham (VGO)[6] are numerically in agreement. Calculating $O(\alpha_s)^2$ calculations to the inclusive thrust distribution for $T < T_o$, they obtain

$$f(T_0) \equiv \frac{1}{\sigma_0}\int_{0.5}^{T_0} \frac{d\sigma}{dT} = B_0 \frac{\alpha_s(s)}{\pi}\left[1 + K\frac{\alpha_s(s)}{\pi}\right] \quad (4)$$

where the coefficient $B_o(T_o = 0.85) = 1.156$[7] and

$$K(T_0 = 0.85) = 17.6 \pm 0.3 \quad (\mathrm{ERT})$$
$$= 17.2 \pm 0.2 \quad (\mathrm{VGO})$$

the numbers again correspond to the $\overline{MS}$ scheme. The corrections are not small and disturbing for some tastes, though one should bear in mind that similar phenomenon in electromagnetic interactions, i.e. the dominance of the $O(\alpha)^2$ effects over the $O(\alpha)$ effects in special kinematic domains, is every day occurrence in e^+e^- annihilation and indeed constitutes a test of the underlying theory of electrons and photons, QED!

Fabricius, Schmidt, Schierholz and Kramer (FSSK)[8] have calculated the analogue of the Sterman-Weinberg formula for 3-jets. They find that the 'thrust' distribution calculated for 3-jet events in well-defined cones in the Sterman-Weinberg variables ε and δ[9], $\frac{d}{dT'}(\varepsilon, \delta)$, has a smaller correction compared to the $O(\alpha_s)$ thrust dist. $\frac{d\sigma}{dT}$. In particular the quantity

$$f(\varepsilon, \delta) = \int_{0.5}^{T_0} \frac{d\sigma}{dT'}(\varepsilon, \delta)\, dT' \quad (5)$$

is stable against $O(\alpha_s)^2$ corrections in their calculations. Note that $f(T_o)$ and $f(\varepsilon, \delta)$ are different quantities and it is a fallacy to compare them directly and draw conclusions about the accuracy of one against the other, a point missed unfortunately by many.

4. It is worthwhile to discuss the separate pieces which comprise the $O(\alpha_s)^2$ calculation of multijet distributions. These can be classified as follows:

(i) contribution of $e^+e^- \to q\bar{q}G$ to $O(\alpha_s)$ (Born diagrams)[10]

(ii) contribution of $e^+e^- \to q\bar{q}GG$, $q\bar{q}q\bar{q}$ to $O(\alpha_s)^2$ (Born diagrams)[11]

(iii) one-loop $O(\alpha_s)^2$ virtual corrections to $e^+e^- \to q\bar{q}G$

and

(iv) the soft-part of the process $e^+e^- \to q\bar{q}GG$, $q\bar{q}q\bar{q}$ contributing to the 3-jet configuration alone.

The processes listed in (i) and (ii) are not new ingredients of the calculations of ERT, VGO and FSSK. They have been previously calculated and verified and hence not controversial. There is agreement on point (iii) among ERT and FSSK, as well as on the cancellation of singular pole terms between (iii) and (iv).[12] It is then clear that the difference has to be traced to the finite parts of the soft process $e^+e^- \to q\bar{q}GG$, $q\bar{q}q\bar{q}$, namely point (iv).

There are several ways to calculate the finite contribution from the 4-parton final states contributing to the 3-jet configuration. The simplest ansatz is to work with invariant masses as done by ERT. The infrared and collinear singularities occur when an invariant mass $s_{ij} = (p_i + p_j)^2$ approaches zero. To be definite let us consider the singularity in y_{13} ($\equiv s_{13}/Q^2$). One could split the y_{13} integration as follows[5]

$$\int dy_{13} \to \theta(1 - y_{123} - y_{134}) \int_0^{y_0} dy_{13} + \theta(y_{123} + y_{134} - 1) \int_{y_{123}+y_{134}-1}^{y_0} dy_{13} \qquad (6)$$

where $y_{ijk} = s_{ijk}/Q^2$, $s_{ijk} = (p_i + p_j + p_k)^2$, $y_0 = y_{123}\, y_{134}$. A term proportional to $\frac{1}{y_{13}}$ gives a singular contribution only in the region of the first integral. In n-dimensions, these singularities appear as pole in the quantity $\epsilon = n-4$. Thus, one has to evaluate the probability density in n dimensions to extract the contribution from the first integral. The second integral being finite, one could use the expressions in 4 space-time dimensions derived earlier. A Monte Carlo integration technique could then be used to evaluate this contribution, which we shall call a genuine 4-jet process defined here by a cut on y_0, i.e. $y_{13} > y_0$ (and similarly for other variables).

Let us now concentrate on the contribution in the region of the first integral. A way to calculate this quantity is to follow ERT by doing the integrations analytically for the singular part of the probability density (in the limit $y_{ij} \to 0$) and evaluating the rest numerically. The singular parts of the relevant matrix element squared are given in Eqs. (3.17), (3.19) and (3.22) of ERT. We have checked the analytic integrations leading to Eq. (3.25) of ERT.

Note that the finite part of Eq. (3.25) of ERT consists of that piece which arises by integrating only the singular part of 4-parton probability density in the first integration region. In addition, the probability density has also non-singular pieces whose contribution in the first integral is hard to evaluate analytically. We evaluate this contribution numerically. Our prescription is straightforward. We introduce a very small invariant mass cut-off parameter, μ_0, and evaluate the $\sigma_{4\text{-partons}}$ in the restricted Kinematic domain defined by $\mu_0 < y < y_0$; $T < T_0$; where the 4 partons masquerade as 3-jets. Then we calculate

$$\sigma^{\text{Born}}_{4\text{-parton}}(\mu_0, y_0, T_0) = \int_{S(\mu_0, y_0, T_0)} dP_S \, |M|^2_{\text{Born}} \qquad (7)$$

$$\sigma^{\text{sing.}}_{4\text{-parton}}(\mu_0, y_0, T_0) = \int_{S(\mu_0, y_0, T_0)} dP_S \, |M|^2_{\text{sing.}}$$

The integrand $|M^2|_{\text{Born}}$ in the above equation is the Born-term matrix element squared for the processes $e^+e^- \to q\bar{q}gg$, $q\bar{q}q\bar{q}$ and $|M^2|_{\text{sing.}}$ is its singular part. The finite contribution is now simply defined as

$$\sigma^{\text{finite}}_{4\text{-parton}} = \lim_{\mu \to 0} \left[\sigma^{\text{Born}}_{4\text{-parton}} - \sigma^{\text{sing.}}_{4\text{-parton}} \right] \qquad (8)$$

We have evaluated this quantity using Monte Carlo techniques. This has to be added to Eq. (3.26) of ERT to define the complete $O(\alpha_s)^2$ corrections to

$\sigma_{3jet}(T_o, y_o)$ which is defined through the thrust-cut T_o (to distinguish between 2- and 3-jets) and an invariant mass cut-off y_o (to distinguish it from the 4-jets). In this sense the y_o cut-off plays the same role as the photon energy resolution ΔE_γ in estimating QED radiative corrections.

We have checked that, within Monte Carlo errors, $\sigma^{finite}_{4\text{-}parton}(T_o, y_o)$ does not depend on μ_o and its dependence on y_o is proportional to the phase space (for $y_o \to \mu_o$, $\sigma^{finite}_{4\text{-}parton}(T_o, \mu_o) \to 0$ in approximately the same ratio as the phase space). In addition, we find that for reasonable values of y_o (or equivalently jet mass m_J) $\sigma^{finite}_{4\text{-}parton}(T_o, y_o)$ is the dominant contribution of the $O(\alpha_s)^2$ corrections to the 3-jet distributions.

The rest of the $O(\alpha_s)^2$ corrections go in defining the genuine 4-jet process $\sigma_{4jet}(T_o, y > y_o)$, which for example is defined by $s_{ij} > y_o$ for all i,j = 1,..4, $i \neq j$. Both $\sigma_{3jet}(T_o, y_o)$ and $\sigma_{4jet}(T_o, y_o)$ depend sensitively on y_o but the sum is independent of y_o and we have checked this numerically. Thus the corrections to an invlusive quantity, like thrust, are independent of y_o (or m_J).

We have calculated the distributions in the variables C and D introduced by ERT and the thrust distribution calculated by Kunszt *), VGO and by Ellis and Ross. Our results are in agreement with these authors both in normalization and shape. Our results for the K-factor corresponding to various thrust cuts are given in table 3. The K-factor depends on T_o and in particular we find

$$K(T_o = 0.85) = 16.5 \pm 0.8$$

*) Kunszt introduced the invariant mass cut-off variable M_J. Our results are not in numerical agreement with his results (ref. (7)). However, since then he has found the errors and his corrected results are in agreement with ours. See Z. Kunszt, CERN Report (to be published).

for the massless quark case with $n_f = 5$. We have also studied the quark mass effects in the Born terms and find them to be small (~ 2 %) at $\sqrt{s} = 35$ GeV. Thus, the entire $O(\alpha_s)^2$ corrections to $\sigma(T_o)$ vary between 57 % (for $T_o = 0.95$) and 75 % (for $T_o = 0.75$) at PETRA energies for $\Lambda_{\overline{MS}} \simeq 100$-150 MeV.

The K-factor is a good measure of the $O(\alpha_s)^2$ corrections if detailed information about the final hadronic states is not completely available, which for example is the case in the Drell-Yan process. In e^+e^- annihilation, however, the final states can be classified as multijet states with well-defined probabilities for each multijet configuration. Thus, at PETRA energies, there is good evidence for 2-, 3- and 4-jets with the rates in approximate agreement with the perturbative QCD estimates presented in table 1. It is prudent to analyse the final states in e^+e^- annihilation in terms of multijet configurations with definite jet multiplicity, and estimate the $O(\alpha_s)^2$ corrections to each jet multiplicity configuration. This of course needs a definition of a jet and it is here that experimental information could be used to define meaningful multijet configurations which could be compared with data --- some day directly.

Since the 2-jets first appeared at SPEAR at $\sqrt{s} \simeq 5$-6 GeV and the average invariant mass of a jet at PETRA energies is ~ 5 GeV, it is not a bad guess to use $M_J = 5$ GeV to define a jet and estimate the jet-multiplicity and the relative probability. With this definition we find that the correction to the genuine 3-jet cross-section at $\sqrt{s} = 35$ GeV is ~ 35 % --- not an alarmingly large number, at the same time $\sigma_{4jet}/\sigma_{tot} \simeq (3$-$5)$ % (see table 3). The frequency of 4-jets is in agreement with experimental data. So, whereas we agree with the algebraic calculations of ERT and VGO, we don't share their skepticism about the convergence of perturbation theory in jet distributions.

5. Having convinced ourselves of the validity of the parton-level calculations for $f(T_o)_{ERT}$, the next step is to make a comparison of the $O(\alpha_s)^2$ distributions with the data and determine the value of $\alpha_s(Q^2)$. However, there is a non-trivial step of converting quarks and gluons into hadrons. This was attempted phenomenologically by using an extended Field-Feynman model [13] described in ref. (14); the model has subsequently been studied by the experimental collaborations at PETRA and PEP and found to be in rather good agreement with their data. The single most important parameter which determines the dominant non-perturbative effects is the intrinsic-p_T of the hadrons. This is assumed to have the form $\exp(-k_T^2/2\sigma_q^2)$ in the Field-Feynman model. Detailed studies of the entire PETRA energy data gives [15]

$$\sigma_q = 0.32 \pm 0.04 \text{ GeV} \tag{9}$$

for the intrinsic transverse momentum of the quarks giving $\langle p_T \rangle_{hadron} \simeq 400$ MeV. We shall use this value to determine the background 2-jet events which have a tail in the thrust (or any other related) distribution. Our calculation show that at $\sqrt{s} = 35$ GeV and σ_q in the range of Eq. (9) 2-jet events don't contribute below a thrust $T_o = 0.82$. This is shown for the value $\sigma_q = 0.32$ GeV in Fig. 1 (dashed curve). Thus, the tail of the distribution in $\frac{1}{\sigma}\frac{d\sigma}{dt}$ receives contribution only through $O(\alpha_s)$ and higher perturbative QCD diagrams. The fraction of events below T_o, $f(T_o)$, can then be used to determine $\alpha_s(Q^2)$. Using the best value $\sigma_q = 0.32$ GeV, we determine $f(T_o)$. The non-perturbative-p_T convoluted $O(\alpha_s)^2$ expression can again be used to express the result as an interpolating function

$$f(T_o) = K_1 \frac{\alpha_s(s)}{\pi}\left[1 + K_2 \frac{\alpha_s(s)}{\pi}\right] \tag{10}$$

and we determine the K-factors to be

$$K_1(T_o = 0.82) = 1.83 \pm 0.1$$
$$K_2(T_o = 0.82) = 16.1 \pm 1.0 \tag{11}$$

where the errors reflect both variation in σ_q and our Monte Carlo errors. It is remarkable that the factor K_2 has, within errors, the same value as in the pure parton level $O(\alpha_s)^2$ QCD calculations. We compare Eq. (9) using the parameters (10) with the TASSO data, shown in Fig. (1). The data has been corrected for acceptance and radiative corrections. Using the TASSO data

$$f(T_o = 0.82) = 0.124 \pm 0.013$$

our best determination of $\alpha_s(Q^2)$ (see fig. 2) is:

$$\alpha_s(Q^2) = 0.128 \pm 0.013 \tag{12}$$

The shape of the distribution $\frac{1}{\sigma}\frac{d\sigma}{dT}$ is also in good agreement with the TASSO data, as shown in Fig. (1) (solid curve).

We have also analysed the Mark-J data, where we chose to compare the theoretical calculations with the fraction of events chosen by putting a cut on oblateness. This was done to minimise the dependence on σ_q as studied by the Mark-J collaboration. Using (O_B = oblateness of the broad jet) the Mark-J data gives $N(O_B > 0.3)/N_{TOT} = 0.127 \pm 0.005$. The prediction of the $O(\alpha_s)^2$ corrected QCD calculations is shown in Fig. (3). Our best fit is

$$\alpha_s(Q^2 = 1225 \text{ GeV}^2) = 0.122 \pm 0.01 \tag{13}$$

Taking the average of the TASSO and Mark-J data we find

$$\alpha_s(Q^2 = 1225\,GeV^2) = 0.125 \pm 0.01 \qquad (14)$$

Corresponding to

$$\Lambda_{\overline{MS}} = 110 \, ^{+70}_{-50} \, MeV \qquad (15)$$

for the QCD scale parameter in the $\overline{MS}$ scheme, with five flavors. The errors don't include the systematic errors on the experimental data. Since the $O(\alpha_s)$ distributions are not significantly changed we expect other data to yield a similar result. The details of the calculations and the comparison with the rest of the data will be published elsewhere.

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$\Lambda_{\overline{MS}}$ (GeV)	$\alpha_s(Q^2)$	$\frac{\sigma_{tot}}{\sigma_{\mu\mu}}$	$\frac{\sigma_2}{\sigma_{tot}}$	$\frac{\sigma_3}{\sigma_{tot}}$	$\frac{\sigma_4}{\sigma_{tot}}$
0.03	0.1033	3.793	0.728	0.243	0.029
0.05	0.1108	3.802	0.702	0.265	0.033
0.075	0.1176	3.809	0.678	0.284	0.038
0.10	0.1229	3.815	0.658	0.301	0.041
0.15	0.1314	3.826	0.627	0.326	0.047
0.20	0.1381	3.835	0.60	0.348	0.052
0.25	0.144	3.842	0.578	0.366	0.056

Table 1

Total and multijet cross sections at $\sqrt{s}$ = 35 GeV. The 3-jet cross-sections are defined with a thrust cut-off $T_o < 0.95$ and the 4-jets with $T_o < 0.95$ and the invariant mass M_{ij} cut-off $M_{ij} > 5$ GeV. $n_f = 5$. Typical errors on σ_3 and σ_4 are ±3 %.

T_o	$K(T_o)$
0.95	14.4 ± 0.7
0.90	15.5 ± 0.8
0.85	16.5 ± 1.0
0.80	18.9 ± 1.0
0.75	19.7 ± 1.0

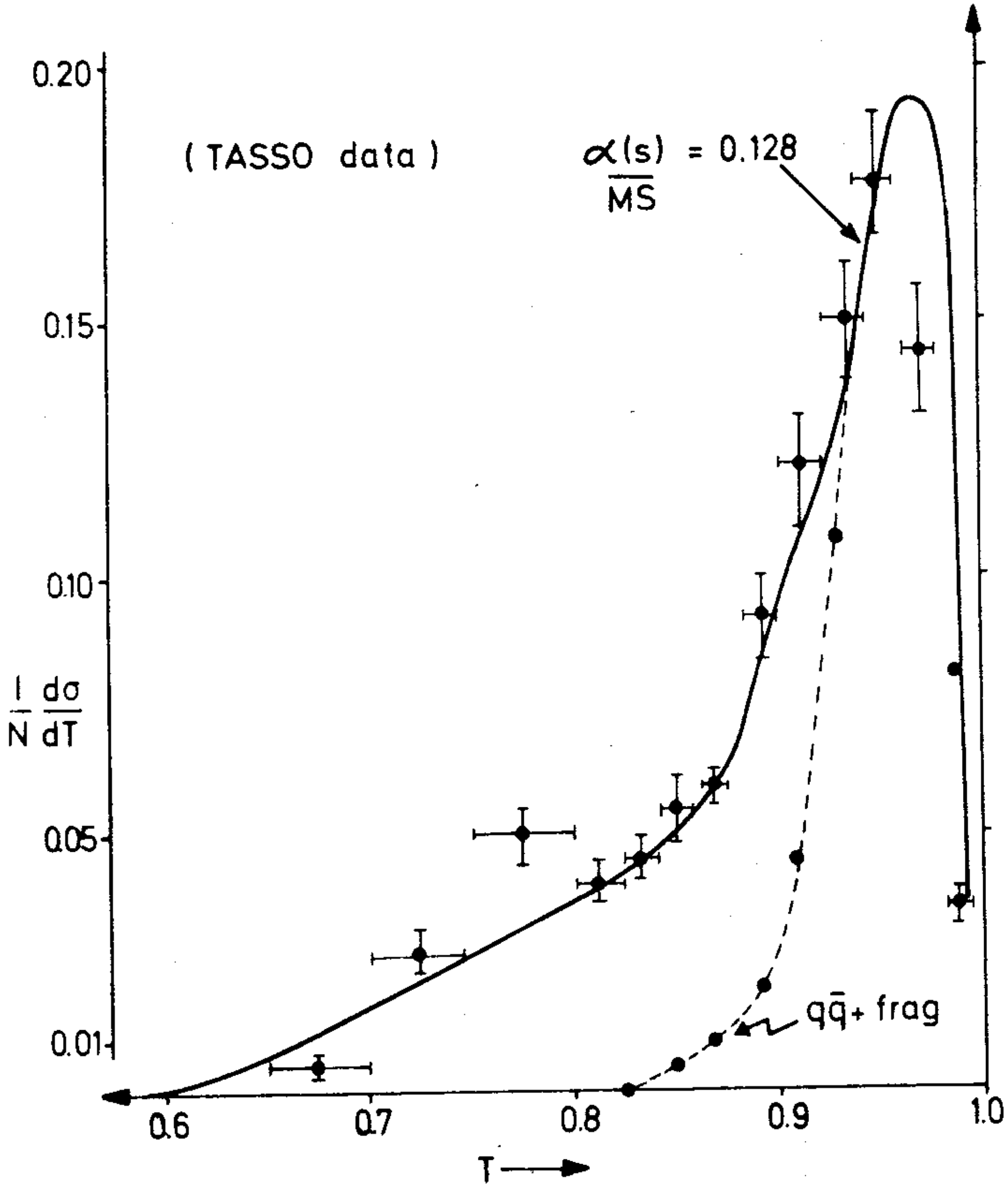
Table 2

The K-factor defined in Eq. (4) for the various thrust cut-off.

α_s	$\frac{\sigma^{finite}_{4\text{-parton}}}{\sigma^3_{Born}}$	$\frac{\sigma_3}{\sigma^3_{Born}}$	$\frac{\sigma_4}{\sigma_3}$
0.103	0.25	1.32	0.12
0.1176	0.284	1.353	0.13
0.1314	0.32	1.40	0.14
0.144	0.35	1.43	0.15

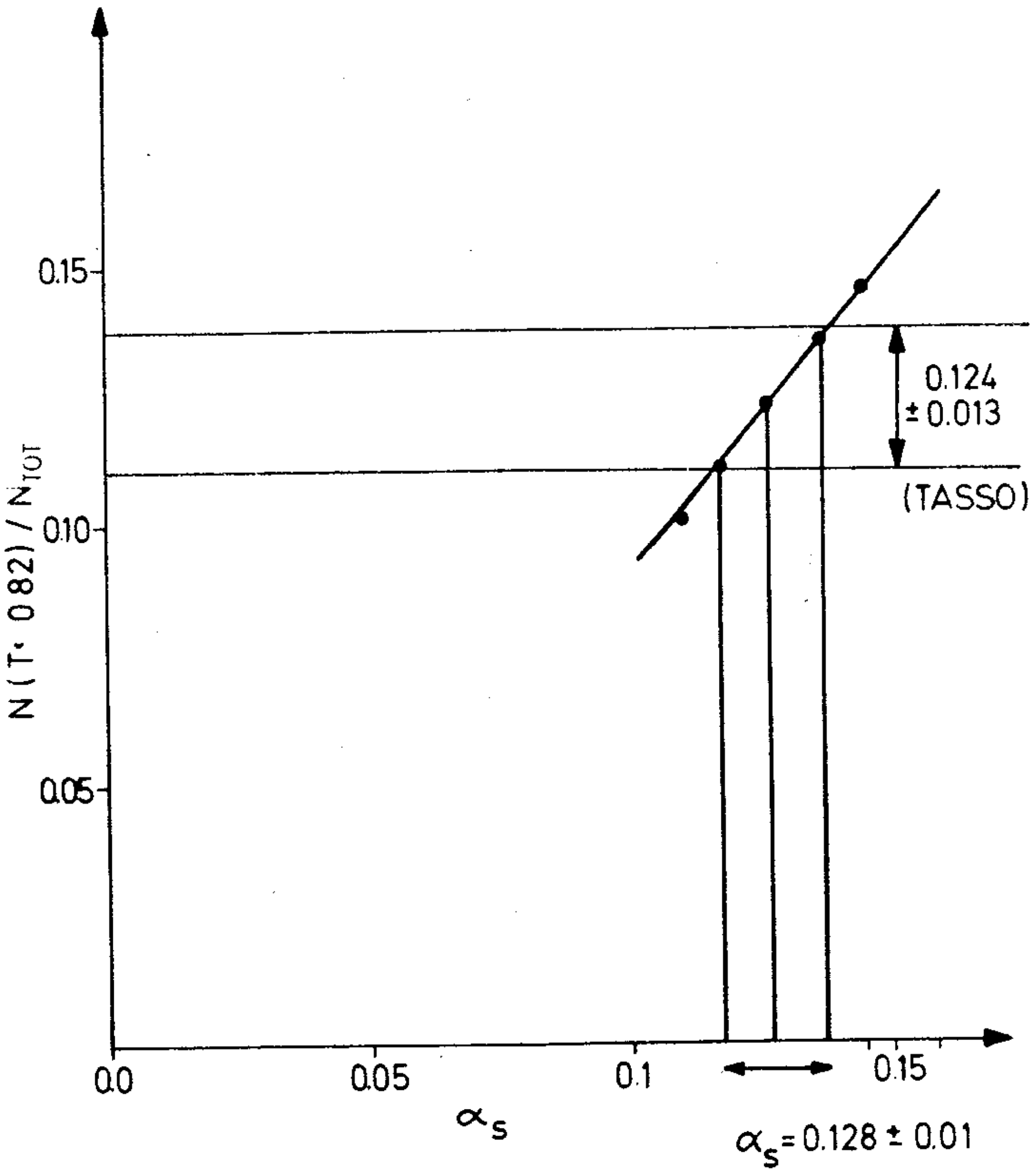
Table 3

$O(\alpha_s)^2$ corrections to the 3 jet cross section at $\sqrt{s}$ = 35 GeV. Also shown are the hard and soft part of the 4-jet cross-section. 3 and 4 jets are defined as in table 1.



(TASSO data)
α(s) = 0.128
MS
q q̄ + frag
1/N dσ/dT
0.20
0.15
0.10
0.05
0.01
0.6
0.7
0.8
0.9
1.0
T

Fig. (1)



0.124
± 0.013
(TASSO)
N(T‹ 0.82) / N_TOT
0.15
0.10
0.05
0.0
0.05
0.1
0.15
α_s
α_s = 0.128 ± 0.01

Fig. (2)

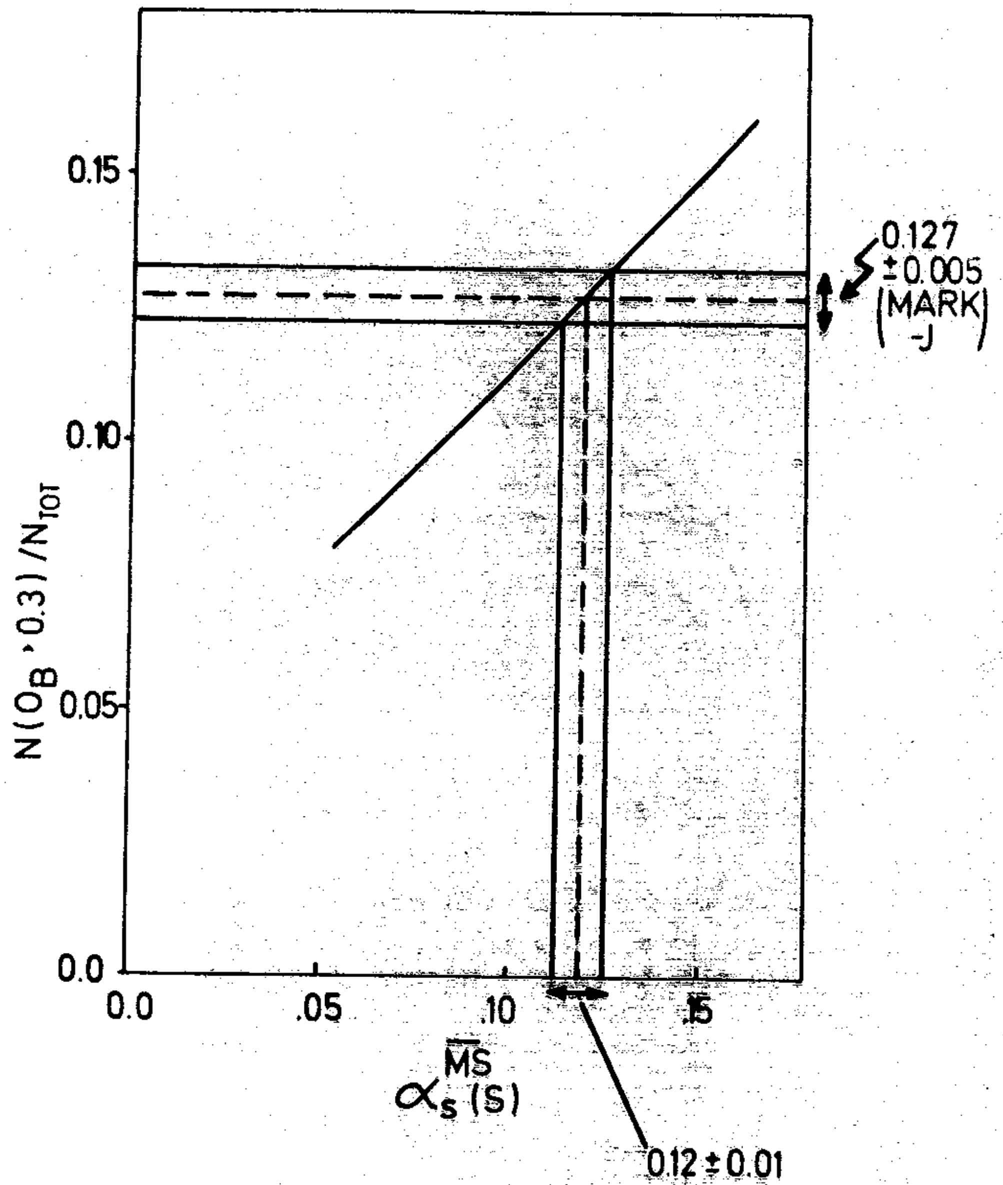

0.15
0.10
0.05
0.0
N(O_B › 0.3) / N_TOT
0.0
.05
.10
.15
MS
αs (S)
0.127
±0.005
(MARK-J)
0.12 ± 0.01

Fig. (3)