DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 80/112
November 1980

LATTICE GAUGE THEORIES, CONFINEMENT, STRINGS AND ALL THAT

by

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DESY 80/112
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Lattice gauge theories, confinement, strings and all that

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Talk given at DESY on November 12th, 1980

In this talk I would like to give an overview over some developments in lattice gauge theory, which might be of some interest for experimental physicists. In particular, I shall try to convince you that lattice gauge theory is not only a play-ground for theorists, but is able to produce numerical results for some non-perturbative quantities. And, of course, I would like to tell you about some work, which has been done here in Hamburg.

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1. Why the lattice

As all of you know perturbation theory has been applied successfully to high energy resp. small distance processes in QCD [1]. This is related to the property of asymptotic freedom [2]. For large momentum transfer Q^2 in a scattering process the effective "running" coupling is small according to

$$g^2(Q^2) = \frac{1}{\beta_0 \ln\frac{Q^2}{\Lambda^2}} - \frac{\beta_1}{\beta_0^3} \frac{\ln\ln\frac{Q^2}{\Lambda^2}}{\ln^2\frac{Q^2}{\Lambda^2}} + \mathcal{O}\left(\frac{1}{\ln^3\frac{Q^2}{\Lambda^2}}\right) \qquad (1)$$

where β_0 and β_1 are the first two coefficients in the Callan-Symanzik β-function.

$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 + O(g^7) \qquad (2)$$

In perturbation theory the in- and outgoing particles are considered to be free coloured quarks and gluons. On the other hand all known physical hadrons are colour-neutral and are believed to be bound-states of quarks. This is the confinement problem. Confinement means that the forces between quarks are such that free quarks are not allowed to exist. This is a strong coupling problem and perturbation theory does not say anything about it.

Confinement in the full QCD is a difficult problem which has not yet been solved. Therefore we shall simplify things in the following by ignoring dynamical quarks. We consider pure gauge theory and add only static quarks by hand. If the potential V(L) between a static quark anti-quark pair at distance L grows linearly with L without bound, we shall speak of confinement, because in this case they feel an attractive force even for large distances.

Perturbation theory gives a $\frac{1}{L}$ potential for small distances L. To calculate V(L) for large L is a non-perturbative strong coupling problem. Therefore we need a method which is suitable for strong coupling. Wilson's lattice gauge theory [3] provides us with such a method.

2. Lattice gauge theory

There are two approaches to define a lattice gauge theory. In the Hamiltonian formalism [4] space is replaced by a threedimensional lattice, while time remains continuous. In this formalism you have a Hamiltonian and a corresponding Schrödinger equation for many degrees of freedom.

The other approach is Euclidean lattice gauge theory [3]. Space and time are on equal footing and are replaced by a four-dimensional lattice with spacing a . Furthermore one performs an analytic continuation of the time variable to imaginary times. This is a well known procedure in quantum field theory [5]. It is a technical trick which makes life easier. Of course, all final results are to be translated back to real times for physical interpretation. In the following we consider the Euclidean approach.

I shall shortly describe what the gauge field is on a lattice without going into details and without giving any motivations [6]. The gauge field variables U(b) are attached to the links b of the lattice. They take values in the

gauge group G = SU(3), SU(2), U(1), etc. The interaction is defined by a coupling between the U(b), such that every four variables U(b) are coupled, if their links form an elementary square, a so-called plaquette. See Fig. 1. In a naive continuum limit you may write

$$U(b) = \mathbb{1} + iagA_{\mu}(x) + O(a^2) \quad \text{for } b = \underset{x}{\bullet}\!\xrightarrow{\mu}\!\bullet \qquad (3)$$

where $A_{\mu}(x)$ is the usual gauge field. Then the lattice action becomes the usual gauge field action

$$S = \frac{1}{4} \int d^4x \quad F^a_{\mu\nu} F^a_{\mu\nu} \qquad (4)$$

The Euclidean trick which I mentioned above leads to an important observation: the equations of lattice gauge theory are formally the same as for a system of statistical mechanics, where g^2 plays the role of temperature T.

$$\text{formally:} \qquad g^2 \mathrel{\hat{=}} T \text{ (temperature)} \qquad (5)$$

Now we are in good luck. Statistical mechanics provides us with some powerful methods to study lattice gauge theory [7,8,9]. These are essentially

a) rigorous inequalities
b) high temperature expansions $\longleftrightarrow$ strong coupling expansions
c) low temperature expansions for some models
d) Monte Carlo studies

Like for ferromagnets, fluids or other systems in statistical mechanics different phases are possible for lattice gauge theory and the phase structure can be studied by the above methods.

3. Confinement

In particular let us look at the behaviour at strong coupling. Consider pure lattice gauge theory and put a static quark anti-quark pair on the lattice. With the help of high temperature expansions or correlation inequalities [10] one can prove: For large g static quarks are confined by an approximately linearly rising potential

$$V(L) \approx \alpha \cdot L \quad , \quad L \text{ large} \tag{6}$$

The high temperature expansion also tells us what happens in the confining phase. The chromo-electric flux lines between the quarks form a string (fig. 2) which carries a constant amount α of energy per unit length, thus leading to the potential (6). α is called the string tension.

At weak coupling other phases are possible. For example in the case of gauge group U(1) the weak coupling phase does not confine [11].

Because we are ultimately interested in the continuum limit of the lattice theory the question arises: what has the continuum limit to do with the confinement phase?

4. The continuum limit and renormalization

Consider SU(3) or SU(2) lattice gauge theory with some lattice spacing a and some coupling constant g. The theory then has some physical length scale l, e.g. the correlation length ξ or $\alpha^{-\frac{1}{2}}$. Measured in units of the lattice spacing the length scale can only depend on g. Therefore

$$l = a \cdot \hat{l}(g) \tag{7}$$

If we want to describe the same physics by a finer lattice, we have to increase $\hat{l}(g)$ simultaneously as a gets smaller, in order to get the same physical length scale l. [1)] (Fig. 3). This is achieved by changing g at the same time, so that g is a function g(a) for fixed l. In the continuum limit finally (if it exists) $\hat{l}(g)$ has to go to infinity. A point $g = g_c$, where this happens, is called a critical point or second order phase transition in statistical mechanics.

All this is a kind of renormalization as you know it. In renormalization theory one first introduces a cutoff in order to make everything finite. In our case the lattice provides us with a non-perturbative gauge-invariant cutoff. Then the cutoff is removed, while some physical quantities are kept fixed. To keep them fixed the bare coupling constant g has to be renormalized. In our case this is the readjustment of g if a goes to zero.

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1) Of course, we have to decide, which scale should be kept fixed in this procedure. Other physical quantities might change somewhat.

Renormalization group equations tell us how g should depend on a if it is small. Due to asymptotic freedom g should also go to zero according to

$$g^2(a) = \frac{1}{\beta_0 \ln \frac{1}{a^2 \Lambda_L^2}} - \frac{\beta_1}{\beta_0^3} \frac{\ln \ln \frac{1}{a^2 \Lambda_L^2}}{\ln^2 \frac{1}{a^2 \Lambda_L^2}} + \dots \qquad (8)$$

where Λ_L is a renormalization scale parameter different from Λ. (Compare Equ. 1). Therefore, if a continuum limit exists, which is confining and has asymptotic freedom at the same time, it is expected to be a $g \to 0$ limit [8]. In other words: It is hoped that the confining phase of SU(3) or SU(2) lattice gauge theory extends down to $g = 0$ and that $g = 0$ is a critical point.

5. The string tension

As length scale 1 we may take $\alpha^{-\frac{1}{2}}$ where α is the string tension [12]. If our assumptions about the continuum limit are correct, then as a consequence of asymptotic freedom (Equ. 8) the string tension measured in units of $\frac{1}{a^2}$ should go to zero exponentially

$$a^2 \alpha = \hat{\alpha}(g) \approx C \, (\beta_0 g^2)^{-\frac{\beta_1}{\beta_0^2}} \exp\left(-\frac{1}{\beta_0 g^2}\right) \qquad (9)$$

as $g \to 0$

where
$$C = \frac{\alpha}{\Lambda_L^2} \qquad (10)$$

is a number, which can be computed in principle.

This problem has been studied by Creutz with the Monte Carlo method [13]. His celebrated results (fig. 4) indicate that indeed the behaviour at weak coupling follows the asymptotic freedom prediction (9) and they allow an estimate of the constant C. Furthermore, they show that a sudden changeover from strong coupling to weak coupling behaviour takes place at a certain value of g (≈ 1 for SU(3)). This sudden changeover is interpreted in Mack's theory of confinement [14].

The string tension can also be studied by the method of strong coupling expansions [12,15,16], which are expansions in powers of $\frac{1}{g^2}$ with a finite radius of convergence. In fig. 4 you see the Monte Carlo data of Creutz and the strong coupling curve up to order g^{-24} for SU(2). One observes that they agree still in the changeover region. For smaller values of g the strong coupling expansion is no longer reliable and goes through zero soon. For SU(3) the situation is similar [12,16].

But strong coupling expansions also allow an estimate of the ratio between Λ_L and $\sqrt{\alpha}$, because this depends essentially only on the location of the changeover region. Using a conversion factor by Hasenfratz [17], the result can be expressed in terms of the more conventional scale parameter Λ^{mom}. One gets for SU(3) (with subjective errors)

$$\Lambda^{mom} = (0.31 \pm 0.1) \sqrt{\alpha} \qquad (11a)$$

from Euclidean calculations [16] and

$$\Lambda^{mom} = (0.40 \pm 0.1) \sqrt{\alpha} \qquad (11b)$$

in the Hamiltonian framework [18]. Creutz's Monte Carlo method yields [13]

$$\Lambda^{mom} = (0.42 \pm 0.1)\sqrt{\alpha} \qquad (11c)$$

Inserting a value of 450 MeV for $\sqrt{\alpha}$ [19] we get a Λ^{mom} parameter between 140 MeV and 190 MeV for the pure gauge theory. It is not known how these numbers change if dynamical quarks are included.

Nevertheless, you see that lattice gauge theory provides us with methods that allow to calculate non-perturbative quantities, which are of physical interest.

6. Roughening

Recently it has been observed [20] that near the changeover region at a certain value of $g = g_R$ a so-called roughening transition takes place. I shall say some words about roughening in the following. For every value of g the confining string between static quarks fluctuates in the transversal directions. This produces an effective string width δ. For large couplings $g > g_R$ the width $\delta(L)$ of a string of length L approaches some limit $\delta(\infty)$ if the quark anti-quark separation goes to infinity. On the other hand if $g < g_R$ one finds that $\delta(L)$ diverges (presumably logarithmically $\sim \ln L$) as L goes to infinity. This is called the roughening of the string. The roughening transition has the following consequences.

i) $\hat{\alpha}(g)$ is expected to have a non-analyticity at g_R. Although the non-analyticity is believed to be very weak (of infinite order), it represents a barrier for strong-coupling expansions for the string tension. Therefore doubt is shed on any numerical estimates which I mentioned in the previous

section. On the other hand it seems to me that the singularity is so weak that it does not much affect these estimates.

ii) The string fluctuations produce a $\frac{1}{L}$ potential in addition to the confining potential [21]. The strength of this Coulomb potential is of the correct order of magnitude extracted from J/Ψ spectrum [19].

7. The glue-ball mass

The pure gauge field should have particle-like colourless excitations which are called glue-balls. In a pioneering work Kogut, Sinclair and Susskind [22] have calculated the mass ratios of glue-balls using strong-coupling expansions in the Hamiltonian framework. One would like to know also the absolute value of the lowest glue-ball mass m_g. For this purpose I have calculated the Euclidean strong-coupling expansion for m_g up to order g^{-12}. See fig.5. Using the method described in sect. 5 one gets the following results.

$$m_g = (2.4 \pm 1.1)\sqrt{\alpha} \quad \text{for } SU(2) \qquad (12\ a)$$

(Compare Berg's Monte Carlo data [23].)

$$m_g = (2.9 \pm 1.2)\sqrt{\alpha} \quad \text{for } SU(3) \qquad (12\ b)$$

With $\sqrt{\alpha} = 450$ MeV this yields for SU(3)

$$m_g \approx (1.3 \pm .5)\ \text{GeV} \qquad (13)$$

which is remarkably close to some speculations [24].

8. Conclusion

I hope, at least some of you are convinced that lattice gauge theory is a promising approach to understand non-perturbative phenomena in gauge field theory.

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Figure captions.

1. Part of a lattice in 3 dimensions with links b and plaquettes p .

2. Chromo-electric string between static quarks .

3. Decreasing the lattice spacing a .

4. String tension α times a^2 as a function of g^{-2} for SU(2) lattice gauge theory in 4 dimensions. The solid line represents the result of 12th order strong coupling expansions [15]. The dots are results of Creutz's Monte Carlo calculation [13] . The dashed lines are the lowest order strong-coupling curve and the weak coupling asymptotic freedom fit according to equ. 9 .

5. Glue-ball mass times lattice spacing a as a function of g^{-2} for SU(2) and SU(3) lattice gauge theory in 4 dimensions. The solid lines show the strong-coupling expansions up to order g^{-12} . The dashed lines are the lowest order strong-coupling curves and the weak coupling fits for SU(2) and SU(3) respectively.

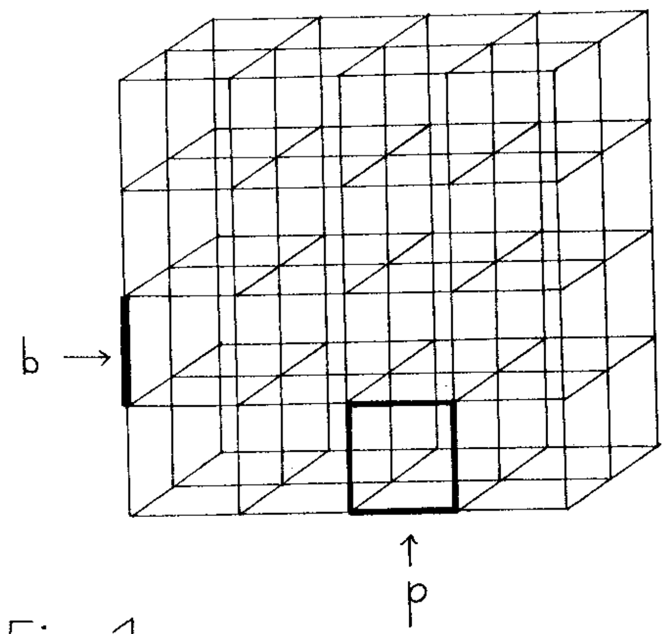

Fig. 1

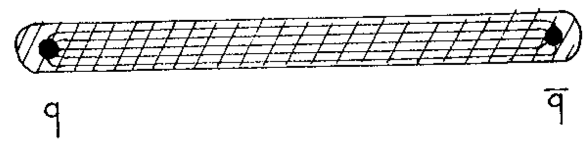

Fig. 2

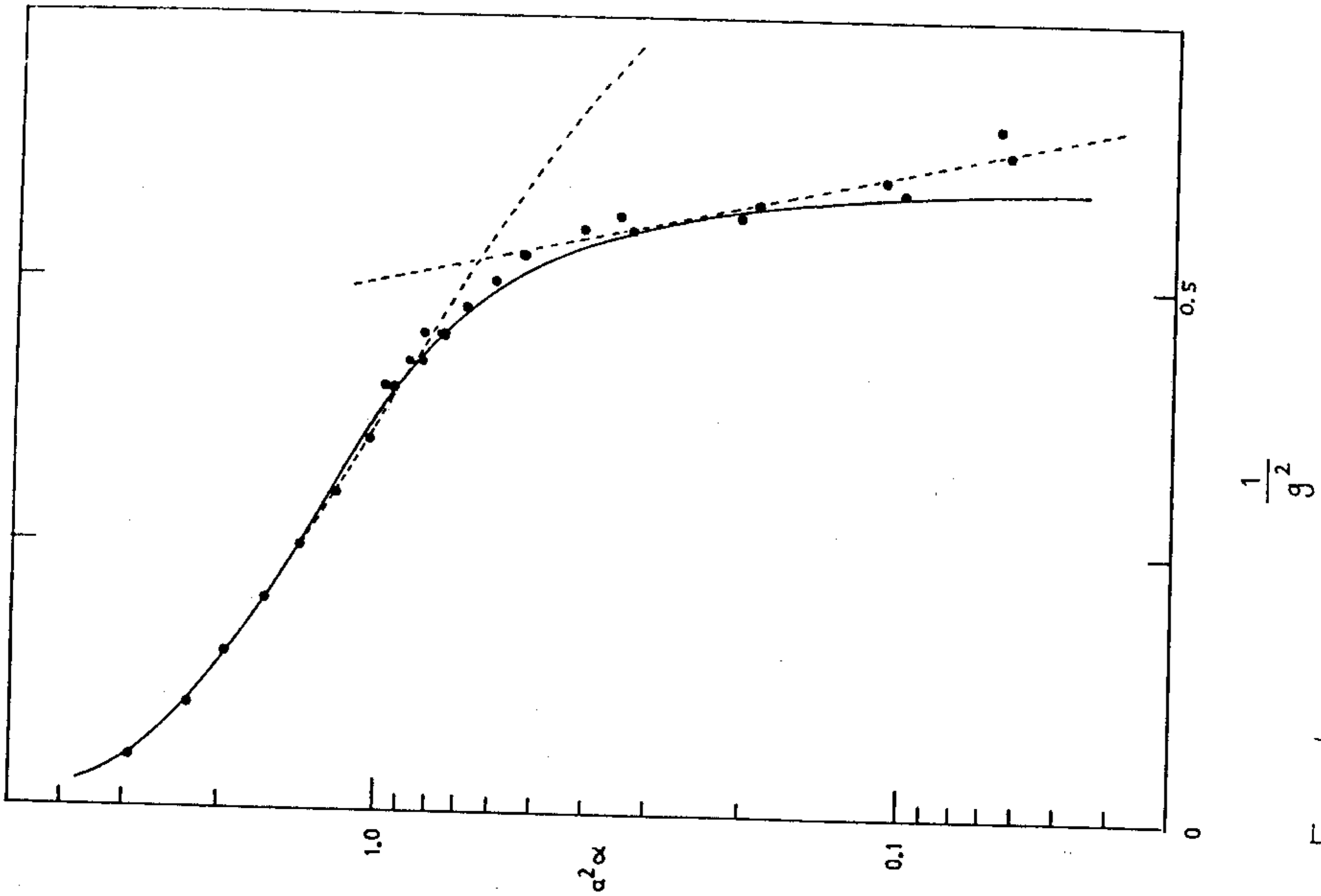

Fig. 4

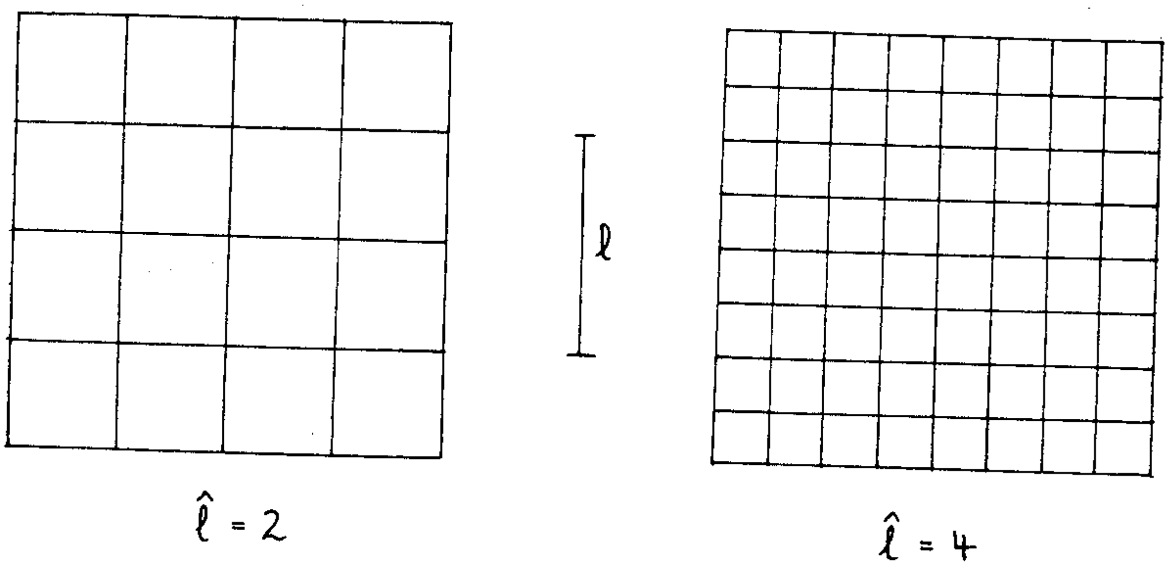

Fig. 3

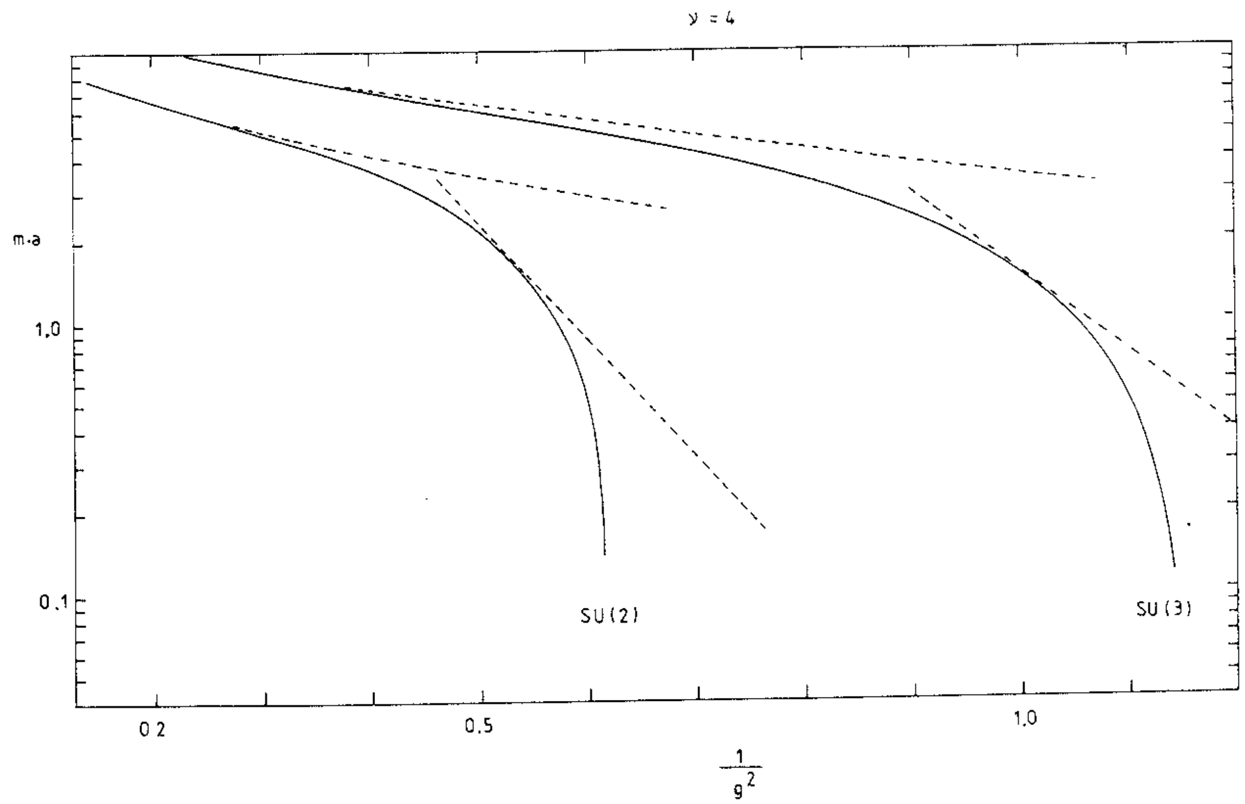

Fig. 5