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ON HIGHER ORDER CORRECTIONS TO THREE-JET CROSS SECTIONS

by

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INTRODUCTION

The observed properties of 3-jet events in e^+e^--annihilation are in good qualitative agreement with lowest order QCD perturbation theory[1]). In order to assure that this agreement is not fortuitous, that is to say that the perturbation series converges reasonably fast, and to be able to make a meaningful quantitative comparison of α_s determined from measurements of the 3-jet cross section to that measured in other processes (such as deep inelastic scattering), it is necessary to compute nonleading higher order corrections.

In this talk I shall report a calculation[2]) of the Sterman-Weinberg type[3]) 3-jet cross section to order α_s^2. We have chosen a Sterman-Weinberg type angle and energy cut off for a variety of reasons[2]). In particular, an acceptable 3-jet measure must be insensitive to the emission of soft and/or collinear radiation and to the process of hadronization which, in contrast to many popular 3-jet measures, is uniquely met by the Sterman-Weinberg definition of 3-jet events.

The talk is divided into three parts. In the first part I shall present the results. The second part discusses an independent (algebraic) test of the cross section formula. Finally, in the third part I shall comment on the contrasting results pioneered by the CALTECH group[4]).

RESULTS

We will call an event a 3-jet event if it has all but a fraction $\varepsilon/2$ of its total energy distributed within three separated cones of (full) opening angle δ for some fixed ε,δ. By x_i, $i = 1,2,3$ we shall denote twice the energy going into the i-th jet cone divided by the total

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energy going into all three cones. The 3-jet cross section for finding a quark jet of energy x_1 and an antiquark jet of energy x_2 (and a gluon jet of energy $x_3 = 2-x_1-x_2$) was found to be[2]) (in the $\overline{MS}$ scheme)

$$\frac{d^2\sigma^{(3)}(\varepsilon,\delta)}{dx_1dx_2} = \sigma_0 \frac{\alpha_s(q^2)}{2\pi} C_F \Big[\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} \Big\{ 1 - \frac{\alpha_s(q^2)}{\pi} \Big[\Big(C_F \ln\frac{\varepsilon}{x_1} + C_F \ln\frac{\varepsilon}{x_2}$$

$$+ N_C \ln\frac{\varepsilon}{x_3} + \frac{3}{2}C_F + \frac{1}{2}\Big(\frac{11}{6}N_C - \frac{1}{3}N_f\Big)\Big) \ln\Big(\frac{1-\cos\delta}{2}\Big) - \ln\varepsilon \Big(N_C \ln\Big(\frac{1-\cos\theta_{13}}{2}\Big)$$

$$+ N_C \ln\Big(\frac{1-\cos\theta_{23}}{2}\Big) + (2C_F - N_C)\ln\Big(\frac{1-\cos\theta_{12}}{2}\Big)\Big) - \Big(C_F\frac{\varepsilon}{x_1} + C_F\frac{\varepsilon}{x_2} + N_C\frac{\varepsilon}{x_3}\Big)\ln\Big(\frac{1-\cos\delta}{2}\Big)$$

$$+ \Big(\frac{11}{6}N_C - \frac{1}{3}N_f\Big)\ln x_3 + R(x_1,x_2) \Big] \Big\} + \frac{\alpha_s(q^2)}{\pi} F(x_1,x_2) \Big] + O(\varepsilon) + O(\delta^2) \qquad (1)$$

where

$$R(x_1,x_2) = \frac{N_C}{2} X_2\Big(\frac{1-\cos\theta_{13}}{2}\Big) + \frac{N_C}{2} X_2\Big(\frac{1-\cos\theta_{23}}{2}\Big) + \frac{1}{2}(2C_F - N_C) X_2\Big(\frac{1-\cos\theta_{12}}{2}\Big)$$

$$- C_F X_2(1-x_1) - C_F X_2(1-x_2) - (2C_F - N_C) X_2(1-x_3) + \frac{1}{4}(2C_F - N_C)\Big[\ln^2(1-x_3)$$

$$- \ln^2\Big(\frac{1-\cos\theta_{12}}{2}\Big)\Big] + \frac{N_C}{4}\Big[\ln^2(1-x_1) + \ln^2(1-x_2) - \ln^2\Big(\frac{1-\cos\theta_{13}}{2}\Big)$$

$$- \ln^2\Big(\frac{1-\cos\theta_{23}}{2}\Big)\Big] - N_C \ln^2 x_3 - (2C_F - N_C)\ln x_3 \ln(1-x_3)$$

$$+ \frac{1}{2}(2C_F - N_C)\ln(1-x_3)\big[\ln(1-x_1) + \ln(1-x_2)\big] + \frac{N_C}{2}\ln(1-x_1)\ln(1-x_2)$$

$$- C_F\big[\ln^2 x_1 + \ln^2 x_2 + \ln x_1 \ln(1-x_1) + \ln x_2 \ln(1-x_2)\big]$$

$$- \frac{3C_F}{2}\big[\ln x_1 + \ln x_2\big] + \frac{C_F}{2}\pi^2 - 3C_F - \frac{32}{9}N_C + \frac{13}{18}N_f \qquad (2)$$

and

$$\mathcal{F}(x_1,x_2) = \frac{x_3^2}{(1-x_1)(1-x_2)} \Big\{ \tfrac{1}{4}(2C_F - N_C)\big[\chi_2(1-x_1) + \chi_2(1-x_2) + 2\chi_2(1-x_3)$$

$$+ \ln x_1 \ln(1-x_1) + \ln x_2 \ln(1-x_2) + 2\ln x_3 \ln(1-x_3) - \ln(1-x_1)\ln(1-x_3)$$

$$- \ln(1-x_2)\ln(1-x_3) - \frac{\pi^2}{3}\big] + C_F - N_C + \frac{N_f}{12}\Big\}$$

$$+ \frac{x_3}{(1-x_1)(1-x_2)} \Big\{ \tfrac{1}{2}(2C_F - N_C)\big[-\chi_2(1-x_1) - \chi_2(1-x_2) - 2\chi_2(1-x_3)$$

$$- \ln x_1 \ln(1-x_1) - \ln x_2 \ln(1-x_2) - 2\ln x_3 \ln(1-x_3) + \ln(1-x_1)\ln(1-x_3)$$

$$+ \ln(1-x_2)\ln(1-x_3) + \frac{\pi^2}{3}\big] - \frac{C_F}{2} + \frac{N_C}{2}\Big\} + \tfrac{1}{2}(2C_F - N_C)\big[\chi_2(1-x_1)$$

$$+ \chi_2(1-x_2) + 2\chi_2(1-x_3) + \ln x_1 \ln(1-x_1) + \ln x_2 \ln(1-x_2)$$

$$+ 2\ln x_3 \ln(1-x_3) - \ln(1-x_1)\ln(1-x_3) - \ln(1-x_2)\ln(1-x_3)$$

$$- \frac{\pi^2}{6}\big] + 2C_F\big[\ln(1-x_1) + \ln(1-x_2)\big] + 2C_F - \frac{5N_f}{6}$$

$$+ \frac{x_1^2 - x_2^2}{(1-x_1)(1-x_2)} \tfrac{1}{4}(2C_F - N_C)\big[\chi_2(1-x_1) - \chi_2(1-x_2) + x_1\ln(1-x_1) - x_2\ln(1-x_2)$$

$$- \ln(1-x_1)\ln(1-x_3) + \ln(1-x_2)\ln(1-x_3)\big] - \frac{C_F}{2}\Big[\frac{1-x_1}{x_2} + \frac{1-x_2}{x_1}$$

$$+ (1-x_1)(1-x_2)\Big(\frac{\ln(1-x_1)}{x_1^2} + \frac{\ln(1-x_2)}{x_2^2}\Big)\Big] - \Big[2C_F(1-x_2)$$

$$- \tfrac{1}{2}(2C_F + N_C)(1-x_1)\Big]\frac{\ln(1-x_1)}{x_1} - \Big[2C_F(1-x_1) - \tfrac{1}{2}(2C_F + N_C)(1-x_2)\Big]\frac{\ln(1-x_2)}{x_2}$$

$$+ (2C_F - N_C)\frac{1-x_3}{x_3}\Big[1 + \frac{1+x_3}{x_3}\ln(1-x_3)\Big] . \tag{3}$$

The angles Θ_{ij} are between the jet cones and are given by

$$\cos\Theta_{ij} = 1 - \frac{2}{x_i x_j}(x_i + x_j - 1) , \tag{4}$$

and χ_2 is the Spence function

$$\chi_2(x) = -\int_0^x dz \, \frac{\ln(1-z)}{z} . \tag{5}$$

In (1) - (3) the group factors have been written out explicity (what was not the case in our letter[2]) which will be important for the later tests. The terms of order ϵ, δ^2 are calculated numerically.

The physical origin of the (large) logarithmic terms

$$- \frac{\alpha_s(q^2)}{\pi} \frac{1}{2}\Big(\frac{11}{6}N_C - \frac{1}{3}N_f\Big)\ln\Big(\frac{1-\cos\delta}{2}\Big) \tag{6}$$

and

$$- \frac{\alpha_s(q^2)}{\pi}\Big(\frac{11}{6}N_C - \frac{1}{3}N_f\Big)\ln x_3 \tag{7}$$

in (1) is that the 4-momentum squared which determines the strength of the strong coupling constant is not q^2 but somewhat smaller. If we perform renormalization at [5]

$$\mathcal{M}^2 = \frac{1-\cos\delta}{2} x_3^2 q^2 \approx \frac{\delta^2}{4} x_3^2 q^2 , \tag{8}$$

the logarithmic terms (6) and (7) get exactly cancelled as can readily be seen. Thus (1) can be written

$$\frac{d^2\sigma^{(3)}(\epsilon,\delta)}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s(\mathcal{M}^2)}{2\pi} C_F \Big[\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\Big\{1 - \frac{\alpha_s(\mathcal{M}^2)}{\pi}\Big[\Big(C_F \ln\frac{\epsilon}{x_1} + C_F \ln\frac{\epsilon}{x_2}$$

$$+ N_C \ln\frac{\epsilon}{x_3} + \frac{3}{2}C_F\Big)\ln\Big(\frac{1-\cos\delta}{2}\Big) - \ln\epsilon\Big(N_C \ln\Big(\frac{1-\cos\Theta_{13}}{2}\Big) + N_C \ln\Big(\frac{1-\cos\Theta_{23}}{2}\Big)$$

$$+ (2C_F - N_C)\ln\left(\frac{1-\cos\theta_R}{2}\right)) - \left(C_F\frac{\epsilon}{x_1} + C_F\frac{\epsilon}{x_2} + N_C\frac{\epsilon}{x_3}\right)\ln\left(\frac{1-\cos\delta}{2}\right)$$

$$+ R(x_1,x_2)\Big]\Big\} + \frac{\alpha_s(M^2)}{\pi} F(x_1,x_2)\Big] + O(\epsilon) + O(\delta^2). \tag{9}$$

This is going to repeat itself in higher orders. In particular, the leading logarithms will sum to [6]

$$\frac{d^2\sigma^{(3)}(\epsilon,\delta)}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s(M^2)}{2\pi} C_F \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} \exp\left\{-\frac{\alpha_s(M^2)}{\pi}(2C_F+N_C)\ln\epsilon\,\ln\left(\frac{1-\cos\delta}{2}\right)\right\}. \tag{10}$$

Since we are not (yet) able to separate between a quark and a gluon jet, and also for reasons of statistics, we shall in the following consider the single inclusive distribution

$$\frac{1}{\sigma}\ \frac{d\sigma^{(3)}(\epsilon,\delta)}{dx_{max}}\ ,\quad x_{max} = \max\{x_1,x_2,x_3\} \tag{11}$$

where σ is given in Ref. 7). This derives from (1) by integrating over one of the jet energies. Note that $2/3 \le x_{max} \le 1$ to all orders. We have fitted (11) to the PLUTO data[9] with $\epsilon = 0.2$, $\delta = 45^{\circ}$ at 30 GeV and obtain for the strong coupling constant in the $\overline{MS}$ scheme

$$\alpha_s = 0.17\,. \tag{12}$$

This is equivalent to

$$\Lambda_{\overline{MS}} = 240\ \text{MeV} \tag{13}$$

in the 1-loop approximation. The corresponding cross section and the PLUTO data are shown in Fig. 1.

The order α_s^2 corrections are small. This can best be seen by comparing (12) to the fit of the order α_s cross section formula which gives [9] $\alpha_s = 0.15$. The Λ value (13)

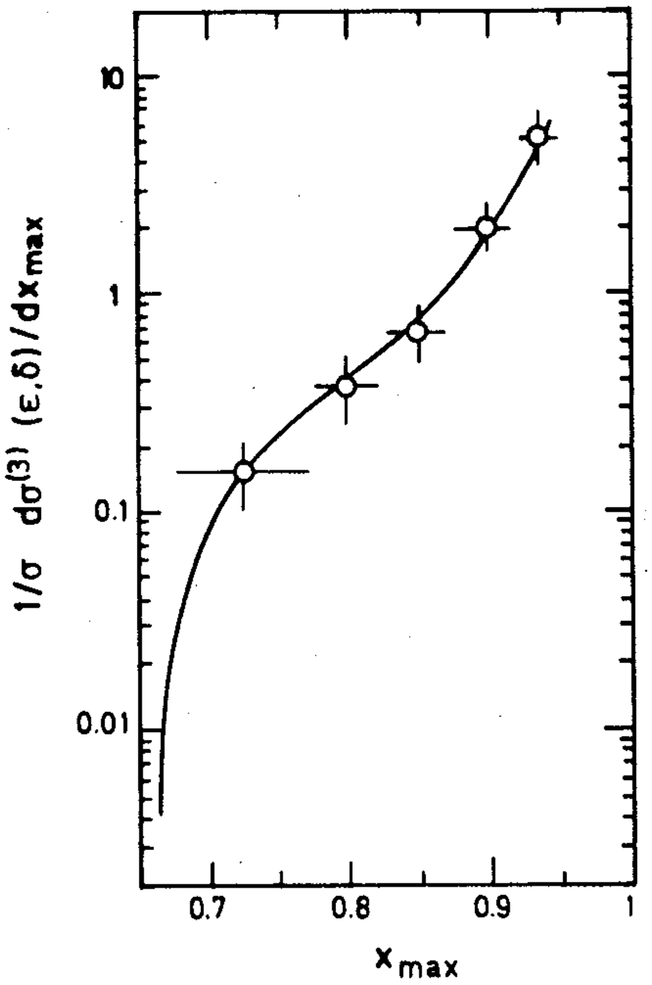

Fig. 1 : Three-jet cross section fitted to PLUTO data[9] with $(\epsilon,\delta) = (0.2, 45^{\circ})$.

agrees very well with the most recent analysis of the large-q^2 neutrino scattering data of the CDHS group[10].

INDEPENDENT TEST OF CROSS SECTION FORMULA

The $\ln\varepsilon$, $\ln(\frac{1-\cos\delta}{2})$ terms in (1) agree with the leading logarithm calculation of Smilga and Vysotsky[11]. This was demonstrated in Ref. 2) and will not be repeated here. In order to check the remaining terms we shall proceed in two steps.

First we set $N_C = N_f = 0$ in (1) - (3) (however, retaining σ_0 as before) which switches off the triple-gluon coupling and the gluon splitting into a $q\bar{q}$ pair. In other words, the gluon jet consists of a single gluon. From the Ward identity we then derive

$$\int_0^{1-\Delta} dx_1 \int_{1-x_1}^{1-\Delta} dx_2 \frac{d^2\sigma^{(3)}(\epsilon,\delta)}{dx_1\, dx_2} \underset{\Delta\to 0}{=} \ln^2\Delta \, \frac{\alpha_s(q^2)}{\pi} C_F \sigma^{(2)}(\epsilon,\delta) \qquad (14)$$

where $\sigma^{(2)}(\varepsilon,\delta)$ is the Sterman-Weinberg cross section for two jets[2),12)]

$$\sigma^{(2)}(\epsilon,\delta) = \sigma_0 \Big\{ 1 - \frac{\alpha_s(q^2)}{\pi} C_F \Big[(2\ln\epsilon + \tfrac{3}{2}) \ln\frac{\delta^2}{4} - 2\epsilon \ln\frac{\delta^2}{4} + \frac{\pi^2}{3} - \frac{5}{2} \Big] \Big\} . \qquad (15)$$

This must be satisfied by our 3-jet cross section (1) - (3).

Only the most singular pieces for $x_3 \to 0$ - and numerically only important terms - of the 3-jet cross section contribute a factor $\ln^2\Delta$. They are (cf. (1) - (3))

$$\frac{d^2\sigma^{(3)}(\epsilon,\delta)}{dx_1\, dx_2} = \sigma_0 \frac{\alpha_s(q^2)}{2\pi} C_F \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} \Big\{ 1 - \frac{\alpha_s(q^2)}{\pi} C_F \Big[(\ln\frac{\epsilon}{x_1} + \ln\frac{\epsilon}{x_2}$$

$$+ \tfrac{3}{2}) \ln(\frac{1-\cos\delta}{2}) - (\frac{\epsilon}{x_1} + \frac{\epsilon}{x_2}) \ln(\frac{1-\cos\delta}{2}) + \chi_2(\frac{1-\cos\theta_{12}}{2})$$

$$- 2\chi_2(1-x_3) + \frac{\pi^2}{2} - 3 - \frac{1}{2} \frac{x_1^2-x_2^2}{x_1^2+x_2^2} (x_1 \ln(1-x_1) - x_2 \ln(1-x_2)) \Big] \Big\} \qquad (16)$$

and give*)

$$\int_0^{1-\Delta} dx_1 \int_{1-x_1}^{1-\Delta} dx_2 \frac{d^2\sigma^{(3)}(\epsilon,\delta)}{dx_1\, dx_2} \underset{\Delta\to 0}{=} \ln^2\Delta \frac{\alpha_s(q^2)}{\pi} C_F \sigma_0 \Big\{ 1 - \frac{\alpha_s(q^2)}{\pi} C_F \Big[(2\ln\epsilon$$

$$+ \tfrac{3}{2}) \ln(\frac{1-\cos\delta}{2}) - 2\epsilon \ln(\frac{1-\cos\delta}{2}) - \chi_2(1) + \frac{\pi^2}{2} - \frac{5}{2} \Big] \Big\} . \qquad (17)$$

The right-hand side of (17) agrees with (14), (15) as it should, realizing that $\chi_2(1) = \pi^2/6$ and $\ln(\frac{1-\cos\delta}{2}) = \ln(\delta^2/4) + O(\delta^2)$.

The second test concerns the contributions proportional to N_C and N_f. Therefore we set $C_F^2 = 0$ but retain $C_F N_C$, $C_F N_f \neq 0$. In this case only the gluon jet is "composite"**). We then consider the limit $x_1, x_3 \to 1$ with $(1-x_3)/(1-x_1) \to 1$. This corresponds to the 2-jet configuration in which the ("composite") gluon jet recoils against the quark jet (and the antiquark energy x_2 goes to zero). In this limit the 3-jet cross section (1) - (3) should be proportional to "one-half" the Sterman-Weinberg type cross section for gluon jets derived by Shizuya and Tye[13]. It should be "one-half" the Shizuya-Tye cross section only because we have one gluon jet in contrast to two back-to-back gluon jets in case of Ref. 13).

For a quantitative comparison we first have to divide out the source dependence in form of the equivalent of what is called the "total cross section" in Ref. 13). This "total cross section" is found to be***)

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*) Noticing that $\lim_{x_1,x_2\to 1} \cos\theta_{12} = -1$.

**) The singularities associated with the quark/antiquark emitting a collinear gluon appearing at intermediate stages of the calculation cancel in the final result.

***) Note that $\cos\theta_{13} \to -1$, $\cos\theta_{23} \to 0$ and $\cos\theta_{12} \to 0$.

$$"\sigma_{tot}" = \lim_{x_1,x_3\to 1} \frac{d^2\sigma^{(3)}(1,180^\circ)}{dx_1\,dx_2} \tag{18}$$

$$= \frac{1}{1-x_1}\,\frac{\alpha_s(q^2)}{2\pi} C_F\,\sigma_0 \left\{ 1 - \frac{\alpha_s(q^2)}{\pi}\left[-\left(\frac{\pi^2}{12}+\frac{19}{8}\right) + \frac{7}{12} N_f \right] \right\}$$

and, in particular, can be read off from (1) - (3), only that we have not stated the terms of order ε, δ^2 explicitly (which in this particular limit can be computed analytically while, generally, they are too involved and can not). Independently, (18) can also be obtained by adding expression (2.3) in Ref. 13), multiplied by the source factor

$$\frac{1}{1-x_1}\,\frac{\alpha_s(q^2)}{2\pi}\,C_F\,, \tag{19}$$

to our $q\bar{q}g$ cross section[2]. We have done this and found the same result.

We then obtain

$$\frac{1}{"\sigma_{tot}"}\,\frac{d^2\sigma^{(3)}(\varepsilon,\delta)}{dx_1\,dx_2} \underset{x_1,x_3\to 1}{=} 1 - \frac{\alpha_s(q^2)}{\pi}\left[\left(N_c \ln\varepsilon + \frac{11}{12}N_c - \frac{1}{6}N_f\right)\ln\left(\frac{1-\cos\delta}{2}\right) + \left(\frac{\pi^2}{6} - \frac{49}{72}\right) N_c + \frac{1}{18} N_f \right]. \tag{20}$$

This agrees with Eq. (3.7) in Ref. 14) taking into account that the square bracket in (20) has to be multiplied by a factor of two for comparison.

We like to emphasize that the untested less singular terms are down by factors of $(1-x_1)$, $(1-x_2)$ and (hence) numerically very small except for the logarithmic term (7) which led to the change of scales (8).

In constrast to Ref. 4) there are no large π^2 terms.

The π^2 terms in (2) are much smaller, have the opposite sign and are nearly cancelled by the constant terms as in the Sterman-Weinberg 2-jet cross section (5).

COMMENTS

We have seen that the order α_s^2 corrections to the 3-jet cross section, which was defined to be the cross section for events having all but a fraction $\varepsilon/2$ of the total energy distributed within three separated cones of opening angle δ, are small beyond all doubts.

The obvious question now is: how does this compare to the results of the other group[4]? These authors have calculated the C and/or ("bare") thrust distributions and found a large order α_s^2 correction. I shall try to answer this question in the rest of this writeup without going into greater details. A somewhat more detailed presentation will be published elsewhere[14].

To begin with, I like to emphasize that the two calculations do not contradict each other as we have calculated different quantities*). The Sterman-Weinberg cross section on the one side puts every 3-jet event with the same (reconstructed) jet energies into the same bin no matter if it proceeds from a 3-parton final state or a 4-parton final state with two partons being at a relative angle $\leq \delta$ and/or one parton being soft ($E \leq \frac{\varepsilon}{2} q$). In this way, and that is most important to notice, the infrared and collinear singularities are fully integrated out bin by bin. This is quite different from the C and ordinary or "bare" thrust distributions on the other side. Here two identically looking 3-jet events can have rather different values of, e.g., "bare" thrust. On top of that, "bare" thrust is rather sensitive to the emission of soft and/or collinear radiation which makes this calculation meaningless.

Let me illustrate this by means of two examples. Suppose we call an event with "bare" thrust $T \leq 0.9$ a 3-jet event (also including four jets) and, accordingly, an event with $T \geq 0.9$ a 2-jet event. The leading 3-jet configuration ($\sim \ln^2(1-T)$) corresponds to the situation where quark and antiquark both have maximum energy which in this case is $\bar{x}_1 = \bar{x}_2 = 0.9$ (by $\bar{x}_i$ we denote the parton energies $\bar{x}_i = 2E_i^{\,parton}/q$; quark and antiquark are labelled 1 and 2, respectively). This is shown in Fig. 2a

*) This is also true for the thrust distribution given in our letter[2]. Forced upon us by the Sterman-Weinberg type angle and energy cut off we have derived thrust from the reconstructed jet axes, that is after smoothing the soft and collinear sinjularities. This is close to what one measures (see following discussion). To go from one distribution to the other is not immediately possible as it means to interchange the order of certain limits and integrations.

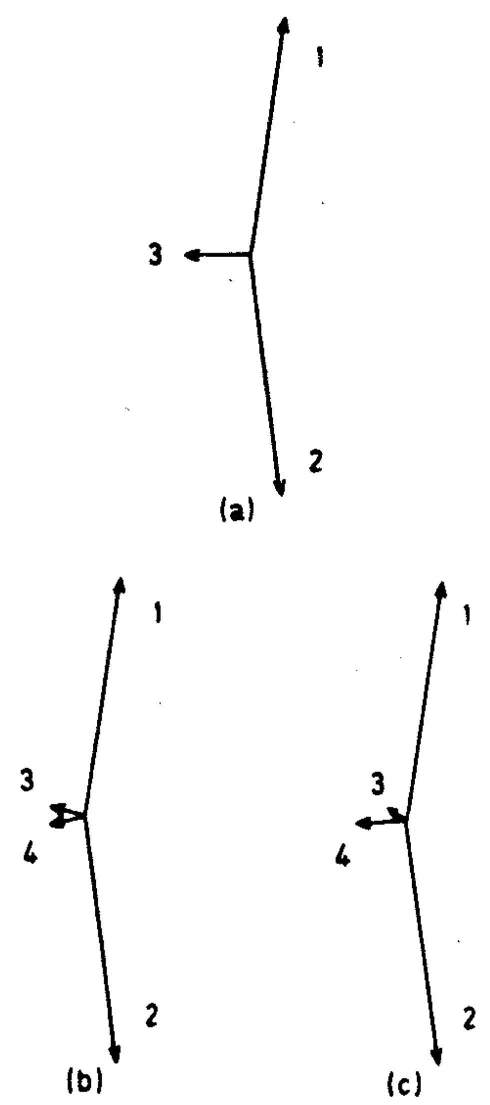

Fig. 2 : Leading 3-jet configurations.

for 3-parton final states and in Figs. 2b and 2c for 4-parton final states. We notice then that in this configuration (and under the above 3-jet classification) the angle between partons 3 and 4 in Figs. 2b and 2c (which can be gluons or a $q\bar{q}$ pair) is limited by the fact that the vector sum of any two parton momenta may not exceed T = 0.9. Otherwise the event will fall into the 2-jet bin.

In the symmetric case of Fig. 2b we find that the angle between partons 3 and 4 must be smaller than $\approx 3^\circ$. Besides the fact that finite order perturbation theory is not applicable at these small angles, this cannot be realized experimentally. At best, we can hope to tell an angle of $\gtrsim 45^\circ$ at present energies (given the fact that the gluon jet has relatively low energy).

In the case of Fig. 2c the angle between, e.g., partons 3 and 1 cannot be smaller than 90°, and 90° only in the limit $\bar{x}_3 \to 0$. This holds for arbitrary $\bar{x}_2$ as well. Since integration over the full solid angle is required to cancel the infrared singularities of the virtual corrections, the double differential cross section in "bare" thrust and $\bar{x}_1$ (i.e., integrated over $\bar{x}_2$) is (positive) infinite at leading $T = \bar{x}_1$, and similarly for the distribution*) in $\bar{x}_2$. The approach to infinity is

$$\sim -\ln(T-\bar{x}_{1,2}) \tag{21}$$

as one can read off from (1) (by setting $\varepsilon = T-\bar{x}_{1,2}$). With the exception of T = 2/3, $d\sigma/dT$ will finally be finite. But as in the previous example, "bare" thrust cuts right through the singular region leaving behind a large 3-jet correction as we shall see.

The circumstance that $d\sigma/dT$ is infinite at T = 2/3 should already be reason enough to discard "bare" thrust as an infrared safe jet measure. In a model like that of Eq. (20) which allows us to measure $\bar{x}_{1,2}$ precisely, "bare" thrust would be meaningless for all $T \gtrsim 2/3$.**)

So much for the qualitative differences. Let me substantiate this by a quantitative comparison now. The configuration of Fig. 2b, where partons 3 and 4 are trapped at a very small (collinearity) angle, contributes a correction to the leading ($\sim \ln(1-T)/(1-T)$) "bare" thrust distribution $d\sigma/dT$:

$$1-\frac{\alpha_s(\xi^2)}{2\pi}\left(\frac{11}{6}N_c-\frac{1}{3}N_f\right)\ln 0.0007 \approx 1+2.8\,\frac{\alpha_s(\xi^2)}{2\pi} \ . \tag{22}$$

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*) In fact, this is also true for nonleading $T = \bar{x}_{3,4}$.

**) With regard to the footnote two pages before it should be said that in particular nonleading configurations the angle may only be zero degrees.

This is exactly the logarithmic term (6) (with $\delta = 3^{\circ}$) which reflects the collinear singularities associated with the gluon jet. For current values of α_s this alone gives a correction of 70 - 80%. The singularity (21), when integrated over $\bar{x}_1$, $\bar{x}_2$, gives rise to a correction to the "3-jet cross section" (defined to be the cross section for events with "bare" thrust $\leqslant$ T):

$$\approx 1 + \frac{\alpha_s(\xi^2)}{2\pi} N_C \left(\frac{2}{3}\pi^2 + 2\ln^2(1-T) - 4X_2(1-T)\right) \left|\ln(\delta)/\ln(1-T)\right|$$

$$\underset{T=0.9}{\approx} \quad 1 + 15\,\frac{\alpha_s(\xi^2)}{2\pi} \quad . \tag{23}$$

This follows from the second term in the inner square bracket of (1) where we have made the approximation $\cos\theta_{13} = \cos\theta_{23} = 0$, $\cos\theta_{12} = -1$. The correction here is 40 - 50%. Both estimates are based on the planar configuration and, hence, are order of magnitude estimates. Note that (23) adds a large π^2 term which brings us close to Ref. 4 (after having put in the correct factor[14])).

We conclude that "bare" thrust divides the small overall order α_s^2 contribution (given by the total cross section[7])) into a large positive 3-jet correction and a large negative 2-jet correction in a highly infrared sensitive and physically irreproducable way.

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