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SUPERWEAK CP-VIOLATION AND RIGHT-HANDED HORIZONTAL INTERACTIONS

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I. Introduction

17 years after its discovery [1] the reason for CP-violation is still mysterious. Up to now it has been experimentally observed only in the subtleties of K-meson decays. This fact follows from its smallness as compared to ordinary weak interactions. Theoretically it is, in fact, not difficult to implement CP- (or equivalently T-) violation. It is, however, harder to understand that if there is a CP-violation at all why is it so small. This applies in particular to most of the currently popular models where the CP-violation is part of the weak interactions described by spontaneously broken gauge theories (see e.g. [2,3]). There exists also the danger of large CP- (and P-) violations in strong interactions, induced by non-perturbative QCD effects [4-6], if the quark mass matrix obtained from spontaneous symmetry breaking is complex.

A possible way to explain the smallness of CP-violation is to associate it to a new superweak interaction due to the repetition of lepton-quark families. (For different recent attempts along this line see [7-10].) Such "horizontal" weak interactions [11-13] have to be superweak anyway because of the observed accuracy of separate electron-, muon and τ -lepton-number conservation and because of the small upper limits on flavour-changing neutral currents [14,15]. The horizontal interactions are associated to a spontaneously broken gauge symmetry, therefore this is a field theoretic realization [16] of the superweak model of CP-violation [17].

In this paper we present a new model of this type, based on the gauge symmetry group $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$. Here $SU(2)_{LV} \otimes U(1)$ is the Weinberg-Salam electroweak symmetry [18,19] and $O(3)_{RH}$ is a right-handed "horizontal" symmetry for the three observed families. (We assume the existence of the t -quark completing the third family.) In our model the CP-violation comes from the spontaneous breaking of the original CP-symmetry of the Lagrangean by the minimum of the Higgs-potential. At the tree level the mixing angles of the left-handed quarks and leptons are real due to the choice of the Higgs-fields even in the case of three families. (Therefore, at least in the tree approximation, the Kobayashi-Maskawa mechanism [20] of CP-violation in ordinary weak interactions is not operating.) The right-handed

Superweak CP-violation and right-handed horizontal interactions.

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Abstract: A horizontal extension of the Weinberg-Salam electroweak theory by a right-handed $O(3)_{RH}$ gauge symmetry for the three fermion families is studied. By an appropriate choice of the Higgs-scalar fields the CP-symmetry of the Lagrangean is spontaneously broken, but the mixing of the left-handed fermion states, and hence the Kobayashi-Maskawa mixing matrix, remains real. The CP-violation is manifested in the superweak horizontal gauge interactions, which are suppressed by the large mass of the corresponding gauge bosons. It is, however, possible that the horizontal boson acting on the second and third families can be considerably lighter than the other two, implying an interesting phenomenology of the related CP-violation effects and flavour-changing neutral currents.

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mixing is however complex, inducing CP-violation in the superweak horizontal interactions, which act only on the right-handed fermion components. The $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ model is a modest extension of the Weinberg-Salam electroweak theory, as the $O(3)_{RH}$ horizontal symmetry affects only the right-handed fermion components, which are scalar under the "vertical" $SU(2)_{LV}$.

In the next section, after defining the model precisely, we construct and diagonalize the mass matrices of fermions and gauge bosons. In section III. we first show (in part referring to the appendix) how CP-violating minima of the Higgs-potential arise and then determine the low-energy, effective, four-fermion interactions containing the CP-violating "horizontal" neutral currents. In the last section we briefly discuss the phenomenological implications of the model and make a few concluding remarks.

II. The $O(3)_{RH}$ family-group

We assume in this paper that there are three lepton-quark families spanning a three-dimensional (fundamental, vector) representation of $O(3)_{RH}$ with the right-handed field components $\psi_{RR}(x)_q$ ($x = 1, 2, 3$; $q = 1, 2, 3, 1/3$). Of course, the Lie-algebra of $O(3)$ is isomorphic to $SU(2)$, therefore we could call the horizontal symmetry $SU(2)_{RH}$ as well. By using the name $O(3)_{RH}$ we want to emphasize that representations with half-integer $SU(2)$ -spin (doublets, quartets etc.) will not be considered. Besides, for a larger number $n > 3$ of families we would take the generalization $O(n)_{RH}$ and its n -dimensional representation. The index q above refers to the electric charge: $q = 1$ stands for charged leptons, $q = 2/3$ for up-quarks and $q = 1/3$ for down-quarks. Right-handed neutrinos will not be considered in the present paper because of their exceptional status.

The left-handed fermion field components $\psi_L^\alpha(x)_h$ ($h = 1, 2, 3$) are doublets (index α) under $SU(2)_{LV}$. The index h stands for the three different families. There are, of course, separate left-handed doublet fields for quarks and leptons but in most cases it will not be necessary to specify whether the quarks or leptons are being considered.

The only difference is in the values of the weak hypercharge Y belonging to $U(1)$: $Y = 1/3$ for the quark doublet and $Y = -1$ for the lepton doublet. (The Y values for the right-handed fields are also determined by the familiar expression for the electric charge: $Q = T_{LV3} + Y/2$.)

The simplest possible set of Higgs scalar fields consists of a complex $SU(2)_{LV}$ doublet, $Y = 1$ and $O(3)_{RH}$ triplet: $\phi_I^\alpha(x)$ and three real $SU(2)_{LV}$ singlets, $Y = 0$ $O(3)_{RH}$ triplets: $\eta_I(x)_J$ ($J = 1, 2, 3$). These latter do not have Yukawa-coupling to fermions. They are needed in order to give large masses to the horizontal gauge bosons. The Yukawa-coupling e.g. to quarks has the form:

$$\begin{aligned} \mathcal{L}_Y = & - \sum_k \left\{ \bar{G}_k \tilde{\psi}_{R+}(x)_{1/3} \phi_{\alpha r}^+(x) \psi_L^y(x)_k + \right. \\ & \left. + \bar{G}_k \tilde{\psi}_{R+}(x)_{2/3} \tilde{\phi}_{\alpha r}^+(x) \psi_L^y(x)_k \right\} + h.c. \end{aligned} \quad (1.1)$$

where the $Y = -1$ field $\tilde{\phi}$ is defined as usual by $\tilde{\phi}^\alpha \equiv \epsilon_{\alpha\beta} \phi_\beta^+$. The Yukawa-coupling constants are real in accordance with the CP-invariance of the Lagrangean. The zeroth order quark mass matrices are given by the vacuum expectation value of the ϕ field, which is of the form

$$\langle 0 | \phi_r^y(x) | 0 \rangle = \delta_{\alpha 2} \frac{1}{\sqrt{2}} (u_r + i v_r) \quad (1.2)$$

This gives the following mass matrix, for instance, for the down-quarks:

$$m_{rk} = \frac{1}{\sqrt{2}} (u_r - i v_r) G_k \quad (1.3)$$

For the charged leptons we have the same form as this, only with different Yukawa-coupling constants G_h' . For the up-quarks, on the other hand, G_h is replaced by \bar{G}_h and the complex conjugate is taken.

From the remarks made after Eq. (1.3), it follows that O_L is, in general, different for different electric charges, whereas O_R is the same and the phases in F are the same for down-quarks and charged leptons and they are opposite for the up-quarks.

It can be shown, that in the parametrization introduced for the three family mixing problem by Maiani [21], we have for the right-handed orthogonal mixing matrix O_R :

$$O_R = \begin{pmatrix} \cos\beta_R & -\sin\gamma_R \sin\beta_R & -\cos\gamma_R \sin\beta_R \\ 0 & \cos\gamma_R & -\sin\gamma_R \\ \sin\beta_R & \sin\gamma_R \cos\beta_R & \cos\gamma_R \cos\beta_R \end{pmatrix} \quad (1.9)$$

where the angles are given by

$$\sin\beta_R = \frac{\rho_1}{\rho} \quad ; \quad \sin\gamma_R = \frac{\rho_2}{\sqrt{\rho^2 - \rho_1^2}} \quad (1.10)$$

The third Maiani-angle θ_R is zero. The horizontal interaction of the physical quarks depends on the right-handed mixing matrix U_R given by Eqs. (1.7-9).

It can be easily shown that the mass matrix m in Eq. (1.3) has two zero eigenvalues (the third, non-zero eigenvalue is $m_3 = G\rho/\sqrt{2}$). This means that the masses of the first two families are zero at the tree-level. In order to obtain non-zero tree-level masses one has to introduce three Yukawa-coupled ϕ -like Higgs-field: $\phi_J^\alpha(x)$ ($J = 1, 2, 3$). (For the possibility of radiative generation of the masses of the first and second family see the remarks made in the last section.) The Higgs-potential with three n -like and three ϕ -like fields is, however, rather complicated, because of the large

In order to obtain the physical fermion states with definite mass the mass matrix m has to be diagonalized by a biunitary transformation:

$$U_R m U_L^{-1} = \text{diag}(m) \quad (1.4)$$

$$U_R m m^+ U_R^{-1} = U_L m^+ m U_L^{-1} = \text{diag}(m^2)$$

(In what follows we shall explicitly consider only the down-quark sector, as the results can be easily transcribed also for the up-quarks and charged leptons.) U_R and U_L are in general 3×3 unitary matrices. The specific form of Eq. (1.3) implies:

$$\begin{aligned} (m^+ m)_{hh'} &= \frac{1}{2} \rho^2 G_h G_{h'} \quad (\rho^2 = \rho_+ \rho_-) \quad (1.5) \\ (m m^+)_{\tau\tau'} &= \frac{1}{2} G^2 \rho_+ \rho_{\tau'} e^{i(\varphi_{\tau'} - \varphi_\tau)} \quad (G^2 = \sum_k G_k^2) \end{aligned}$$

where we used the notation

$$u_+ + i v_+ \equiv \rho_+ e^{i\varphi_+} \quad (1.6)$$

From Eq. (1.5) it follows, that

$$U_L = O_L \quad ; \quad U_R = O_R F \quad (1.7)$$

where O_L and O_R are real orthogonal matrices and F is a diagonal matrix containing the phases:

$$F = \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix} \quad (1.8)$$

number of independent invariants. Therefore, in the next section we shall consider in detail only the simpler case with one ϕ -field. Note, that if the three ϕ -fields have the vacuum expectation values

$$\langle 0 | \phi_r^\alpha(x) | 0 \rangle = \delta_{\alpha 2} \delta_{r 7} \frac{1}{\sqrt{2}} (u_r + i v_r), \quad (1.11)$$

then both the gauge boson matrix and the structure of the fermion mixing matrices in (1.7) remain the same. As a consequence, from the point of view of CP-violation nothing essential is changed. The down-quark mass matrix, for instance, following from Eq. (1.11) is

$$m_{rh} = \frac{1}{\sqrt{2}} (u_r - i v_r) g_{rh}, \quad (1.12)$$

This implies

$$\begin{aligned} (m^+ m)_{rh} &= \frac{1}{2} \sum_{\tau=1}^3 \rho_\tau^2 g_{\tau h} g_{\tau r} \\ (m^+ m)_{r+1} &= \frac{1}{2} \sum_{h=1}^3 g_{rh} g_{+h} \rho_{+h} e^{i(\varphi_r - \varphi_{+h})} \end{aligned} \quad (1.13)$$

The relations (1.7-8) are valid as before, but the explicit form of O_R is more complicated than (1.9). In fact, O_R is now different for different electric charges (like O_L , too). The reality of the left-handed mixings implies that the Cabibbo-matrix entering the left-handed, vertical interactions

$$C_L \equiv U_L (q = 2/3) U_L (q = 1/3)^{-1} \quad (1.14)$$

is real. Therefore the Kobayashi-Maskawa CP-violation phase [20] vanishes. There is, however, CP-violation in the right-handed horizontal couplings because of the phases in U_R . These phases are opposite for the up-quarks and down-quarks. This means that $\det m$ has

opposite phase for up- and down-quarks, therefore the total phase of the determinant of the quark mass matrix on the whole flavour space is zero at the tree-level. As a consequence, there is probably no problem with the induced strong CP-(and P-) violation as it can arise only through the radiative corrections, similarly to the model in [22,23].

Let us consider the mass matrix of gauge bosons following from the vacuum expectation values in (1.11). (This contains also Eq. (1.2) with a single ϕ -field as a special case.) Corresponding to the symmetry group $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ there are altogether 7 gauge bosons: $W_{1,2,3}(x)_\mu$ for $SU(2)_{LV}$, $B(x)_\mu$ for $U(1)$ and $H_{1,2,3}(x)_\mu$ for $O(3)_{RH}$. The experimental upper limits on the flavour-changing (in fact, generally "family changing") neutral currents [4,15] tell us that the masses of the horizontal gauge bosons are at least a factor 10^2 larger than the masses of the Weinberg-Salam gauge bosons. These high masses are provided by the vacuum expectation values of the Higgs-fields $\eta_J(x)_J$ ($J = 1,2,3$), which are scalar with respect to $SU(2)_{LV} \otimes U(1)$. Therefore, our model has two different intrinsic mass scales set by the magnitudes of the vacuum expectation values of ϕ -fields and η -fields, respectively, the latter being at least a factor 10^2 larger than the former.

In order to break down the $O(3)_{RH}$ symmetry completely we need at least two η -fields. The third one is needed for the spontaneous breaking of the CP-symmetry. As we shall see in the appendix, it seems to be impossible to have a CP-violating minimum of the Higgs-potential with two η -fields. By the $O(3)_{RH}$ symmetry-transformation freedom the vacuum expectation values of the three η -fields can always be transformed into the position:

$$\begin{aligned} \langle 0 | \eta_r(x) | 0 \rangle &= \delta_{r1} z_1 \equiv z_{1r}, \\ \langle 0 | \eta_r(x) | 0 \rangle &= (\delta_{r2} \cos \alpha + \delta_{r2} \sin \alpha) z_2 \equiv z_{2r}, \end{aligned} \quad (1.15)$$

$$\langle 0 | \eta_r(x) | 0 \rangle = [(\delta_{r1} \cos \alpha' + \delta_{r2} \sin \alpha') \sin \beta' + \delta_{r3} \cos \beta'] z_3 \equiv z_{3r},$$

Here g_V denotes the "vertical" $SU(2)_{UV}$ coupling constant. The coupling constant belonging to the weak-hypercharge group $U(1)$ will be denoted below by g' . The photon and Z -fields are the usual linear combinations of W_3 and B with the Weinberg-angle Θ_W :

$$\begin{aligned} A_\mu &= \sin\Theta_W W_{3\mu} + \cos\Theta_W B_\mu \\ Z_\mu &= -\cos\Theta_W W_{3\mu} + \sin\Theta_W B_\mu \end{aligned} \quad \left(\sin\Theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \right). \quad (1.20)$$

The photon field A_μ is, of course, massless, whereas the diagonal element in the mass-squared matrix belonging to Z is:

$$M_{ZZ}^2 = \frac{1}{4} (g_V^2 + g'^2) \rho^2. \quad (1.21)$$

This is, however, not the physical mass-squared of the Z -boson, because the vacuum expectation value of the ϕ -fields induces mixing terms between the horizontal gauge bosons and the Z -field in (1.20), too. Using the generators in Eq. (1.16), together with the generators of $SU(2)_{UV} \otimes U(1)$, it can be easily shown that the H_a - Z mixing term is ($a = 1, 2, 3$):

$$M_{aZ}^2 = M_{Za}^2 = \frac{1}{2} g_H \sqrt{g_V^2 + g'^2} g_\phi \rho \cos(\varphi_c - \varphi_\phi), \quad (1.22)$$

abc being a cyclic permutation of 123.

The diagonalization of the 4×4 mass-squared matrix of the neutral (Z and H) gauge bosons is facilitated by the hierarchy of the vacuum expectation values. Because of $|z_j| \gg \rho$ we can use e.g. ordinary time-independent perturbation theory [24] for the determination of the eigenvalues and eigenvectors, taking the contributions of the ϕ -fields (proportional to ρ) as the perturbation. The diagonalization of the "unperturbed" matrix M_{ab}^2 ($a, b = 1, 2, 3$) is possible analytically. The general formulae are, however, rather lengthy and later on we do not need them, as we shall assume that one of the horizontal gauge bosons (namely H_1) is much lighter than the other two. This

where the absolute values are ordered according to $|z_1| \geq |z_2| \geq |z_3|$. The gauge boson H_a ($a = 1, 2, 3$) belongs to the rotation in the $O(3)_{RH}$ -space around the a 'th axis, hence to the generator T_a , where

$$T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad T_2 = \begin{pmatrix} 0 & -i & 0 \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (1.16)$$

This implies that for a generic vacuum expectation value z_r of an n -like Higgs-field the mass-squared matrix of the horizontal bosons is (with g_H the $O(3)_{RH}$ coupling constant):

$$M_{ab}^2 = \frac{1}{4} g_H^2 (z_r z_r \delta_{ab} - z_a z_b). \quad (1.17)$$

Adding together the contributions of the three n -like fields in Eq. (1.15) and the similar contributions of the (complex) fields $\phi_r^\alpha(x)_J$, having the vacuum expectation values (1.11), we have

$$M_{ab}^2 = \frac{g_H^2}{4} \left[\sum_{J=1}^3 (z_r z_r \delta_{ab} - z_a z_b) + (u_r u_r \delta_{ab} - u_a u_b) \right]. \quad (1.18)$$

The last terms due to the ϕ -fields are, of course, only small corrections to the contribution of the n -fields.

The Weinberg-Salam $SU(2)_{UV} \otimes U(1)$ gauge bosons get their masses from the vacuum expectation values of the $SU(2)_{UV}$ doublet fields $\phi_r^\alpha(x)_J$ ($J = 1, 2, 3$), therefore the mass of the charged W -bosons is the standard one:

$$M_W^2 = \frac{1}{4} g_V^2 \rho^2 \quad (\rho^2 = S_r \rho_r = u_r u_r + v_r v_r). \quad (1.19)$$

means that it is enough to consider the special case $|z_1\rangle \gg |z_2\rangle \gg |z_3\rangle$. In this case one can neglect z_3 completely in the mass matrix (not, however, in minimizing the Higgs-potential: see next section). Putting $z_3 = 0$, H_3 becomes diagonal and the small mixing angle θ_H among H_1 and H_2 is given by:

$$\tan 2\theta_H = \frac{-\sin 2\alpha \cdot z_2^2}{z_1^2 + z_2^2 \cos 2\alpha} \quad (1.23)$$

The "unperturbed" (i.e. $\rho = 0$) masses of the neutral gauge bosons are the following:

$$\begin{aligned} M_Z^2 &= 0, & M_{H4}^2 &= \frac{1}{4} g_H^2 w_2^2, \\ M_{H2}^2 &= \frac{1}{4} g_H^2 w_1^2, & M_{H3}^2 &= \frac{1}{4} g_H^2 (z_1^2 + z_2^2), \end{aligned} \quad (1.24)$$

where we introduced the notation

$$\begin{aligned} w_1^2 &= \frac{1}{2} \left[z_1^2 + z_2^2 + \sqrt{(z_1^2 - z_2^2)^2 + 4z_1^2 z_2^2 \cos^2 \alpha} \right], \\ w_2^2 &= \frac{1}{2} \left[z_1^2 + z_2^2 - \sqrt{(z_1^2 - z_2^2)^2 + 4z_1^2 z_2^2 \cos^2 \alpha} \right]. \end{aligned} \quad (1.25)$$

This shows, that $|z_1\rangle > |z_2\rangle > |z_3\rangle$ implies, in general, $M_{H3} > M_{H2} > M_{H1}$.

Taking into account the first order perturbation due to the terms of order ρ we have:

$$\begin{aligned} M_Z^2 &= \frac{1}{4} (g_V^2 + g^2) \rho^2, & M_{H3}^2 &= \frac{1}{4} g_H^2 (z_1^2 + z_2^2 + \rho_1^2 + \rho_2^2), \\ M_{H1}^2 &= \frac{1}{4} g_H^2 \left[w_2^2 + \rho_3^2 + \rho_2^2 \cos^2 \theta_H + \rho_1^2 \sin^2 \theta_H + \rho_1 \rho_2 \sin 2\theta_H \cos(\theta_H - \theta_2) \right], & (1.26) \\ M_{H2}^2 &= \frac{1}{4} g_H^2 \left[w_1^2 + \rho_3^2 + \rho_2^2 \sin^2 \theta_H + \rho_1^2 \cos^2 \theta_H - \rho_1 \rho_2 \sin 2\theta_H \cos(\theta_H - \theta_2) \right]. \end{aligned}$$

It can be seen that the Z-boson mass remains up to first order at its canonical value $M_Z = M_W / \cos \theta_W$. In the next order it is shifted slightly upwards due to the mixing with H-bosons. The situation is phenomenologically similar to the case of the neutral SU(2), which was considered in a different context recently by Claudson, Georgi and Yildiz [25]. The eigenvectors corresponding to the eigenvalues in (1.26) can also be determined by perturbation theory. Transforming this information, also up to first order, into a rotation matrix with respect to the unperturbed coordinate system, one can obtain the inverse mass-squared matrix M^{-2} of the HZ-system, which enters the low-energy effective neutral current interactions. The final result is given in Table I. It is remarkable, that the off-diagonal terms in the $H_{1,2,3}$ subspace are a factor ρ^2/z^2 smaller than the diagonal ones. Similarly, the HZ-mixing terms are smaller than $M_Z^{-2} \gg M_{H\alpha}^{-2}$ by the same factor.

III. CP-violating effective four-fermion interactions.

As we discussed in the previous section, from the point of view of CP-violation it is enough to consider the simpler case with a single Yukawa-coupled field $\phi_I^\alpha(x)$. A necessary condition for spontaneous CP-violation is that $\phi_I^\alpha(x)$ develops complex vacuum expectation values in the minimum of the Higgs-potential. This conditions is, however, not always sufficient. The overall phase in Eq. (1.2) can be arbitrarily adjusted by the $SU(2)_{UV}$ -freedom left over after transforming the vacuum expectation values in the $\alpha = 2$ direction. As we shall see below, even a non-zero relative phase between the different components does not always imply CP-violation.

A convenient complete set of independent invariants for a single ϕ and three different n-fields is the following:

$$\begin{aligned} B_{\mathbb{Z}} &= \eta_{+3} \eta_{+1} - z_{\mathbb{Z}}^2 \equiv \eta_{\mathbb{Z}} \cdot \eta_{\mathbb{Z}} - z_{\mathbb{Z}}^2 \quad (\mathbb{Z} = 1, 2, 3), \\ B_{3+3} &= \eta_K \cdot \eta_L - z_K z_L \cos \alpha_{KL} \quad (K \neq L \neq \mathbb{Z}; \mathbb{Z}, K, L = 1, 2, 3), \\ B_{\mathbb{Z}} &= \phi_{\alpha+}^+ \phi_{+}^\alpha - \frac{1}{2} \rho^2 = \phi \cdot \phi - \frac{1}{2} \rho^2, \end{aligned}$$

$$Q_7 = 2\gamma_{r\bar{r}} \gamma_{s\bar{s}} \phi_{\alpha r}^+ \phi_{\beta s}^\alpha \quad (\bar{r} = 1, 2, 3),$$

$$Q_{3+\bar{r}} = (\gamma_{rK} \gamma_{sL} + \gamma_{sK} \gamma_{rL}) \phi_{\alpha r}^+ \phi_{\beta s}^\alpha \quad (K \neq L \neq \bar{r}, \bar{r}, K, L = 1, 2, 3),$$

$$Q_7 = (\phi_{\alpha r}^+ \phi_{\beta s}^\alpha)^2 - \phi_{\alpha r}^+ \phi_{\beta s}^\alpha \phi_{\beta s}^\alpha \phi_{\alpha r}^+,$$

$$Q_8 = \phi_{\alpha r}^+ \phi_{\beta s}^\alpha \phi_{\beta r}^+ \phi_{\alpha s}^\beta. \quad (2.1)$$

The most general renormalizable invariant potential is therefore:

$$V(\eta, \phi) = \sum_{j_1, k=1}^7 \lambda_{j_1 k} \rho_{j_1} B_k + \sum_{j=1}^8 \alpha_j Q_j. \quad (2.2)$$

Here λ_{jk} and α_j are dimensionless coupling constants, such that the first term with λ_{jk} is a positive definite quadratic form of the B 's and α_j ($j = 1, \dots, 8$) is positive. The parameters with the dimension of a mass ($z_{1,2,3}$ and ρ) are put into the definition of the bilinear invariants B_j ($j = 1, \dots, 7$).

It is shown in the appendix that for a finite range of the parameters this potential has its absolute minimum at the point of the vacuum expectation values given by Eqs. (1.15) and (1.2). The angle α in (1.15) is equal to α_{12} contained in B_6 and α' and β' can also be expressed by α_{12} , α_{13} and α_{31} . In the present paper we shall not need α' and β' explicitly, therefore this relation is omitted. (In general, α_{KL} is the angle between the directions of the vacuum expectation values z_{Kr} and z_{Lr} .) The important point is that the phases $\phi_{1,2,3}$ in the vacuum expectation value of ϕ are completely general and none of the $\rho_{1,2,3}$ vanishes. As we shall see below, this insures CP-non-conservation in the horizontal interactions.

Our next task is to determine the low-energy effective four-fermion interactions among the physical fermions. The charged current is the standard one with a real Cabibbo-Kobayashi-Maskawa matrix, therefore we shall concentrate on the neutral current interactions. As was shown in the previous section, the four neutral gauge bosons $H_1, 2, 3$ and Z are mixed by the spontaneous symmetry breaking. The corresponding inverse-mass-squared matrix is given by Table I. in the case $M_Z^{-2} \gg M_{H1}^{-2}, M_{H2}^{-2}, M_{H3}^{-2}$ (equivalent to $\rho \ll z_j^2$). The neutral currents have to be given in terms of the physical fermion fields $\psi_{L,R}(x)$ obtained from $\phi_{L,R}(x)$ by the unitary transformations in (1.7), which diagonalize the mass matrix:

$$\chi_{L,R}(x) \equiv U_{L,R} \psi_{L,R}(x). \quad (2.3)$$

The "family-changing" (horizontal) neutral current belonging to H_a ($a = 1, 2, 3$) is then:

$$J_{a, q}^\mu(x) = (U_{Rq}^T U_{L,R}^{-1})_{rs} \bar{\chi}_{Rr}(x) \gamma_\mu \chi_{Rs}(x) q. \quad (2.4)$$

Here T_a is the horizontal generator given by Eq. (1.16) and the index $q = 1, 2/3, 1/3$ is also included in order to distinguish the different electric charges. The matrices $U_{Rq}^T U_{L,R}^{-1}$ are displayed in Table II. for U_R given by Eqs. (1.7-8) and (1.9). The neutral current belonging to Z is the standard one:

$$J_Z^\mu(x) = \sin^2 \theta_w J_{em}^\mu - \sum_{k=1}^3 \bar{\chi}_{Lk}(x) \gamma_\mu \frac{T_3}{2} \chi_{Lk}(x), \quad (2.5)$$

where h is the family index and J_{em} is the electromagnetic current.

The full low-energy effective neutral current interaction Hamiltonian is:

$$\begin{aligned} \mathcal{H}(x)_{eff} = & \frac{(g' + g)^2}{2M_Z^2} J_Z^\mu(x) J_Z^\mu(x) + \frac{g_H^2 \sqrt{g'^2 + g^2}}{2} J_Z^\mu(x) \sum_{a=1}^3 M_a^{-2} \sum_{\alpha, \beta} \bar{f}_{a, \beta}(x) f_{a, \alpha}(x) + \\ & + \frac{g_H^2}{8} \sum_{a, b=1}^3 M_a^{-2} M_b^{-2} \sum_{q, q'} \bar{f}_{a, q}(x) f_{b, q'}(x). \end{aligned} \quad (2.6)$$

The first term is the standard Weinberg-Salam neutral current interaction. The third term is the "family-changing" (flavour changing) neutral current interaction between two horizontal currents, and the second one is a mixed interaction involving the Z-neutral-current and the horizontal currents. The inverse gauge boson masses are, given by Table I. Because of $M_Z^{-2} \gg M_{ab}^{-2}, M_{Za}^{-2}$ the family-changing pieces are in general small. As shown by Table I., the off-diagonal (a ≠ b) pieces in the last term of (2.6) have an extra suppression factor compared to the diagonal (a = b) terms, which are similar in order of magnitude to the mixed ZH-terms.

The CP-violation in \mathcal{H}_{eff} is caused by the matrices $U_R^T U_R^{-1}$ (see Table II.) appearing in the horizontal currents. It can be seen from Eq. (1.10), that for general non-zero $\rho_{1,2,3}$ the mixing angles β_R and γ_R do not take on any special value, therefore in general neither $\sin\beta_R, \sin\gamma_R$ nor $\cos\beta_R, \cos\gamma_R$ vanish. Similarly, as is shown in the appendix, the phase differences ($\varphi_{1-3} - \varphi_k$) are also arbitrary, hence the off-diagonal elements in $U_R^T U_R^{-1}$ are general complex numbers, which appear in \mathcal{H}_{eff} . This violates the CP-symmetry (and time-reversal symmetry), which would require the coefficients of the current-current terms in \mathcal{H}_{eff} to be real.

IV. Discussion and concluding remarks.

In the previous sections we have seen, that in the $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ model the spontaneous breaking of CP-invariance is manifested at the tree level only in the right-handed horizontal interactions suppressed by the large masses of the horizontal gauge bosons. This is the consequence of the peculiar chiral structure of interactions (only left-handed "vertically" and only right-handed "horizontally") and of the form of vacuum expectation values in Eqs. (1.2) or (1.11). This form is achieved in the simpler case (1.2), considered in detail in section III., by taking a single ϕ -field. In this case the masses of the first two families vanish at the tree level. These masses have to be given by the loop corrections. An interesting possibility for this is to embed the $O(3)_{RH}$ horizontal symmetry in a left-right symmetric $O(3)_{LH} \otimes O(3)_{RH}$, similarly to a recent paper on radiative quark mass

generation [26]. The radiative corrections generally do not preserve the specific form of the fermion mass matrix m given by (1.3), therefore the Kobayashi-Maskawa phase δ in the left-handed mixing matrix (1.14) is expected to get a small, non-zero value at the one loop level. This implies a small CP-violation also in vertical interactions, where the smallness is explained by the fact that $\delta \neq 0$ comes from radiative corrections. The presence of CP-violations in both horizontal and vertical interactions, of course, complicates the phenomenology. In the discussion below we shall assume, for simplicity, that the horizontal CP-violation dominates. (If the vertical CP-violation dominates, the phenomenology is of the conventional Kobayashi-Maskawa type [20,27].)

The CP-non-conserving, low energy effective four-fermion interaction containing the horizontal neutral currents was determined in the previous section. The dominant piece of the $dd \rightarrow ss$ transition relevant for the neutral kaon system is, from Eqs. (2.6), (2.4) and Table II. the following:

$$\mathcal{H}_{dd \rightarrow ss} (x) \cong \frac{1}{8} g_H^2 \tilde{\gamma}_R(x) \tilde{\gamma}_R(x) d_R(x) \tilde{\gamma}_R(x) \tilde{\gamma}_R(x) d_R(x) \cdot \left\{ M_{H1}^{-2} \sin^2 \beta_R \left[\sin^2(\varphi_2 - \varphi_3) \cos^2 \gamma_R - \cos^2(\varphi_2 - \varphi_3) - i \sin 2(\varphi_2 - \varphi_3) \cdot \cos 2\gamma_R \right] + M_{H2}^{-2} \sin^2 \gamma_R \cos^2 \beta_R \left[-\cos 2(\varphi_3 - \varphi_1) - i \sin 2(\varphi_3 - \varphi_1) \right] + M_{H3}^{-2} \cos^2 \gamma_R \cos^2 \beta_R \left[-\cos 2(\varphi_1 - \varphi_2) + i \sin 2(\varphi_1 - \varphi_2) \right] \right\}. \quad (4.1)$$

According to Eq. (1.26) the horizontal gauge boson masses satisfy $M_{H1} < M_{H2} < M_{H3}$. This favours the first term, but the mixing angles β_R and γ_R can be small, typically one can take e.g. $\beta_R = 10^{-2}$ and $\gamma_R = 10^{-1}$. (This is satisfied if in Eq. (1.10) $\rho_1/\rho = 10^{-2}$ and $\rho_2/\rho = 10^{-1}$. Note that β_R is the mixing angle between the first

and third families, whereas γ_R is the one between the second and third families, therefore it is plausible to take β_R smaller.) The effect of these mixing angles in (4.1) is, that if all the three terms in the curly brackets are roughly the same, then $M_{H1} \approx 10^{-1} M_{H2} \approx 10^{-2} M_{H3}$. This leaves open the interesting possibility, that the gauge boson H_1 is considerably lighter than H_3 . Consequently, the horizontal transitions between the second and third family are larger than those between the first and second one [5]. Replacing $\cos\beta_R$ and $\cos\gamma_R$ by 1, and considering the three contributions in (4.1) separately, the measured value of the imaginary part of the K_{OL}, K_{OS} mass difference (or the conventional ϵ -parameter of CP-violation) gives for $\sin 2\Delta\phi$ of order 1:

$$\begin{aligned} \sin^2 \beta_R M_W^2 / M_{H1}^2 &\leq 10^{-9} , \\ \sin^2 \gamma_R M_W^2 / M_{H2}^2 &\leq 10^{-9} , \\ M_W^2 / M_{H3}^2 &\leq 10^{-9} . \end{aligned} \quad (4.2)$$

Here one of the inequalities has to be an equality. For the above values of the mixing angles ($\beta_R = 10^{-2}$, $\gamma_R = 10^{-1}$) this means $M_{H1} \geq 30$ TeV, $M_{H2} \geq 300$ TeV, $M_{H3} \geq 3000$ TeV. Therefore, as already noted in another horizontal CP-violation model by Davidson and Wali [8] the mass of the horizontal gauge boson H_3 comes out from the CP-violation about a factor 10^2 larger than the lower limits obtained from the absence of the family-changing decays in the first two families [14].

The prediction for the CP-violating parameter ϵ' in our model is smaller than the one in the Kobayashi-Maskawa model [28]. ϵ' is proportional to the imaginary part of the so called "penguin graphs" [29]. In the standard Kobayashi-Maskawa scheme the penguin graphs induced by the W-boson contribute to ϵ' and give $\epsilon'/\epsilon \approx 1\%$ [28]. In our model these graphs are purely real since the Kobayashi-Maskawa phase is zero. However, the horizontal bosons induce extra

"penguin graphs" (Fig. 1), which are non-real and hence contribute to ϵ' . These graphs are finite because the analogue of GIM mechanism is operative. (This is a consequence of the relation $\sum_a \hat{T}_a^T = 2$ satisfied by the matrices \hat{T}_a of Table II.) The detailed prediction depends upon what we assume for various mixing angles. Roughly, we find the ratio

$$\left| \frac{\epsilon'_H}{\epsilon'_{KM}} \right| \approx 10^{-4} - 10^{-6} \quad (4.3)$$

between ϵ' in our scheme (ϵ'_H) and in the Kobayashi-Maskawa scheme (ϵ'_{KM}). Within the present experimental accuracy this prediction is indistinguishable from the prediction of the superweak model [17].

The electric dipole moment of the neutron is in our model smaller than the "usual" superweak value $D_n = 10^{-29} e \cdot \text{cm}$ [30]. This is due to the extra suppression factor in the imaginary part of the graph in Fig. 2.. Namely, the diagonal part of the H-boson inverse mass-squared matrix gives a pure real contribution (see Table II.). As already noted previously, the off-diagonal terms of M^{-2} in Table I. have additional suppression factors like $\rho_j \rho_K / z_j^2$.

Another phenomenological consequence of the $SU(2)_L \otimes U(1) \otimes O(3)_R$ model is the small mixing of the Z-boson with the three horizontal gauge bosons. According to Eq. (2.6) and Table I. the corresponding decays of the Z are suppressed, compared to the normal neutral current decays, by factors like $\rho_j \rho_K / z_j^2$. Again, the H_3 -channels are suppressed by the smallest factor: $\rho_1 \rho_2 / z_1^2$ and the H_1 -channels by the least small one: $\rho_2 \rho_3 / w_2^2$. Hence there are small CP-non-conserving, flavour-changing decays of the Z to the first two families at the level of 10^{-12} and to the second and third families at the level of 10^{-6} (in the amplitude, for the above values of β_R and γ_R). These are very small, but perhaps the mixing angles are different. For $\sin \gamma_R = O(1)$ and $\beta_R = 10^{-3}$ the suppression factor $\rho_2 \rho_3 / w_2^2$ is only 10^{-3} and then the observation of the CP-violating, flavour changing decay $Z \rightarrow \mu \tau$ is not hopeless.

As mentioned above, the possibility of a relatively light horizontal gauge boson H_1 acting on the second and third families is not ruled out by the present data. $M_{H1} \approx 3$ TeV, for instance, would require in Eq. (4.2) $\beta_R \approx 10^{-3}$. (An even smaller value of M_{H1} is allowed if in Eqs. (4.1-2) $\sin 2(\phi_2 - \phi_3) < 1$ is taken into account.) Perhaps the best place to search for such effects is offered by possible rare r -decays [15], but rare decays of the b -quark may also give interesting information. The CP-violating character of the horizontal interactions implies that these rare decay channels would show large CP-violating effects.

In summary: the predictions of the $SU(2)_{LV} \otimes U(1) \otimes O(3)_{RH}$ model for the conventional CP-violation parameters are typically of "superweak" magnitude (or even smaller). This is due to the large mass ($\approx 10^6$ GeV) of the horizontal gauge boson acting on the first two families. It is, however, possible that the horizontal gauge boson acting on the heavy families is much lighter ($\approx 10^3$ GeV). Therefore, the CP-violation in the third family can be of "milliweak" strength.

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Table I: The inverse-mass-squared matrix M^{-2} of neutral gauge bosons up to the first order in ρ^2/z^2 . M^{-2} is symmetric therefore only the upper part is filled in. M_{Ha}^2 and M_Z^2 is given in Eq. (1.26), for other notations see the text.

M_{H1}^{-2}	$(M_{H1}^{-2} - M_{H2}^{-2}) \left[\frac{1}{2} (\rho_1^2 - \rho_2^2) \sin 2\theta_H + \rho_1 \rho_2 \cos 2\theta_H \cos(\phi_1 - \phi_2) \right] \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right)^{-1}$	$(M_{H1}^{-2} - M_{H3}^{-2}) \left[\rho_1 \rho_3 \cos \theta_H \cos(\phi_1 - \phi_3) - \rho_2 \rho_3 \sin \theta_H \cos(\phi_2 - \phi_3) \right] \omega_1^{-2}$	$(M_Z^{-2} - M_{H1}^{-2}) \left[\rho_2 \rho_3 \cos \theta_H \sin(\phi_2 - \phi_3) - \rho_1 \rho_3 \sin \theta_H \sin(\phi_1 - \phi_3) \right] \frac{2\sqrt{g_V^2 + g^2}}{g_H \omega_2^2}$
M_{H2}^{-2}		$(M_{H2}^{-2} - M_{H3}^{-2}) \left[\rho_2 \rho_3 \cos \theta_H \cos(\phi_2 - \phi_3) + \rho_1 \rho_3 \sin \theta_H \cos(\phi_3 - \phi_1) \right] \omega_2^{-2}$	$(M_Z^{-2} - M_{H2}^{-2}) \left[\rho_1 \rho_3 \cos \theta_H \sin(\phi_3 - \phi_1) + \rho_2 \rho_3 \sin \theta_H \sin(\phi_2 - \phi_3) \right] \frac{2\sqrt{g_V^2 + g^2}}{g_H \omega_1^2}$
		M_{H3}^{-2}	$(M_Z^{-2} - M_{H3}^{-2}) \rho_1 \rho_2 \sin(\phi_1 - \phi_2) \cdot \frac{2\sqrt{g_V^2 + g^2}}{g_H (\omega_1^2 + \omega_2^2)}$
			M_Z^{-2}

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Table II: The matrices $\hat{T}_a = U_R^T U_a U_R^{-1}$ in the horizontal currents (2.4) for the electric charge $q = 1$ and $1/3$ (charged leptons and down-quarks). For $q = 2/3$ (up-quarks) everything remains the same only the phases have to be reversed: $\varphi_j \rightarrow -\varphi_j$.

$$\hat{T}_1 = \begin{bmatrix} \sin(\varphi_2 - \varphi_3) \sin 2\gamma_R \sin^2 \beta_R & -i \cos(\varphi_2 - \varphi_3) \sin \beta_R - \sin(\varphi_2 - \varphi_3) \sin \beta_R \cos 2\gamma_R & -\frac{1}{2} \sin(\varphi_2 - \varphi_3) \sin^2 \beta_R \sin 2\gamma_R \\ i \cos(\varphi_2 - \varphi_3) \sin \beta_R - \sin(\varphi_2 - \varphi_3) \sin \beta_R \cos 2\gamma_R & -\sin(\varphi_2 - \varphi_3) \sin 2\gamma_R & -i \cos(\varphi_2 - \varphi_3) \cos \beta_R + \sin(\varphi_2 - \varphi_3) \cos \beta_R \cos 2\gamma_R \\ -\frac{1}{2} \sin(\varphi_2 - \varphi_3) \sin 2\gamma_R \sin 2\beta_R & i \cos(\varphi_2 - \varphi_3) \cos \beta_R + \sin(\varphi_2 - \varphi_3) \cos \beta_R \cos 2\gamma_R & \sin(\varphi_2 - \varphi_3) \cos^2 \beta_R \sin 2\gamma_R \end{bmatrix}$$

$$\hat{T}_2 = \begin{bmatrix} -\sin(\varphi_3 - \varphi_1) \cos \gamma_R \sin 2\beta_R & [-i \cos(\varphi_3 - \varphi_1) - \sin(\varphi_3 - \varphi_1)] \sin \gamma_R \cos \beta_R & i \cos(\varphi_3 - \varphi_1) \cos \gamma_R + \sin(\varphi_3 - \varphi_1) \cos \gamma_R \cos 2\beta_R \\ [i \cos(\varphi_3 - \varphi_1) - \sin(\varphi_3 - \varphi_1)] \sin \gamma_R \cos \beta_R & 0 & [i \cos(\varphi_3 - \varphi_1) - \sin(\varphi_3 - \varphi_1)] \sin \beta_R \sin \gamma_R \\ -i \cos(\varphi_3 - \varphi_1) \cos \gamma_R + \sin(\varphi_3 - \varphi_1) \cos \gamma_R \cos 2\beta_R & [-i \cos(\varphi_3 - \varphi_1) - \sin(\varphi_3 - \varphi_1)] \sin \beta_R \sin \gamma_R & \sin(\varphi_3 - \varphi_1) \cos \gamma_R \sin 2\beta_R \end{bmatrix}$$

$$\hat{T}_3 = \begin{bmatrix} -\sin(\varphi_1 - \varphi_2) \sin \gamma_R \sin 2\beta_R & [-i \cos(\varphi_1 - \varphi_2) + \sin(\varphi_1 - \varphi_2)] \cos \gamma_R \cos \beta_R & -i \cos(\varphi_1 - \varphi_2) \sin \gamma_R + \sin(\varphi_1 - \varphi_2) \sin \gamma_R \cos 2\beta_R \\ [i \cos(\varphi_1 - \varphi_2) + \sin(\varphi_1 - \varphi_2)] \cos \gamma_R \cos \beta_R & 0 & [i \cos(\varphi_1 - \varphi_2) + \sin(\varphi_1 - \varphi_2)] \cos \gamma_R \sin \beta_R \\ i \cos(\varphi_1 - \varphi_2) \sin \gamma_R + \sin(\varphi_1 - \varphi_2) \sin \gamma_R \cos 2\beta_R & [-i \cos(\varphi_1 - \varphi_2) + \sin(\varphi_1 - \varphi_2)] \cos \gamma_R \sin \beta_R & \sin(\varphi_1 - \varphi_2) \sin \gamma_R \sin 2\beta_R \end{bmatrix}$$

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Figures

Figure 1 The "penguin" graph induced by the horizontal gauge boson H (G is a gluon).

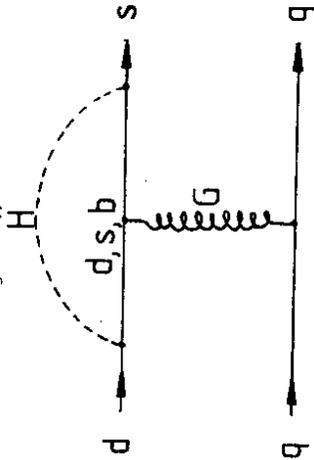
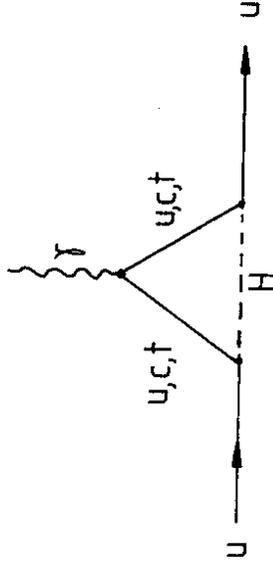


Figure 2 The graph with a horizontal gauge boson H contributing to the electric dipole moment of the u-quark. (The wavy line γ is a photon.)



Appendix

In this appendix we show that for an appropriate range of parameter values the Higgs-potential (22) has its absolute minimum in the point of the vacuum expectation values given by Eqs. (1.2) and (1.15). Let us first ignore the "mixed" quadrilinear couplings and (1.15). Let us first ignore the "mixed" quadrilinear couplings Q_1, \dots, Q_6 connecting the ϕ -field with the η -fields. We first determine the minimum in this case and then "switch on" the couplings Q_1, \dots, Q_6 keeping the corresponding parameters χ_1, \dots, χ_6 small compared to the others. We shall see that the absolute minimum obtained by this "perturbative" procedure fulfills all our requirements.

The parameters λ_{jk} determine, by assumption, a positive definite quadratic form which has its (zero) absolute minimum if all B_j^i ($j = 1, \dots, 7$) are zero. This can be fulfilled only if $\eta_r(x)_j$ ($J = 1, 2, 3$) is equal to the three-vector with r-component z_{Jr} , having the length z_j and relative angles α_{JK} to the other z_{Kr} 's. By an appropriate O(3)_{RH} transformation z_{Jr} can be brought to the form given in (1.15) where $\alpha = \alpha_{12}$ and α', β' can also be expressed by α_{12}, α_{23} and α_{31} . The requirement $B_7 = 0$ implies that the ϕ -field has a length $\rho/\sqrt{2}$.

Using the Cauchy-Schwarz inequality it can be easily shown that the invariants Q_7 and Q_8 are non-negative. By explicit calculation or, more elegantly, with the help of the hermitian matrix $V_{rs} = \phi_{or}^+ \phi_s^+$ one can show that

$$Q_7 = 2 \{ \phi_{1r}^+ \phi_r^+ \phi_{2s}^+ \phi_s^+ - \phi_{2r}^+ \phi_r^+ \phi_{1s}^+ \phi_s^+ \} \geq 0, \quad (A.1)$$

This is zero if and only if $\phi_r^+ = \lambda \phi_r^2$ (λ independent of r). By an appropriate SU(2)_{LV} transformation it is always possible to achieve e.g. $\phi_{r=1}^{\alpha=1} = 0$ implying $\lambda = 0$ and hence $\phi_r^+ = 0$ ($r = 1, 2, 3$). This insures that the photon remains massless (the electric charge will be conserved) and the minimum in ϕ has the form required by Eq. (1.2). In order to fulfill also the previous condition we have to

put

$$\phi^2 = \phi_r^+ \phi_r = u_r u_r + v_r^+ v_r, \quad (A.2)$$

The invariant Q_8 has in this minimum the form

$$Q_8 = \frac{1}{4} (u_r u_r - v_r^+ v_r)^2 + (u_r v_r)^2 \geq 0, \quad (A.3)$$

This shows that the absolute minimum of the potential V without the mixed quadrilinear terms is equal to zero and it is in the set of points satisfying

$$u_r u_r = v_r^+ v_r = \frac{\rho^2}{2}, \quad u_r v_r = 0 \quad (A.4)$$

As far as the ϕ -dependent part is concerned this is consistent with a result in Ref. [31].

The relative orientation of the equal length orthogonal vector pair u, v to the three vectors $z_{1,2,3}$ is up to now undetermined. There is also the freedom of choosing the overall phase left over from the SU(2)_{LV} transformation (resulting in $\phi_r^1 = 0$). A common phase change $\phi_r \rightarrow \phi_r + \varphi$ in the three components of ϕ_r^2 is equivalent to the transformation

$$u_r^1 = u_r \cos \varphi - v_r^+ \sin \varphi, \quad v_r^1 = u_r \sin \varphi + v_r^+ \cos \varphi. \quad (A.5)$$

This leaves the plane of u, v always invariant, and it is a rigid rotation of the u, v pair if Eq. (A.4) holds. In other words, the whole potential V can only depend on the normal vector n of the u, v -plane because it is independent of φ .

The effect of the mixed terms Q_1, \dots, Q_6 is to fix the relative orientation of the normal vector n with respect to the three vectors $z_{1,2,3}$. Explicitly, the "mixed" part of V in the minimum obtained above is

$$V_{\text{mixed}} = \sum_{j=1}^3 \chi_j^2 \left[(u_r z_{jr}^+)^2 + (v_r^+ z_{jr}^+)^2 \right] + \sum_{\substack{j=1 \\ k \neq l \neq j}}^3 \chi_{3+j}^2 \left[u_r z_{kr}^+ u_l z_{lr}^+ + v_r^+ z_{kr}^+ v_l^+ z_{lr}^+ \right]. \quad (A.6)$$

If the parameters χ_1, \dots, χ_6 are small compared to λ_{jk} and χ_7, χ_8 , then in first approximation the vectors $z_{1,2,3}$ remain unchanged and u, v still satisfy Eq. (A.4). After some trigonometric work one can show that in this case

$$V_{mix} = \sum_{j=1}^3 \int_j \sin^2 \omega_j + \sum_{k \neq l=1,2,3} \int_{kl} \chi_k \chi_l \sin \omega_k \sin \omega_l \cos(\delta_k - \delta_l), \quad (A.7)$$

where

$$\int_j = \sum_{k=1}^2 \chi_k z_k z_j^2 \quad (j=1,2,3),$$

$$\int_{kl} = \chi_{3+j} z_k z_l z_j^2 \quad (k \neq l \neq j). \quad (A.8)$$

ω_j is the angle between n and z_j and $\delta_k - \delta_l$ is the angle between the projections of the vectors z_k and z_l onto the u, v -plane with normal vector n . It follows from a spherical triangle that we have also

$$\cos(\delta_k - \delta_l) = \frac{\cos \alpha_{kl} - \cos \omega_1 \cos \omega_2}{\sin \omega_1 \sin \omega_2}, \quad (A.9)$$

where α_{kl} is the angle between z_k and z_l .

Combining (A.7-8) and (A.9) one can see that the minimum of V_{mix} as a function of the orientation of n relative to the vectors z_j ($j = 1, 2, 3$) is at general values of the angles $\omega_{1,2,3}$. For instance, taking $\cos \alpha_{kl} = 0$ (z_k orthogonal to z_l) for all k, l and $\Gamma_{12} = \Gamma_{23} = \Gamma_{31} > \Gamma_{1,2,3}$ the minimum is somewhere in the middle between the three orthogonal directions of z_j 's. This means that the phases φ_j given by

$$\text{tg } \varphi_j = \frac{v_j}{u_j} \quad (A.10)$$

are general, resulting in CP-violation. This conclusion is not changed if the small changes in the vectors z_j and u, v due to the small mixed couplings are also taken into account. One can, for instance, show that in the plane perpendicular to n the vectors u and v will not remain exactly orthogonal. The deviation of their angle from $\pi/2$ will be of the order $\chi_j / \chi_8 \ll 1$ ($j = 1, \dots, 6$). The position of the absolute minimum of V is an analytic function of χ_1, \dots, χ_6 in some neighbourhood of the point $\chi_1 = \dots = \chi_6 = 0$. This insures that within this neighbourhood the minimum constructed above remains the absolute minimum. In the whole parameter space there can be, of course, also other regions with different absolute minima. There might be CP-conserving minima and also other minima with CP-violation.

From the above construction it is also clear, that with two η -fields it is impossible to obtain, at least by this perturbative method, a CP-violating minimum. Namely, in the form (A.7) of V_{mix} there are then only the terms with Γ_1, Γ_2 and Γ_{12} . In this case one can show that in the minimum the n -vector is always in the z_1, z_2 -plane. By the use of the phase freedom (A.5) one can always rotate u or v in the z_1, z_2 -plane. Taking, for instance, v we have $v_3 = 0$ and $u_1 = u_2 = 0$. Therefore, the phase differences $\Delta\varphi_{jk} = \varphi_j - \varphi_k$ given by Eq. (A.10) are either 0 or $\pi/2$. Due to the specific structure of the matrices in Table II. the CP-violation in the horizontal neutral current interaction is proportional to $\sin 2\Delta\varphi_{jk}$, therefore it vanishes for $\Delta\varphi_{jk} = 0$ and $\Delta\varphi_{jk} = \pi/2$.