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A CLUSTER ALGORITHM FOR JET STUDIES

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Abstract

A procedure is described which determines the number of jets in hadronic final states by means of a cluster algorithm. In addition it yields a measurement of the energy and the direction of each jet. The properties of this method are studied using Monte Carlo simulations of different types of e^+e^- - annihilation final states. It is shown that in the case of 3-jet events direct comparison with the underlying parton structure can be made. Possible further applications of this method are discussed.

1. Introduction

The phenomenology of high energy reactions of hadrons can to a large extent be described in terms of models in which all hadronic matter is composed of point-like constituents, called partons /1/. In particular the elementary process of large energy transfer collisions is assumed to be a two-body parton parton (e.g. quark quark) interaction. Partons have not been directly observed so far. However, in our present understanding they fragment into ordinary hadrons. If the energy of the partons is high enough, on average their direction is preserved by the hadrons.

Qualitatively this has been confirmed by the observation of 2-jet-structures in e^+e^- - annihilation /2/. The observation of jets also in hard scattering proton proton collisions /3/ and in deep inelastic lepton scattering processes /4/ supports the picture of hadronic jets as a universal phenomenon.

Deviations from the 2-jet picture are expected to appear at higher energies. In the framework of Quantum Chromodynamics (QCD) strongly accelerated partons may radiate additional partons (gluons) /5/ which again form hadronic jets. This process of gluon bremsstrahlung has recently been observed in e^+e^-- annihilation /6/. Another source of multijet final states are very heavy hadrons like quark-antiquark resonances decaying into gluons /7/, which subsequently fragment into hadrons. First evidence for the latter type of process has come from studies of the decay of the $\Upsilon(9.46)$ resonance produced in e^+e^-- annihilation /8/.

To describe and to identify jets in a hadronic event an increasing number of methods have been developed. These methods determine the direction of the jets (jet axes) and define scalar measures to describe the topology of an event, e.g. the 'sphericity' method and the 'thrust' and 'triplicity' methods. The former procedure diagonalizes the momentum tensor /9/. Assuming 2-jet events, the jet axis is determined by the principal axis of the tensor which has the smallest eigenvalue. The eigenvalues are used to define a scalar measure, the sphericity, which describes the topology of the event. Thrust /10,7/ and triplicity /11/ methods maximize the longitudinal momenta with respect to the jet axes by combinatorial algorithms. The scalar measures are thrust in the 2-jet case and triplicity in the 3-jet case. In contrast with these methods one can avoid determining axes by describing the shape of the event with an expansion of the momentum volume in spherical harmonics /12/.

Studying events which do not contain a priori a fixed number of jets, the former methods are disadvantageous because they assume two or three jet structure thereby imposing strong kinematical constraints, i.e. collinearity for 2-jet events and planarity for 3-jet events. Thus it would be beneficial to have a procedure which determines the number of jets event by event rather than fixing this number by the method.

We describe a simple and pragmatic algorithm to find an arbitrary number of jets in a given event. Furthermore this procedure is able to determine for each jet its energy and direction. It collects neighbouring particles (charged and neutrals) into clusters. The algorithm reflects the experimental fact that hadrons of the same jet cluster together. Recently methods of jet analysis have been published /13/ or sketched out /14/, which are also able to determine an arbitrary number of jets per event.

A detailed description of our algorithm is given in section 2, tests of the method are presented in section 3 and section 4 shows to what extent the underlying parton configuration can be reconstructed in the 3 parton case. The results of section 3 and 4 are obtained by Monte Carlo studies. In the last section we discuss possible physics analyses which could be done by means of this method.

2. Definition of a cluster algorithm

In the framework of the quark parton model the characteristics of the fragmentation are given by the prescription

(1) the transverse momentum $p_\perp$ of the hadrons with respect to the original parton direction which is strongly limited,

(2) the distribution of the longitudinal momentum fraction $z = p_{\parallel} / p_0$ carried by a hadron which is independent of the parton energy p_0.

These imply that the hadrons appear in a cone of opening angle $\langle p_\perp \rangle / (\langle z \rangle \cdot p_0)$. Thus if $\langle p_\perp \rangle \ll p_0$ on average the hadrons preserve the direction and energy of the original parton. Consequences for the topological structure of jets are obvious:

(i) the structure becomes evident using only the direction of the particles, and

(ii) the angle between a particle and its nearest neighbour in the same jet is small.

Using the standard Monte Carlo model /15/ for fragmentation (ii) can be verified easily. Fig. 1 shows the distribution of the minimum angle between two hadrons of the same jet at center of mass energies E_{cm} = 13 GeV (Fig. 1a) and at 30 GeV (Fig. 1b). For both energies these distributions peak at small angles ($\sim 10^0$). Thus the underlying idea of the algorithm proposed is to link hadrons to clusters which have at least one neighbour with a similar direction.

We use a two step algorithm to define jets as clusters with the aim of minimising the influence of fluctuations in the fragmentation processes. In the first step we define preclusters. A precluster is a set of particles, where each particle fulfills a vicinity relation which depends only on its direction. From the momentum sum of the particles a direction of the precluster is derived.

The second step treats these preclusters like particles in the first step and links them to clusters. Thus a cluster is a set of preclusters. Finally a cluster is called a jet if it fulfils some criteria.

More precisely we use the following two step algorithm in which

(1) all particles - charged and neutrals - are used.
The particle i is taken to have momentum $\vec{p}_i$, direction $\vec{n}_i = \vec{p}_i / |\vec{p}_i|$ and energy E_i.

In the first step preclusters are defined such that

(2) each particle is a member of only one precluster,

(3) any two particles i, k ($i \neq k$) belong to the same precluster if $\vec{n}_i \cdot \vec{n}_k > \cos\alpha$ for a predefined value of α.

The direction of the precluster D_i is defined by

$$\vec{n}_{D_i} = \sum_{k \in D_i} \vec{p}_k \Big/ \sum_{k \in D_i} |\vec{p}_k| \quad .$$

In the second step the preclusters are concatenated to clusters

(4) each precluster is member of exactly one cluster,

(5) any two preclusters D_i, D_k ($i \neq k$) belong to the same cluster if $\vec{n}_{Di} \cdot \vec{n}_{Dk} > \cos\beta$ for a predefined value of $\beta \geq \alpha$.

The energy and the direction of a cluster C_i are defined by

$$E_{C_i} = \sum_{k \in C_i} E_k \qquad \text{and} \qquad \vec{n}_{C_i} = \sum_{k \in C_i} \vec{p}_k \Big/ \sum_{k \in C_i} |\vec{p}_k| \ .$$

The momentum of a cluster is defined assuming zero mass : $\vec{P}_{Ci} = \vec{n}_{Ci} \cdot E_{Ci}$.

Finally some cuts are made to define a jet:

(6) the multiplicity of clusters n_c is defined by the minimal number of clusters which fulfill

$$\sum_{i=1}^{n_c} E_{C_i} > E_{tot} \cdot (1 - \varepsilon)$$

for a predefined value of ε, where E_{tot} is the energy sum of all particles. The clusters are accepted if they belong to the set of n_c most energetic clusters;

(7) each accepted cluster is called a jet if $E_{Ci} > E_{th}$ for a predefined value of E_{th}

All jets found by this procedure are ordered according to their energy so that

$$E_{J_i} \geq E_{J_{i+1}}$$

The number of jets found in an event is called n_j.

Technically this algorithm is used in a recursive procedure, which works as follows:

Starting with an arbitrary particle, e.g. $i = 1$, which forms an elementary precluster D_i^o, any particle k is added to this precluster if the vicinity relation (3) for 0 and k holds. The particles now in the precluster are, for example, $\{1,2,\ldots n_o\}$. Then this procedure is repeated looking for particles k which fulfil the vicinity relation (3) with respect to particle 2. This procedure is repeated as long as particles k' are found, which fulfil the vicinity condition with any of the particles which are already members in the precluster. At

this point the precluster D_1 is complete. Any particle which is not a member of D_1 can be the elementary precluster D_2^o, which itself can grow as described above to the complete precluster D_2. After repeating this procedure all particles are assigned to preclusters, which may contain only one single particle. The number of preclusters found depends on the choice of the parameter α.

The second step works like the first one with particles and their directions being replaced by preclusters and their directions as defined above. In this way all particles are assigned to clusters, which also may contain only one particle. This second step is necessary to reduce the influence of the parton fragmentation. It may happen that the angle between two preclusters is small, thus fulfilling (5), but the vicinity relation (3) does not hold for any pair of particles i, k where i and k belong to different preclusters. At this step for the first time the momenta of the particles are taken into account. This is done in a collective way via the definition of the direction of a precluster in the algorithm and not by weighting each particle separately.

The cut (6) allows a fraction less than ε of the total energy not to be assigned to clusters according to the picture that always a few particles appear far away from the direction of the original parton. Thus normally $\varepsilon \ll 1$. The final cut requiring a minimal jet energy E_{th} is necessary to fulfil the condition of jet formation, i.e. $E_{th} \gg \langle p_\perp \rangle$. It is evident that the energy sum of the n_j observed jets can be smaller than $(1-\varepsilon) \cdot E_{tot}$.

It turns out that this scheme of jet identification is able to minimize the influence of strong fluctuations in the fragmentation process without any cut in the particle momenta. Defining jets by means of the algorithm described above the four parameters (α, β, ε, E_{th}) must be optimized to the appropriate problem, e.g. analysing a specific n_j-class. Once a set of parameters is chosen, this algorithm has no ambiguity and the assignment of a particle to a jet is unambiguously determined.

3. Test of the cluster method

Since it was our aim to develop a method which is especially applicable to analyse e^+e^- - annihilation final states at PETRA energies, we tested the proposed algorithm using Monte Carlo simulations of four different final states:

1.) a 2-jet model ($q\bar{q}$) /15/ including c- and b-quark production. The values for the fragmentation parameters were fixed to the values of ref. 15; for the c- and b-quark fragmentation standard assumptions were made /16/.

2.) a gluon bremsstrahlungs model ($q\bar{q}g$) taking into account the first order term of the QCD matrix element /17/. In this model the gluons fragment like quarks in model (1). To avoid singularities an energy dependent thrust cut has to be made, namely $T < 0.95$ at 30 GeV and $T < 0.82$ at 13 GeV cm energy (thrust is in this context the normalized energy x_1 of the most energetic parton (p_1): $T = x_1 = 2E_{p1}/E_{cm}$).

3.) a model to describe the decay of a hypothetical heavy $q\bar{q}$-resonance into three gluons (ggg). This model was developed to describe the Υ (9.46) decay using a matrix element similar to that of orthopositronium decay /18/. The particle multiplicity fitting the data at the Υ-resonance /8/ was scaled logarithmically to a resonance mass of 30 GeV.

4.) an isotropic multiparticle phase space model (PS) at 30 GeV cm energy which provides an extreme model with no dynamics allowing studies of the decay of two heavy objects which are generated almost at rest.

Radiative corrections of the initial state /19/, which can influence the event topologies, are included for models (1) and (2). In our algorithm the definition of a jet depends on four parameters, namely α, β, ε, E_{th}. The two 'collecting angles' α and β are the most sensitive parameters of this procedure, whereas the fraction of energy ε allowed to be outside of the clusters and the minimal energy of a jet are less sensitive but depend on the physical process to be studied.

To obtain useful values for α and β we studied $q\bar{q}g$ events at $E_{cm} = 30$ GeV. First we used all events and equal angles $\alpha = \beta$. The probability to find an event (which is by definition a 3-jet event) to be reconstructed as 3-jet event is plotted in Fig. 2 as a function of α (triangles). It is obvious that for large values of α different jets tend to be merged into one cluster and that for small α one jet splits into several clusters. Thus in both cases the probability to properly reconstruct the 3-jet structure becomes small. In the range $20^0 < \alpha < 40^0$ the probability, however, is fairly constant.

In order to select a clean sample of 3 jet candidates we required that exactly three jets contain at least 90% of the total energy. With this reduced sample the probability of finding 3-jets is also plotted in Fig. 2. Applying the same collecting angle for clusters and preclusters as done before (open circles), the maximum probability is obtained only in a small region around 35^0 while the probability drops very fast with smaller values of α. The full points show the efficiency variation for different values of α while the value of β is kept constant (45^0). Finally for all our further analyses we choose $\alpha = 30^0$, $\beta = 45^0$.

The distribution of the normalized cluster energy is shown in Fig. 3a and 3b for $q\bar{q}$ and $q\bar{q}g$ events at a cm energy of 30 GeV. Both distributions show beside their expected peaks at the average cluster energy an additional enhancement at very small energies. This peak is most probably produced by low energy particles which are so far away from the jet axes that they could not be attributed to any of them. To suppress these low energy clusters we introduce condition (6) which allows for a certain fraction ε of the energy to be outside the jets. If ε is chosen too large, the clusters of low energetic partons, especially in the three jet case, (Fig. 3b) are cut out. So we fixed $\varepsilon = 0.1$, which has a small effect on the cluster of low energy partons for $q\bar{q}g$ events but suppresses nicely the low energy clusters, as indicated in Fig. 3a and 3b (hatched area for accepted clusters).

To fix the fourth parameter of this procedure, E_{th}, one should take into account the fact that in e^+e^--annihilation the 2-jet structure becomes visible for cm-energies above $\sim$5 GeV /2/. Below these energies the jets are so broad that no structure could be observed. From the MC studies we find $E_{th} = 2$ GeV to be a reasonable choice.

Fig. 4a - d show the distributions of the number of jets per event for the four different processes mentioned above. It should be pointed out that in the context of the procedure described here a jet is defined as a certain amount of energy which is clearly separated from other energy streams in an event. Therefore it cannot be expected that the number of jets equals always the number of partons.

The $q\bar{q}$ events produce a clear two jet structure (Fig. 4a), and three jet configurations are faked with a reasonably small probability. Only the $b\bar{b}$ events, indicated by the shaded area, produce about equal fractions of 2- and 3-jet events. This effect is a consequence of the relatively high quark mass

($M_b \cong 5$ GeV) and the assumed fragmentation of the b quarks /16/.

The multiplicity of jets n_j for the $q\bar{q}g$ events (Fig. 4b) and ggg events (Fig.4c) clearly favours three jets. About 30% of the events of both models produce a two jet structure: either the two backward jets are merged to one broad jet, or the third cluster contains only a small fraction of the parton energy which is not sufficient to be counted as a jet (cf. condition (7)). It should be mentioned that $q\bar{q}g$ events have been generated up to $x_1 = 0.95$ and that for the ggg events the maximum value is $x_1 = 1$.

In principle the collecting angles α and β are functions of the cm energy. However, keeping α and β constant, it turns out that the results shown in Fig. 4a and b do not change significantly for a lower cm energy of 13 GeV. Furthermore reasonable changes of the parton fragmentation parameters /15/ do not affect these results.

For the isotropic phase space model the multiplicity of jets is centred at ~ 5. This distribution is completely different from that obtained for three parton events. Less than 10% of the phase space events show up as a three jet structure (Fig. 4d).

4. Reconstruction of the partons

In the following the three parton processes $q\bar{q}g$ and ggg are studied in more detail to investigate to what extent the properties of the underlying partons can be reconstructed by this method. To do so we used only events generated with an evident three-jet structure, i.e. we used events satisfying the following criteria:

(i) the energy of the least energetic (third) parton is larger than E_{th}

(ii) he angle between any two partons is larger than β.

In Fig. 5 the distributions of the angle θ_k between the found jet and the related parton are plotted for the most energetic parton (k = 1) and least energetic parton (k = 3) for the $q\bar{q}g$ events at cm-energies of 13 and 30 GeV. We identify a jet i with the parton k if the angle θ_k between them is minimal for all k = 1,2,3. We find that even the reconstruction of the third parton is fairly good, although at 13 GeV the average energy of this parton is only 2.5 GeV.

The ratio of the reconstructed energy to the generated parton energy is shown in Fig. 6 for both $q\bar{q}g$ (a-c) and ggg (d-f) events at 30 GeV cm energy. For the gluon bremsstrahlung process the third jet shows a weaker correlation with the related parton energy (Fig. 6c), while this correlation is stronger for the three gluon decay (Fig. 6f). In the latter process events, in which the energy of all three partons is equal, are produced more often and, in addition, the distribution of the angle between the second and third parton is different. Thus in the three gluon decay the complete separation of the third jet is more probable than in the $q\bar{q}g$ process. All distributions peak at ~ 0.9 due to particles not attributed to jets (cf. conditions (6) and (7) of the cluster algorithm in ch. 2).

To measure x_1, the normalized energy of the most energetic parton, we use the thrust method /10,7/ for the three jet momenta, i.e. we maximize their longitudinal momenta by means of the thrust method

$$x_1 = \max \{ \sum_{i=1}^{3} |P''_{Ji}| \,/\, \sum_{i=1}^{3} |\vec{P}_{Ji}| \}$$

where P''_{Ji} denotes the momentum component of the jet J_i with respect to the thrust axis. Since the momenta of the 3 jets are not constrained by energy and momentum conservation (and since real data is not always completely measured) this definition of x_1 turns out to be the best approximation of the normalized energy of the most energetic parton. In Fig. 7 we compare measured and generated x_1 for the $q\bar{q}g$ and the ggg processes. The cluster method has a tendency to overestimate the x_1-values. The reason for this behaviour is the partial merging of the two backward jets, which results in a smaller angle between the related two jets and a shrinking of the transverse momenta with respect to the leading jet. Thus a larger x_1 is measured. At $E_{cm} = 30$ GeV the cluster method is able to measure x_1 up to T = 0.97.

Also the reconstruction of the event plane of the 3 partons for both $q\bar{q}g$ and ggg events is possible. We define the normal of the event plane by

$$\vec{N} = \frac{(\vec{P}_{J_2} \times \vec{P}_{J_3}) + (\vec{P}_{J_3} \times \vec{P}_{J_1})}{|(\vec{P}_{J_2} \times \vec{P}_{J_3}) + (\vec{P}_{J_3} \times \vec{P}_{J_1})|}$$

i.e. we average the two planes defined by $(\vec{P}_{J_2} \times \vec{P}_{J_3})$ and by $(\vec{P}_{J_3} \times \vec{P}_{J_1})$

which are not necessarily identical since the three jets are not constrained to a plane. Although the correlation of $\vec{p}_{J_3}$ to the generated third parton is the weakest one as shown above, this jet, however, is the most relevant in this definition, because by this third jet the planar structure of an event becomes evident independent of fluctuations caused by the fragmentation. Fig. 8 shows the distribution of the angle χ between the generated and measured normals of the event plane. For $q\bar{q}g$ events at E_{cm} = 13 GeV (Fig. 8a) and at E_{cm} = 30 GeV (Fig. 8b) a reasonably good reconstruction of this plane is possible.

So far in our measurement of parton properties we have assumed an ideal detector. To study possible influences of a real detector, we used the Monte Carlo simulation program of the PLUTO detector as an example. The PLUTO detector /20/ is able to measure particle directions better than their momenta. It turns out that the distributions of Fig.5,7 and 8 are slightly broadened, but the detector effects do not prevent a clean jet identification as described above. The probability for the full reconstruction of a generated 3-jet event (Fig. 2) becomes slightly smaller due to the limited acceptance of the detector.

5. Applications of the cluster method

The proposed algorithm is able to determine the number of jets in an event, so it is obvious that it adds a new piece of information. It should be noted that this method does not require a priori energy and momentum conservation in the event to be analyzed. Thus, no special corrections for radiation and acceptance losses have to be applied before submitting the event to this algorithm. Since the solid angle for collecting particles in a cluster is small ($\sim$7% of 4π using the parameters as defined in sec. 3) this method can be used for events with a fairly large number of jets, probably up to 10.

The algorithm is very simple and can be translated into any computer code easily. The execution of the program is rather fast, the cpu-time to analyse an event of n particles is proportional to n^2, whereas combinatorial methods become very slow for high multiplicities.

It is possible to define a scalar measure T_n which describes the quality of the event to fit to the found n-jet structure like thrust for n = 2 or triplicity for n = 3:

$$T_n = \{ \sum_{k=1}^{n} (\sum_{\ell \in J_k} |p''_\ell|) \} / \sum_{i=1}^{n_0} |\vec{p}_i|$$

p''_ℓ is the longitudinal momentum component of the particle ℓ assigned to the jet J_k with respect to its direction $\vec{n}_{Jk}$. The summation in the nominator runs first over all particles of the jet J_k and then over all jets k, the summation of the denominator runs over all particles n_0. $T_n \to 0$ for an isotropic event with infinite particle multiplicity and $T_n = 1$ for an event in which all particle directions coincide with the directions of the jets they are assigned to.

It has been shown in the previous section that by means of this cluster method studies of 3-jet final states are feasible. This analysis can be carried out without applying strong cuts, e.g. in x_1, which especially in the $q\bar{q}g$ model would reduce the number of events considerably. Recently this method has been applied by the PLUTO collaboration to study high energy e^+e^--annihilation events /21/. The authors claim that this method can be used up to relative high x_1-values and their results are rather insensitive against changes of the parametes of the parton fragmentation model.

In future experiments with the next generation of colliding beam machines ($p\bar{p}$, LEP, HERA ...) multijet structures are expected from several sources (e.g. $e^+e^- \to W^+W^- \to$ 4 jets). An analysis using this cluster algorithm could be very useful for the study of these multijet events. Probably some of the conditions will have to be modified and fitted to the specific experiment, but we believe that this algorithm can always be used provided the number of jets is not too large.

6. Summary

We have developed a rigorous algorithm for finding jets in hadronic multiparticle final states. Using a simple vicinity relation this algorithm determines the number of jets in an event and measures their energy and momenta. To a certain extent the jets can be identified with the original parton (assuming zero mass kinematics). This has been demonstrated in 3-jet final states as an example. Measuring the number of jets in an event adds a new piece of information to that obtained by the well known and established methods. Reconstructing the underlying parton configuration allows, e.g., direct comparison with theoretical predictions in the framework of QCD, which does not take into account the parton fragmentation to real hadrons.

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Figure Captions

1. Distribution of the minimal angle θ_i for $q\bar{q}$ model at (a) E_{cm} = 13 GeV and (b) E_{cm} = 30 GeV. θ_i is the angle between particle i and its nearest neighbour: $\theta_i = \min \measuredangle (\vec{p}_i, \vec{p}_j)$ for all $j \neq i$.

2. Probability to reconstruct a $q\bar{q}g$ event as a 3-jet event at E_{cm} = 30 GeV as a function of the 'collecting angle' α (Triangles: $\alpha = \beta$; circles : selected 3-jet events, open circles: $\alpha = \beta$, closed circles: $\beta = 45^0$).

3. Relative cluster energy E_{Ci}/E_{cm} (E_{cm} = 30 GeV) for (a) $q\bar{q}$ and (b) $q\bar{q}g$ models. The shaded area represents the accepted clusters (fulfilling condition 6 in the text).

4. Distribution of numbers of jets per event (n_j) at E_{cm} = 30 GeV
   a) $q\bar{q}$ model, the shaded area corresponds to the contribution of $b\bar{b}$-events;
   b) $q\bar{q}g$ model, the shaded area corresponds to events with $x_1 < 0.9$;
   c) ggg and d) isotropic phase space model.

5. Distribution of the angle θ_i between jet i and related parton (i = 1 most energetic jet, i = 3 least energetic jet) for E_{cm} = 13 GeV (a,b) and E_{cm} = 30 GeV (c,d).

6. Distribution of the reconstructed jet energy (in units of the energy of the related parton). i = 1 denotes the most energetic, i = 3 the least energetic jet for ggg (a-c) and $q\bar{q}g$ (d-f) model at E_{cm} = 30 GeV.

7. Correlation between generated and measured normalized energy of the most energetic parton (x_1) for (a) $q\bar{q}g$ and (b) ggg model.

8. Distribution of the angle χ between the normals of the generated and reconstructed event plane at E_{cm} = 13 GeV (a) and 30 GeV (b) for $q\bar{q}g$ model.

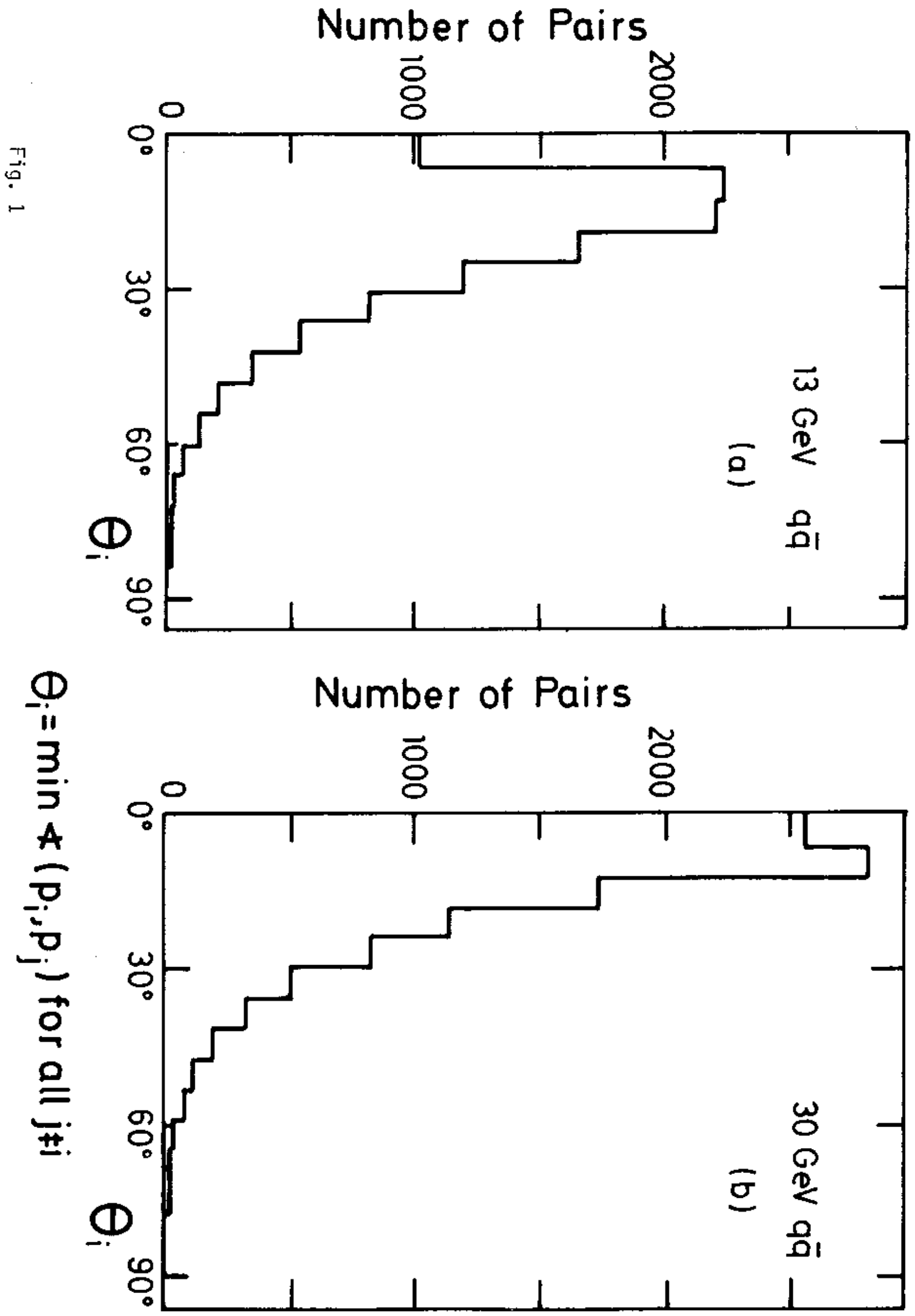


Fig. 1

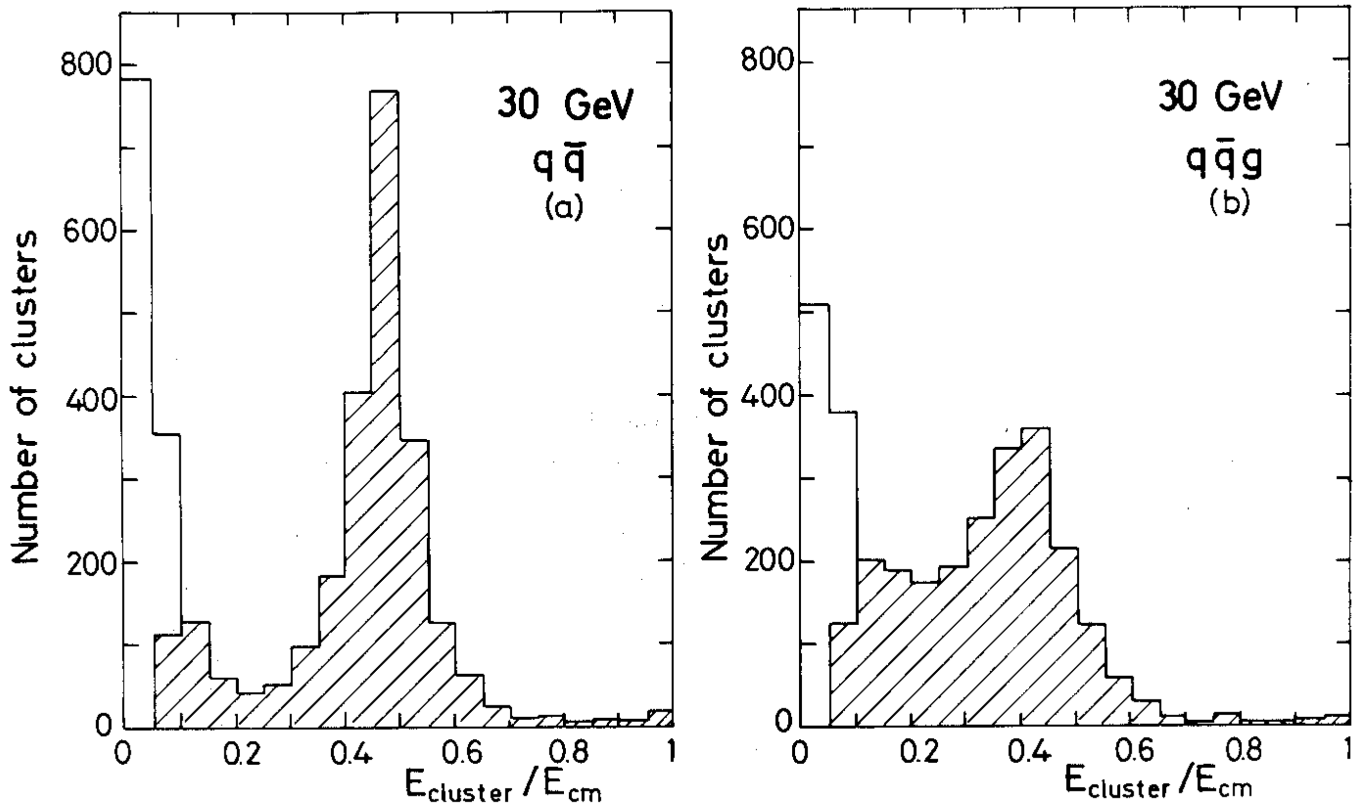

Number of clusters
30 GeV
$q\bar{q}$
(a)
$E_{cluster}/E_{cm}$
30 GeV
$q\bar{q}g$
(b)

Fig. 3

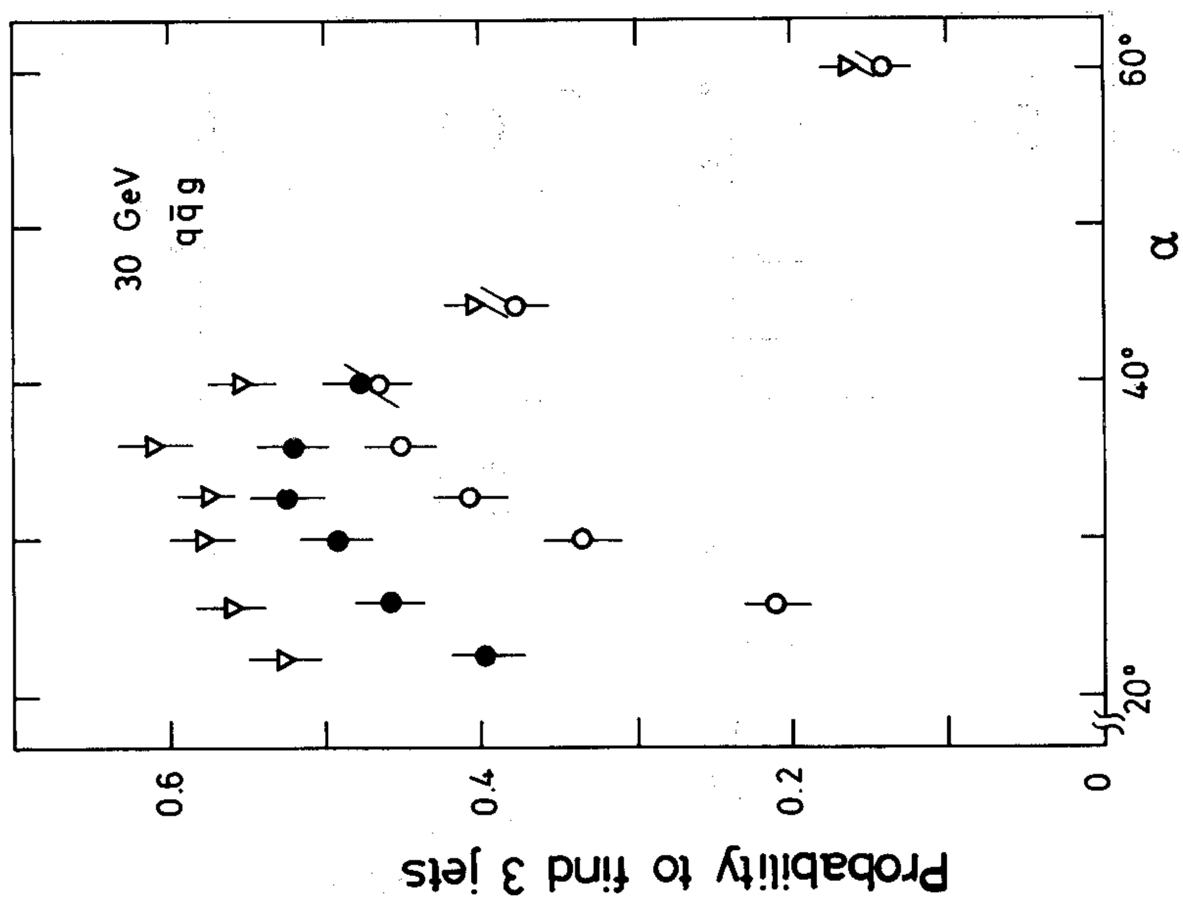
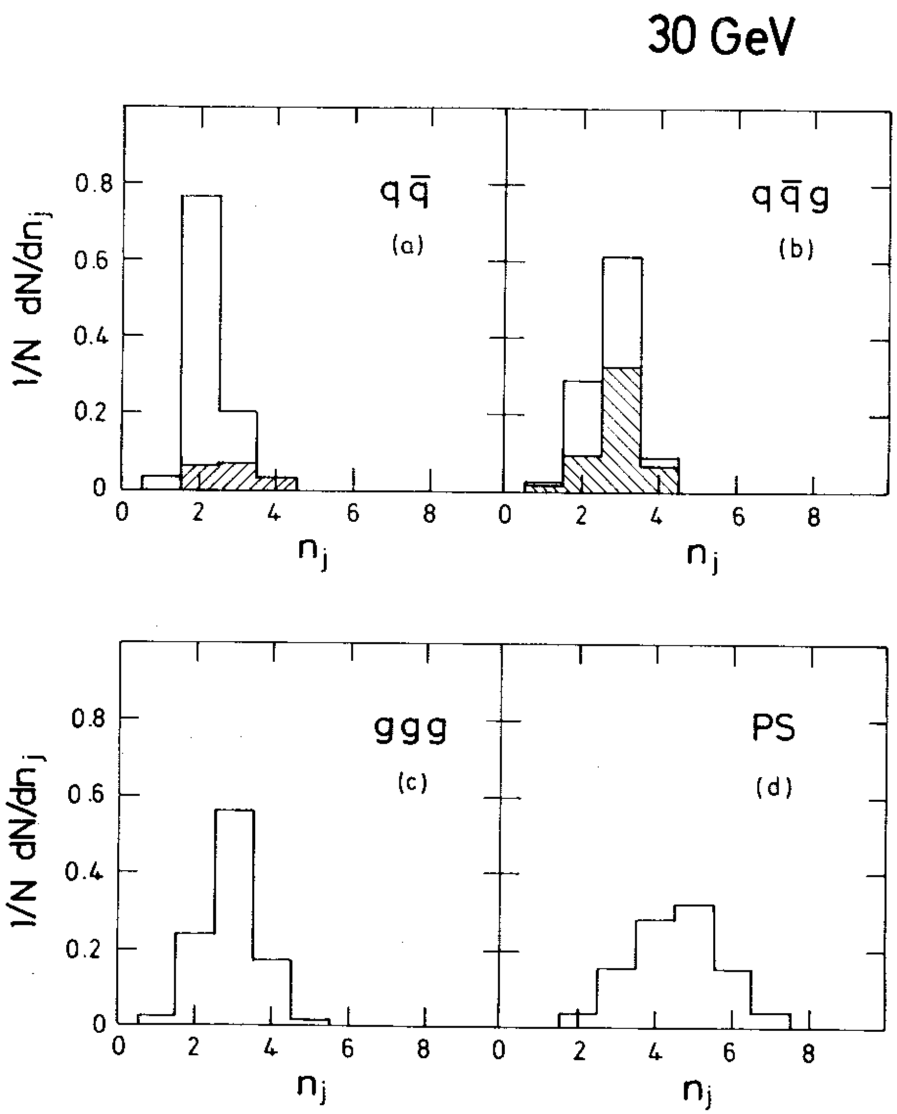
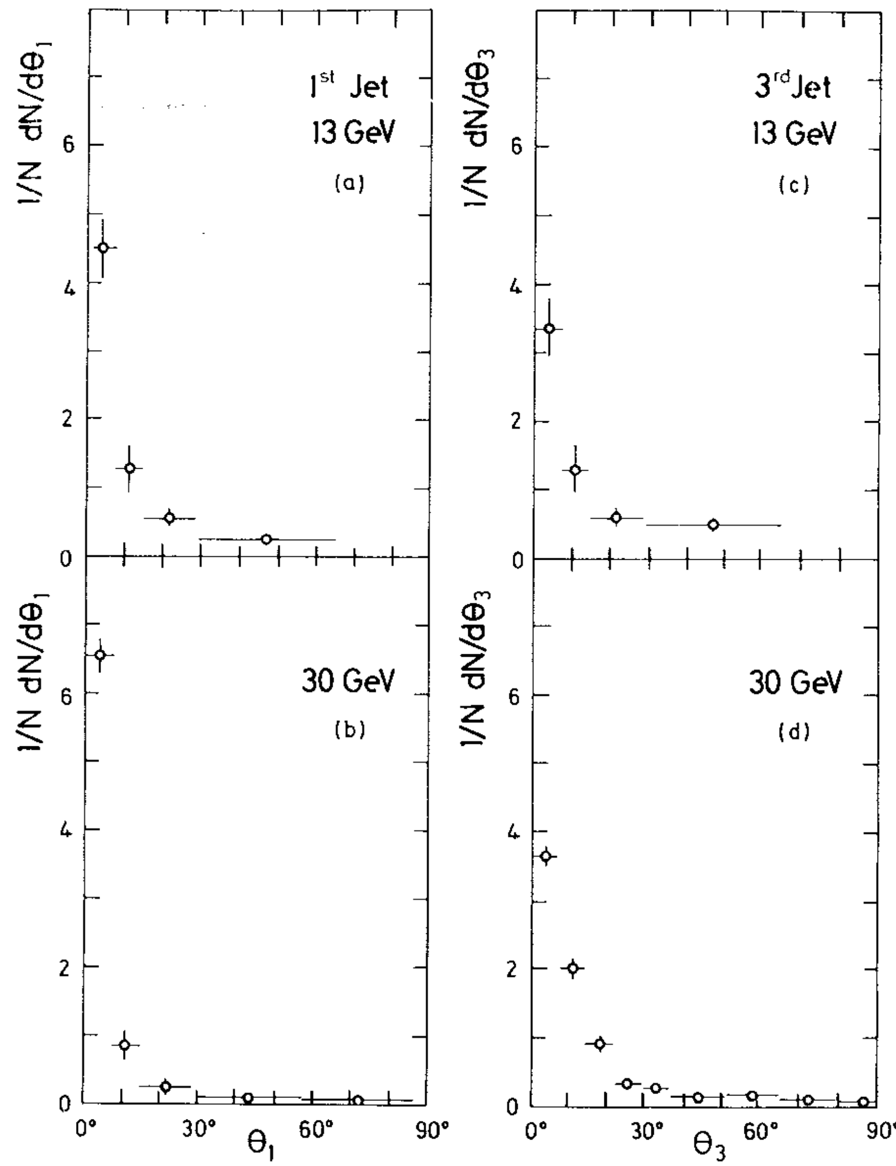

30 GeV
$q\bar{q}g$
Probability to find 3 jets
α

Fig. 2



30 GeV
1/N dN/dnj
q q̄
(a)
q q̄ g
(b)
g g g
(c)
PS
(d)
nj

Fig. 4



1/N dN/dθ1
1st Jet
13 GeV
(a)
30 GeV
(b)
1/N dN/dθ3
3rd Jet
13 GeV
(c)
30 GeV
(d)
θ1
θ3

Fig. 5

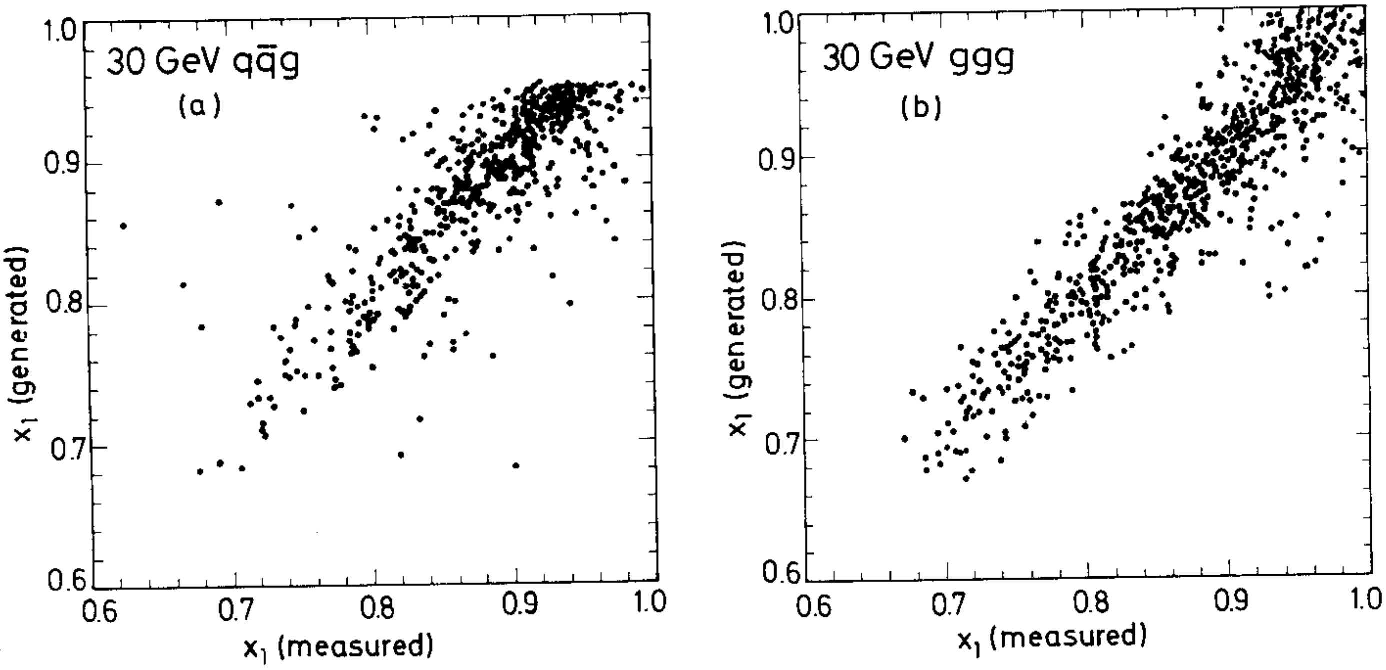

30 GeV $q\bar{q}g$
(a)
30 GeV ggg
(b)
x_1 (generated)
x_1 (measured)
0.6
0.7
0.8
0.9
1.0

Fig. 7

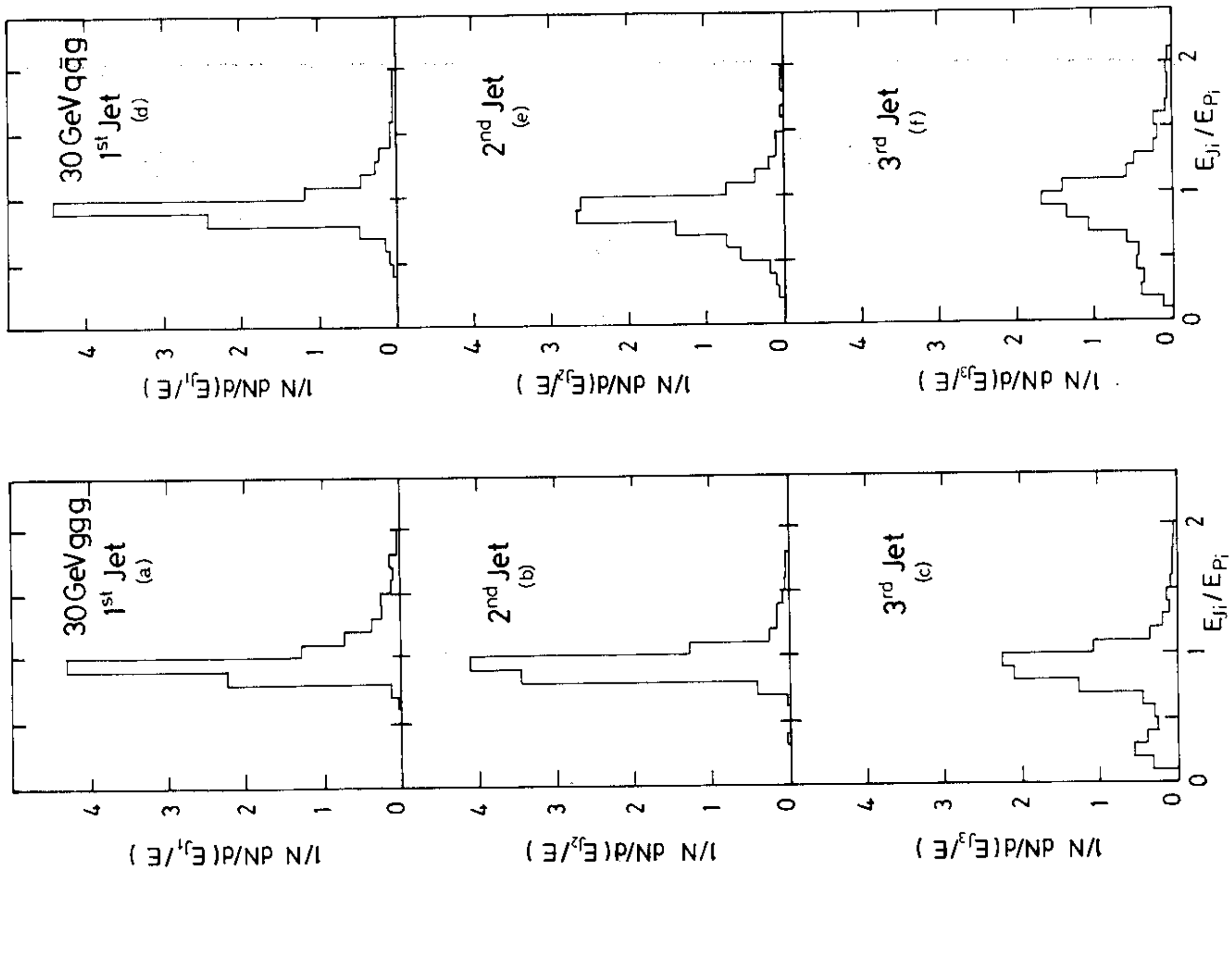

30 GeV $q\bar{q}g$
1^{st} Jet
(d)
2^{nd} Jet
(e)
3^{rd} Jet
(f)
30 GeV ggg
1^{st} Jet
(a)
2^{nd} Jet
(b)
3^{rd} Jet
(c)
E_{ji} / E_{P_i}
$1/N\ dN/d(E_{j1}/E)$
$1/N\ dN/d(E_{j2}/E)$
$1/N\ dN/d(E_{j3}/E)$

Fig. 6

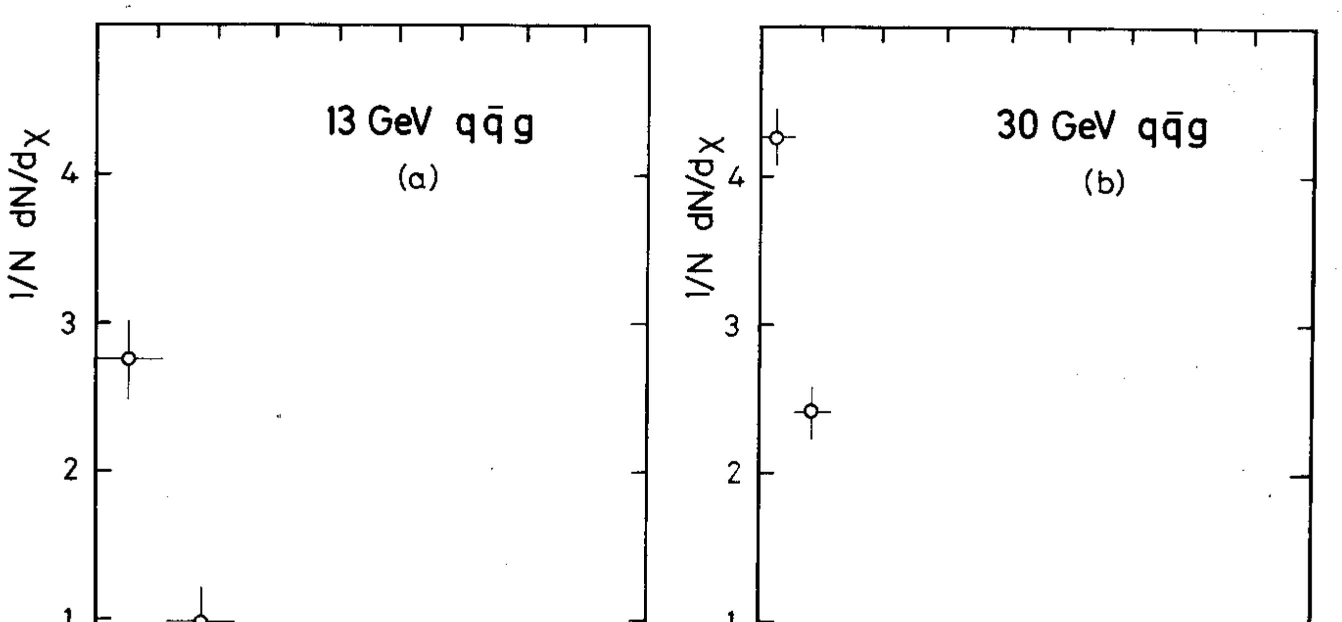


Fig. 8