DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 81-050
August 1981

MULTIPLE BREMSSTRAHLUNG IN GAUGE THEORIES AT HIGH ENERGIES

I: GENERAL FORMALISM FOR QUANTUM ELECTRODYNAMICS

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NOTKESTRASSE 85 · 2 HAMBURG 52

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I: General Formalism for Quantum Electrodynamics *

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* Work supported in part by the NATO Research Grant No. RG 079.80.

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§ Work supported in part by the United States Department of Energy under Contract No. DE-AS02-76-ER03227.

Abstract

Multiple bremsstrahlung is studied on the level of tree diagrams for gauge theories. At high energies and in most of the kinematic region, the fermion mass can be neglected. In this case, it is natural to introduce helicity states for both fermions and gauge particles. Our general formalism is given in detail for quantum electrodynamics. In particular, it is expedient to use photon polarization vectors which depend on the fermion helicities. In this way, extensive cancellations between Feynman diagrams are accomplished automatically.

1. Introduction

Perturbation calculations in quantum electrodynamics can be found in a number of textbooks; those without ultraviolet divergences have been known for about half a century. Yet, there is something mysterious and unsettling about such calculations: the result is often much simpler than the intermediate steps. It is the purpose of this series of papers to investigate, for the ultra-relativistic limit, the use of helicity states to avoid much of the intermediate steps [1]. Although the motivation for this investigation comes from QCD, involving especially the very light u and d quarks, the first five papers in this series will be devoted to quantum electrodynamics.

Consider as an example the bremsstrahlung process

$$e^+e^- \longrightarrow e^+e^- \gamma , \tag{1.1}$$

which is important in luminosity measurements at high-energy e^+e^- accelerators such as PETRA. At high energies and for photons radiated in directions away from the electrons and positrons, the electron mass may be neglected, at least as a first approximation. In lowest order, there are eight Feynman diagrams, and hence 36 traces for the differential cross section. It was recently realized that, when these numerous terms are combined and put on a common denominator, a large number of cancellations occur, leading to an elegant factorization [2]. This result was first obtained by brute force, and gives a nice illustration of the observation in the first paragraph.

Once the result is known, verification through computer programs such as REDUCE or SCHOONSCHIP is quite easy.

It is natural to inquire whether similar simplifications also occur in double bremsstrahlung processes, such as

$$e^+e^- \longrightarrow e^+e^- \gamma\gamma . \tag{1.2}$$

Although it is inconceivable not to have great simplifications, all attempts to get a relatively simple answer through computer programs have failed. It is for the reason that the present method of helicity states is devised.

In this paper I, the general methodology is discussed for quantum electrodynamics. Bremsstrahlung processes such as (1.1) are studied in paper II, and double bremsstrahlung processes, such as

$$e^+e^- \longrightarrow 4\gamma , \tag{1.3}$$

$$e^+e^- \longrightarrow \mu^+\mu^- \gamma\gamma , \tag{1.4}$$

and (1.2) in papers III to V.

2. Polarization Vectors

Consider a photon with four-momentum k_μ . In quantum electrodynamics, a real photon is always attached to an electron line. In lowest order, where only tree diagrams are considered, this electron line eventually ends as two external particles. For definiteness, let these be an outgoing positron and an outgoing electron, with four-momenta $q_{+\mu}$ and $q_{-\mu}$ respectively such that

$$q_+^2 = q_-^2 = m^2 , \tag{2.1}$$

where m is the electron mass. A massless photon has two polarization states, and an especially convenient choice of the polarization vectors is

$$\varepsilon_\mu'^{\parallel} = N\left[(q_+ k)\, q_{-\mu} - (q_- k)\, q_{+\mu}\right], \tag{2.2}$$

and
$$\varepsilon_\mu^{\perp} = N\, \varepsilon_{\mu\alpha\beta\gamma}\, q_+^\alpha\, q_-^\beta\, k^\gamma , \tag{2.3}$$

where
$$N = \left[2\,(q_+ q_-)\,(q_+ k)\,(q_- k) - m^2 (q_+ k)^2 - m^2 (q_- k)^2 \right]^{-1/2}. \tag{2.4}$$

In (2.2), the prime for ε_μ' merely indicates that this is not quite the polarization vector to be used later on. Note that the normalization factor N is the same for both polarization vectors, and that

$$(\varepsilon'^{\parallel})^2 = (\varepsilon^{\perp})^2 = -1 \tag{2.5}$$
$$(k\, \varepsilon'^{\parallel}) = (k\, \varepsilon^{\perp}) = (\varepsilon'^{\parallel}\, \varepsilon^{\perp}) = 0 .$$

From (2.4) it is clear that the situation is especially simple if the electron mass can be neglected. This is the case if the energy is sufficiently high and the photon direction is significantly different from those of the

charged particles. This will be assumed to be the case in the rest of this paper. Therefore (2.4) reduces to

$$N = \left[2(q_+ q_-)(q_+ k)(q_- k) \right]^{-1/2} , \tag{2.6}$$

and (2.1) to

$$q_+^2 = q_-^2 = 0. \tag{2.7}$$

The next step is to take into account the fact that the electron is a fermion of spin 1/2. When only gauge interactions are present, Eq. (2.7) implies that the right-handed and left-handed electrons interact separately.

Formally, the argument is as follows. From (2.2) and (2.3), the circularly polarized states are characterized by

$$\varepsilon_\mu'^{\pm} = (\varepsilon_\mu'^{\parallel} \pm i\, \varepsilon_\mu^{\perp}) / \sqrt{2} . \tag{2.8}$$

Using the identity

$$i\, \gamma^\mu \varepsilon_{\mu\alpha\beta\gamma} = (\gamma_\alpha \gamma_\beta \gamma_\gamma - \gamma_\alpha g_{\beta\gamma} + \gamma_\beta g_{\alpha\gamma} - \gamma_\gamma g_{\alpha\beta})\, \gamma_5 , \tag{2.9}$$

we get from (2.8)

$$\not{\varepsilon}'^{\pm} = -\frac{1}{2\sqrt{2}} N \left[\not{k} \not{q}_- \not{q}_+ (1 \pm \gamma_5) - \not{q}_- \not{q}_+ \not{k} (1 \mp \gamma_5) \mp 2 (q_+ q_-) \not{k}\, \gamma_5 \right] . \tag{2.10}$$

Since the last term here is proportional to $\not k$, it can be omitted because of current conservation. Accordingly, the polarization vectors $\varepsilon_\mu'^{\pm}$ are modified to be $\varepsilon_\mu^{\pm}$ so that effectively

$$\not\varepsilon^{\pm} = -\frac{1}{2\sqrt{2}} N [\not k \not q_- \not q_+ (1 \pm \gamma_5) - \not q_- \not q_+ \not k (1 \mp \gamma_5)] . \qquad (2.11)$$

This is the basic formulae for the present formalism. Since the omitted term depends on γ_5, the $\varepsilon_\mu^{\pm}$ of (2.11) depends on the helicity of the electron line.

3. Remarks

There are several reasons why (2.11) leads to great simplifications.

(A) If the photon line is next to the external electron or positron line, only one of the terms on the right-hand side of (2.11) contributes. The reason is simply due to

$$\not q_+ v(q_+) = 0 \qquad \text{and} \qquad \bar{u}(q_-) \not q_- = 0 , \qquad (3.1)$$

where u and v are the usual spinors for massless electrons and positrons.

(B) Whenever there is a photon line next to the external electron or positron line, by (A) either a factor $1 + \gamma_5$ or a factor $1 - \gamma_5$ appears. This factor ensures that, for every other real photon line attached to this electron line, only one of the two terms on the right-hand side of (2.11) survives. This is another way of saying that the right-handed and the left-

handed electrons interact separately.

(C) When the photon line is next to the external electron line, there is a cancellation of the denominator due to (3.1):

$$\begin{aligned} &\bar{u}(q_-) \not\varepsilon^{\pm} \frac{\not q_- + \not k}{2 (q_- k)} \\ &= -\frac{1}{2\sqrt{2}} N \bar{u}(q_-) \not k \not q_- \not q_+ (1 \pm \gamma_5) \frac{\not q_- + \not k}{2 (q_- k)} \\ &= -\frac{1}{2\sqrt{2}} N \bar{u}(q_-) \not q_+ (\not q_- + \not k) (1 \mp \gamma_5) . \end{aligned} \qquad (3.2)$$

As we shall see in more detail in II, this denominator cancellation is to a large extent responsible for the simplicity of the answers obtained previously by brute force [2].

Although (2.11) is sufficient and $\varepsilon_\mu^{\pm}$ themselves are not needed for quantum electrodynamics, it is nevertheless useful to have the polarization vectors explicitly. They are given by

$$\varepsilon_\mu^{\pm} = (\varepsilon_\mu^{\parallel} \pm i \varepsilon_\mu^{\perp}) / \sqrt{2} , \qquad (3.3)$$

where $\varepsilon_\mu^{\parallel} = N [(q_+ k) q_{-\mu} - (q_- k) q_{+\mu} \pm (q_+ q_-) k_\mu]$ (3.4)

when $(1 \pm \gamma_5) v(q_+) = 0$. (3.5)

It should be emphasized that the step from (2.10) to (2.11), while simple, is the central idea of the present formalism. In the language of diagrams, the verification of current conservation requires cancellations between various diagram. This step accomplishes in a simple way the extensive cancellation seen previously [2] on the level of the cross section. The desirable properties (A) - (C) appear only with (2.11).

4. Spinor States

The formalism developed so far is sufficient for single bremsstrahlung to be studied in II. To a large extent, it is also sufficient for double bremsstrahlung. However, in general the formalism can be carried further, as shown in this section.

The basic fact to be used here is that, since right-handed and left-handed massless electrons interact separately, there is essentially no internal degree of freedom. This is to be used as follows. Consider a massless electron of four-momentum q; there are two possible helicity states $u_+(q)$ and $u_-(q)$ specified by

$$u_\pm(q) = \frac{1}{2}(1 \pm \gamma_5)\, u_\pm(q) \quad . \tag{4.1}$$

Then
$$\bar{u}_\pm(q) = \bar{u}_\pm(q)\, \frac{1}{2}(1 \mp \gamma_5) \quad . \tag{4.2}$$

By normalizing the spinors such that

$$u_+(q)\, \bar{u}_+(q) + u_-(q)\, \bar{u}_-(q) = \not{q} \quad , \tag{4.3}$$

it follows that

$$u_\pm(q)\, \bar{u}_\pm(q) = \frac{1}{2}(1 \pm \gamma_5)\, \not{q} \quad . \tag{4.4}$$

It is therefore possible to go back and forth between the two sides of (4.4), without any summation over internal degrees of freedom.

For the positron of momentum q, the relations are similar:

$$v_\pm(q) = \frac{1}{2}(1 \mp \gamma_5)\, v_\pm(q) \quad , \tag{4.5}$$

$$\bar{v}_\pm(q) = \bar{v}_\pm(q)\, \frac{1}{2}(1 \pm \gamma_5) \quad , \tag{4.6}$$

and
$$v_\pm(q)\, \bar{v}_\pm(q) = \frac{1}{2}(1 \mp \gamma_5)\, \not{q} \quad . \tag{4.7}$$

It is therefore convenient to use a Dirac bracket notation. Let $|q, +\rangle$ denote either $u_+(q)$ or $v_-(q)$, $|q, -\rangle$ either $u_-(q)$ or $v_+(q)$, $\langle q, +|$ either $\bar{u}_+(q)$ or $\bar{v}_-(q)$, and $\langle q, -|$ either $\bar{u}_-(q)$ or $\bar{v}_+(q)$. Then,

$$|q, +\rangle\langle q, +| = \frac{1}{2}(1 + \gamma_5)\, \not{q}$$

and
$$|q, -\rangle\langle q, -| = \frac{1}{2}(1 - \gamma_5)\, \not{q} \quad . \tag{4.8}$$

Of course,

$$\langle q, + | q, + \rangle = \langle q, - | q, - \rangle = 0 \quad . \tag{4.9}$$

More generally, for arbitrary q_1 and q_2 satisfying $q_1^2 = q_2^2 = 0$,

$$\langle q_1 , \pm | q_2 , \pm \rangle = 0. \tag{4.10}$$

Since these states can each carry an arbitrary phase, products like $\langle q_1 , + | q_2 , - \rangle$ are not well defined, but the magnitude is

$$|\langle q_1 , + | q_2 , - \rangle|^2 = \mathrm{Tr}\, \frac{1}{2} (1 - \gamma_5) \not{q}_2 \frac{1}{2} (1 + \gamma_5) \not{q}_1 = 2 (q_1 q_2) . \tag{4.11}$$

The advantage of the Dirac notation is to detach the spinor from the nature of the particle, the only important property being masslessness, $q^2 = 0$. Since the photon is also massless, it is sometimes convenient to express in terms of spinors:

$$\frac{1}{2} (1 + \gamma_5) \not{k} = | k, + \rangle \langle k, + |$$

and $$\frac{1}{2} (1 - \gamma_5) \not{k} = | k, - \rangle \langle k, - | . \tag{4.12}$$

With (2.6), the substitution of (4.8) and (4.12) into (2.11) gives

$$\not{\varepsilon}^{\pm} = - \frac{1}{2} [(q_+ q_-) (q_+ k) (q_- k)]^{-1/2} \langle q_- , \pm | q_+ , \mp \rangle$$

$$[| k, \mp \rangle \langle k, \mp | q_- , \pm \rangle \langle q_+ , \mp | - | q_- , \pm \rangle \langle q_+ , \mp | k, \pm \rangle \langle k, \pm |] . \tag{4.13}$$

This already shows that the denominator $(q_+ q_-)$ actually does not appear.

If it is possible to choose the phases so that the brackets are all positive,

then (4.13) simplifies further by (4.11) to

$$\not{\varepsilon}^{\pm} = - (q_+ k)^{-1/2} | k, \mp \rangle \langle q_+ , \mp | + (q_- k)^{-1/2} | q_- , \pm \rangle \langle k, \pm | . \tag{4.14}$$

5. Discussion

The key step here is to use photon polarization vectors which depend on the helicities of the electron lines. In order to have the denominator cancellation of Sec. 3 (C), it is essential to use the external momenta of the electron line to which the photon is attached. This leads to the following complication: different polarization vectors are used for photons radiated from different electron lines. Consider the simplest example

$$e^+(p_+) + e^-(p_-) \longrightarrow \mu^+(q_+) + \mu^-(q_-) + \gamma(k) . \tag{5.1}$$

If the photon is radiated by the muon, then (2.8) with (2.2) and (2.3) is used; if the photon comes from the electron line, then the q's must be replaced by the p's. Since a photon can have only two polarizations, the polarization vectors must be related by a phase

$$\varepsilon'^{\pm}_{\mu}(q_+ , q_-) = e^{i\phi^{\pm}} \varepsilon'^{\pm}_{\mu}(p_+ , p_-) + A_{\pm} k_{\mu} . \tag{5.2}$$

As the muon and electron currents are separately conserved, the $A_{\pm}$ are constants of no relevance. The phases $\phi^{\pm}$ are given by the dot products of the ε', and must be properly taken into account. For multiple bremsstrahlung processes, these phases can be fairly complicated.

This formalism will be applied first to electrodynamic processes and then to QCD [3].

Acknowledgements

One of us (TTW) thanks Professor H. Joos, Professor E. Lohrmann, Professor V. Soergel, and Professor K. Symanzik for their kind hospitality at DESY.

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