HALF-FLAT STRUCTURES ON INDECOMPOSABLE LIE GROUPS

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ABSTRACT. This article can be viewed as a continuation of the articles [12] and [5] where the decomposable Lie algebras admitting half-flat SU(3)-structures are classified. The new main result is the classification of the indecomposable six-dimensional Lie algebras with five-dimensional nilradical which admit a half-flat SU(3)-structure. As an important step of the proof, a considerable refinement of the classification of six-dimensional Lie algebras with five-dimensional non-Abelian nilradical is established. Additionally, it is proved that all non-solvable six-dimensional Lie algebras admit half-flat SU(3)-structures.

1. Introduction

SU(3)-structures on a real six-manifold M are reductions of the frame bundle of M to SU(3) and can equivalently be described as quadruples (g, J, ω, Ψ) consisting of an almost Hermitian structure (g, J, ω) and a unit $(3,0)$-form Ψ. Such a structure is called half-flat if it satisfies

$$d\,\mathrm{Re}\Psi = 0\,, \quad d(\omega \wedge \omega) = 0.$$

A left-invariant half-flat SU(3)-structure on a six-dimensional Lie group can be characterised by a pair $(\omega, \rho) \in \Lambda^2\mathfrak{g}^* \times \Lambda^3\mathfrak{g}^*$ of a non-degenerate two-form ω and a three-form ρ of specific type on the associated Lie algebra $\mathfrak{g}$ satisfying certain compatibility relations and $d\rho = 0 = d(\omega \wedge \omega)$. We say that such a pair (ω, ρ) is a half-flat SU(3)-structure on the Lie algebra $\mathfrak{g}$.

The problem of determining the six-dimensional Lie algebras which admit a half-flat SU(3)-structure has been solved for the nilpotent case by Conti [2]. In [12], one of the authors has classified direct sums of two three-dimensional Lie algebras which admit such structures. The remaining decomposable six-dimensional Lie algebras which admit half-flat SU(3)-structures have been classified by the authors [5].

In this article we continue the work of [5] and tackle the problem for indecomposable six-dimensional Lie algebras. Non-solvable indecomposable six-dimensional Lie algebras have been classified by Turkowski [14]. The resulting list is given in Table 1 also including the only indecomposable six-dimensional simple Lie algebra. We obtain the following result by giving explicit examples in all cases in Table 2 and by considering the known results in the decomposable case [12], [5].

Theorem 1.1. *Let $\mathfrak{g}$ be a non-solvable six-dimensional Lie algebra. Then $\mathfrak{g}$ admits a half-flat* SU(3)*-structure.*

In the solvable case, we restrict our attention to indecomposable six-dimensional Lie algebras with five-dimensional nilradical. For the algebras with Abelian nilradical, we can exploit the relation between half-flat SU(3)-structures and cocalibrated G_2-structures and solve the existence problem by using a result of the first author [4]. For the algebras with non-Abelian nilradical, we have to resort to the classification of the indecomposable six-dimensional Lie algebras with five-dimensional nilradical. The slightly cumbersome classification was first established by Mubarakzyanov [10] in 1963 and was recently corrected by Shabanskaya [13].

In order to prove the main classification result, we need to subdivide Mubarakzyanov's classes according to the dimensions $\mathrm{h}^k(\mathfrak{g})$ of the Lie algebra cohomology groups and the dimension of the centre similar as in [5]. Although we exclude the Lie algebras with Abelian nilradical, the number of naturally appearing subclasses turns out to be larger than we expected, cf. Table 3. Nevertheless, the new data contributes to the understanding of low-dimensional Lie algebras and may be useful in the study of further problems concerning six-dimensional Lie algebras. As a first application, apart from our main result, we classify the (2,3)-trivial six-dimensional Lie algebras in section 4.

Since the Lie algebras with five-dimensional nilradical admitting a half-flat SU(3)-structure have hardly anything in common, a simple characterisation seems not possible and we have to state our main result in the following form.

Theorem 1.2. *A solvable indecomposable six-dimensional Lie algebra with five-dimensional nilradical admits a half-flat* SU(3)*-structure if and only if it is contained in Table 4.*

Note that Table 4 also contains an explicit example for each case.

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By a result of Mubarakzyanov [10], a solvable six-dimensional Lie algebra which is neither nilpotent nor decomposable has four- or five-dimensional nilradical. Hence, the classification remains open only for the case of solvable indecomposable six-dimensional Lie algebras with four-dimensional nilradical.

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2. Obstruction theory for half-flat SU(3)-structures

2.1. **Known obstructions.** A half-flat SU(3)-structure on a Lie algebra $\mathfrak{g}$ can be described within the framework of stable forms on real vector spaces developed by Hitchin [6] and thoroughly discussed e.g. in [3].

A *stable* k-form $\rho \in \Lambda^k V^*$ on a vector space V is a k-form which lies in an open $\mathrm{GL}(V)$ orbit for the natural action of $\mathrm{GL}(V)$ on V. A two-form in even dimensions is stable if and only if it is non-degenerate. To characterise stability of three-forms $\rho \in \Lambda^3 V^*$ on an oriented six-dimensional vector space V, Hitchin [7] introduced

$$K_\rho(v) := \kappa\left((v \lrcorner \rho) \wedge \rho\right) \in V \otimes \Lambda^6 V^*, \quad \lambda(\rho) := \frac{1}{6}\mathrm{tr} K_\rho^2 \in \left(\Lambda^6 V^*\right)^{\otimes 2}, \quad J_\rho := \frac{1}{\sqrt{|\lambda(\rho)|}} K_\rho \in V \otimes V^*,$$

where κ denotes the natural isomorphism $\Lambda^5 V^* \cong V \otimes \Lambda^6 V^*$ and $\sqrt{|\lambda(\rho)|} \in \Lambda^6 V^*$ denotes the positively oriented root of $|\lambda(\rho)| \in \left(\Lambda^6 V^*\right)^{\otimes 2}$. A three-form $\rho \in \Lambda^3 \mathfrak{g}^*$ in dimension six is stable if and only if $\lambda(\rho) \neq 0$ and the induced endomorphism J_ρ of V is a complex structure on V if and only if $\lambda(\rho) < 0$.

A *half-flat* SU(3)*-structure* on a six-dimensional Lie algebra $\mathfrak{g}$ is a pair of stable forms $(\omega, \rho) \in \Lambda^2 \mathfrak{g}^* \times \Lambda^3 \mathfrak{g}^*$ such that $\lambda(\rho) < 0$, $\omega \wedge \rho = 0$ and $d\omega^2 = 0 = d\rho$ and such that $g(\cdot, \cdot) := \omega(J_\rho \cdot, \cdot)$ is a Euclidean metric. A half-flat SU(3)-structure $(\omega, \rho) \in \Lambda^2 \mathfrak{g}^* \times \Lambda^3 \mathfrak{g}^*$ is called *normalised* if $J_\rho^* \rho \wedge \rho = \frac{2}{3}\omega^3$.

In [5, Corollary 3.2] we already developed an obstruction to the existence of a half-flat SU(3)-structure which is a refinement of the obstructions developed by Conti [2]. We state it here again denoting by $Z^p(\mathfrak{g})$ the space of all closed p-forms on $\mathfrak{g}$.

Proposition 2.1. *Let $\mathfrak{g}$ be a six-dimensional Lie algebra with a volume form $\nu \in \Lambda^6 \mathfrak{g}^*$. If there is a non-zero one-form $\alpha \in \mathfrak{g}^*$ satisfying*

$$\alpha \wedge \tilde{J}_\rho^* \alpha \wedge \sigma = 0 \tag{2.1}$$

for all $\rho \in Z^3(\mathfrak{g})$ and all $\sigma \in Z^4(\mathfrak{g})$, where $\tilde{J}_\rho^* \alpha$ is defined for $X \in \mathfrak{g}$ by

$$\tilde{J}_\rho^* \alpha(X)\, \nu \quad = \quad \alpha \wedge (X \lrcorner \rho) \wedge \rho, \tag{2.2}$$

then $\mathfrak{g}$ does not admit a half-flat SU(3)*-structure.*

2.2. **Obstructions from the relation to cocalibrated G_2-structures.** The obstruction obtained above is not the only tool we need for the proof of Theorem 1.2. Additionally, we exploit the relation between half-flat SU(3)-structures on six-dimensional Lie algebras $\mathfrak{g}$ and cocalibrated G_2-structures on seven-dimensional Lie algebras $\mathfrak{g} \oplus \mathbb{R}$. We roughly sketch this relation which is discussed in detail e.g. in [3] and [11].

Let $(\omega, \rho) \in \Lambda^2 \mathfrak{g}^* \times \Lambda^3 \mathfrak{g}^*$ be a half-flat SU(3)-structure on a six-dimensional Lie algebra $\mathfrak{g}$. Denote by J_ρ the complex structure on $\mathfrak{g}$ induced by ρ. Choose some non-zero element $\alpha \in \mathfrak{g}^0 \backslash \{0\}$ in the annihilator $\mathfrak{g}^0$ of $\mathfrak{g}$ in $\mathfrak{h} := \mathfrak{g} \oplus \mathbb{R}$. Then

$$\varphi := \omega \wedge \alpha + J_\rho^* \rho$$

is a G_2*-structure* on $\mathfrak{h} := \mathfrak{g} \oplus \mathbb{R}$, i.e. the stabiliser of φ under the natural action of $\mathrm{GL}(\mathfrak{h})$ on $\Lambda^3 \mathfrak{h}^*$ is isomorphic to G_2. Since $\mathrm{G}_2 \subseteq \mathrm{SO}(7)$, the G_2-structure φ induces a Euclidean metric g_φ and an orientation on $\mathfrak{h}$ such that the subspaces $\mathfrak{g}$ and $\mathbb{R}$ are orthogonal with respect to g_φ. Hence, it induces a Hodge star operator $\star_\varphi : \Lambda^3 \mathfrak{h}^* \to \Lambda^4 \mathfrak{h}^*$. A straightforward computation shows that the Hodge dual $\star_\varphi \varphi$ of φ is given by

$$\star_\varphi \varphi = \rho \wedge \alpha + \frac{1}{2}\omega^2. \tag{2.3}$$

Since the forms ρ, $\frac{1}{2}\omega^2$ and $\alpha \in \mathfrak{g}^0$ are all closed, the four-form $\star_\varphi \varphi$ is closed, as well. A G_2-structure φ with closed Hodge dual $\star_\varphi \varphi$ is called *cocalibrated*.

Thus, each half-flat SU(3)-structure on a six-dimensional Lie algebra induces a cocalibrated G_2-structure on the seven-dimensional Lie algebra $\mathfrak{h} = \mathfrak{g} \oplus \mathbb{R}$.

Proposition 2.2. *Let $\mathfrak{g}$ be an indecomposable six-dimensional Lie algebra and assume that the nilradical* $\mathrm{Nil}(\mathfrak{g})$ *of $\mathfrak{g}$ is isomorphic to $\mathbb{R}^5$. Then $\mathfrak{g}$ does not admit a half-flat* SU(3)*-structure.*

Proof. Let $\mathfrak{g}$ be an indecomposable six-dimensional Lie algebra with five-dimensional Abelian nilradical. Assume that $\mathfrak{g}$ admits a half-flat SU(3)-structure. Then the seven-dimensional Lie algebra $\mathfrak{h} := \mathfrak{g} \oplus \mathbb{R}$ has to admit a cocalibrated G_2-structure. In [4], the seven-dimensional Lie algebras $\mathfrak{h}$ with Abelian codimension one ideals which admit cocalibrated G_2-structures have been classified. Note that $\mathfrak{u} := \mathrm{Nil}(\mathfrak{g}) \oplus \mathbb{R}$ is an Abelian ideal of codimension one in $\mathfrak{h}$ which allows us to apply the results of [4] as follows. The indecomposability of $\mathfrak{g}$ implies that the complex Jordan normal form of $\mathrm{ad}(e_7)|_{\mathfrak{u}}$ for some $e_7 \in \mathfrak{h}\backslash\mathfrak{u}$ has to contain exactly one Jordan block of size one with 0 on the diagonal. However, this statement contradicts [4, Theorem 1.1] stating that in the complex Jordan normal form of $\mathrm{ad}(e_7)|_{\mathfrak{u}}$ the number of Jordan blocks of size one with zero on the diagonal has to be even. □

In the proof of the main result, we use another obstruction obtained by the relation between half-flat SU(3)-structures and cocalibrated G_2-structures. The following result on G_2-structures can be found in [4, Lemma 3.9, Remark 3.10].

Lemma 2.3. *Let V be a seven-dimensional vector space and $\varphi \in \Lambda^3 V^*$ be a G_2-structure on V. Let $v \in V$, $v \neq 0$ be arbitrary and U be an arbitrary complement of* $\mathrm{span}(v)$ *in V. Then the three-form $\tilde{\rho} := (v \lrcorner \star_\varphi \varphi)|_U \in \Lambda^3 U^*$ is a stable three-form on U with $\lambda(\tilde{\rho}) < 0$.*

As a consequence, we can prove the following obstruction condition.

Proposition 2.4. *Let $\mathfrak{g}$ be a six-dimensional Lie algebra and set $\mathfrak{h} := \mathfrak{g} \oplus \mathbb{R}$. Choose a non-zero one-form $\alpha \in \mathfrak{g}^0$ in the annihilator $\mathfrak{g}^0$ of $\mathfrak{g}$ in $\mathfrak{h}$. For each pair $(\rho, \sigma) \in Z^3(\mathfrak{g}) \times Z^4(\mathfrak{g})$ of a closed four-form and a closed three-form on $\mathfrak{g}$ we define a four-form $\Omega(\rho, \sigma) \in \Lambda^4 \mathfrak{h}^*$ on $\mathfrak{h}$ as follows:*

$$\Omega(\rho, \sigma) := \rho \wedge \alpha + \sigma.$$

If there exists a non-zero element $X \in \mathfrak{h}$ and a complement W of $\mathrm{span}(X)$ *in $\mathfrak{h}$ such that for all pairs $(\rho, \sigma) \in Z^3(\mathfrak{g}) \times Z^4(\mathfrak{g})$ the three-form $\tilde{\rho}(\rho, \sigma) := (X \lrcorner \Omega(\rho, \sigma))|_W \in \Lambda^3 W^*$ on W fulfils $\lambda(\tilde{\rho}) \geq 0$, then $\mathfrak{g}$ does not admit any half-flat* SU(3)*-structure.*

Proof. Let $\mathfrak{g}$, $\mathfrak{h}$, $\alpha \in \mathfrak{g}^0$ as in the statement. Assume that $X \in \mathfrak{h}$ and $W \subseteq \mathfrak{h}$ as in the statement exist and that, nevertheless, $\mathfrak{g}$ admits a half-flat SU(3)-structure $(\omega, \rho) \in \Lambda^2 \mathfrak{g}^* \times \Lambda^3 \mathfrak{g}^*$. Set $\sigma := \frac{1}{2}\omega^2$. Then $(\rho, \sigma) \in Z^3(\mathfrak{g}) \times Z^4(\mathfrak{g})$. As aforementioned, the half-flat SU(3)-structure (ω, ρ) induces a cocalibrated G_2-structure φ on $\mathfrak{h}$ and by equation (2.3), the Hodge dual is given by

$$\star_\varphi \varphi = \rho \wedge \alpha + \sigma = \Omega(\rho, \sigma).$$

Applying Lemma 2.3 for $0 \neq X \in \mathfrak{h}$ and the complement W of $\mathrm{span}(X)$ in $\mathfrak{h}$ yields that the three-form $\tilde{\rho}(\rho, \sigma) = (X \lrcorner \star_\varphi \varphi)|_W = (X \lrcorner \Omega(\rho, \sigma))|_W \in \Lambda^3 W^*$ on W has to fulfil $\lambda(\tilde{\rho}) < 0$, a contradiction. □

3. Proof of Theorem 1.2

First of all, Proposition 2.2 yields that all Lie algebras with Abelian nilradical do not admit a half-flat SU(3)-structure. As Table 3 contains all indecomposable Lie algebras with non-Abelian five-dimensional nilradical according to the classification of Mubarakzyanov [10] and Shabanskaya [13], it suffices to prove existence or non-existence in each case contained in the list. The existence problem is completely solved by the explicit examples given in Table 4. In the following, we prove the non-existence for the remaining Lie algebras.

For all Lie algebras, except $A_{6,39}^{-1,-1}$, $A_{6,41}^{-1}$ $A_{6,76}^{-1}$, $A_{6,78}$, $B_{6,3}$ and $B_{6,4}^{0}$, we apply Proposition 2.1. In each case, we work in the basis $(e^1, \ldots, e^6)$ of $\mathfrak{g}^*$ given in Table 3. Analogously to the proof of [5, Proposition 3.3], we show that $\alpha = e^6$ is a one-form fulfilling equation (2.1) for all $\rho \in Z^3(\mathfrak{g})$ and all $\sigma \in Z^4(\mathfrak{g})$. More precisely, we start with a pair $(\rho, \sigma) \in \Lambda^3 \mathfrak{g}^* \times \Lambda^4 \mathfrak{g}^*$ of a three-form ρ and a four-form σ expressed with respect to the induced basis on forms using 35 coefficients in total. The general solution of the equations $d\rho = 0$ and $d\omega = 0$ can be obtained by eliminating a certain amount of coefficients due to the separation of the classes in Table 3 by different Lie algebra cohomology. The computation of $\tilde{J}_\rho$ with respect to the given basis by equation (2.2) allows us to verify equation (2.1) for $\alpha = e^6$ and all $(\rho, \sigma) \in Z^3(\mathfrak{g}) \times Z^4(\mathfrak{g})$. All calculations can be executed conveniently in a computer algebra system.

Unfortunately, Proposition 2.1 cannot be applied to the Lie algebras $A_{6,39}^{-1,-1}$, $A_{6,41}^{-1}$ $A_{6,76}^{-1}$, $A_{6,78}$, $B_{6,3}$ and $B_{6,4}^{0}$. The following proof uses Proposition 2.4. Again, we compute the general closed three-form $\rho \in Z^3(\mathfrak{g})$ and the general closed four-form $\sigma \in Z^4(\mathfrak{g})$ with respect to the basis $(e^1, \ldots, e^6)$ given in Table 3. We choose $e^7 \in (\mathfrak{g} \oplus \mathbb{R})^*$ with $de^7 = 0$ such that $(e^1, \ldots, e^7)$ is a basis of $(\mathfrak{g} \oplus \mathbb{R})^* \cong \mathfrak{g}^* \oplus \mathbb{R}^*$ and compute $\Omega(\rho, \sigma) := \rho \wedge e^7 + \sigma \in \Lambda^4(\mathfrak{g} \oplus \mathbb{R})^*$. Defining $X := e_3$ and $W := \mathrm{span}(e_1, e_2, e_4, e_5, e_6, e_7)$, we compute for each of the four Lie algebras the three-form $\tilde{\rho}(\rho, \sigma) \in \Lambda^3 W$, $\tilde{\rho}(\rho, \sigma) := (X \lrcorner \Omega(\rho, \sigma))|_W$. When we compute $\lambda(\tilde{\rho}(\rho, \sigma))$ with respect to W, it turns out that it is in each case the square of a polynomial in the coefficients of the general closed three-form $\rho \in Z^3(\mathfrak{g})$ and of the general closed four-form $\sigma \in Z^4(\mathfrak{g})$. Thus $\lambda(\tilde{\rho}) \geq 0$ and none of the four Lie algebras admits a half-flat SU(3)-structure according to Proposition 2.4.

4. $(2,3)$-trivial Lie algebras of dimension six

An interesting application of Table 3 is the classification of six-dimensional Lie algebras which are $(2,3)$-*trivial*, i.e. whose second and third Lie algebra cohomology vanishes. These Lie algebras play an analogous role for the study of multi-moment maps as semi-simple Lie algebras do for the study of moment maps in symplectic geometry, see [8] and [9].

A classification of $(2,3)$-trivial Lie algebras up to dimension five has been established by Madsen and Swann in [8]. The most important tool for the classification is the following theorem proved in [9].

Theorem 4.1 (Madsen, Swann)**.** *A Lie algebra* $\mathfrak{g}$ *is* $(2,3)$*-trivial if and only if* $\mathfrak{g}$ *is solvable, the derived Lie algebra* $\mathfrak{n} = [\mathfrak{g},\mathfrak{g}]$ *is nilpotent of codimension one in* $\mathfrak{g}$ *and* $H^i(\mathfrak{n})^{\mathfrak{g}} = \{0\}$ *for* $i = 1,2,3$*.*

In particular, $(2,3)$-trivial Lie algebras are indecomposable. Thus, Theorem 4.1 implies the following result on $(2,3)$-trivial six-dimensional Lie algebras.

Corollary 4.2. *A six-dimensional Lie algebra* $\mathfrak{g}$ *is* $(2,3)$*-trivial if and only if it is one of the Lie algebras in Table 3 with* $h^2(\mathfrak{g}) = h^3(\mathfrak{g}) = 0$ *or if the nilradical* $\mathfrak{n}$ *of* $\mathfrak{g}$ *is* $\mathbb{R}^5$ *and the induced endomorphism* $\mathrm{ad}(v)\,|_{\Lambda^i\mathfrak{n}}$ *for an arbitrary* $v \in \mathfrak{g}\backslash\mathfrak{n}$ *has trivial kernel for* $i = 1,2,3$*.*

Appendix

Table 1 contains all non-solvable indecomposable six-dimensional Lie algebras and Table 3 contains all indecomposable six-dimensional Lie algebras with five-dimensional, non-Abelian nilradical. Table 3 is further subdivided by the different non-Abelian nilradicals which appear.

The notation and the Lie brackets in Table 1 are taken literally from [14]. Table 3 is based on the original list by Mubarakzyanov [10] and, apart from the obvious subdivision according to the number of free parameters and the Lie algebra cohomology, the list is modified as follows. On the one hand, some of Mubarakzyanov's classes $g_{6,n}$ are redundant since there is an isomorphism to one of the other classes for certain parameter values. On the other hand, Shabanskaya [13] found 6 new classes which are fitted in Table 1 according to their nilradical and denoted by $B_{6,i}$, $i = 1,\ldots,6$. Moreover, a large number of isomorphisms for certain parameter values has been discovered by Shabanskaya [13] and the authors resulting in a range restriction or vanishing of certain parameters. It turns out to be hard to assure that no further isomorphisms are possible due to the complexity and large amount of data. Lastly, a few parameter values are excluded because the corresponding Lie algebra is decomposable or nilpotent. Note that the reason for excluding parameter values is usually obvious when considering the matrix representing ad_{e_6} whereas non-obvious modifications are explained in footnotes. The names of the classes are modified such that the remaining parameters are written as exponents of the class symbol A and are denoted by a, b, c if continuous and by ε if discrete.

The Lie brackets in both tables are written in the well-known dual notation. Thereby, $\mathrm{e}^1,\ldots,\mathrm{e}^6$ is a basis of $\mathfrak{g}^*$ and the images of e^i for $i = 1,\ldots,6$ under the exterior derivative d are given with rising i from left to right. We use the abbreviation e^{ij} for $\mathrm{e}^i \wedge \mathrm{e}^j$. In the column labelled $\mathfrak{z}$ the dimension of the centre of the corresponding Lie algebra is given. The column labelled $\mathrm{h}^*(\mathfrak{g})$ contains the vector $(\mathrm{h}^1(\mathfrak{g}),\ldots,\mathrm{h}^6(\mathfrak{g}))$ of the dimensions of the Lie algebra cohomology groups, where $\mathrm{h}^0(\mathfrak{g})$ is omitted since it always equals one. Notice that unimodular Lie algebras are characterised by the non-vanishing of the top-dimensional cohomology group. In order to emphasise the unimodular entries in the lists, the non-zero $\mathrm{h}^6(\mathfrak{g})$ are written bold and underlined. The last column, labelled half-flat, is checked if and only if the Lie algebra under consideration admits a half-flat SU(3)-structure. Note that all Lie algebras in Table 1 admit half-flat SU(3)-structures.

Table 2 contains one example $(\omega,\rho) \in \Lambda^2\mathfrak{g}^* \times \Lambda^3\mathfrak{g}^*$ of a normalised half-flat SU(3)-structure for each non-solvable indecomposable six-dimensional Lie algebra. Similarly, Table 4 contains one example $(\omega,\rho) \in \Lambda^2\mathfrak{g}^* \times \Lambda^3\mathfrak{g}^*$ of a normalised half-flat structure for each indecomposable six-dimensional Lie algebra with five-dimensional nilradical which admits such a structure. These structures are given in both cases in the corresponding basis $(e^1,\ldots,e^6)$ of $\mathfrak{g}^*$ of Table 1 or 3, respectively. Moreover, the Euclidean metric induced by these forms on $\mathfrak{g}$ is added. The label ONB indicates that the considered basis is orthonormal. Similarly, OB indicates that the considered basis is orthogonal. In this case, the norms of the non-unit basis vectors are given explicitly.

Note that the collection of invariants in Table 3 is complemented by [1] where the invariants of the coadjoint representation for the Lie algebras in question are determined.

Table 1: Non-solvable indecomposable six-dimensional Lie algebras

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
$L_{6,1}$	$(e^{23}, -e^{13}, e^{12}, e^{26} - e^{35}, -e^{16} + e^{34}, e^{15} - e^{24})$	0	$(0,0,2,0,0,\underline{\mathbf{1}})$	✓
$L_{6,2}$	$(e^{23}, 2e^{12}, -2e^{13}, e^{14} + e^{25}, -e^{15} + e^{34}, e^{45})$	1	$(0,0,2,0,0,\underline{\mathbf{1}})$	✓
$L_{6,3}$	$(e^{23}, 2e^{12}, -2e^{13}, e^{14} + e^{25} + e^{46}, -e^{15} + e^{34} + e^{56}, 0)$	0	$(1,0,1,1,0,0)$	✓
$L_{6,4}$	$(e^{23}, 2e^{12}, -2e^{13}, 2e^{14} + 2e^{25}, e^{26} + e^{34}, -2e^{16} + 2e^{35})$	0	$(0,0,2,0,0,\underline{\mathbf{1}})$	✓
$\mathfrak{so}(3,1)$	$(e^{23} - e^{56}, -e^{13} + e^{46}, e^{12} - e^{45}, e^{26}, e^{34}, e^{15})$	0	$(0,0,2,0,0,\underline{\mathbf{1}})$	✓

Table 2: Half-flat SU(3)-structures on non-solvable indecomposable six-dimensional Lie algebras

Lie algebra	Normalised half-flat SU(3)-structure
$L_{6,1}$, $\mathfrak{so}(3,1)$	$\omega = e^{14} + e^{25} - e^{36}, \quad \rho = -e^{126} - e^{135} + e^{234} + e^{456}$, ONB
$L_{6,2}$	$\omega = -e^{14} + e^{25} + e^{36}, \quad \rho = -2e^{125} - 2e^{126} + 3e^{135} + 2e^{136} + e^{156} + 2e^{234} + e^{245} + e^{345} - e^{346} - \frac{19}{4}e^{456}$,
	$g = 2(e^1)^2 + 4(e^2)^2 + 6(e^3)^2 + \frac{17}{2}(e^4)^2 + \frac{53}{4}(e^5)^2 + \frac{19}{2}(e^6)^2 - 8e^1 \cdot e^4 - 8e^2 \cdot e^3 - 4e^2 \cdot e^6 - 10e^3 \cdot e^5 - 4e^3 \cdot e^6 + 21e^5 \cdot e^6$
$L_{6,3}$	$\omega = e^{15} + e^{24} - e^{26} + e^{36} + e^{56}, \quad \rho = 3e^{124} + 11e^{126} - e^{134} - 2e^{136} - 2e^{156} + e^{235} - e^{246} + e^{346} + e^{456}$,
	$g = 5(e^1)^2 + 14(e^2)^2 + (e^3)^2 + (e^4)^2 + (e^5)^2 + 18(e^6)^2 - 4e^1 \cdot e^4 - 18e^1 \cdot e^6 - 6e^2 \cdot e^3 - 4e^2 \cdot e^5 + 8e^4 \cdot e^6$
$L_{6,4}$	$\omega = -\frac{1}{2}\sqrt{3}\,(e^{14} + e^{15} - 2e^{24} - e^{25} + e^{36}), \quad \rho = \frac{1}{2}\sqrt{3}\,(e^{123} + e^{126} + e^{134} + e^{235} - e^{456})$,
	$g = (e^1)^2 + 2(e^2)^2 + (e^3)^2 + 2(e^4)^2 + (e^5)^2 + (e^6)^2 - 2e^1 \cdot e^2 + e^1 \cdot e^4 + e^1 \cdot e^5 - 2e^2 \cdot e^4 - e^2 \cdot e^5 + e^3 \cdot e^6 + 2e^4 \cdot e^5$

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	Nilradical $\mathfrak{h}_3 \oplus \mathbb{R}^2$			
$A_{6,13}^{a,b,c}$	$((a+b)e^{16} + e^{23}, ae^{26}, be^{36}, e^{46}, ce^{56}, 0)$			
[1]	$0 < \lvert a\rvert \leq \lvert b\rvert,\ -1 < c \leq 1,\ a \neq -1,\ b \notin \{-1, -a, -2a, -(2a+1), -\frac{1}{2}(a+1), -(a+\frac{1}{2})\}$, $c \notin \{0, -(a+1), -(b+1), -(2a+b+1), -(2a+2b+1)\}$	0	$(1,0,0,0,0,0)$	–
	$a = 0,\ b \notin \{-1, -\frac{1}{2}, 0\},\ -1 < c \leq 1,\ c \notin \{0, -b, -2b, -b-1, -2b-1\}$	0	$(2,1,0,0,0,0)$	–
	$a = -1,\ b \notin \{-1, 0, \frac{1}{2}, 1, 2\}$, $c \notin \{-1, 0, 1, -b, -2b, -b-1, -b+1, -b+2, -2b+1, -2b+2\}$	0	$(1,1,1,0,0,0)$	–
	$c = -1,\ 0 < \lvert a\rvert \leq b,\ a \neq \pm 1,\ b \notin \{1, -a, -2a, -2a \pm 1, -\frac{1}{2}a \pm \frac{1}{2}, -a \pm \frac{1}{2}\}$	0	$(1,1,1,0,0,0)$	–
	$b = -2a,\ a \notin \{-1, 0, \frac{1}{3}, \frac{1}{2}\},\ -1 < c \leq 1,\ c \notin \{0, -a, 2a, 3a, -1-a, -1+2a, -1+3a\}$	0	$(1,1,1,0,0,0)$	–
	$b = -(2a+1),\ a \notin \{-1, -\frac{2}{3}, -\frac{1}{2}, -\frac{1}{3}, 0\},\ c \notin \{-1, 0, 1, -a-1, 2a, 2a+2, 3a+1, 3a+2\}$	0	$(1,0,1,1,0,0)$	–
	$c = -(a+1),\ a \notin \{-1, 0\}$, $b \notin \{-1, 0, \frac{1}{2}, \pm a, -\frac{a}{2}, -2a, -(2a+1), -\frac{a}{2} \pm \frac{1}{2}, \pm a + 1, -(a+\frac{1}{2})\}$	0	$(1,0,1,1,0,0)$	–
	$b = -a,\ a > 0,\ a \neq 1,\ -1 < c \leq 1,\ c \notin \{0, \pm a, -1 \pm a\}$	1	$(1,0,1,1,0,0)$	–
	$b = -(a+\frac{1}{2}),\ a > -\frac{1}{4},\ a \notin \{0, \frac{1}{2}\},\ c \notin \{0, \pm 1, \pm a, \pm(a+\frac{1}{2}), \pm(a-\frac{1}{2}), \pm(1+a)\}$	0	$(1,0,0,1,1,0)$	–
	$c = -(2a+b+1),\ a \notin \{-1, -\frac{1}{2}, 0\}$, $b \notin \{-1, 0, 1, -\frac{1}{2}a, \pm a, -2a, -(2a+1), -\frac{1}{2}(a+1), -(a+\frac{1}{2}), \pm(a+1)\}$	0	$(1,0,0,1,1,0)$	–
	$c = -(2a+2b+1),\ a \notin \{-1, 0\}$, $b \notin \{-1, 0, -\frac{1}{2}a, -a, -2a, -(2a+1), -\frac{1}{2}(a+1), -(a+\frac{1}{2})\}$	0	$(1,0,0,0,1,\underline{\mathbf{1}})$	–
	$(a,b) = (0,-1),\ c \notin \{-1, 1, 2\}$	0	$(2,2,2,1,0,0)$	–
	$(a,c) = (0,-1),\ b > 0,\ b \notin \{\frac{1}{2}, 1\}$	0	$(2,2,2,1,0,0)$	–
	$(a,b) = (0,-\frac{1}{2}),\ c \notin \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$	0	$(2,1,1,2,1,0)$	–
	$(a,c) = (0,-b-1),\ -2 \leq b < 0,\ b \notin \{-1, -\frac{1}{2}\}$	0	$(2,1,1,2,1,0)$	–
	$(a,c) = (0,-2b-1),\ -1 < b < 0,\ b \neq -\frac{1}{2}$	0	$(2,1,0,1,2,\underline{\mathbf{1}})$	–

[1] $A_{6,13}^{a,b,c} \cong A_{6,13}^{b,a,c} \cong A_{6,13}^{a/c,b/c,1/c}$, $A_{6,13}^{0,0,c}$ and $A_{6,13}^{a,b,0}$ are decomposable.

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$(a,b)=(-1,\frac{1}{2}),\ c\notin\{-\frac{3}{2},-1,-\frac{1}{2},0,\frac{1}{2},1,\frac{3}{2}\}$	0	(1,2,2,1,1,0)	–
	$(b,c)=(-2a,-1),\ a>0,\ a\notin\{\frac{1}{3},\frac{1}{2},1\}$	0	(1,2,2,1,1,0)	–
	$(a,b)=(-1,-1),\ c\notin\{-1,0,1,2,3,4\}$	0	(1,2,2,0,0,0)	–
	$(a,b)=(-1,2),\ c\notin\{-4,-3,-2,-1,0,1\}$	0	(1,2,2,0,0,0)	–
	$(a,c)=(-1,-1),\ b\notin\{-1,0,\frac{1}{2},1,\frac{3}{2},2,3\}$	0	(1,2,2,0,0,0)	–
	$(a,c)=(-1,1),\ b\notin\{-2,-1,-\frac{1}{2},0,\frac{1}{2},1,2\}$	0	(1,2,2,0,0,0)	–
	$(a,c)=(-1,-b),\ -1<b<1,\ b\notin\{0,\frac{1}{2}\}$	0	(1,2,2,0,0,0)	–
	$(a,b)=(-1,1),\ c\notin\{-2,-1,0,1\}$	1	(1,1,3,2,0,0)	–
	$(a,c)=(-1,-2b+1),\ b\notin\{-1,0,\frac{1}{2},1,2\}$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(b,c)=(-2a,2a-1),\ a\notin\{-1,0,\frac{1}{3},\frac{1}{2}\}$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(b,c)=(-a,-1),\ a>0,\ a\neq 1$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(a,c)=(-1,-b-1),\ b\notin\{-2,-1,0,\frac{1}{2},1,2,3\}$	0	(1,1,2,1,0,0)	–
	$(a,c)=(-1,-b+2),\ b\notin\{-2,-1,0,\frac{1}{2},1,3\}$	0	(1,1,2,1,0,0)	–
	$(a,b)=(-\frac{2}{3},\frac{1}{3}),\ c\notin\{-\frac{4}{3},-1,-\frac{1}{3},0,\frac{2}{3},1\}$	0	(1,1,2,1,0,0)	–
	$(b,c)=(-2a-1,-1),\ a\notin\{-\frac{3}{2},-1,-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},0\}$	0	(1,1,2,1,0,0)	–
	$(b,c)=(-2a,-1-a),\ a\notin\{-1,-\frac{1}{3},-\frac{1}{4},0,\frac{1}{3},\frac{1}{2}\}$	0	(1,1,2,1,0,0)	–
	$(a,c)=(-1,-2b),\ b\notin\{-2,-1,-\frac{1}{2},0,\frac{1}{2},1,2\}$	0	(1,1,1,1,1,0)	–
	$(a,c)=(-1,-b+1),\ b\notin\{-1,0,\frac{1}{2},1,2\}$	0	(1,1,1,1,1,0)	–
	$(a,c)=(-1,-2b+2),\ b\notin\{-1,0,\frac{1}{2},1,\frac{3}{2},2,3\}$	0	(1,1,1,1,1,0)	–
	$(b,c)=(-2a,3a-1),\ a\notin\{-1,0,\frac{1}{4},\frac{1}{3},\frac{1}{2},1\}$	0	(1,1,1,1,1,0)	–
	$(b,c)=(-a-\frac{1}{2},-1),\ a>-\frac{1}{4}\ a\notin\{0,\frac{1}{2},1,\frac{3}{2}\}$	0	(1,1,1,1,1,0)	–
	$(b,c)=(-a,-1-a),\ a\notin\{-1,-\frac{1}{2},0,1\}$	1	(1,0,2,3,1,0)	–
	$(a,b)=(-\frac{1}{3},-\frac{1}{3}),\ c\notin\{-1,-\frac{2}{3},0,\frac{1}{3},1,\frac{4}{3}\}$	0	(1,0,2,2,0,0)	–
	$(b,c)=(-2a-1,1),\ a\notin\{-2,-1,-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},0,\frac{1}{2}\}$	0	(1,0,2,2,0,0)	–
	$(b,c)=(-2a-1,-a-1),\ a\notin\{-2,-1,-\frac{3}{4},-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},0\}$	0	(1,0,2,2,0,0)	–
	$(b,c)=(-2a-1,2a),\ a\notin\{-2,-1,-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},0,\frac{1}{2}\}$	0	(1,0,2,2,0,0)	–
	$(b,c)=(a,-a-1),\ a\notin\{-1,-\frac{1}{3},-\frac{1}{4},0,\frac{1}{3},\frac{1}{2}\}$	0	(1,0,2,2,0,0)	–
	$(b,c)=(-2a-1,3a+2),\ -1<a<-\frac{1}{3},\ a\notin\{-\frac{3}{4},-\frac{2}{3},-\frac{1}{2}\}$	0	(1,0,2,2,0,0)	✓
	$(b,c)=(-2a-1,3a+1),\ \ a\notin\{-1,-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},-\frac{1}{4},0,1\}$	0	(1,0,1,2,1,0)	✓
	$(b,c)=(-2a-1,2a+2),\ a\notin\{-\frac{3}{2},-1,-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},0,1\}$	0	(1,0,1,2,1,0)	–
	$(b,c)=(-a-\frac{1}{2},-a-1),\ a\notin\{-2,-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2}\}$	0	(1,0,1,2,1,0)	–
	$(b,c)=(-a-\frac{1}{2},1),\ a>-\frac{1}{4},\ a\notin\{0,\frac{1}{2},1,\frac{3}{2}\}$	0	(1,0,0,2,2,0)	–
	$(b,c)=(-a-\frac{1}{2},a),\ a\notin\{-1,-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\}$	0	(1,0,0,2,2,0)	–
	$(b,c)=(a,-3a-1),\ a\notin\{-1,-\frac{1}{2},-\frac{1}{3},-\frac{1}{4},0,1\}$	0	(1,0,0,2,2,0)	–
	$(a,b,c)=(0,-1,1)$	0	(2,3,4,3,2,$\underline{\mathbf{1}}$)	✓
	$(a,b,c)=(0,-1,-1)$	0	(2,3,4,2,0,0)	–
	$(a,b,c)=(0,-\frac{1}{2},-1)$	0	(2,2,3,3,1,0)	–
	$(a,b,c)=(-\frac{1}{2},0,\frac{1}{2})$	0	(2,2,3,3,1,0)	✓
	$(a,b,c)=(0,-\frac{1}{2},1)$	0	(2,1,2,4,2,0)	–
	$(a,b,c)=(-\frac{1}{2},0,-\frac{1}{2})$	0	(2,1,2,4,2,0)	✓
	$(a,b,c)=(-1,-1,1)$	0	(1,4,4,0,0,0)	–

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$(a,b,c)=(-1,1,-1)$	1	(1,3,6,3,1,$\underline{\mathbf{1}}$)	✓
	$(a,b,c)\in\{(-1,\frac{1}{2},-1),\ (-1,\frac{1}{2},1)\}$	0	(1,3,3,2,2,0)	–
	$(a,b,c)\in\{(-1,\frac{1}{2},-\frac{1}{2}),\ (-1,2,-1)\}$	0	(1,3,3,1,1,0)	–
	$(a,b,c)\in\{(-1,-1,-1),\ (-1,2,1)\}$	0	(1,3,3,0,0,0)	–
	$(a,b,c)=(-1,1,1)$	1	(1,2,5,3,0,0)	–
	$(a,b,c)\in\{(-1,-1,3),\ (-1,2,-3)\}$	0	(1,2,4,2,1,$\underline{\mathbf{1}}$)	–
	$(a,b,c)\in\{(-1,\frac{1}{2},-\frac{3}{2}),\ (-1,\frac{1}{2},\frac{3}{2})\}$	0	(1,2,3,2,1,0)	–
	$(a,b,c)=(-\frac{2}{3},\frac{1}{3},-1)$	0	(1,2,3,2,1,0)	✓
	$(a,b,c)\in\{(-1,3,-1),\ (-1,-2,1)\}$	0	(1,2,3,1,0,0)	–
	$(a,b,c)\in\{(-1,\frac{1}{2},\frac{1}{2}),\ (-1,-1,2)\}$	0	(1,2,2,2,2,0)	–
	$(a,b,c)\in\{(-1,-1,4),\ (-1,2,-4),(-1,\frac{3}{2},-1),(-1,-\frac{1}{2},1)\}$	0	(1,2,2,1,1,0)	–
	$(a,b,c)=(-1,1,-2)$	1	(1,1,4,4,1,0)	✓
	$(a,b,c)\in\{(-\frac{2}{3},\frac{1}{3},1),\ (-\frac{2}{3},\frac{1}{3},-\frac{4}{3}),\ (-\frac{1}{3},-\frac{1}{3},-1)\}$	0	(1,1,3,2,0,0)	–
	$(a,b,c)\in\{(-1,-2,4),\ (-1,3,-4),\ (-\frac{3}{2},2,-1)\}$	0	(1,1,2,2,1,0)	–
	$(a,b,c)=(-\frac{1}{4},-\frac{1}{2},\frac{1}{4})$	0	(1,1,2,2,1,0)	✓
	$(a,b,c)=(-\frac{1}{3},-\frac{1}{3},-\frac{2}{3})$	0	(1,0,4,4,0,0)	–
	$(a,b,c)=(-\frac{1}{3},-\frac{1}{3},1)$	0	(1,0,4,4,0,0)	✓
	$(a,b,c)\in\{(-2,3,1),\ (\frac{1}{2},-2,1)\}$	0	(1,0,3,3,0,0)	–
	$(a,b,c)=(-\frac{3}{4},\frac{1}{2},-\frac{1}{4})$	0	(1,0,3,3,0,0)	✓
	$(a,b,c)\in\{(-\frac{1}{4},-\frac{1}{4},\frac{3}{4}),\ (-\frac{1}{4},-\frac{1}{4},-\frac{3}{4})\}$	0	(1,0,2,3,1,0)	–
	$(a,b,c)=(\frac{3}{2},-2,1)$	0	(1,0,1,3,2,0)	–
	$(a,b,c)=(1,-3,4)$	0	(1,0,1,3,2,0)	✓
	$(a,b,c)\in\{(1,-\frac{3}{2},1),\ (1,1,-4)\}$	0	(1,0,0,3,3,0)	–
$A_{6,14}^{a,b}$	$((a+b)e^{16}+e^{23}+e^{56},ae^{26},be^{36},e^{46},(a+b)e^{56},0)$			
[2]	$\lvert a\rvert\leq\lvert b\rvert,\ a\notin\{-1,0\},\ b\notin\{-1,0,-a,-\frac{3}{2}a,-2a,\ -(a+1),\ -(a+\frac{1}{2}),\ -(a+\frac{1}{3}),$ $-(2a+1),\ -\frac{1}{2}(a+1),\ -\frac{1}{2}(3a+1),\ -\frac{1}{3}(2a+1)\}$	0	(1,0,0,0,0,0)	–
	$b=-a,\ a>0,\ a\neq 1$	1	(2,1,1,2,1,0)	–
	$b=0,\ a\notin\{0,-1,-\frac{1}{2},-\frac{1}{3}\}$	0	(2,1,0,0,0,0)	–
	$b=-1,\ a\notin\{-1,0,\frac{1}{3},\frac{1}{2},\frac{2}{3},1,\frac{3}{2},2\}$	0	(1,1,1,0,0,0)	–
	$b=-2a,\ a\notin\{-1,0,\frac{1}{4},\frac{1}{3},\frac{1}{2},1\}$	0	(1,1,1,0,0,0)	–
	$b=-(a+1),\ a\geq-\frac{1}{2},\ a\notin\{0,1,2\}$	0	(1,1,1,0,0,0)	–
	$b=-\frac{3}{2}a,\ a\notin\{-2,-1,0,\frac{1}{2},\frac{2}{3},\frac{2}{5},1,2\}$	0	(1,0,1,1,0,0)	–
	$b=-(2a+1),\ a\notin\{-2,-1,-\frac{3}{4},-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},0\}$	0	(1,0,1,1,0,0)	–
	$b=-\frac{1}{2}(3a+1),\ a\notin\{-1,-\frac{3}{5},-\frac{1}{2},-\frac{1}{3},-\frac{1}{5},0,\frac{1}{3},1\}$	0	(1,0,0,1,1,0)	–
	$b=-(a+\frac{1}{2}),\ a\geq-\frac{1}{4},\ a\notin\{0,\frac{1}{2},1\}$	0	(1,0,0,1,1,0)	–
	$b=-(a+\frac{1}{3}),\ a\geq-\frac{1}{6},\ a\notin\{0,\frac{1}{3},\frac{2}{3}\},$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(0,0)$	1	(4,5,5,4,1,0)	–
	$(a,b)=(0,-1)$	0	(2,3,3,1,0,0)	–
	$(a,b)=(-1,1)$	1	(2,2,3,4,2,0)	–
	$(a,b)=(0,-\frac{1}{2})$	0	(2,1,1,3,2,0)	–

[2] $A_{6,14}^{a,b}\cong A_{6,14}^{b,a}$

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$(a, b) = (0, -\frac{1}{3})$	0	(2,1,0,1,2,$\underline{\mathbf{1}}$)	–
	$(a, b) \in \{(-1, \frac{1}{2}),\ (1, -2)\}$	0	(1,2,2,1,1,0)	–
	$(a, b) \in \{(-1, -1),\ (-1, 2)\}$	0	(1,2,2,0,0,0)	–
	$(a, b) \in \{(-1, \frac{2}{3}),\ (\frac{1}{3}, -\frac{2}{3})\}$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(a, b) \in \{(-1, \frac{3}{2}),\ (2, -3)\}$	0	(1,1,2,1,0,0)	–
	$(a, b) \in \{(\frac{1}{4}, -\frac{1}{2}),\ (-1, \frac{1}{3})\}$	0	(1,1,1,1,1,0)	–
	$(a, b) \in \{(-2, 3),\ (\frac{1}{2}, -\frac{3}{4})\}$	0	(1,0,2,2,0,0)	–
	$(a, b) \in \{(\frac{2}{5}, -\frac{3}{5})\}$	0	(1,0,1,2,1,0)	✓
	$(a, b) = (1, -\frac{3}{2})$	0	(1,0,1,2,1,0)	–
	$(a, b) \in \{(-\frac{1}{3}, -\frac{1}{3}),\ (-\frac{1}{5}, -\frac{1}{5})\}$	0	(1,0,0,2,2,0)	–
$A^{a}_{6,15}$ [3]	$((a+1)e^{16} + e^{23}, e^{26}, ae^{36}, e^{26} + e^{46}, e^{36} + ae^{56}, 0)$			
	$-1 < a \leq 1,\ a \notin \{0, -\frac{1}{3}, -\frac{1}{2}, -\frac{2}{3}\}$	0	(1,0,0,0,0,0)	–
	$a = 0$	1	(2,2,1,0,0,0)	–
	$a = -1$	1	(1,2,4,2,1,$\underline{\mathbf{1}}$)	✓
	$a = -2$	0	(1,1,2,1,0,0)	–
	$a = -3$	0	(1,0,1,1,0,0)	–
	$a = -\frac{3}{2}$	0	(1,0,0,1,1,0)	–
$A_{6,16}$	$(e^{16} + e^{23} + e^{46}, e^{26}, 0, e^{26} + e^{46}, e^{36}, 0)$	1	(2,2,1,0,0,0)	–
$A^{\varepsilon,a}_{6,17}$	$(ae^{16} + e^{23} + \varepsilon e^{46}, ae^{26}, 0, e^{36}, e^{56}, 0)$			
	$\varepsilon = 0,\ a \notin \{-1, -\frac{1}{2}, 0\}$	1	(2,2,1,0,0,0)	–
	$\varepsilon = 0,\ a = 0$	2	(3,6,6,3,1,0)	–
	$\varepsilon = 1,\ a = 0$	1	(3,4,4,3,1,0)	–
	$\varepsilon = 0,\ a = -1$	1	(2,3,4,3,1,0)	–
	$\varepsilon = 0,\ a = -\frac{1}{2}$	1	(2,2,2,2,2,$\underline{\mathbf{1}}$)	–
$A^{a,b}_{6,18}$	$((a+1)e^{16} + e^{23}, ae^{26}, e^{36}, e^{36} + e^{46}, be^{56}, 0)$			
	$a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0\}$, $b \notin \{-2, -1, 0, -a, -(a+1), -(a+2), -(a+3), -(2a+1), -(2a+2), -(2a+3)\}$	0	(1,0,0,0,0,0)	–
	$a = 0,\ b \notin \{-3, -2, -1, 0\}$	0	(2,1,0,0,0,0)	–
	$a = -1,\ b \notin \{-2, -1, 0, 1\}$	1	(1,1,2,1,0,0)	–
	$a = -2,\ b \notin \{-2, -1, 0, 1, 2, 3\}$	0	(1,1,1,0,0,0)	–
	$a = -\frac{1}{2},\ b \notin \{-\frac{5}{2}, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}\}$	0	(1,1,1,0,0,0)	–
	$b = -1,\ a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, 1\}$	0	(1,1,1,0,0,0)	–
	$b = -a,\ a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, 1, 2\}$	0	(1,1,1,0,0,0)	–
	$a = -3,\ b \notin \{-2, -1, 0, 1, 2, 3, 4, 5\}$	0	(1,0,1,1,0,0)	–
	$b = -2,\ a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2\}$	0	(1,0,1,1,0,0)	–
	$b = -(a+1),\ a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, 1\}$	0	(1,0,1,1,0,0)	–
	$b = -(a+2),\ a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, 1\}$	0	(1,0,1,1,0,0)	–
	$b = -(2a+1),\ a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2\}$	0	(1,0,1,1,0,0)	–
	$a = -\frac{3}{2},\ b \notin \{-2, -\frac{3}{2}, -1, 0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$	0	(1,0,0,1,1,0)	–
	$b = -(a+3),\ a \notin \{-3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, 1, 2\}$	0	(1,0,0,1,1,0)	–

[3] $A^{a}_{6,15} \cong A^{1/a}_{6,15}$

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$b=-(2a+2),\ a\notin\{-3,-2,-\frac{3}{2},-1,-\frac{1}{2},0,1\}$,	0	(1,0,0,1,1,0)	–
	$b=-(2a+3),\ a\notin\{-3,-2,-\frac{3}{2},-1,-\frac{1}{2},0\}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(0,-1)$	0	(2,2,3,2,0,0)	–
	$(a,b)=(0,-2)$	0	(2,1,2,3,1,0)	–
	$(a,b)=(0,-3)$	0	(2,1,0,1,2,$\underline{\mathbf{1}}$)	–
	$(a,b)=(-1,-1)$	1	(1,2,4,2,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(-1,1)$	1	(1,2,4,2,0,0)	–
	$(a,b)\in\{(-\frac{1}{2},-1),\ (-2,-1),\ (-2,-1)\}$	0	(1,2,2,1,1,0)	–
	$(a,b)\in\{(-\frac{1}{2},\frac{1}{2}),\ (1,-1)\}$	0	(1,2,2,0,0,0)	–
	$(a,b)=(-1,-2)$	1	(1,1,3,3,1,0)	–
	$(a,b)\in\{(-3,3),\ (-\frac{1}{2},-2),\ (-2,1)\}$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(a,b)\in\{(-\frac{1}{2},-\frac{3}{2}),(-\frac{1}{2},-\frac{1}{2}),(2,-2),(-3,-1),(-2,-2),(-2,3)\}$	0	(1,1,2,1,0,0)	–
	$(a,b)\in\{(-\frac{1}{2},-\frac{5}{2}),\ (-\frac{3}{2},-1),\ (-\frac{3}{2},\frac{3}{2})\}$	0	(1,1,1,1,1,0)	–
	$(a,b)\in\{(\frac{1}{2},-2),(1,-2),(1,-3),(-3,-2),(-3,1),(-3,2)\}$	0	(1,0,2,2,0,0)	–
	$(a,b)=(-3,5)$	0	(1,0,2,2,0,0)	✓
	$(a,b)\in\{(-\frac{3}{2},-2),(-\frac{3}{2},2),(-\frac{3}{2},\frac{1}{2}),(-\frac{3}{2},-\frac{1}{2}),(-3,4)\}$	0	(1,0,1,2,1,0)	–
	$(a,b)=(2,-5)$	0	(1,0,1,2,1,0)	✓
	$(a,b)\in\{(-\frac{3}{2},-\frac{3}{2}),\ (-\frac{3}{2},1),\ (1,-4)\}$	0	(1,0,0,2,2,0)	–
$A^{a}_{6,19}$	$((a+1)e^{16}+e^{23}+e^{56},ae^{26},e^{36},e^{36}+e^{46},(a+1)e^{56},0)$			
	$a\notin\{-3,-2,-\frac{3}{2},-\frac{4}{3},-1,-\frac{2}{3},-\frac{1}{2},0\}$	0	(1,0,0,0,0,0)	–
	$a=-1$	1	(2,2,3,3,1,0)	–
	$a=0$	0	(2,1,0,0,0,0)	–
	$a=-2$	0	(1,2,2,1,1,0)	–
	$a=-\frac{1}{2}$	0	(1,1,1,0,0,0)	–
	$a=-\frac{3}{2}$	0	(1,0,1,2,1,0)	–
	$a\in\{-3,\ -\frac{2}{3}\}$	0	(1,0,1,1,0,0)	–
	$a=-\frac{4}{3}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A^{a}_{6,20}$	$(e^{16}+e^{23}+e^{46},0,e^{36},e^{36}+e^{46},ae^{56},0)$			
	$a\notin\{0,-1,-2,-3\}$	0	(2,1,0,0,0,0)	–
	$a=-1$	0	(2,2,2,1,0,0)	–
	$a=-2$	0	(2,1,1,2,1,0)	–
	$a=-3$	0	(2,1,0,1,2,$\underline{\mathbf{1}}$)	–
$A^{a,b}_{6,21}$	$(2ae^{16}+e^{23},ae^{26},e^{26}+ae^{36},e^{46},be^{56},0)$			
[4]	$-1<b\leq 1,\ b\neq 0,$ $a\notin\{-1,-\frac{1}{3},-\frac{1}{4},0,-b,-\frac{1}{3}b,-\frac{1}{4}b,-(b+1),-\frac{1}{3}(b+1),-\frac{1}{4}(b+1)\}$	0	(1,0,0,0,0,0)	–
	$a=0,\ -1<b\leq 1,\ b\neq 0$	1	(2,2,2,1,0,0)	–
	$a=-1,\ b\notin\{-1,0,1,2\}$	0	(1,1,1,0,0,0)	–
	$b=-1,\ a>0,\ a\notin\{\frac{1}{4},\frac{1}{3},1\}$	0	(1,1,1,0,0,0)	–
	$b=-(a+1),\ -2\leq a<0,\ a\notin\{-1,-\frac{1}{3},-\frac{1}{4}\}$	0	(1,0,1,1,0,0)	–
	$a=-\frac{1}{3},\ b\notin\{-1,-\frac{2}{3},0,\frac{1}{3}\}$	0	(1,0,1,1,0,0)	–

[4] $A^{a,b}_{6,21}\cong A^{a/b,1/b}_{6,21}$, $A^{a,0}_{6,21}$ is decomposable.

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$a=-\frac{1}{4}$, $b\notin\left\{-1,-\frac{3}{4},-\frac{1}{4},\frac{1}{4},\frac{3}{4},1\right\}$	0	(1,0,0,1,1,0)	–
	$b=-(3a+1)$, $-\frac{2}{3}\leq a<0$, $a\notin\left\{-\frac{1}{2},-\frac{1}{3},-\frac{1}{4}\right\}$	0	(1,0,0,1,1,0)	–
	$b=-(4a+1)$, $-\frac{1}{2}\leq a<0$, $a\notin\left\{-\frac{1}{3},-\frac{1}{4}\right\}$	0	(1,0,0,0,1,**1**)	–
	$(a,b)=(0,-1)$	1	(2,3,4,3,2,**1**)	–
	$(a,b)\in\{(-1,-1),\,(-1,1)\}$	0	(1,2,2,0,0,0)	–
	$(a,b)\in\left\{(-1,3),\,(-\frac{1}{3},\frac{1}{3})\right\}$	0	(1,1,2,1,1,**1**)	–
	$(a,b)=(-\frac{1}{3},-1)$	0	(1,1,2,1,0,0)	–
	$(a,b)\in\left\{(-1,2),\,(-1,4),\,(-\frac{1}{4},-1)\right\}$	0	(1,1,1,1,1,0)	–
	$(a,b)\in\left\{(-\frac{1}{3},-\frac{2}{3}),\,(-\frac{1}{3},1)\right\}$	0	(1,0,2,2,0,0)	–
	$(a,b)\in\left\{(-\frac{1}{4},-\frac{3}{4}),\,(-\frac{1}{3},\frac{4}{3})\right\}$	0	(1,0,1,2,1,0)	–
	$(a,b)\in\left\{(-\frac{1}{4},-\frac{1}{4}),\,(-\frac{1}{4},1)\right\}$	0	(1,0,0,2,2,0)	–
$A^{a}_{6,22}$	$(2ae^{16}+e^{23}+e^{56},ae^{26},e^{26}+ae^{36},e^{46},2ae^{56},0)$			
	$a\notin\{-1,-\frac{1}{2},-\frac{1}{3},-\frac{1}{4},-\frac{1}{5},-\frac{1}{6},0\}$	0	(1,0,0,0,0,0)	–
	$a=0$	1	(3,4,4,3,1,0)	–
	$a=-\frac{1}{2},\,-1$	0	(1,1,1,0,0,0)	–
	$a=-\frac{1}{3}$	0	(1,0,2,2,0,0)	–
	$a=-\frac{1}{5},-\frac{1}{4}$	0	(1,0,0,1,1,0)	–
	$a=-\frac{1}{6}$	0	(1,0,0,0,1,**1**)	–
$A^{\varepsilon,a}_{6,23}$ [5]	$(2e^{16}+e^{23}+\varepsilon e^{56},e^{26},e^{26}+e^{36},e^{36}+e^{46},(2+a)e^{56},0)$			
	$\varepsilon=0$, $a\notin\{-7,-6,-5,-4,-3,-2\}$	0	(1,0,0,0,0,0)	–
	$\varepsilon=0$, $a=-3$	0	(1,1,1,0,0,0)	–
	$\varepsilon=0$, $a\in\{-4,-5\}$	0	(1,0,1,1,0,0)	–
	$\varepsilon=0$, $a=-6$	0	(1,0,0,1,1,0)	–
	$\varepsilon=0$, $a=-7$	0	(1,0,0,0,1,**1**)	–
	$\varepsilon=1$, $a=0$	0	(1,0,0,0,0,0)	–
$A^{\varepsilon}_{6,24}$	$(e^{23}+\varepsilon e^{46},0,e^{26},e^{36},e^{56},0)$			
	$\varepsilon=0$	2	(2,3,3,2,1,0)	–
	$\varepsilon=1$	1	(2,3,3,2,1,0)	–
$A^{a,b}_{6,25}$ [6]	$((b+1)e^{16}+e^{23},e^{26},be^{36},ae^{46},e^{46}+ae^{56},0)$			
	$-1<b\leq 1$, $b\notin\left\{-\frac{1}{2},0\right\}$, $a\notin$ $\left\{-1,-\frac{1}{2},0,-b,-\frac{1}{2}b,-2(b+1),-(b+1),-(2b+1),-\frac{1}{2}(2b+1),-(b+2),-\frac{1}{2}(b+2)\right\}$	0	(1,0,0,0,0,0)	–
	$a=0$, $-1<b\leq 1$, $b\notin\left\{0,-\frac{1}{2}\right\}$	1	(2,2,1,0,0,0)	–
	$b=0$, $a\notin\left\{-2,-1,-\frac{1}{2},0\right\}$	0	(2,1,0,0,0,0)	–
	$a=-1$, $b\notin\left\{-2,-1,-\frac{1}{2},0,\frac{1}{2},1,2\right\}$	0	(1,1,1,0,0,0)	–
	$b=-2$, $a\notin\left\{-1,-\frac{1}{2},0,1,\frac{3}{2},2,3\right\}$	0	(1,1,1,0,0,0)	–
	$a=-\frac{1}{2}$, $b\notin\left\{-2,-\frac{3}{2},-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2},1\right\}$	0	(1,0,1,1,0,0)	–
	$b=-1$, $a>0$, $a\notin\left\{\frac{1}{2},1\right\}$	1	(1,0,1,1,0,0)	–
	$a=-b-2$ (isomorphic to $a=-2b-1$), $b\notin\left\{-4,-2,-\frac{3}{2},-1,-\frac{1}{2},0,1\right\}$	0	(1,0,1,1,0,0)	–
	$a=-\frac{1}{2}b-1$ (isomorphic to $a=-b-\frac{1}{2}$), $b\notin\left\{-2,-1,-\frac{2}{3},-\frac{1}{2},0,1,2\right\}$	0	(1,0,0,1,1,0)	–

[5] The parameter α in $g_{6,23}$ can be normalised to 1 since $g_{6,23}$ is nilpotent for $\alpha=0$, $\varepsilon=0$ and $g_{6,23}\cong A^{0}_{6,24}$ for $\alpha=0$, $h=0$.
[6] $A^{a,b}_{6,25}\cong A^{a/b,1/b}_{6,25}$

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$a=-2b-2,\ -1<b\le 1,\ b\notin\{-\frac{3}{4},-\frac{2}{3},-\frac{1}{2},0\}$	0	(1,0,0,1,1,0)	–
	$a=-b-1,\ -1<b\le 1,\ b\notin\{-\frac{1}{2},0\}$	0	(1,0,0,0,1,**1**)	–
	$(a,b)=(0,0)$	1	(3,4,3,1,0,0)	–
	$(a,b)=(0,-\frac{1}{2})$	1	(2,3,3,2,1,0)	–
	$(a,b)=(0,-1)$	2	(2,2,2,2,2,**1**)	–
	$(a,b)=(-1,0)$	0	(2,2,2,2,2,**1**)	✓
	$(a,b)\in\{(-\frac{1}{2},0),\ (-2,0)\}$	0	(2,1,1,2,1,0)	–
	$(a,b)=(-1,-\frac{1}{2})$	0	(1,2,2,1,1,0)	–
	$(a,b)=(-\frac{1}{2},-\frac{1}{2})$	0	(1,1,2,1,1,**1**)	–
	$(a,b)\in\{(-1,2),\ (-\frac{1}{2},-2),\ (3,-2)\}$	0	(1,1,2,1,0,0)	–
	$(a,b)\in\{(-1,-2),\ (-1,1)\}$	0	(1,2,2,0,0,0)	–
	$(a,b)=(-1,-1)$	1	(1,1,3,2,0,0)	–
	$(a,b)\in\{(-1,\frac{1}{2}),\ (\frac{3}{2},-2)\}$	0	(1,1,1,1,1,0)	–
	$(a,b)=(-\frac{1}{2},-1)$	1	(1,0,2,3,1,0)	–
	$(a,b)\in\{(-\frac{1}{2},-\frac{3}{2}),(-\frac{1}{2},-\frac{1}{4}),(-\frac{1}{2},1),(-3,1)\}$	0	(1,0,2,2,0,0)	–
	$(a,b)=(-\frac{1}{2},-\frac{3}{4})$	0	(1,0,1,2,1,0)	–
	$(a,b)\in\{(-\frac{3}{2},1),\ (-\frac{2}{3},-\frac{2}{3})\}$	0	(1,0,0,2,2,0)	–
$A^{a}_{6,26}$ [7]	$((a+1)e^{16}+e^{23}+e^{56},e^{26},ae^{36},(a+1)e^{46},e^{46}+(a+1)e^{56},0)$			
	$-1<a\le 1,\ a\notin\{0,-\frac{1}{2},-\frac{2}{3},-\frac{3}{4}\}$	0	(1,0,0,0,0,0)	–
	$a=-1$	1	(2,2,2,2,2,**1**)	–
	$a=0$	0	(2,1,0,0,0,0)	–
	$a=-\frac{1}{2}$	0	(1,1,1,0,0,0)	–
	$a=-\frac{2}{3}$	0	(1,0,1,1,0,0)	–
	$a=-\frac{3}{4}$	0	(1,0,0,1,1,0)	–
$A^{\varepsilon_1,\varepsilon_2,a}_{6,27}$ [8]	$((\varepsilon_2+a)e^{16}+e^{23}+\varepsilon_1e^{56},\varepsilon_2e^{26},ae^{36},e^{36}+ae^{46},e^{46}+ae^{56},0)$			
	$\varepsilon_1=0,\quad \varepsilon_2=1,\ a>0$	0	(1,0,0,0,0,0)	–
	$\varepsilon_1=0,\quad \varepsilon_2=1,\ a=0$	1	(2,2,2,1,0,0)	–
	$\varepsilon_1\in\{0,1\},\ \varepsilon_2=0,\ a=1$	0	(2,1,0,0,0,0)	–
$A^{a}_{6,28}$	$(2e^{16}+e^{23},e^{26},e^{26}+e^{36},ae^{46},e^{46}+ae^{56},0)$			
	$a\notin\{-4,-3,-2,-\frac{3}{2},-1,-\frac{1}{2},0\}$	0	(1,0,0,0,0,0)	–
	$a=0$	1	(2,2,1,0,0,0)	–
	$a=-1$	0	(1,2,2,0,0,0)	–
	$a=-3$	0	(1,0,2,2,0,0)	✓
	$a=-\frac{1}{2}$	0	(1,0,1,1,0,0)	–
	$a\in\{-4,-\frac{3}{2}\}$	0	(1,0,0,1,1,0)	–
	$a=-2$	0	(1,0,0,0,1,**1**)	–
$A_{6,29}$	$(2e^{16}+e^{23}+e^{56},e^{26},e^{26}+e^{36},2e^{46},e^{46}+2e^{56},0)$	0	(1,0,0,0,0,0)	–
$A_{6,30}$	$(e^{23},0,e^{26},e^{46},e^{46}+e^{56},0)$	1	(2,2,2,1,0,0)	–
$A_{6,31}$	$(2e^{16}+e^{23},e^{26},e^{26}+e^{36},e^{36}+e^{46},e^{46}+e^{56},0)$	0	(1,0,0,0,0,0)	–

[7] $A^{a}_{6,26}\cong A^{1/a}_{6,26}$
[8] $A^{\varepsilon_1,\varepsilon_2,a}_{6,27}\cong A^{-\varepsilon_1,-\varepsilon_2,-a}_{6,27}$, $A^{\varepsilon_1,0,0}_{6,27}$ is nilpotent.

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
$A_{6,32}^{a,b,c}$	$(2ae^{16}+e^{23}, ae^{26}-e^{36}, e^{26}+ae^{36}, be^{46}, ce^{56}, 0)$			
[9]	$a>0,\ \lvert b\rvert \le \lvert c\rvert,\ b\notin\{0,-4a\},\ c\notin\{0,-4a,-b,-(4a+b)\}$	0	(1,0,0,0,0,0)	–
	$c=-b,\ a>0,\ b>0,\ b\neq 4a$	0	(1,1,1,0,0,0)	–
	$a=0,\ 0<b\le\lvert c\rvert,\ c\neq -b$	1	(1,0,1,1,0,0)	–
	$b=-4a,\ a>0,\ c\notin\{0,\pm 4a\}$	0	(1,0,0,1,1,0)	–
	$c=-(4a+b),\ a>0,\ b\ge -2a,\ b\neq 0$	0	(1,0,0,0,1,**1**)	–
	$(a,c)=(0,-b),\ b>0$	1	(1,1,2,1,1,**1**)	–
	$(b,c)=(-4a,4a),\ a>0$	0	(1,1,1,1,1,0)	–
	$(b,c)=(-4a,-4a),\ a>0$	0	(1,0,0,2,2,0)	–
$A_{6,33}^{a,b}$	$(2ae^{16}+e^{23}+e^{56}, ae^{26}-e^{36}, e^{26}+ae^{36}, be^{46}, 2ae^{56}, 0)$			
	$a>0,\ b\notin\{0,-2a,-4a,-6a\}$	0	(1,0,0,0,0,0)	–
	$a=0,\ b>0$	1	(2,1,1,2,1,0)	–
	$b=-2a,\ a>0$	0	(1,1,1,0,0,0)	–
	$b=-4a,\ a>0$	0	(1,0,0,1,1,0)	–
	$b=-6a,\ a>0$	0	(1,0,0,0,1,**1**)	–
$A_{6,34}^{\varepsilon,a,b}$	$(2ae^{16}+e^{23}+\varepsilon e^{56}, ae^{26}-e^{36}, e^{26}+ae^{36}, (2a+b)e^{46}, e^{46}+(2a+b)e^{56}, 0)$			
	$\varepsilon=0,\ a>0,\ b\notin\{-2a,-4a,-6a\}$	0	(1,0,0,0,0,0)	–
	$\varepsilon=0,\ b=-2a,\ a>0$	1	(2,2,1,0,0,0)	–
	$\varepsilon=0,\ a=0,\ b>0$	1	(1,0,1,1,0,0)	–
	$\varepsilon=0,\ b=-6a,\ a>0$	0	(1,0,0,1,1,0)	–
	$\varepsilon=0,\ b=-4a,\ a>0$	0	(1,0,0,0,1,**1**)	–
	$\varepsilon=1,\ b=0,\ a>0$	0	(1,0,0,0,0,0)	–
	$\varepsilon\in\{0,1\},\ (a,b)=(0,0)$	2	(2,2,2,2,2,**1**)	–
$A_{6,35}^{a,b,c}$	$((a+b)e^{16}+e^{23}, ae^{26}, be^{36}, ce^{46}-e^{56}, e^{46}+ce^{56}, 0)$			
[10]	$0<a\le\lvert b\rvert,\ b\notin\{0,-a,-2a,\}\ c\notin\left\{0,-\frac{1}{2}a,-\frac{1}{2}b,-(\frac{1}{2}a+b),-(\frac{1}{2}b+a),-(a+b)\right\}$	0	(1,0,0,0,0,0)	–
	$a=0,\ b>0,\ c\notin\left\{0,-b,-\frac{1}{2}b\right\}$	0	(2,1,0,0,0,0)	–
	$b=-2a,\ a>0,\ c\notin\left\{0,-\frac{1}{2}a,a,\frac{3}{2}a\right\}$	0	(1,1,1,0,0,0)	–
	$c=0,\ 0<a\le\lvert b\rvert,\ b\notin\{-a,-2a\}$	0	(1,1,1,0,0,0)	–
	$b=-a,\ a>0,\ c>0,\ c\notin\left\{\frac{1}{2}a\right\}$	1	(1,0,1,1,0,0)	–
	$c=-\frac{1}{2}a,\ a>0,\ b\notin\left\{0,-2a,-a,-\frac{1}{2}a,a\right\}$	0	(1,0,1,1,0,0)	–
	$c=-(\frac{1}{2}a+b),\ a>0,\ b\notin\left\{0,-2a,-a,-\frac{1}{2}a,a\right\}$	0	(1,0,0,1,1,0)	–
	$c=-(a+b),\ 0<a\le\lvert b\rvert,\ b\notin\{0,-2a,-a\}$	0	(1,0,0,0,1,**1**)	–
	$(a,c)=(0,0),\ b>0$	0	(2,2,2,1,0,0)	–
	$(a,c)=(0,-\frac{1}{2}b),\ b>0$	0	(2,1,1,2,1,0)	–
	$(a,c)=(0,-b),\ b>0$	0	(2,1,0,1,2,**1**)	–
	$(b,c)=(-2a,0),\ a>0$	0	(1,2,2,1,1,0)	–
	$(b,c)=(-2a,a),\ a>0$	0	(1,1,2,1,1,**1**)	–
	$(b,c)=(-a,0),\ a>0$	1	(1,1,2,1,1,**1**)	–
	$(b,c)=(-2a,-\frac{1}{2}a),\ a>0$	0	(1,1,2,1,0,0)	–

[9] $A_{6,32}^{a,b,c}\cong A_{6,32}^{a,c,b}\cong A_{6,32}^{-a,-b,-c}$, $A_{6,32}^{a,0,c}\cong A_{6,32}^{a,b,0}$ is decomposable, the parameter ε in $g_{6,32}$ is redundant since $g_{6,32}\cong A_{6,33}$ for $\varepsilon\neq 0$.
[10] $A_{6,35}^{a,b,c}\cong A_{6,35}^{b,a,c}\cong A_{6,35}^{-a,-b,-c}$, $A_{6,35}^{0,0,c}$ is decomposable.

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$(b,c)=(-2a,\frac{3}{2}a),\ a>0$	0	(1,1,1,1,1,0)	–
	$(b,c)=(-a,\frac{1}{2}a),\ a>0$	1	(1,0,2,3,1,0)	–
	$(b,c)=(a,-\frac{1}{2}a),\ a>0$	0	(1,0,2,2,0,0)	–
	$(b,c)=(a,-\frac{3}{2}a),\ a>0$	0	(1,0,0,2,2,0)	–
$A_{6,36}^{a,b}$	$(2ae^{16}+e^{23},ae^{26},e^{26}+ae^{36},be^{46}-e^{56},e^{46}+be^{56},0)$			
	$a>0,\ b\notin\{0,-2a,-\frac{3}{2}a,-\frac{1}{2}a\}$	0	(1,0,0,0,0,0)	–
	$a=0,\ b>0$	1	(2,2,2,1,0,0)	–
	$b=0,\ a>0$	0	(1,1,1,0,0,0)	–
	$b=-\frac{1}{2}a,\ a>0$	0	(1,0,1,1,0,0)	–
	$b=-\frac{3}{2}a,\ a>0$	0	(1,0,0,1,1,0)	–
	$b=-2a,\ a>0$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(0,0)$	1	(2,3,4,3,2,$\underline{\mathbf{1}}$)	–
$A_{6,37}^{a,b,c}$ [11]	$(2ae^{16}+e^{23},ae^{26}-e^{36},e^{26}+ae^{36},be^{46}-ce^{56},ce^{46}+be^{56},0)$			
	$a>0,\ b\notin\{0,-2a\},\ c>0,\ (b,c)\notin\{(-a,1),(-3a,1)\}$	0	(1,0,0,0,0,0)	–
	$b=0,\ a>0,\ c>0$	0	(1,1,1,0,0,0)	–
	$a=0,\ b>0,\ c>0$	1	(1,0,1,1,0,0)	–
	$b=-2a,\ a>0,\ c>0$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
	$(b,c)=(-a,1),\ a>0$	0	(1,2,2,0,0,0)	–
	$(a,b)=(0,0),\ c>0,\ c\neq 1$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(b,c)=(-3a,1),\ a>0$	0	(1,0,2,2,0,0)	✓
	$(a,b,c)=(0,0,1)$	1	(1,3,6,3,1,$\underline{\mathbf{1}}$)	✓
$A_{6,38}^{a}$	$(2ae^{16}+e^{23},ae^{26}-e^{36},e^{26}+ae^{36},e^{26}+ae^{46}-e^{56},e^{36}+e^{46}+ae^{56},0)$			
	$a>0$	0	(1,0,0,0,0,0)	–
	$a=0$	1	(1,2,4,2,1,$\underline{\mathbf{1}}$)	✓
$B_{6,1}$ [12]	$(e^{16}+e^{23}+e^{56},e^{26},0,e^{36},e^{56},0)$	1	(2,2,1,0,0,0)	–
	Nilradical $\mathfrak{n}_4\oplus\mathbb{R}$			
$A_{6,39}^{a,b}$	$((b+1)e^{16}+e^{45},e^{15}+(b+2)e^{26},ae^{36},be^{46},e^{56},0)$			
	$a\notin\{-1,0\}$, $b\notin\{-3,-2,-\frac{4}{3},-1,-\frac{1}{2},0,-a,-(a+3),-\frac{1}{3}(a+3),-\frac{1}{3}(a+4),-\frac{1}{2}(a+1),-\frac{1}{2}(a+4)\}$	0	(1,0,0,0,0,0)	–
	$b=0,\ a\notin\{-4,-3,-1,0\}$	0	(2,1,0,0,0,0)	–
	$a=-1,\ b\notin\{-3,-2,-\frac{3}{2},-\frac{4}{3},-1,-\frac{2}{3},-\frac{1}{2},0,1\}$	0	(1,1,1,0,0,0)	–
	$b=-3,\ a\notin\{-1,0,2,3,5,6\}$	0	(1,1,1,0,0,0)	–
	$b=-\frac{1}{2},\ a\notin\{-3,-\frac{5}{2},-\frac{3}{2},-1,0,\frac{1}{2}\}$	0	(1,1,1,0,0,0)	–
	$b=-a,\ a\notin\{-1,0,\frac{1}{2},1,\frac{4}{3},\frac{3}{2},2,3,4\}$	0	(1,1,1,0,0,0)	–
	$b=-2,\ a\notin\{-1,0,2,3\}$	1	(1,0,1,1,0,0)	–
	$b=-(a+3),\ a\notin\{-5-3,-\frac{5}{2},-2,-\frac{5}{3},-1,0\}$	0	(1,0,1,1,0,0)	–
	$b=-\frac{1}{2}(a+1),\ a\notin\{-5,-1,0,1,\frac{5}{3},3,5\}$	0	(1,0,1,1,0,0)	–
	$b=-1,\ a\notin\{-2,-1,0,1\}$	0	(1,0,1,1,0,0)	✓
	$b=-\frac{1}{2}(a+4),\ a\notin\{-6,-4,-3,-2,-\frac{4}{3},-1,0,2,4\}$	0	(1,0,0,1,1,0)	–

[11] $A_{6,37}^{a,b,0}\cong A_{6,32}^{a,b,b}$, $A_{6,37}^{a,b,c}\cong A_{6,37}^{a,b,-c}\cong A_{6,37}^{-a,-b,c}$, $A_{6,37}^{a,0,0}$ is decomposable.
[12] $B_{6,1}=n_{6,8}$ in [13]

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$b=-\frac{4}{3}$, $a\notin\{-\frac{5}{3},-\frac{4}{3},-1,0,1,\frac{4}{3},\frac{5}{3}\}$	0	(1,0,0,1,1,0)	–
	$b=-\frac{1}{3}(a+1)$, $a\notin\{-6,-3,-\frac{3}{2},-1,0,1,\frac{3}{2},3,6\}$	0	(1,0,0,1,1,0)	✓
	$b=-\frac{1}{3}(a+4)$, $a\notin\{-4,-\frac{5}{2},-1,0,2,5\}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(-1,0)$	0	(2,2,2,1,0,0)	–
	$(a,b)=(-3,0)$	0	(2,1,1,2,1,0)	✓
	$(a,b)=(-4,0)$	0	(2,1,0,1,2,$\underline{\mathbf{1}}$)	–
	$(a,b)\in\{(-1,-3),\ (-1,-\frac{1}{2}),\ (-1,1),\ (\frac{1}{2},-\frac{1}{2}),\ (3,-3)\}$	0	(1,2,2,0,0,0)	–
	$(a,b)=(-1,-2)$	1	(1,1,3,2,0,0)	–
	$(a,b)=(1,-1)$	0	(1,1,3,2,0,0)	✓
	$(a,b)\in\{(-1,-1),\ (-\frac{5}{2},-\frac{1}{2}),\ (5,-3)\}$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(2,-2)$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(a,b)\in\{(-1,-\frac{4}{3}),\ (-1,-\frac{3}{2}),\ (-3,-\frac{1}{2}),\ (2,-3),\ (\frac{4}{3},-\frac{4}{3}),\ (4,-4)\}$	0	(1,1,1,1,1,0)	–
	$(a,b)\in\{(-\frac{3}{2},-\frac{1}{2}),\ (-1,-\frac{2}{3}),\ (\frac{3}{2},-\frac{3}{2}),\ (6,-3)\}$	0	(1,1,1,1,1,0)	✓
	$(a,b)=(3,-2)$	1	(1,0,2,3,1,0)	✓
	$(a,b)=(-2,-1)$	0	(1,0,2,3,1,0)	✓
	$(a,b)=(-5,2)$	0	(1,0,2,2,0,0)	–
	$(a,b)=\{(\frac{5}{3},-\frac{4}{3}),(-\frac{5}{3},-\frac{4}{3})\}$	0	(1,0,1,2,1,0)	–
	$(a,b)=(-\frac{4}{3},-\frac{4}{3})$	0	(1,0,0,2,2,0)	–
	$(a,b)=(-6,1),\ (1,-\frac{4}{3})$	0	(1,0,0,2,2,0)	✓
$A^{a}_{6,40}$	$((a+1)e^{16}+e^{45},e^{15}+(a+2)e^{26}+e^{36},(a+2)e^{36},ae^{46},e^{56},0)$			
	$a\notin\{-3,-\frac{5}{2},-2,-\frac{3}{2},-\frac{4}{3},-\frac{5}{4},-1,-\frac{1}{2},0\}$	0	(1,0,0,0,0,0)	–
	$a=-2$	1	(2,1,1,2,1,0)	–
	$a=0$	0	(2,1,0,0,0,0)	–
	$a=-1$	0	(1,1,2,1,0,0)	–
	$a\in\{-3,\ -\frac{1}{2}\}$	0	(1,1,1,0,0,0)	–
	$a=-\frac{5}{2}$	0	(1,0,1,1,0,0)	–
	$a=-\frac{4}{3}$	0	(1,0,0,1,1,0)	–
	$a=-\frac{5}{4}$	0	(1,0,0,1,1,0)	✓
	$a=-\frac{3}{2}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A^{a}_{6,41}$	$((a+1)e^{16}+e^{45},e^{15}+(a+2)e^{26},ae^{36}+e^{46},ae^{46},e^{56},0)$			
	$a\notin\{-3,-2,-\frac{3}{2},-\frac{4}{3},-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{3},0\}$	0	(1,0,0,0,0,0)	–
	$a=0$	1	(2,2,1,0,0,0)	–
	$a=-1$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$a\in\{-3,-\frac{1}{2}\}$	0	(1,1,1,0,0,0)	–
	$a=-2$	1	(1,0,1,1,0,0)	–
	$a\in\{-\frac{3}{2},-\frac{1}{3}\}$	0	(1,0,1,1,0,0)	–
	$a=-\frac{4}{3}$	0	(1,0,0,1,1,0)	–
	$a=-\frac{3}{4}$	0	(1,0,0,1,1,0)	✓
$A^{a}_{6,42}$	$((a+1)e^{16}+e^{45},e^{15}+(a+2)e^{26},e^{36}+e^{56},ae^{46},e^{56},0)$			
	$a\notin\{-4,-3,-\frac{5}{2},-2,-\frac{5}{3},-\frac{4}{3},-1,-\frac{1}{2},0\}$	0	(1,0,0,0,0,0)	–
	$a=0$	0	(2,1,0,0,0,0)	–

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$a=-1$	0	(1,1,3,2,0,0)	✓
	$a\in\left\{-3,-\frac{1}{2}\right\}$	0	(1,1,1,0,0,0)	–
	$a=-4$	0	(1,0,1,1,0,0)	–
	$a=-2$	1	(1,0,1,1,0,0)	–
	$a\in\left\{-\frac{5}{2},-\frac{4}{3}\right\}$	0	(1,0,0,1,1,0)	–
	$a=-\frac{5}{3}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A_{6,43}$	$(e^{45},e^{15}+e^{26}+e^{36},e^{36}+e^{56},-e^{46},e^{56},0)$	0	(1,1,2,1,0,0)	–
$A_{6,44}^{a}$	$(2e^{16}+e^{45},e^{15}+3e^{26},ae^{36},e^{46}+e^{56},e^{56},0)$			
	$a\notin\{0,-1,-3,-4,-6,-7\}$	0	(1,0,0,0,0,0)	–
	$a=-1$	0	(1,1,1,0,0,0)	–
	$a\in\{-4,-3\}$	0	(1,0,1,1,0,0)	–
	$a=-6$	0	(1,0,0,1,1,0)	–
	$a=-7$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A_{6,45}$	$(2e^{16}+e^{45},e^{15}+3e^{26}+e^{36},3e^{36},e^{46}+e^{56},e^{56},0)$	0	(1,0,0,0,0,0)	–
$A_{6,46}$	$(2e^{16}+e^{45},e^{15}+3e^{26},e^{36}+e^{46},e^{46}+e^{56},e^{56},0)$	0	(1,0,0,0,0,0)	–
$A_{6,47}^{\varepsilon,a}$	$(e^{16}+e^{45},e^{15}+e^{26}+\varepsilon e^{46},ae^{36},e^{46},0,0)$			
	$\varepsilon\in\{0,\pm1\}$, $a\notin\{0,-1,-2,-3\}$	0	(2,1,0,0,0,0)	–
	$\varepsilon\in\{0,\pm1\}$, $a=-1$	0	(2,2,2,1,0,0)	–
	$\varepsilon\in\{0,\pm1\}$, $a=-2$	0	(2,1,1,2,1,0)	✓
	$\varepsilon\in\{0,\pm1\}$, $a=-3$	0	(2,1,0,1,2,$\underline{\mathbf{1}}$)	✓
$A_{6,48}$	$(e^{16}+e^{45},e^{15}+e^{26}+e^{36},e^{36},e^{46},0,0)$			
$A_{6,49}^{\varepsilon}$	$(e^{16}+e^{45},e^{15}+e^{26}+\varepsilon e^{46},e^{56},e^{46},0,0)$, $\varepsilon\in\{0,\pm1\}$	1	(2,2,1,0,0,0)	–
$A_{6,50}^{\varepsilon}$	$(e^{16}+e^{45},e^{15}+e^{26}+\varepsilon e^{36},e^{36}+e^{46},e^{46},0,0)$, $\varepsilon\in\{0,\pm1\}$	0	(2,1,0,0,0,0)	–
$A_{6,51}^{\varepsilon}$	$(e^{45},e^{15}+\varepsilon e^{46},e^{36},0,0,0)$, $\varepsilon=\pm1$	1	(3,4,4,3,1,0)	✓
$A_{6,52}^{\varepsilon}$	$(e^{45},e^{15}+\varepsilon e^{46},e^{36},e^{56},0,0)$, $\varepsilon\in\{0,\pm1\}$	1	(2,3,3,2,1,0)	–
	Nilradical $A_{5,1}$			
$A_{6,53}$	$(e^{35},e^{45},e^{36},e^{46},-e^{56},0)$	2	(1,0,3,5,2,0)	–
$A_{6,54}^{a,b}$	$(e^{16}+e^{35},be^{26}+e^{45},(1-a)e^{36},(b-a)e^{46},ae^{56},0)$			
[13]	$-1<b\leq1$, $b\notin\left\{-\frac{1}{2},0\right\}$, $a\notin\left\{-1,0,1,2,\pm b,2b,\frac{1}{2}(b+1),\pm(b+1),b+\frac{1}{2},\frac{1}{2}b+1,2(b+1)\right\}$	0	(1,0,0,0,0,0)	–
	$a=0$, $-1<b\leq1$, $b\notin\left\{-\frac{1}{2},-1,0\right\}$	0	(2,1,0,0,0,0)	–
	$a=1$, $b\notin\left\{-2,-1,-\frac{1}{2},0,\frac{1}{2},1\right\}$	0	(2,1,0,0,0,0)	–
	$a=b+1$, $-1<b<1$, $b\notin\left\{-\frac{1}{2},0\right\}$	0	(1,1,1,1,1,0)	✓
	$a=-1$, $b\notin\left\{-4,-3,-2,-\frac{3}{2},-1,-\frac{1}{2},0,1\right\}$	0	(1,1,1,0,0,0)	–
	$a=2$, $b\notin\left\{-3,-2,-1,-\frac{1}{2},0,1,\frac{3}{2},2,3\right\}$	0	(1,1,1,0,0,0)	–
	$a=\frac{1}{2}(b+1)$, $-1<b<1$, $b\notin\left\{-\frac{1}{2},-\frac{1}{3},0,\frac{1}{3}\right\}$	0	(1,1,1,0,0,0)	–
	$b=0$, $a\notin\left\{-1,0,\frac{1}{2},1,2\right\}$	1	(1,0,1,1,0,0)	–
	$a=-(b+1)$, $-1<b\leq1$, $b\notin\left\{-\frac{3}{4},-\frac{1}{2},-\frac{1}{3},0\right\}$	0	(1,0,1,1,0,0)	–
	$b=-1$, $a>0$, $a\notin\left\{\frac{1}{2},1,2\right\}$	0	(1,0,1,1,0,0)	–
	$a=b+\frac{1}{2}$ (isomorphic to $a=\frac{1}{2}b+1$), $b\notin\left\{-2,-\frac{3}{2},-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2},1,\frac{3}{2}\right\}$	0	(1,0,1,1,0,0)	–

[13] $A_{6,54}^{a,b}\cong A_{6,54}^{a/b,1/b}$

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$b=-2,\ a\notin\left\{-4,-2,-\frac{3}{2},-1,-\frac{1}{2},0,1,2\right\}$	0	(1,0,0,1,1,0)	–
	$a=2(b+1),\ -1<b\leq 1,\ b\notin\left\{-\frac{2}{3},-\frac{1}{2},0\right\}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(0,0)$	1	(3,4,3,1,0,0)	–
	$(a,b)=(1,1)$	0	(3,3,1,0,0,0)	–
	$(a,b)=(0,-1)$	0	(2,3,4,3,2,$\underline{\mathbf{1}}$)	✓
	$(a,b)=(1,0)$	1	(2,2,3,3,1,0)	✓
	$(a,b)\in\{(1,-1),\ (2,2)\}$	0	(2,2,2,1,0,0)	–
	$(a,b)\in\{(1,-2),\ (0,-2)\}$	0	(2,1,1,2,1,0)	–
	$(a,b)=(1,-\frac{1}{2})$	0	(2,1,0,1,2,$\underline{\mathbf{1}}$)	–
	$(a,b)=(2,1)$	0	(1,3,3,1,1,0)	✓
	$(a,b)=(2,-2)$	0	(1,2,2,1,1,0)	–
	$(a,b)=(-1,-2)$	0	(1,2,2,2,2,0)	✓
	$(a,b)\in\{(2,3),\ (-1,-3),\ (-1,1)\}$	0	(1,2,2,0,0,0)	–
	$(a,b)\in\left\{(-1,0),\ (\frac{1}{2},0)\right\}$	1	(1,1,3,2,0,0)	–
	$(a,b)\in\left\{(-1,-\frac{3}{2}),\ (2,0)\right\}$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$(a,b)\in\left\{(-1,-4),(2,-3),(2,-1),(2,\frac{3}{2})\right\}$	0	(1,1,2,1,0,0)	–
	$(a,b)\in\left\{(2,-\frac{1}{2}),\ (\frac{1}{4},-\frac{1}{2})\right\}$	0	(1,1,1,1,1,0)	–
	$(a,b)\in\left\{(\frac{3}{2},1),\ (\frac{1}{2},-1)\right\}$	0	(1,0,2,2,0,0)	–
	$(a,b)=(\frac{1}{3},-\frac{4}{3})$	0	(1,0,2,2,0,0)	✓
	$(a,b)=(-\frac{3}{2},-2)$	0	(1,0,1,2,1,0)	✓
$A^{a}_{6,55}$	$(e^{16}+e^{35}+e^{46},(a+1)e^{26}+e^{45},(1-a)e^{36},e^{46},ae^{56},0)$			
	$a\notin\left\{-4,-3,-2,-\frac{3}{2},-1,-\frac{1}{2},0,1,2,3\right\}$	0	(1,0,0,0,0,0)	–
	$a\in\{0,1\}$	0	(2,1,0,0,0,0)	–
	$a=-2$	0	(1,1,2,1,0,0)	–
	$a=-1$	1	(1,1,2,1,0,0)	–
	$a\in\left\{-\frac{1}{2},2\right\}$	0	(1,1,1,0,0,0)	–
	$a=3$	0	(1,0,1,1,0,0)	–
	$a\in\left\{-3,-\frac{3}{2}\right\}$	0	(1,0,0,1,1,0)	–
	$a=-4$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A^{a}_{6,56}$	$(e^{16}+e^{35},(1-b)e^{26}+e^{36}+e^{45},(1-a)e^{36},(1-2a)e^{46},ae^{56},0)$			
	$a\notin\left\{-1,0,\frac{1}{2},\frac{2}{3},\frac{3}{4},1,\frac{4}{3},\frac{3}{2},2,3\right\}$	0	(1,0,0,0,0,0)	–
	$a=1$	1	(2,2,3,3,1,0)	✓
	$a\in\left\{0,\frac{1}{2}\right\}$	0	(2,1,0,0,0,0)	–
	$a=2$	0	(1,1,2,1,0,0)	–
	$a\in\left\{-1,\frac{2}{3}\right\}$	0	(1,1,1,0,0,0)	–
	$a=\frac{3}{4}$	0	(1,0,1,1,0,0)	–
	$a\in\left\{\frac{3}{2},3\right\}$	0	(1,0,0,1,1,0)	–
	$a=\frac{4}{3}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A^{a}_{6,57}$	$(e^{16}+e^{35},2ae^{26}+e^{45},(1-a)e^{36},ae^{46}+e^{56},ae^{56},0)$			
	$a\notin\left\{-1,-\frac{2}{3},-\frac{1}{2},-\frac{1}{3},-\frac{1}{4},0,1,2\right\}$	0	(1,0,0,0,0,0)	–
	$a=0$	1	(2,2,2,1,0,0)	–

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$a = 1$	0	(2,1,0,0,0,0)	–
	$a = -1$	0	(1,1,1,1,1,0)	–
	$a = 2$	0	(1,1,1,0,0,0)	–
	$a \in \{-\frac{1}{2}, -\frac{1}{3}\}$	0	(1,0,1,1,0,0)	–
	$a = -\frac{1}{4}$	0	(1,0,0,1,1,0)	–
	$a = -\frac{2}{3}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A^{\varepsilon}_{6,58}$	$(3e^{16} + e^{35}, 2e^{26} + e^{36} + e^{45}, 2e^{36}, e^{46} + \varepsilon e^{56}, e^{56}, 0)$, $\varepsilon \in \{0, 1\}$	0	(1,0,0,0,0,0)	–
$A_{6,59}$ [14]	$(e^{16} + e^{35}, e^{45} + e^{46}, e^{36}, e^{56}, 0, 0)$	1	(2,2,2,1,0,0)	–
$A_{6,60}$ [15]	$(e^{16} + e^{35} + e^{46}, 2e^{26} + e^{45}, 0, e^{46} + e^{56}, e^{56}, 0)$	0	(2,1,0,0,0,0)	–
$A^{a}_{6,61}$	$(2e^{16} + e^{35}, 2ae^{26} + e^{45}, e^{36} + e^{56}, (2a-1)e^{46}, e^{56}, 0)$			
	$a \notin \{-2, -\frac{3}{2}, -1, -\frac{3}{4}, -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}\}$	0	(1,0,0,0,0,0)	–
	$a = \frac{1}{2}$	0	(2,1,0,0,0,0)	–
	$a = 0$	1	(1,1,2,1,0,0)	–
	$a = -\frac{1}{2}$	0	(1,1,1,1,1,0)	–
	$a = \frac{1}{4}$	0	(1,1,1,0,0,0)	–
	$a \in \{-1, -\frac{3}{2}\}$	0	(1,0,1,1,0,0)	–
	$a = -2$	0	(1,0,0,1,1,0)	–
	$a = -\frac{3}{4}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A^{\varepsilon}_{6,62}$	$(2e^{16} + e^{35}, e^{26} + e^{36} + e^{45}, e^{36} + \varepsilon e^{56}, 0, e^{56}, 0)$, $\varepsilon \in \{0, 1\}$	0	(2,1,0,0,0,0)	–
$A^{a}_{6,63}$	$(e^{16} + e^{35}, ae^{26} + e^{45} + e^{46}, e^{36}, ae^{46}, 0, 0)$			
	$a \notin \{-2, -1, -\frac{1}{2}, 0\}$	0	(2,1,0,0,0,0)	–
	$a = 0$	1	(3,4,3,1,0,0)	–
	$a = -1$	0	(2,2,2,2,2,$\underline{\mathbf{1}}$)	✓
	$a \in \{-2, -\frac{1}{2}\}$	0	(2,1,1,2,1,0)	–
$A^{\varepsilon}_{6,64}$ [16]	$(e^{16} + e^{35} + e^{46}, e^{26} + \varepsilon e^{36} + e^{45}, e^{36}, e^{46}, 0, 0)$, $\varepsilon = \pm 1$	0	(2,1,0,0,0,0)	–
$A^{\varepsilon,a}_{6,65}$	$(\varepsilon e^{16} + e^{35}, e^{16} + \varepsilon e^{26} + e^{45}, (\varepsilon - a)e^{36}, e^{36} + (\varepsilon - a)e^{46}, ae^{56}, 0)$			
	$\varepsilon = 1$, $a \notin \{-2, -1, 0, 1, \frac{3}{2}, 2\}$	0	(1,0,0,0,0,0)	–
	$\varepsilon = 1$, $a = 1$	0	(2,2,1,0,0,0)	–
	$\varepsilon = 1$, $a = 0$	0	(2,1,0,0,0,0)	–
	$\varepsilon = 1$, $a = 2$	0	(1,1,1,1,1,0)	✓
	$\varepsilon = 1$, $a = -1$	0	(1,1,1,0,0,0)	–
	$\varepsilon = 0$, $a = 1$	1	(1,0,1,2,1,0)	–
	$\varepsilon = 1$, $a \in \{-2, \frac{3}{2}\}$	0	(1,0,1,1,0,0)	–
$A_{6,66}$	$(2e^{16} + e^{35}, e^{16} + 2e^{26} + e^{45}, e^{36} + e^{56}, e^{36} + e^{46}, e^{56}, 0)$	0	(1,0,0,0,0,0)	–
$A^{\varepsilon}_{6,68}$ [17]	$(e^{16} + e^{35} + ae^{46}, e^{16} + e^{26} + e^{45}, e^{36}, e^{36} + e^{46}, 0, 0)$, $\varepsilon \in \{0, 1\}$	0	(2,1,0,0,0,0)	–
$A_{6,69}$	$(e^{16} + e^{35}, e^{16} + e^{26} + e^{45} + e^{46}, e^{36}, e^{36} + e^{46}, 0, 0)$	0	(2,1,0,0,0,0)	–
$A^{a,b}_{6,70}$	$(be^{16} - e^{26} + e^{35}, e^{16} + be^{26} + e^{45}, (b-a)e^{36} - e^{46}, e^{36} + (b-a)e^{46}, ae^{56}, 0)$			
	$a > 0$, $b \notin \{-\frac{1}{2}a, 0, \frac{1}{4}a, \frac{1}{2}a, a\}$	0	(1,0,0,0,0,0)	–

[14] The parameter h in $g_{6,59}$ is redundant since it can be normalised for $h \neq 0$ and $g_{6,59} = A^{0}_{6,63}$ for $h = 0$.
[15] The parameter ω in $g_{6,60}$ is redundant since $g_{6,60} = A^{1}_{6,55}$ for $\omega = 0$.
[16] $A^{0}_{6,64} = A^{0}_{6,55}$
[17] $g_{6,67}$ is redundant since $g^{h}_{6,67} \cong A^{1,1/2}_{6,65}$ for all $h \in \mathbb{R}$.

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	$a=0,\ b>0$	0	(2,1,0,0,0,0)	–
	$b=\frac{1}{2}a,\ a>0,$	0	(1,1,1,1,1,0)	✓
	$b=a,\ a>0$	0	(1,1,1,0,0,0)	–
	$b=-\frac{1}{2}a,\ a>0$	0	(1,0,1,1,0,0)	–
	$b=0,\ a>0$	0	(1,0,1,1,0,0)	–
	$b=\frac{1}{4}a,\ a>0$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
	$(a,b)=(0,0)$	0	(2,3,4,3,2,$\underline{\mathbf{1}}$)	✓
	Nilradical $A_{5,2}$			
$A^{a}_{6,71}$	$((a+3)e^{16}+e^{25},(a+2)e^{26}+e^{35},(a+1)e^{36}+e^{45},ae^{46},e^{56},0)$			
	$a\notin\left\{-4,-3,-\frac{7}{3},-2,-\frac{7}{4},-\frac{3}{2},-1,-\frac{1}{2},0\right\}$	0	(1,0,0,0,0,0)	–
	$a=0$	0	(2,1,0,0,0,0)	–
	$a=-\frac{3}{2}$	0	(1,1,1,1,1,0)	✓
	$a\in\left\{-4,-\frac{1}{2}\right\}$	0	(1,1,1,0,0,0)	–
	$a=-3$	1	(1,0,1,1,0,0)	–
	$a=-2$	0	(1,0,1,1,0,0)	–
	$a=-1$	0	(1,0,1,1,0,0)	✓
	$a=-\frac{7}{3}$	0	(1,0,0,1,1,0)	–
	$a=-\frac{7}{4}$	0	(1,0,0,0,1,$\underline{\mathbf{1}}$)	–
$A_{6,72}$	$(4e^{16}+e^{25},3e^{26}+e^{35},2e^{36}+e^{45},e^{46}+e^{56},e^{56},0)$	0	(1,0,0,0,0,0)	–
$A^{\varepsilon}_{6,73}$	$(e^{16}+e^{25}+ae^{36},e^{26}+e^{35}+ae^{46},e^{36}+e^{45},e^{46},0,0),\ \varepsilon=\pm1$	0	(2,1,0,0,0,0)	–
$A_{6,74}$	$(e^{16}+e^{25},e^{26}+e^{35},e^{36}+e^{45},e^{46},0,0)$	0	(2,1,0,0,0,0)	–
$A_{6,75}$	$(e^{16}+e^{25}+e^{46},e^{26}+e^{35},e^{36}+e^{45},e^{46},0,0)$	0	(2,1,0,0,0,0)	–
$B^{\varepsilon,a}_{6,2}$ [18]	$(e^{16}+e^{25}+\varepsilon e^{36}+ae^{46},e^{26}+e^{35}+\varepsilon e^{46},e^{36}+e^{45},e^{46},0,0),\ \varepsilon=\pm1,\ a\neq0$	0	(2,1,0,0,0,0)	–
	Nilradical $A_{5,3}$			
$A^{a}_{6,76}$	$((2a+1)e^{16}+e^{25},(a+1)e^{26}+e^{45},e^{24}+(a+2)e^{36},e^{46},ae^{56},0)$			
[19]	$-1<a\leq1,\ a\notin\left\{0,-\frac{1}{3},-\frac{1}{2},-\frac{4}{5}\right\}$	0	(1,0,0,0,0,0)	✓
	$a=0$	0	(2,1,0,0,0,0)	✓
	$a=-1$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
	$a=-3$	0	(1,1,1,0,0,0)	✓
	$a=-2$	1	(1,0,1,1,0,0)	✓
	$a=-\frac{4}{5}$	0	(1,0,0,1,1,0)	✓
$A^{\varepsilon}_{6,77}$	$(e^{16}+e^{25}+\varepsilon e^{46},e^{26}+e^{45},e^{24}+2e^{36},e^{46},0,0),\ \varepsilon=\pm1$	0	(2,1,0,0,0,0)	✓
$A_{6,78}$	$(-e^{16}+e^{25},e^{45},e^{24}+e^{36}+e^{46},e^{46},-e^{56},0)$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
$A_{6,79}$	$(3e^{16}+e^{25}+e^{36},2e^{26}+e^{45},e^{24}+3e^{36},e^{46},e^{46}+e^{56},0)$	0	(1,0,0,0,0,0)	✓
$B^{a}_{6,3}$	$(2ae^{16}+e^{45},e^{15}+3ae^{26}+e^{36},e^{14}-e^{26}+3ae^{36},ae^{46}-e^{56},e^{46}+ae^{56},0)$			
[20]	$a\neq0$	0	(1,0,0,0,0,0)	✓
	$a=0$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
$B^{\varepsilon}_{6,4}$	$(e^{45},e^{15}+e^{36},e^{14}-e^{26}+\varepsilon e^{56},-e^{56},e^{46},0),\ \varepsilon=\pm1$	0	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–

[18] $B_{6,2}=n_{6,76}$ in [13]

[19] $A^{a}_{6,76}\cong A^{1/a}_{6,76}$, $A^{0}_{6,76}=A^{0}_{6,77}$

[20] $B_{6,3}=n_{6,83}$, $B_{6,4}=n_{6,84}$ in [13], $g_{6,80}$ and $g_{6,81}$ are redundant since $g_{6,80}\cong A^{0}_{6,76}$ and $g^{\varepsilon}_{6,81}\cong A^{\varepsilon}_{6,77}$.

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	Nilradical $A_{5,4}$			
$A_{6,82}^{\varepsilon,a,b}$ [21]	$(2\varepsilon e^{16}+e^{24}+e^{35},(\varepsilon+a)e^{26},(\varepsilon+b)e^{36},(\varepsilon-a)e^{46},(\varepsilon-b)e^{56},0)$			
	$\varepsilon=1,\ 0\le a\le b,\ a\notin\{1,5\},\ b\notin\{1,5,2\pm a,4\pm a\}$	0	(1,0,0,0,0,0)	–
	$\varepsilon=1,\ a=1,\ b\ge 0,\ b\notin\{1,3,5\}$	0	(2,1,0,0,0,0)	–
	$\varepsilon=1,\ b=a+2,\ a>-1,\ a\notin\{0,1,3,5\}$	0	(1,1,1,0,0,0)	–
	$\varepsilon=1,\ b=a+4,\ a\ge -2,\ a\notin\{-1,0,1,5\}$	0	(1,0,1,1,0,0)	–
	$\varepsilon=1,\ a=5,\ b\ge 0,\ b\notin\{1,3,5,7,9\}$	0	(1,0,0,1,1,0)	–
	$\varepsilon=1,\ (a,b)=(1,1)$	0	(3,3,1,0,0,0)	–
	$\varepsilon=1,\ (a,b)=(1,3)$	0	(2,2,2,1,0,0)	–
	$\varepsilon=1,\ (a,b)=(1,5)$	0	(2,1,1,2,1,0)	–
	$\varepsilon=1,\ (a,b)=(0,2)$	0	(1,2,2,0,0,0)	–
	$\varepsilon=1,\ (a,b)\in\{(5,3),\ (5,7)\}$	0	(1,1,1,1,1,0)	–
	$\varepsilon=1,\ (a,b)=(0,4)$	0	(1,0,2,2,0,0)	–
	$\varepsilon=1,\ (a,b)=(5,9)$	0	(1,0,1,2,1,0)	✓
	$\varepsilon=1,\ (a,b)=(5,5)$	0	(1,0,0,2,2,0)	–
	$\varepsilon=0,\ a=1,\ 0<b<1$	1	(1,1,2,1,1,**1**)	✓
	$\varepsilon=0,\ (a,b)=(1,0)$	1	(3,3,2,3,3,**1**)	✓
	$\varepsilon=0,\ (a,b)=(1,1)$	1	(1,3,6,3,1,**1**)	✓
$A_{6,83}^{\varepsilon,a}$	$(2\varepsilon e^{16}+e^{24}+e^{35},(\varepsilon+a)e^{26},e^{26}+(\varepsilon+a)e^{36},(\varepsilon-a)e^{46}-e^{56},(\varepsilon-a)e^{56},0)$			
	$\varepsilon=1,\ a\ge 0,\ a\notin\{1,2,5\}$	0	(1,0,0,0,0,0)	–
	$\varepsilon=1,\ a=1$	0	(2,2,1,0,0,0)	–
	$\varepsilon=1,\ a=2$	0	(1,0,1,1,0,0)	–
	$\varepsilon=1,\ a=5$	0	(1,0,0,1,1,0)	–
	$\varepsilon=0,\ a=1$	1	(1,1,2,1,1,**1**)	–
$A_{6,84}$	$(e^{24}+e^{35},e^{26},e^{56},-e^{46},0,0)$	1	(2,2,2,2,2,**1**)	✓
$A_{6,85}^{a}$	$(2e^{16}+e^{24}+e^{35},(a+1)e^{26},e^{36}+e^{56},(1-a)e^{46},e^{56},0)$			
	$a\ge 0,\ a\notin\{1,2,4,5\}$	0	(1,0,0,0,0,0)	–
	$a=1$	0	(2,1,0,0,0,0)	–
	$a=2$	0	(1,1,1,0,0,0)	–
	$a=4$	0	(1,0,1,1,0,0)	–
	$a=5$	0	(1,0,0,1,1,0)	–
$A_{6,87}$ [22]	$(2e^{16}+e^{24}+e^{35},e^{26},e^{36}+e^{56},e^{36}+e^{46},e^{26}+e^{56},0)$	0	(1,0,0,0,0,0)	–
$A_{6,88}^{\varepsilon,a,b}$ [23]	$(2\varepsilon e^{16}+e^{24}+e^{35},(\varepsilon+a)e^{26}-be^{36},be^{26}+(\varepsilon+a)e^{36},(\varepsilon-a)e^{46}-be^{56},be^{46}+(\varepsilon-a)e^{56},0)$			
	$\varepsilon=1,\ a\ge 0,\ a\notin\{1,2\},\ b>0$	0	(1,0,0,0,0,0)	–
	$\varepsilon=1,\ a=1,\ b>0$	0	(1,1,1,0,0,0)	–
	$\varepsilon=1,\ a=2,\ b>0$	0	(1,0,1,1,0,0)	–
	$\varepsilon=0,\ a=1,\ b>0$	1	(1,1,2,1,1,**1**)	✓
	$\varepsilon=0,\ (a,b)=(0,1)$	1	(1,3,6,3,1,**1**)	✓

[21] $A_{6,82}^{\varepsilon,a,b}\cong A_{6,82}^{\varepsilon,b,a}\cong A_{6,82}^{\varepsilon,-a,b}\cong A_{6,82}^{\varepsilon,a,-b}$

[22] $g_{6,86}$ is redundant since $g_{6,86}=A_{6,83}^{1,0}$.

[23] $A_{6,88}^{\varepsilon,a,0}\cong A_{6,82}^{\varepsilon,a,a}$ and $A_{6,88}^{\varepsilon,a,b}\cong A_{6,88}^{\varepsilon,-a,b}\cong A_{6,88}^{\varepsilon,a,-b}$

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$\mathrm{h}^*(\mathfrak{g})$	half-flat
$A_{6,89}^{\varepsilon,\mathrm{a},\mathrm{b}}$	$(2\varepsilon e^{16}+e^{24}+e^{35},(\varepsilon+b)e^{26},\varepsilon e^{36}-ae^{56},(\varepsilon-b)e^{46},ae^{36}+\varepsilon e^{56},0)$			
[24]	$\varepsilon=1,\ a>0,\ b\geq 0,\ b\notin\{1,5\}$	0	(1,0,0,0,0,0)	–
	$\varepsilon=1,\ b=1,\ a>0$	0	(2,1,0,0,0,0)	–
	$\varepsilon=1,\ b=5,\ a>0$	0	(1,0,0,1,1,0)	–
	$\varepsilon=0,\ a=1,\ b\neq 0$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	✓
	$\varepsilon=0,\ (a,b)=(1,0)$	1	(3,3,2,3,3,$\underline{\mathbf{1}}$)	✓
$A_{6,90}^{\varepsilon,\mathrm{a}}$	$(2\varepsilon e^{16}+e^{24}+e^{35},\varepsilon e^{26}+e^{46},\varepsilon e^{36}+ae^{56},\varepsilon e^{46},-ae^{36}+\varepsilon e^{56},0)$			
[25]	$\varepsilon=1,\ a\in\mathbb{R}$	0	(1,0,0,0,0,0)	–
	$\varepsilon=0,\ a=\pm 1$	1	(2,2,2,2,2,$\underline{\mathbf{1}}$)	✓
$A_{6,93}^{\varepsilon,\mathrm{a}}$	$(2\varepsilon e^{16}+e^{24}+e^{35},\varepsilon e^{26}-ae^{56},\varepsilon e^{36}-ae^{46}-e^{56},e^{26}+ae^{36}+\varepsilon e^{46},ae^{26}+\varepsilon e^{56},0)$			
[26]	$\varepsilon=1,\ a\geq 0$	0	(1,0,0,0,0,0)	–
	$\varepsilon=0,\ a=1$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	–
$B_{6,5}^{\mathrm{a},\mathrm{b}}$ [27]	$(2e^{16}+e^{24}+e^{35},e^{26}+ae^{46},e^{36}+be^{56},-ae^{26}+e^{46},-be^{36}+e^{56},0),\ a>0,\ \lvert b\rvert<\lvert a\rvert$	0	(1,0,0,0,0,0)	–
$B_{6,6}^{\mathrm{a}}$	$(e^{24}+e^{35},e^{46},ae^{56},-e^{26},-ae^{36},0)$			
[28]	$-1<a<1,\ a\neq 0$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	✓
	$a=\pm 1$	1	(1,3,6,3,1,$\underline{\mathbf{1}}$)	✓
	Nilradical $A_{5,5}$			
$A_{6,94}^{\mathrm{a}}$	$((a+2)e^{16}+e^{25}+e^{34},(a+1)e^{26}+e^{35},ae^{36},2e^{46},e^{56},0)$			
	$a\notin\left\{-5,-3,-2,-\frac{5}{3},-\frac{3}{2},-\frac{4}{3},-1,-\frac{1}{2},0\right\}$	0	(1,0,0,0,0,0)	–
	$a=0$	0	(2,1,0,0,0,0)	–
	$a=-2$	1	(1,1,2,1,1,$\underline{\mathbf{1}}$)	✓
	$a=-3$	0	(1,1,1,1,1,0)	✓
	$a=-\frac{1}{2}$	0	(1,1,1,0,0,0)	–
	$a=-1$	0	(1,0,1,1,0,0)	✓
	$a\in\left\{-5,-\frac{3}{2}\right\}$	0	(1,0,1,1,0,0)	–
	$a=-\frac{5}{3}$	0	(1,0,0,1,1,0)	✓
	$a=-\frac{4}{3}$	0	(1,0,0,1,1,0)	–
$A_{6,95}$	$(2e^{16}+e^{25}+e^{34}+e^{46},e^{26}+e^{35},0,2e^{46},e^{56},0)$	0	(2,1,0,0,0,0)	–
$A_{6,96}$	$(3e^{16}+e^{25}+e^{34},2e^{26}+e^{35}+e^{46},e^{36}+e^{56},2e^{46},e^{56},0)$	0	(1,0,0,0,0,0)	–
$A_{6,97}$	$(4e^{16}+e^{25}+e^{34},3e^{26}+e^{35},2e^{36},e^{36}+2e^{46},e^{56},0)$	0	(1,0,0,0,0,0)	–
$A_{6,98}^{\varepsilon}$	$(e^{16}+e^{25}+\varepsilon e^{26}+e^{34},e^{26}+e^{35},e^{36},\varepsilon e^{56},0,0)$			
	$\varepsilon=0$	0	(3,3,1,0,0,0)	–
	$\varepsilon=1$	0	(2,2,1,0,0,0)	–

[24] $A_{6,89}^{\varepsilon,0,\mathrm{b}}\cong A_{6,82}^{\varepsilon,\mathrm{b},0}$ and $A_{6,89}^{\varepsilon,\mathrm{a},\mathrm{b}}\cong A_{6,89}^{\varepsilon,-\mathrm{a},\mathrm{b}}\cong A_{6,89}^{\varepsilon,\mathrm{a},-\mathrm{b}}$

[25] The Lie brackets of $g_{6,90}$, $g_{6,91}=A_{6,90}^{0,1}$ and $g_{6,93}$ are corrected in [13, Appendix G].

[26] $g_{6,92}$ is redundant since

$$g_{6,92}^{\alpha,\mu_0,\nu_0}\cong g_{6,82}^{\alpha,\sqrt{-\mu_0\nu_0},\sqrt{-\mu_0\nu_0}} \quad \text{for } \mu_0\nu_0<0 \text{ and } \mu_0=0,\ \nu_0=0,$$
$$g_{6,92}^{\alpha,\mu_0,\nu_0}\cong g_{6,88}^{\alpha,0,\sqrt{\mu_0\nu_0}} \quad \text{for } \mu_0\nu_0>0,$$
$$g_{6,92}^{\alpha,\mu_0,\nu_0}\cong g_{6,83}^{\alpha,0} \quad \text{for } \mu_0=0,\ \nu_0\neq 0 \text{ and } \mu_0\neq 0,\ \nu_0=0.$$

[27] $B_{6,5}=n_{6,95}$ in [13], $B_{6,5}^{\mathrm{a},\mathrm{b}}\cong B_{6,5}^{\mathrm{b},\mathrm{a}}\cong B_{6,5}^{-\mathrm{a},-\mathrm{b}}$, $B_{6,5}^{\mathrm{a},\mathrm{a}}\cong A_{6,92^*}^{1/\mathrm{a}}$ where $A_{6,92^*}$ is the class mentioned in [1].

[28] $B_{6,6}=n_{6,96}$ in [13], $B_{6,6}^{\mathrm{a}}\cong B_{6,6}^{1/\mathrm{a}}$, $B_{6,6}^{0}\cong A_{6,89}^{0,1,0}$, $B_{6,6}^{1}\cong A_{6,92^*}^{0}$

Table 3: Indecomposable six-dimensional Lie algebras with five-dimensional non-Abelian nilradical – continued

$\mathfrak{g}$	Lie bracket	$\mathfrak{z}$	$h^*(\mathfrak{g})$	half-flat
	Nilradical $A_{5,6}$			
$A_{6,99}$	$(5e^{16}+e^{25}+e^{34}, 4e^{26}+e^{35}, 3e^{36}+e^{45}, 2e^{46}, e^{56}, 0)$	0	(1,0,0,0,0,0)	✓

Table 4: Indecomposable six-dimensional Lie algebras with five-dimensional nilradical admitting a half-flat SU(3)-structure

Lie algebra	Normalised half-flat SU(3)-structure
$A_{6,13}^{a,-(2a+1),3a+1}$, $a \neq 0$	$\omega = -e^{13}+2ae^{26}+e^{45}$, $\rho = e^{124}+2ae^{156}-e^{235}-2ae^{346}$, OB, $\|e_6\| = 2\|a\|$
$A_{6,13}^{0,-1,1}$	$\omega = e^{15}-e^{26}+e^{34}$, $\rho = -e^{124}-e^{136}-e^{235}-e^{456}$, ONB
$A_{6,13}^{a,-(2a+1),3a+2}$, $a \neq -1$	$\omega = -(2a+2)e^{16}-e^{23}+e^{45}$, $\rho = e^{124}+e^{135}+(2a+2)e^{256}-(2a+2)e^{346}$, OB, $\|\|e_6\|\| = 2\|a+1\|$
$A_{6,13}^{-1,1,-1}$	$\omega = -e^{16}-e^{25}+e^{34}$, $\rho = -e^{124}-e^{135}+e^{236}-e^{456}$, ONB
$A_{6,14}^{\frac{2}{5},-\frac{3}{5}}$	$\omega = -e^{13}+\frac{4}{5}e^{26}+e^{45}$, $\rho = e^{125}-\frac{4}{5}e^{146}+e^{234}-\frac{4}{5}e^{356}$, OB, $\|\|e_6\|\| = \frac{4}{5}$
$A_{6,15}^{-1}$	$\omega = e^{16}-e^{24}+e^{35}$, $\rho = -e^{125}-e^{134}-e^{236}-e^{456}$, ONB
$A_{6,18}^{-3,5}$	$\omega = 4e^{16}+e^{23}+e^{45}$, $\rho = e^{125}+e^{134}+4e^{246}-4e^{356}$, OB, $\|\|e_6\|\| = 4$
$A_{6,18}^{2,-5}$	$\omega = -e^{13}+4e^{26}+e^{45}$, $\rho = -e^{125}+4e^{146}-e^{234}+4e^{356}$, OB, $\|\|e_6\|\| = 4$
$A_{6,25}^{-1,0}$	$\omega = -e^{15}+e^{24}-e^{36}$, $\rho = e^{126}+e^{134}-e^{146}+e^{235}+e^{256}-2e^{456}$,
	$g = (e^1)^2+(e^2)^2+(e^3)^2+2(e^4)^2+2(e^5)^2+(e^6)^2-2e^1\cdot e^5-2e^2\cdot e^4$
$A_{6,28}^{-3}$	$\omega = -e^{16}+e^{24}+e^{35}-4e^{45}$, $\rho = -e^{125}+e^{134}+e^{236}-4e^{246}-e^{456}$,
	$g = (e^1)^2+(e^2)^2+(e^3)^2+17(e^4)^2+(e^5)^2+(e^6)^2-8e^3\cdot e^4$
$A_{6,37}^{a,-3a,1}$, $a \neq 0$	$\omega = -4ae^{16}+e^{23}-e^{45}$, $\rho = e^{125}-e^{134}+4ae^{246}+4ae^{356}$, OB, $\|\|e_6\|\| = 4\|a\|$
$A_{6,37}^{0,0,1}$	$\omega = -e^{16}+e^{24}+e^{35}$, $\rho = -e^{125}+e^{134}+e^{236}-e^{456}$, ONB
$A_{6,38}^{0}$	$\omega = -e^{16}-e^{25}+e^{34}$, $\rho = -e^{124}-e^{135}+e^{236}-e^{456}$, ONB
$A_{6,39}^{a,-\frac{1}{3}(a+3)}$, $a \notin \{\frac{3}{2}, 0\}$	$\omega = -e^{14}+e^{23}+\frac{2a+3}{2a-3}e^{34}+(\frac{2}{3}a-1)e^{56}$, $\rho = (-\frac{2}{3}a+1)e^{126}-e^{135}+(-\frac{2}{3}a-1)e^{236}-e^{245}$ $+(\frac{2}{3}a-1)e^{346}$, $g = (e^1)^2+(e^2)^2+\frac{8a^2+18}{(2a-3)^2}(e^3)^2+(e^4)^2+(e^5)^2+\frac{1}{9}(2a-3)^2(e^6)^2+\frac{-4a-6}{2a-3}e^1\cdot e^3$
$A_{6,39}^{\frac{3}{2},-\frac{3}{2}}$	$\omega = -e^{12}-e^{34}+2e^{56}$, $\rho = e^{135}+2e^{146}+2e^{236}-e^{245}$
	$g = (e^1)^2+(e^2)^2+(e^3)^2+(e^4)^2+(e^5)^2+4(e^6)^2$
$A_{6,39}^{a,-1}$, $a \notin \{-1, 0\}$	$\omega = (1+a)e^{16}+e^{23}-e^{34}+e^{45}$, $\rho = -e^{124}+e^{135}+(1+a)e^{236}+(1+a)e^{256}+(1+a)e^{346}$
	$g = (e^1)^2+(e^2)^2+2(e^3)^2+(e^4)^2+(e^5)^2+(1+a)^2(e^6)^2+2e^3\cdot e^5$
$A_{6,40}^{-\frac{5}{4}}$	$\omega = -e^{13}-e^{14}+e^{23}-3e^{34}-\frac{1}{2}e^{56}$
	$\rho = \frac{1}{2}e^{126}-e^{135}+e^{136}+e^{145}+\frac{1}{2}e^{146}-\frac{3}{2}e^{236}-e^{245}-\frac{1}{2}e^{246}-\frac{1}{2}e^{346}$
	$g = 2(e^1)^2+(e^2)^2+10(e^3)^2+(e^4)^2+(e^5)^2+\frac{1}{2}(e^6)^2-2e^1\cdot e^2+6e^1\cdot e^3+e^5\cdot e^6$
$A_{6,41}^{-\frac{3}{4}}$	$\omega = -e^{14}+e^{23}-2e^{24}-\frac{2}{3}e^{34}-\frac{3}{2}e^{56}$
	$\rho = \frac{3}{2}e^{126}-e^{135}+\frac{1}{4}e^{136}+2e^{145}-\frac{1}{2}e^{236}-e^{245}+\frac{1}{2}e^{345}-\frac{3}{2}e^{346}$
	$g = (e^1)^2+(e^2)^2+\frac{41}{36}(e^3)^2+5(e^4)^2+(e^5)^2+\frac{9}{4}(e^6)^2+\frac{2}{3}e^1\cdot e^3+\frac{1}{3}e^2\cdot e^3-4e^3\cdot e^4$
$A_{6,42}^{-1}$	$\omega = 2e^{16}+e^{23}-e^{34}+e^{45}$, $\rho = -e^{124}+e^{135}+2e^{236}+2e^{256}+2e^{346}$
	$g = (e^1)^2+(e^2)^2+2(e^3)^2+(e^4)^2+(e^5)^2+4(e^6)^2+2e^3\cdot e^5$
$A_{6,47}^{\varepsilon,-3}$, $\varepsilon \in \{0, \pm 1\}$	$\omega = e^{12}+e^{23}+e^{34}+2e^{56}$, $\rho = -2e^{126}-e^{135}+2e^{146}+2e^{236}+e^{245}$,
	$g = (e^1)^2+2(e^2)^2+(e^3)^2+(e^4)^2+(e^5)^2+4(e^6)^2-2e^2\cdot e^4$
$A_{6,47}^{-1,-2}$	$\omega = e^{12}-e^{23}+e^{35}-e^{46}+7e^{56}$, $\rho = 2e^{126}-e^{134}+7e^{135}-e^{156}-7e^{236}-e^{245}-3e^{356}$
	$g = (e^1)^2+5(e^2)^2+10(e^3)^2+(e^4)^2+50(e^5)^2+(e^6)^2+6e^1\cdot e^3-4e^2\cdot e^5-14e^4\cdot e^5$
$A_{6,47}^{0,-2}$	$\omega = e^{12}-2e^{15}-e^{35}+e^{46}+e^{56}$, $\rho = 2e^{126}+e^{134}+e^{135}+e^{156}-e^{236}-e^{245}$,

Table 4: Indecomposable six-dimensional Lie algebras with five-dimensional nilradical admitting a half-flat SU(3)-structure

Lie algebra	Normalised half-flat SU(3)-structure
	$g=5(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+2(\mathrm{e}^5)^2+(\mathrm{e}^6)^2+4\mathrm{e}^1\cdot\mathrm{e}^3+2\mathrm{e}^4\cdot\mathrm{e}^5$
$A_{6,47}^{1,-2}$	$\omega=-\mathrm{e}^{15}+\mathrm{e}^{23}+\mathrm{e}^{46}$, $\rho=-\sqrt{2}\mathrm{e}^{126}-\frac{1}{2}\sqrt{2}\mathrm{e}^{134}+\frac{1}{2}\sqrt{2}\mathrm{e}^{245}+\sqrt{2}\mathrm{e}^{356}$, OB, $\lvert\lvert e_4\rvert\rvert=\frac{1}{\sqrt{2}}$, $\lvert\lvert e_6\rvert\rvert=\sqrt{2}$
$A_{6,51}^{\varepsilon}$, $\varepsilon=\pm1$	$\omega=\mathrm{e}^{16}+\mathrm{e}^{23}-\mathrm{e}^{34}+\mathrm{e}^{45}$, $\rho=-\mathrm{e}^{124}+\mathrm{e}^{135}+\mathrm{e}^{236}+\mathrm{e}^{256}+\mathrm{e}^{346}$
	$g=(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+2(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+(\mathrm{e}^6)^2+2\mathrm{e}^3\cdot\mathrm{e}^5$
$A_{6,54}^{0,-1}$	$\omega=\mathrm{e}^{12}+\mathrm{e}^{34}+\mathrm{e}^{56}$, $\rho=\mathrm{e}^{136}+\mathrm{e}^{145}+\mathrm{e}^{235}-\mathrm{e}^{246}$, ONB
$A_{6,54}^{\frac{1}{3},-\frac{4}{3}}$	$\omega=2\mathrm{e}^{16}+\mathrm{e}^{24}-\mathrm{e}^{35}$, $\rho=-\mathrm{e}^{125}-\frac{25}{16}\mathrm{e}^{134}+\frac{3}{2}\mathrm{e}^{146}+2\mathrm{e}^{236}+\frac{3}{4}\mathrm{e}^{345}+2\mathrm{e}^{456}$
	$g=\frac{25}{16}(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+\frac{25}{16}(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+4(\mathrm{e}^6)^2-\frac{3}{2}\mathrm{e}^1\cdot\mathrm{e}^5+3\mathrm{e}^3\cdot\mathrm{e}^6$
$A_{6,54}^{-\frac{3}{2},-2}$	$\omega=\mathrm{e}^{13}+\mathrm{e}^{25}+\mathrm{e}^{46}-\mathrm{e}^{56}$, $\rho=\mathrm{e}^{126}-\mathrm{e}^{145}-\mathrm{e}^{234}+\mathrm{e}^{235}-\mathrm{e}^{356}$,
	$g=(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+2(\mathrm{e}^5)^2+(\mathrm{e}^6)^2-2\mathrm{e}^4\cdot\mathrm{e}^5$
$A_{6,54}^{\mathrm{a},\mathrm{a}-1}$, $\mathrm{a}\neq0$	$\omega=-\mathrm{e}^{13}-\mathrm{e}^{24}-\mathrm{a}\mathrm{e}^{56}$, $\rho=-\mathrm{a}\mathrm{e}^{126}-\mathrm{e}^{145}+\mathrm{e}^{235}+\mathrm{a}\mathrm{e}^{346}$, OB, $\lvert\lvert e_6\rvert\rvert=\lvert\mathrm{a}\rvert$
$A_{6,56}^{1}$	$\omega=-\mathrm{e}^{13}-\mathrm{e}^{24}-\mathrm{e}^{56}$, $\rho=-\mathrm{e}^{126}-\mathrm{e}^{145}+\mathrm{e}^{235}+\mathrm{e}^{346}$, ONB
$A_{6,63}^{-1}$	$\omega=\mathrm{e}^{12}+\mathrm{e}^{34}+\mathrm{e}^{56}$, $\rho=\frac{5}{4}\mathrm{e}^{136}+\mathrm{e}^{145}-\frac{1}{2}\mathrm{e}^{146}+\mathrm{e}^{235}+\frac{1}{2}\mathrm{e}^{236}-\mathrm{e}^{246}$,
	$g=\frac{5}{4}(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+\frac{5}{4}(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+(\mathrm{e}^6)^2+\mathrm{e}^1\cdot\mathrm{e}^2-\mathrm{e}^3\cdot\mathrm{e}^4$
$A_{6,65}^{1,2}$	$\omega=-\mathrm{e}^{13}-\mathrm{e}^{24}-2\mathrm{e}^{56}$, $\rho=-2\mathrm{e}^{126}-\mathrm{e}^{145}+\mathrm{e}^{235}+2\mathrm{e}^{346}$, OB, $\lvert\lvert e_6\rvert\rvert=2$
$A_{6,70}^{\mathrm{a},\frac{\mathrm{a}}{2}}$, $\mathrm{a}\neq0$	$\omega=\mathrm{e}^{13}+\mathrm{e}^{24}+\mathrm{a}\mathrm{e}^{56}$, $\rho=-\mathrm{a}\mathrm{e}^{126}-\mathrm{e}^{145}+\mathrm{e}^{235}+\mathrm{a}\mathrm{e}^{346}$, OB, $\lvert\lvert e_6\rvert\rvert=\lvert\mathrm{a}\rvert$
$A_{6,70}^{0,0}$	$\omega=-\mathrm{e}^{12}+\mathrm{e}^{34}-\mathrm{e}^{56}$, $\rho=-\mathrm{e}^{136}+\mathrm{e}^{145}-\mathrm{e}^{235}-\mathrm{e}^{246}$, ONB
$A_{6,71}^{-1}$	$\omega=\mathrm{e}^{12}+\mathrm{e}^{25}-3\mathrm{e}^{36}-\mathrm{e}^{45}+18\mathrm{e}^{56}$, $\rho=3\mathrm{e}^{126}+\mathrm{e}^{135}-6\mathrm{e}^{146}-\mathrm{e}^{234}-6\mathrm{e}^{245}-3\mathrm{e}^{456}$,
	$g=2(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+(\mathrm{e}^3)^2+2(\mathrm{e}^4)^2+37(\mathrm{e}^5)^2+9(\mathrm{e}^6)^2-2\mathrm{e}^1\cdot\mathrm{e}^5-2\mathrm{e}^2\cdot\mathrm{e}^4-12\mathrm{e}^3\cdot\mathrm{e}^5$
$A_{6,71}^{-\frac{3}{2}}$	$\omega=\mathrm{e}^{12}-\mathrm{e}^{23}+\mathrm{e}^{34}+\mathrm{e}^{56}$, $\rho=-\mathrm{e}^{136}-\mathrm{e}^{145}-\mathrm{e}^{235}+\mathrm{e}^{246}-\mathrm{e}^{345}$,
	$g=(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+2(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+(\mathrm{e}^6)^2+2\mathrm{e}^1\cdot\mathrm{e}^3$
$A_{6,76}^{\mathrm{a}}$, $\mathrm{a}\neq-1$	$\omega=\mathrm{e}^{13}+(4\mathrm{a}+4)\mathrm{e}^{26}-\frac{3}{4}\mathrm{e}^{34}+\mathrm{e}^{45}$, $\rho=\mathrm{e}^{124}+(3\mathrm{a}+3)\mathrm{e}^{136}+(4\mathrm{a}+4)\mathrm{e}^{156}+\mathrm{e}^{235}+(4\mathrm{a}+4)\mathrm{e}^{346}$,
	$g=(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+\frac{25}{16}(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+16\,(\mathrm{a}+1)^2\,(\mathrm{e}^6)^2+\frac{3}{2}\mathrm{e}^3\cdot\mathrm{e}^5$
$A_{6,77}^{\varepsilon}$, $\varepsilon=\pm1$	$\omega=-\mathrm{e}^{13}-4\mathrm{e}^{26}+\frac{3}{4}\mathrm{e}^{34}-\mathrm{e}^{45}$, $\rho=\mathrm{e}^{124}+3\mathrm{e}^{136}+4\mathrm{e}^{156}+\mathrm{e}^{235}+4\mathrm{e}^{346}$
	$g=(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+\frac{25}{16}(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+16(\mathrm{e}^6)^2+\frac{3}{2}\mathrm{e}^3\cdot\mathrm{e}^5$
$A_{6,79}$	$\omega=\mathrm{e}^{13}+8\mathrm{e}^{26}-\frac{3}{4}\mathrm{e}^{34}+\mathrm{e}^{45}$, $\rho=\mathrm{e}^{124}+6\mathrm{e}^{136}+8\mathrm{e}^{156}+\mathrm{e}^{235}+8\mathrm{e}^{346}$
	$g=-(\mathrm{e}^1)^2-(\mathrm{e}^2)^2-\frac{25}{16}(\mathrm{e}^3)^2-(\mathrm{e}^4)^2-(\mathrm{e}^5)^2-64(\mathrm{e}^6)^2-\frac{3}{2}\mathrm{e}^3\cdot\mathrm{e}^5$
$B_{6,3}^{\mathrm{a}}$, $\mathrm{a}\neq0$	$\omega=8\mathrm{a}\mathrm{e}^{16}+\mathrm{e}^{23}-\frac{3}{4}\mathrm{e}^{34}+\mathrm{e}^{45}$, $\rho=-\mathrm{e}^{124}+\mathrm{e}^{135}+6\mathrm{a}\mathrm{e}^{236}+8\mathrm{a}\mathrm{e}^{256}+8\mathrm{a}\mathrm{e}^{346}$
	$g=(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+\frac{25}{16}(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+64\mathrm{a}^2(\mathrm{e}^6)^2+\frac{3}{2}\mathrm{e}^3\cdot\mathrm{e}^5$
$A_{6,82}^{0,1,\mathrm{b}}$, $0\leq\mathrm{b}\leq1$, $A_{6,84}$, $A_{6,89}^{0,1,\mathrm{b}}$, $\mathrm{b}\geq0$, and $A_{6,90}^{0,\mathrm{a}}$, $\mathrm{a}=\pm1$	$\omega=\mathrm{e}^{16}+\mathrm{e}^{23}+\mathrm{e}^{45}$, $\rho=-\mathrm{e}^{124}+\mathrm{e}^{135}+\mathrm{e}^{256}+\mathrm{e}^{346}$, ONB
$A_{6,82}^{1,5,9}$	$\omega=\mathrm{e}^{14}-3\mathrm{e}^{24}-12\mathrm{e}^{26}-\mathrm{e}^{35}$, $\rho=\mathrm{e}^{125}-12\mathrm{e}^{136}-\mathrm{e}^{234}+36\mathrm{e}^{236}-12\mathrm{e}^{456}$
	$g=(\mathrm{e}^1)^2+10(\mathrm{e}^2)^2+(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+144(\mathrm{e}^6)^2-6\mathrm{e}^1\cdot\mathrm{e}^2$
$A_{6,88}^{0,1,\mathrm{b}}$, $\mathrm{b}>0$, $A_{6,88}^{0,0,1}$	$\omega=-\mathrm{e}^{16}-\mathrm{e}^{23}+\mathrm{e}^{45}$, $\rho=-\mathrm{e}^{125}+\mathrm{e}^{134}+\mathrm{e}^{246}+\mathrm{e}^{356}$, ONB
$B_{6,6}^{\mathrm{a}}$, $-1\leq\mathrm{a}\leq1$, $\mathrm{a}\neq0$	$\omega=\mathrm{e}^{16}+\mathrm{e}^{23}+\mathrm{e}^{45}$, $\rho=-\mathrm{e}^{124}+\mathrm{e}^{135}+\mathrm{e}^{256}+\mathrm{e}^{346}$, ONB
$A_{6,94}^{-1}$	$\omega=\mathrm{e}^{14}+\mathrm{e}^{15}-3\mathrm{e}^{16}-3\mathrm{e}^{26}+\mathrm{e}^{34}$, $\rho=\mathrm{e}^{123}-3\mathrm{e}^{146}+3\mathrm{e}^{156}-\mathrm{e}^{245}+3\mathrm{e}^{246}+3\mathrm{e}^{356}$,
	$g=2(\mathrm{e}^1)^2+(\mathrm{e}^2)^2+(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+(\mathrm{e}^5)^2+18(\mathrm{e}^6)^2+2\mathrm{e}^1\cdot\mathrm{e}^3-6\mathrm{e}^5\cdot\mathrm{e}^6$
$A_{6,94}^{-\frac{5}{3}}$	$\omega=\mathrm{e}^{12}-\frac{1}{7}\mathrm{e}^{23}+\frac{7}{2}\mathrm{e}^{25}+\mathrm{e}^{34}-\frac{7}{3}\mathrm{e}^{36}+\frac{7}{3}\mathrm{e}^{56}$,
	$\rho=\frac{1}{3}\mathrm{e}^{126}-\mathrm{e}^{135}+\frac{7}{3}\mathrm{e}^{146}+\mathrm{e}^{234}+\frac{7}{3}\mathrm{e}^{236}+\mathrm{e}^{245}+\frac{7}{6}\mathrm{e}^{256}+\frac{49}{6}\mathrm{e}^{456}$,
	$g=(\mathrm{e}^1)^2+\frac{50}{49}(\mathrm{e}^2)^2+2(\mathrm{e}^3)^2+(\mathrm{e}^4)^2+\frac{53}{4}(\mathrm{e}^5)^2+\frac{49}{9}(\mathrm{e}^6)^2-7\mathrm{e}^1\cdot\mathrm{e}^5+\frac{2}{7}\mathrm{e}^2\cdot\mathrm{e}^4-2\mathrm{e}^3\cdot\mathrm{e}^5$

Table 4: Indecomposable six-dimensional Lie algebras with five-dimensional nilradical admitting a half-flat SU(3)-structure

Lie algebra	Normalised half-flat SU(3)-structure
$A_{6,94}^{-2}$	$\omega = \mathrm{e}^{14} - \mathrm{e}^{16} + 2\mathrm{e}^{24} - \mathrm{e}^{26} + \mathrm{e}^{35}$, $\rho = -\mathrm{e}^{125} + \mathrm{e}^{134} + \mathrm{e}^{236} - \mathrm{e}^{456}$,
	$g = (\mathrm{e}^1)^2 + 2(\mathrm{e}^2)^2 + (\mathrm{e}^3)^2 + 2(\mathrm{e}^4)^2 + (\mathrm{e}^5)^2 + (\mathrm{e}^6)^2 + 2\mathrm{e}^1 \cdot \mathrm{e}^2 - 2\mathrm{e}^4 \cdot \mathrm{e}^6$
$A_{6,94}^{-3}$	$\omega = -\mathrm{e}^{14} - \mathrm{e}^{25} - \frac{3}{2}\mathrm{e}^{34} - 3\mathrm{e}^{36}$, $\rho = 3\mathrm{e}^{126} - \mathrm{e}^{135} + \mathrm{e}^{234} - \frac{9}{2}\mathrm{e}^{236} - 3\mathrm{e}^{456}$,
	$g = (\mathrm{e}^1)^2 + (\mathrm{e}^2)^2 + \frac{13}{4}(\mathrm{e}^3)^2 + (\mathrm{e}^4)^2 + (\mathrm{e}^5)^2 + 9(\mathrm{e}^6)^2 + 3\mathrm{e}^1 \cdot \mathrm{e}^3$
$A_{6,99}$	$\omega = \frac{4}{3}\mathrm{e}^{12} + \mathrm{e}^{14} + \frac{42}{19}\mathrm{e}^{23} + \mathrm{e}^{25} - \frac{63}{38}\mathrm{e}^{34} - 9\mathrm{e}^{36} - \frac{729}{38}\mathrm{e}^{56}$,
	$\rho = -9\mathrm{e}^{126} - \mathrm{e}^{135} + \mathrm{e}^{234} - \frac{567}{38}\mathrm{e}^{236} - \frac{81}{38}\mathrm{e}^{245} + 12\mathrm{e}^{256} + 9\mathrm{e}^{456}$,
	$g = (\mathrm{e}^1)^2 + \frac{25}{9}(\mathrm{e}^2)^2 + \frac{5413}{1444}(\mathrm{e}^3)^2 + (\mathrm{e}^4)^2 + \frac{8005}{1444}(\mathrm{e}^5)^2 + 81(\mathrm{e}^6)^2 - \frac{63}{19}\mathrm{e}^1 \cdot \mathrm{e}^3 + \frac{8}{3}\mathrm{e}^2 \cdot \mathrm{e}^4 + \frac{81}{19}\mathrm{e}^3 \cdot \mathrm{e}^5$

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