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FINAL STATE POLARIZATION EFFECTS IN $e^-e^+ \to \gamma, Z \to q\bar{q}g$

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Abstract:
We discuss the polarization of the quark and/or antiquark in $e^-e^+ \to \gamma, Z \to q\bar{q}g$, and of the hadrons (baryons or vector mesons) belonging to the most energetic jet initiated either by the quark or the antiquark.

1. Introduction

According to our present understanding of fundamental interactions, neutral current effects are expected to show up at high energies in electron-positron annihilation. Neutral currents have so far been established only in the space-like region of lepton-lepton and lepton-nucleon scattering [1] and the data are consistent with the Glashow-Weinberg-Salam model [2] for $\sin^2\theta_W = 0.23$. This model then predicts sizable electromagnetic - weak interference effects also in the time-like region accessible in e^+e^--annihilation [3] and Drell-Yan processes [4]. It should be possible to measure these effects with the next generation of accelerators and thus to learn more about the neutral current structure. For this purpose it will be helpful to have experiments which use polarized beams or which measure final-state polarization. In fact, such experiments will be required for an unambiguous determination of the weak coupling constants.

Here we discuss final state polarization effects in e^+e^--annihilation into hadrons in the framework of electroweak gauge theories with a single neutral vector boson (Z) in addition to the photon (γ). The hadrons are assumed to be produced, not directly, but via the fragmentation of quarks and gluons. The polarization of the quark (q) and antiquark ($\bar{q}$) in $e^-e^+ \to \gamma, Z \to q\bar{q}$ and the way this polarization is transmitted to final state baryons or vector mesons has been discussed recently by Nieves [5], Augustin and Renard [6], and Bartl, Fraas and Majerotto [7]. Here we consider the same topic for the case when an additional gluon (g) is emitted (3-jet events):

$$e^-(p_-,\vec{\xi}^-) + e^+(p_+,\vec{\xi}^+) \to \gamma, Z \to q(p_1,h_1) + \bar{q}(p_2,h_2) + g(p_3) . \qquad (1)$$

In sect.2 we give the cross-section for arbitrarily polarized beams ($\vec{\xi}^\pm$) and longitudinal polarization $h_1(h_2)$ of $q(\bar{q})$. The calculation is done in lowest order QCD with quark masses neglected and for arbitrary vector and axial vector couplings of the Z^o. The polarization of the quark (or antiquark) is then given for the case when q (or $\bar{q}$) determines the thrust T. For $T \to 1$ we recover the 2-jet results. In sect.3 we calculate the density matrix of an inclusively produced hadron H belonging to the most energetic jet initiated either by the polarized quark or antiquark. The fragmentation

process is described by spin-dependent fragmentation functions [8]. The general formulas of this section are then applied to baryons in sect. 4 and to vector mesons in sect.5. Under plausible assumptions regarding the fragmentation process, the polarization of these particles turns out to be quite large in the standard model [2]. In principle, it should be possible to measure at least the polarization of the baryons via their parity-violating decays. Finally, we collect some useful formulas in Appendix A and give, in Appendix B, the general expression for the polarization vector of the quark. This expression is relevant when massive quarks, higher order QCD corrections or the radiation of scalar gluons are considered.

2. Cross sections and polarization of the quarks.

To study process (1) for polarized quarks and/or antiquarks we use the method proposed in [9]*). There the angular and energy (or thrust) dependence, as well as the dependence on beam polarization and on the properties and couplings of the Z^0 have been worked out explicitly in terms of the hadron tensors associated with the final state. The process (1) for unpolarized final state particles has been chosen to illustrate the method. For polarized quarks and/or antiquarks we have to reconsider the hadron tensors only, but not the way they enter the various distributions. This is an advantage of the method used in [9], as it avoids redoing each time the lengthy calculations implied by Υ- and Z^0-exchange. A method based on the helicity formalism will be presented in [10]. Some of the results for 1Υ-exchange can be found in the standard references [11].

In the case of polarized quarks and antiquarks the hadron tensors denoted now by $\mathcal{H}_r^{\mu\nu}$ are given in the high energy limit (quark masses neglected) by

$$\mathcal{H}_1^{\mu\nu} = \frac{1}{4}\left[(1-h_1h_2)H_1^{\mu\nu} + (h_1-h_2)H_4^{\mu\nu}\right],$$
$$\mathcal{H}_2^{\mu\nu} = \mathcal{H}_3^{\mu\nu} = 0,$$
$$\mathcal{H}_4^{\mu\nu} = \frac{1}{4}\left[(1-h_1h_2)H_4^{\mu\nu} + (h_1-h_2)H_1^{\mu\nu}\right], \qquad (2)$$

where $H_1^{\mu\nu}$ and $H_4^{\mu\nu}$ are the hadron tensors for the unpolarized case, as given in I(4.8)-(4.9), $h_i = \vec{\xi}_i \cdot \hat{p}_i$ is the longitudinal component ("helicity") of the rest frame polarization vector $\vec{\xi}_i$ of the quark (i=1) and antiquark (i=2). The transverse parts $\vec{\xi}_{i\perp}$ have been disregarded, since they contribute only to quark-antiquark spin correlations not considered here. The gluon spin and all SU(3)-color indices have been summed over. The expressions (2) simply follow from I(4.4)-(4.7) if the polarization of the quark and antiquark is taken into account by making the following substitutions in I(4.4):

*) In the following we refer to this paper as I for notations, conventions and details of the calculation.

$$(\not p_1+m_f) \rightarrow \frac{1}{2}(\not p_1+m_f)(1+\gamma_5 \not s_1) \approx \frac{1}{2}\not p_1(1-h_1\gamma_5),$$
$$(\not p_2-m_f) \rightarrow \frac{1}{2}(\not p_2-m_f)(1+\gamma_5 \not s_2) \approx \frac{1}{2}\not p_2(1+h_2\gamma_5), \qquad (3)$$

where the last expressions apply to the case $m_f = 0$ with $\vec{\xi}_{i\perp}$ disregarded (compare I(A.4)). In this case we have, for example,

$$\mathcal{H}_{JJ}^{\mu\nu} = -16\pi\alpha_s g_{\rho\sigma} \frac{1}{4} \mathrm{Tr}\left[O^{\rho\mu}\not p_2(1+h_2\gamma_5)O^{\nu\sigma}\not p_1(1-h_1\gamma_5)\right]$$
$$= \frac{1}{4}\left(H_{JJ}^{\mu\nu} - h_2 H_{AJ}^{\mu\nu} + h_1 H_{JA}^{\mu\nu} - h_1h_2 H_{AA}^{\mu\nu}\right)$$
$$= \frac{1}{4}\left[(1-h_1h_2)H_1^{\mu\nu} + (h_1-h_2)H_4^{\mu\nu}\right],$$

and similarly for the other tensors. From I(B.2) we then obtain (2) immediately. Since $H_1^{\mu\nu}$ is real (symmetric) and $H_4^{\mu\nu}$ is imaginary (antisymmetric) we see that, compared with the unpolarized case, $\mathcal{H}_1^{\mu\nu}$ develops an imaginary and $\mathcal{H}_4^{\mu\nu}$ a real part for $h_1 \neq h_2$.

According to (2) the hadron tensors $\mathcal{H}_r^{\mu\nu}$ for the polarized case are fully determined by those for the unpolarized case, H_r^{mn}. Consequently, the partial cross sections $\sigma_\alpha(\mathcal{H}_r)$, α = 1(U), 2(L), 3(T), 4,...9 [or $\sigma_K(\mathcal{H}_r)$, K = A, B, C, D] introduced in[9] can be expressed in terms of $\sigma_\alpha(H_r) \equiv \sigma_{\alpha r}$ [or $\sigma_K(H_r) \equiv \sigma_{Kr}$] by relations similar to (2). Since H_1^{mn} (H_4^{mn}) is symmetric (antisymmetric) it contributes only to $\sigma_{\alpha 1}$ ($\sigma_{\alpha 4}$) for α = 1,...,6 (α = 7, 8, 9), i.e. to σ_{K1} (σ_{K4}) for K = A, B, C (D). We thus obtain

$$\sigma_K(\mathcal{H}_1) = \frac{1}{4}\begin{cases}(1-h_1h_2)\,\sigma_{K1} & K = A,B,C,\\ (h_1-h_2)\,\sigma_{D4} & K = D;\end{cases}$$
$$\sigma_K(\mathcal{H}_2) = \sigma_K(\mathcal{H}_3) = 0 \qquad K = A,B,C,D;$$
$$\sigma_K(\mathcal{H}_4) = \frac{1}{4}\begin{cases}(h_1-h_2)\,\sigma_{K1} & K = A,B,C,\\ (1-h_1h_2)\,\sigma_{D4} & K = D.\end{cases} \qquad (4)$$

Using (4) and I(4.11) in I(2.16) and I(2.17), the differential cross section for the process (1) can be written as

$$\frac{(2\pi)^2}{\sigma_{pt}} \frac{d^5\sigma_f(h_1,h_2)}{d\varphi\, d\cos\theta\, d\chi\, dx_1\, dx_2} = \frac{1}{4}\left[(1-h_1h_2)\,\sigma_f^{(0)} + (h_1-h_2)\,\sigma_f^{(1)}\right], \tag{5}$$

where $\sigma_{pt} = 4\pi\alpha^2/(3s)$ is the point-like cross section for $e^+e^- \to \gamma \to \mu^+\mu^-$ and

$$\sigma_f^{(0)} = (1-h_-h_+)\left(g_{11}^f\sigma_{A1} + g_{44}^f\sigma_{D4}\right) + (h_- - h_+)\left(g_{41}^f\sigma_{A1} + g_{14}^f\sigma_{D4}\right)$$
$$+ X(\varphi)\left(g_{21}^f\sigma_{B1} + g_{31}^f\sigma_{C1}\right) + Y(\varphi)\left(g_{21}^f\sigma_{C1} - g_{31}^f\sigma_{B1}\right),$$

$$\sigma_f^{(1)} = (1-h_-h_+)\left(g_{14}^f\sigma_{A1} + g_{41}^f\sigma_{D4}\right) + (h_- - h_+)\left(g_{44}^f\sigma_{A1} + g_{11}^f\sigma_{D4}\right)$$
$$+ X(\varphi)\left(g_{24}^f\sigma_{B1} + g_{34}^f\sigma_{C1}\right) + Y(\varphi)\left(g_{24}^f\sigma_{C1} - g_{34}^f\sigma_{B1}\right). \tag{6}$$

Here $\sigma_f^{(1)}$ follows from $\sigma_f^{(0)}$, the cross section for unpolarized q and $\bar{q}$, by letting $g_{r1}^f \to g_{r4}^f$ and $g_{r4}^f \to g_{r1}^f$ for $r = 1, 2, 3, 4$. The flavour and energy dependent coefficients $g_{rr'}^f(s)$ are given explicitly in (A.1). The cross sections $\sigma_{K1}(\theta,\chi)$, $K = A, B, C$ and $\sigma_{D4}(\theta,\chi)$ follow from I(2.15) and I(4.12) and will be given below. The $e^\pm$-beams are allowed to have longitudinal ($h_\pm$) as well as transverse polarization (taken into account by $X(\varphi)$ and $Y(\varphi)$ according to (A.3)). Finally, x_1 and x_2 in (5) are the scaled energies of the quark and antiquark ($x_1 + x_2 + x_3 = 2$), whereas the labo-
φ, θ and χ are the Euler angles of a rotation by which the labo-
ratory frame Ox'y'z' is carried over into a frame Oxyz conveniently fixed to the final state configuration (Fig.2 in [9]). Out of the many possible choices of Oxyz we consider in the following only the one corresponding to the so called thrust frame (Fig.3b in [9]). In this case the event plane (defined by $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$) is chosen as the Oxz plane ($\to p_{iy} = 0$, $i = 1, 2, 3$), with Oz along the thrust axis (most energetic parton) and Ox pointing

into the half-plane of the second most energetic parton. The lab frame Ox'y'z' has Oz' along the e^--beam and Ox' along the Lorentz force. The angles φ, θ and χ now have the following meaning: φ is the (azimuthal) angle (around the beam axis) between the plane of the storage ring ($\varphi = 0$) and the scattering plane Oz'z defined by the beam and thrust axes; θ is the (polar) angle between the beam and thrust axis; χ is the (azimuthal) angle (around the thrust axis) between the scattering plane Oz'z and the event plane Ozx.

The angular and thrust (T) distribution of a polarized quark (regardless of the antiquark polarization) is now given by

$$\frac{(2\pi)^2}{\sigma_{pt}} \frac{d^4\sigma_q(h_q)}{d\varphi\, d\cos\theta\, d\chi\, dT} = \frac{1}{2}\left(\sigma_q^{(0)} + h_q\,\sigma_q^{(1)}\right), \tag{7}$$

where $\sigma_q^{(0)}$ and $\sigma_q^{(1)}$ refer to the quantities (6) evaluated for the following expressions of σ_{Kr} (compare I(3.4), I(3.5)):

$$\sigma_{A1} = \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma_U}{dT} + \frac{3}{4}\sin^2\theta\,\frac{d\sigma_L}{dT}$$
$$+\frac{3}{4}\sin^2\theta\cos 2\chi\,\frac{d\sigma_T}{dT} - \frac{3}{2\sqrt{2}}\sin 2\theta\cos\chi\,\frac{d\sigma_I}{dT},$$

$$\sigma_{B1} = \frac{3}{8}\sin^2\theta\,\frac{d\sigma_U}{dT} - \frac{3}{4}\sin^2\theta\,\frac{d\sigma_L}{dT}$$
$$+\frac{3}{4}(1+\cos^2\theta)\cos 2\chi\,\frac{d\sigma_T}{dT} + \frac{3}{2\sqrt{2}}\sin 2\theta\cos\chi\,\frac{d\sigma_I}{dT},$$

$$\sigma_{C1} = \frac{3}{2}\cos\theta\sin 2\chi\,\frac{d\sigma_T}{dT} + \frac{3}{\sqrt{2}}\sin\theta\sin\chi\,\frac{d\sigma_I}{dT}, \tag{8}$$

$$\sigma_{D4} = \frac{3}{4}\cos\theta\,\frac{d\sigma_7}{dT} - \frac{3}{\sqrt{2}}\sin\theta\cos\chi\,\frac{d\sigma_8}{dT}.$$

Here the cross sections $d\sigma_\alpha/dT$ are given by

$$\frac{d(\sigma_U+\sigma_L)}{dT} = \frac{2\alpha_s}{\pi}\left\{-\frac{(4-T)(3T-2)}{2(1-T)} + \frac{1+T^2}{1-T}\ln\left(\frac{2T-1}{1-T}\right)\right\},$$

$$\frac{d\sigma_L}{dT} = \frac{2\alpha_s}{\pi}\left\{\frac{3T-2}{T}\right\},$$

$$\frac{d\sigma_T}{dT} = \frac{1}{2}\frac{d\sigma_L}{dT},$$

$$\frac{d\sigma_I}{dT} = \frac{2\alpha_s}{\pi}\left\{\frac{1}{\sqrt{2}}\left(\frac{2}{T}\sqrt{2T-1} - \frac{1}{\sqrt{1-T}}\right)\right\},$$

$$\frac{d\sigma_F}{dT} = \frac{2\alpha_s}{\pi}\left\{-\frac{(4-2T+T^2)(3T-2)}{2T(1-T)} + \frac{1+T^2}{1-T}\ln\left(\frac{2T-1}{1-T}\right)\right\},$$

$$\frac{d\sigma_8}{dT} = \frac{d\sigma_I}{dT}, \tag{9}$$

and can be read off as the coefficients (apart from G^f_{Kr}) of the fragmentation functions $D^h_q(z)$ in I(4.27); $\alpha_s = g_s^2/4\pi$ is the QCD running coupling constant.

Similarly, the angular and thrust distribution of a polarized antiquark (regardless of the quark polarization) is given by

$$\frac{(2\pi)^2}{\sigma_{pt}}\frac{d^4\sigma_{\bar q}(h_{\bar q})}{d\varphi\, d\cos\theta\, d\chi\, dT} = \frac{1}{2}\left(\sigma^{(0)}_{\bar q} - h_{\bar q}\sigma^{(1)}_{\bar q}\right), \tag{10}$$

where $\sigma^{(0)}_{\bar q}$ and $\sigma^{(1)}_{\bar q}$ are the same as $\sigma^{(0)}_q$ and $\sigma^{(1)}_q$ apart from the change of sign $\sigma_{D4} \to -\sigma_{D4}$ (compare the coefficients of $D^h_{\bar q}(z)$ in I(4.27)). Thus, if we denote $\sigma^{(i)}_q = \sigma^{(i)}_q(\sigma_{A1}, \sigma_{B1}, \sigma_{C1}, \sigma_{D4})$, then

$$\sigma^{(i)}_{\bar q} = \sigma^{(i)}_q(\sigma_{A1}, \sigma_{B1}, \sigma_{C1}, -\sigma_{D4}), \qquad (i=0,1) \tag{11}$$

with σ_{A1}, σ_{B1}, σ_{C1} and σ_{D4} given by (8) and (9).

It follows from (7) that the outgoing quark has a longitudinal polarization P_q given by

$$P_q = \sigma^{(1)}_q/\sigma^{(0)}_q = P_q(\sigma_{A1}, \sigma_{B1}, \sigma_{C1}, \sigma_{D4}), \tag{12}$$

whereas, from (10) and (11), the polarization of the antiquark is

$$P_{\bar q} = -\sigma^{(1)}_{\bar q}/\sigma^{(0)}_{\bar q} = -P_q(\sigma_{A1}, \sigma_{B1}, \sigma_{C1}, -\sigma_{D4}). \tag{13}$$

Thus $P_{\bar q} \neq P_q$ and, in principle, polarization measurements can be used to discriminate between a quark and its antiquark at energies where Z^0-exchange becomes sizable. The flavour dependence through $g^f_{rr'}$, however, is the same for P_q and $P_{\bar q}$.

At low energies Z^0-exchange can be neglected and from (6) and (A.1) we obtain in the 1γ-exchange approximation

$$\sigma^{(0)}_q[1\gamma] = Q_q^2\left[(1-h_-h_+)\sigma_{A1} + X(\varphi)\sigma_{B1} + Y(\varphi)\sigma_{C1}\right],$$

$$\sigma^{(1)}_q[1\gamma] = Q_q^2\left[(h_- - h_+)\sigma_{D4}\right], \tag{14}$$

with $\sigma_{Kr}(\theta,\chi,T)$ taken from (8) and (9). Using (11) to obtain $\sigma^{(i)}_{\bar q}[1\gamma]$, we get the energy and flavour independent expression

$$P_q^{[1\gamma]} = P_{\bar q}^{[1\gamma]} = \frac{(h_- - h_+)\sigma_{D4}}{(1-h_-h_+)\sigma_{A1} + X(\varphi)\sigma_{B1} + Y(\varphi)\sigma_{C1}}. \tag{15}$$

Thus the quark (or antiquark) can be (longitudinally) polarized only if at least one of the beams has a longitudinal polarization. For unpolarized beams a non-vanishing $P_q^{[1\gamma]}$ can be obtained by taking quark masses and higher order QCD corrections into account [12].

The angular distributions considered so far are too differential to be of practical interest. We thus integrate over φ and χ to obtain

$$\frac{1}{\sigma_{pt}}\frac{d^2\sigma_q(h_q)}{d\cos\theta\, dT} = \frac{3}{16}\left(\frac{d\sigma_U}{dT} + 2\frac{d\sigma_L}{dT}\right)\left(1+\alpha(T)\cos^2\theta\right)$$

$$\cdot\left[G^q_{A1} + G^q_{D4}\,P^\gamma(\cos\theta,T) + h_q\left(G^q_{A4} + G^q_{D1}\,P^\gamma(\cos\theta,T)\right)\right], \qquad (16)$$

where we have introduced G^q_{A1} and G^q_{D4} from (A.2) and have defined

$$P^\gamma(\cos\theta,T) \equiv \frac{\sigma_{D4}}{\sigma_{A1}} = \frac{2\epsilon(T)\cos\theta}{1+\alpha(T)\cos^2\theta}, \qquad (17)$$

with

$$\alpha(T) = \left(\frac{d\sigma_U}{dT} - 2\frac{d\sigma_L}{dT}\right)\Big/\left(\frac{d\sigma_U}{dT} + 2\frac{d\sigma_L}{dT}\right),$$

$$\epsilon(T) = \frac{d\sigma_F}{dT}\Big/\left(\frac{d\sigma_U}{dT} + 2\frac{d\sigma_L}{dT}\right), \qquad (18)$$

and $d\sigma_\alpha/dT$ given by (9). P^γ is directly related, on one side, to the forward-backward asymmetry A_q of the (unpolarized) quark [3]

$$A_q(s;\cos\theta,T) = \left(G^q_{D4}/G^q_{A1}\right) P^\gamma(\cos\theta,T) \qquad (19)$$

and, on the other side, to the polarization of the quark in the 1γ-exchange approximation

$$P_q^{[1\gamma]} = P_{\bar q}^{[1\gamma]} = h\,P^\gamma(\cos\theta,T), \qquad h \equiv \frac{h_- - h_+}{1-h_-h_+}. \qquad (20)$$

With Z^o-exchange included, the quark polarization becomes

$$P_q(s;\cos\theta,T) = \frac{G^q_{A4}(s) + G^q_{D1}(s)\,P^\gamma(\cos\theta,T)}{G^q_{A1}(s) + G^q_{D4}(s)\,P^\gamma(\cos\theta,T)}. \qquad (21)$$

The cross section for producing a polarized antiquark follows from (16) by replacing $h_q \to -h_{\bar q}$ and $P^\gamma \to -P^\gamma$. This then yields

$$A_{\bar q}(s;\cos\theta,T) = -A_q(s;\cos\theta,T),$$

$$P_{\bar q}(s;\cos\theta,T) = -P_q(s;-\cos\theta,T). \qquad (22)$$

The dependence of A_q and P_q on thrust (T) and cosΘ is only through $P^\gamma(\cos\Theta,T)$, on energy (s) and flavour only through G^q_{Ar} and G^q_{Dr} given in (A.2), and on (longitudinal) beam polarization through $h = (h_- - h_+)/(1 - h_-h_+)$, with $h_\pm = 0$ for unpolarized beams. The only effect of gluon radiation in (1) on P^γ, P_q and $P_{\bar q}$ is the T-dependence of these quantities through α(T) and ε(T) given in (18); the QCD running coupling constant α_s drops out. For $T \to 1$ we have $\alpha(T) \to 1$ and $\epsilon(T) \to 1$, so that P^γ, P_q and $P_{\bar q}$ reduce to the known expressions [6] corresponding to $e^-e^+ \to \gamma, Z \to q\bar q$ without gluon radiation.

In Fig.1 we have shown $P^\gamma(\cos\Theta,T) = -P^\gamma(-\cos\Theta,T)$ as a function of cosΘ for different thrust values. P^γ gives the maximal (h = 1 in (20)) polarization of the quark (antiquark) in the 1γ-exchange approximation. As expected, it is largest for T = 1. Thus the effect of gluon radiation in (1) is to reduce the polarization of the outgoing quark (antiquark) with decreasing thrust. There is, however, no energy dependence of P^γ. If Z^o-exchange is included, P_q from (21) and $P_{\bar q}$ from (22) show a characteristic s-dependence as illustrated in Figs. 2 (h = +1), 3 (h = 0) and 4 (h = -1) for the Weinberg-Salam model [2] with $\sin^2\Theta_W = 0.23$ (see Appendix A). The five curves shown on each figure refer to constant values of $P^\gamma(\cos\Theta,T)$ = 0.8 (....), 0.4 (-·-·-), 0.0 (——), -0.4 (---), -0.8 (-··-··-) and start, at low energies, at the corresponding values hP^γ. Since in the 1γ-exchange approximation we would have $P_q^{[1\gamma]} = P_{\bar q}^{[1\gamma]} = hP^\gamma$ independent of energy, the s-dependence of the curves shown is due solely to Z^o-exchange. Note that $P^{[1\gamma]} = 0$ for h = 0

3. Polarization of hadrons.

We have seen in the previous section that the quark and/or antiquark produced in reaction (1) might have a sizable polarization. The question then arises whether and to what extent this polarization is transmitted during fragmentation (hadronization) to the hadrons, especially baryons and vector mesons of the corresponding jet. Since very little is known on the fragmentation process itself, the best one can do is to parametrize it in a rather model-independent way by so called spin-dependent fragmentation functions [5,7,8] and to discuss various (model-dependent) assumptions regarding these functions [13]. It is, however, very difficult experimentally to test these assumptions, even if the hadrons turn out to be substantially polarized, because parity violating decays are needed in order to reconstruct the density matrix of the hadron from the angular distribution of its decay products. This is almost hopeless for vector mesons, but might become feasible for Λ and Σ hyperons (if copiously produced near the Z^o-pole). A more direct method would be to look for parity-odd correlations among the fastest particles of the jet [14].

The problems just mentioned have been considered recently [5,6,7] for 2-jet events generated in $e^-e^+ \to \gamma, Z \to q\bar{q}$. Here we consider the case of 3-jet events initiated by $q\bar{q}g$. We study the polarization of a hadron H belonging to the most energetic jet originating either in a polarized quark or a polarized antiquark*). Let $D^{H\lambda}_{q\varkappa}(z)$ be the probability that a quark q with helicity $\varkappa$ will form a hadron H with helicity λ and momentum fraction $z = p_H/p_q = x_H/x_q$. From parity conservation in the fragmentation process we then have

$$D^{H,-\lambda}_{q,-\varkappa}(z) = D^{H,\lambda}_{q,\varkappa}(z) \,, \tag{23}$$

leading to the following relations:

$$\begin{aligned} \tfrac{1}{2}\left(D^{H,\lambda}_{q,+\frac{1}{2}} + D^{H,\lambda}_{q,-\frac{1}{2}}\right) &\equiv D^{H,\lambda}_{q} = D^{H,-\lambda}_{q} \,, \\ \tfrac{1}{2}\left(D^{H,\lambda}_{q,+\frac{1}{2}} - D^{H,\lambda}_{q,-\frac{1}{2}}\right) &\equiv \Delta^{H,\lambda}_{q} = -\Delta^{H,-\lambda}_{q} \,, \\ \sum_\lambda D^{H,\lambda}_{q,+\frac{1}{2}} = \sum_\lambda D^{H,\lambda}_{q,-\frac{1}{2}} &= \tfrac{1}{2}\sum_{\varkappa,\lambda} D^{H,\lambda}_{q,\varkappa} \equiv D^{H}_{q} \,, \end{aligned} \tag{24}$$

*) We disregard the small [15] fraction of events in which the gluon jet is the most energetic.

where D^H_q is the usual fragmentation function of a quark q into a hadron H, regardless of helicities (the factor 1/2 is just the statistical spin factor $1/(2s_q+1)$ of the fragmenting quark). Similar definitions and relations hold for $D^{H,\lambda}_{\bar{q}\,\bar{\varkappa}}(z)$, the probability that an antiquark $\bar{q}$ with helicity $\bar{\varkappa}$ forms a hadron H with helicity λ and momentum fraction $z = x_H/x_{\bar{q}}$.

The differential cross section for finding in the quark or antiquark jet of thrust T a hadron H with helicity λ, fractional momentum $z = x_H/T$ and direction (Θ,φ) is then given by

$$\frac{d^4\sigma_H(\lambda)}{d\varphi\, d\cos\theta\, dT\, dz} = \sum_{q\varkappa} \frac{d^3\sigma_q(h_q=2\varkappa)}{d\varphi\, d\cos\theta\, dT} D^{H\lambda}_{q\varkappa}(z) + \sum_{\bar{q}\bar{\varkappa}} \frac{d^3\sigma_{\bar{q}}(h_{\bar{q}}=2\bar{\varkappa})}{d\varphi\, d\cos\theta\, dT} D^{H\lambda}_{\bar{q}\bar{\varkappa}}(z) \,, \tag{25}$$

where we sum over all quarks and antiquarks that can fragment into H and where $d^3\sigma_q$ and $d^3\sigma_{\bar{q}}$ follow from (7) and (10) by integration over χ. Strictly speaking the relation (25) is valid only in the high energy limit, when quark and hadron masses, as well as the transverse momentum of the hadron relative to the thrust direction can be neglected. In this case $d^4\sigma_H(\lambda)$ is just the diagonal element $\rho^H_{\lambda\lambda}$ of the unnormalized hadron density matrix, whose off-diagonal elements are zero [5]. In the same limit $\Theta_H = \Theta_q (=\Theta_{\bar{q}}) = \Theta$ and $\varphi_H = \varphi_q (=\varphi_{\bar{q}}) = \varphi$, as already assumed in (25). Using now (7) and (10), integrated over χ, in (25) and introducing the definitions

$$\begin{aligned} \sigma^{(0)}_{H,\lambda} &\equiv \sum_q \sigma^{(0)}_q D^{H,\lambda}_q + \sum_{\bar{q}} \sigma^{(0)}_{\bar{q}} D^{H,\lambda}_{\bar{q}} = \sigma^{(0)}_{H,-\lambda} \,, \\ \sigma^{(1)}_{H,\lambda} &\equiv \sum_q \sigma^{(1)}_q \Delta^{H,\lambda}_q - \sum_{\bar{q}} \sigma^{(1)}_{\bar{q}} \Delta^{H,\lambda}_{\bar{q}} = -\sigma^{(1)}_{H,-\lambda} \,, \end{aligned} \tag{26}$$

the cross section (25) can be written as

$$\frac{(2\pi)}{\sigma_{jet}} \frac{d^4\sigma_H(\lambda)}{d\varphi\, d\cos\theta\, dT\, dz} = \sigma^{(0)}_{H,\lambda} + \sigma^{(1)}_{H,\lambda} \,. \tag{27}$$

Summing over λ and using (24) we obtain the cross section for observing the hadron H regardless of its polarization

$$\frac{(2\pi)}{\sigma_{pt}}\frac{d^4\sigma_H}{d\varphi\, d\cos\theta\, dT\, dz} = \sum_q \sigma_q^{(0)} D_q^H + \sum_{\bar q}\sigma_{\bar q}^{(0)} D_{\bar q}^H \equiv \sigma_H^{(0)} . \qquad (28)$$

In the 1γ-exchange approximation the expressions (26) simplify according to (14) and (11):

$$\sigma_{H,\lambda}^{(0)}[1\gamma] = \left[(1-h_-h_+)\sigma_{A1}(\theta) + X(\varphi)\sigma_{B1}(\theta)\right]\sum_q Q_q^2\left(D_q^{H,\lambda} + D_{\bar q}^{H,\lambda}\right),$$

$$\sigma_{H,\lambda}^{(1)}[1\gamma] = (h_- - h_+)\,\sigma_{D4}(\theta)\sum_q Q_q^2\left(\Delta_q^{H,\lambda} + \Delta_{\bar q}^{H,\lambda}\right), \qquad (29)$$

where now only the χ-integrated cross sections (8) appear.

From (27) and (28) we obtain the normalized density matrix of the hadron H

$$\hat\rho^H = \frac{1}{\sigma_H^{(0)}}\,\mathrm{diag}\left[\sigma_{H,s}^{(0)} + \sigma_{H,s}^{(1)},\ \sigma_{H,s-1}^{(0)} + \sigma_{H,s-1}^{(1)},\ \dots,\ \sigma_{H,s}^{(0)} - \sigma_{H,s}^{(1)}\right], \qquad (30)$$

where s is the spin of H and we have written $\hat\rho^H_{\lambda\lambda} \sim \sigma^{(0)}_{H,\lambda} + \sigma^{(1)}_{H,\lambda} = \sigma^{(0)}_{H,-\lambda} - \sigma^{(1)}_{H,-\lambda}$ for $\lambda \leq 0$. Now the expectation value of various spin operators (polarization vector, alignment, etc.) can be easily determined, particularly since only multipole moments q_M^L($L = 0, 1, \dots, 2s$; $M = L, L-1, \dots, -L$) with $M = 0$ appear in the expansion of (the diagonal matrix) $\hat\rho^H$[16]. Although the discussion of this arbitrary spin case is straightforward, we shall not elaborate more on it, but consider instead the particular cases $s = 1/2$ and $s = 1$.

4. Polarization of baryons.

For spin-1/2 baryons B the density matrix (30) takes on the form

$$\hat\rho^B = \tfrac{1}{2}\,\mathrm{diag}\left(1+P_B,\ 1-P_B\right) = \tfrac{1}{2}\left(1 + P_B\sigma_z\right), \qquad (31)$$

where P_B is the longitudinal polarization of B

$$P_B = \sigma^{(1)}_{B,\frac12}/\sigma^{(0)}_{B,\frac12} = 2\sigma^{(1)}_{B,\frac12}/\sigma^{(0)}_B = P_B(s;\varphi,\cos\theta,T,z). \qquad (32)$$

A similar expression holds for antibaryons ($B \to \bar B$). Any further discussion has to rely on some model for the spin-dependent fragmentation functions. The following two assumptions lead to great simplifications:

i) Valence-quark dominance: only those terms in (26) and (28) are retained for which q (or $\bar q$) is a valence quark of H ($\rightarrow$ "favoured" fragmentation functions);

ii) Helicity conservation in the fragmentation process [6]: for baryons this means $D^{B,\lambda}_{q,\varkappa} = 0$ if $\varkappa \neq \lambda$, implying

$$D_q^{B,\frac12} = \Delta_q^{B,\frac12} = \tfrac12 D_q^B . \qquad (33)$$

The spin-dependent fragmentation functions then no longer appear.

For a baryon B(q,q',q'') composed of the quarks q, q' and q'' the first assumption implies (Σ' means sum over q,q' and q'' only)

$$P_B = \left(2\Sigma'\sigma_q^{(1)}\Delta_q^{B,\frac12}\right)\Big/\left(\Sigma'\sigma_q^{(0)}D_q^B\right), \qquad (34)$$

and the second then gives

$$P_B = \left(\Sigma'\sigma_q^{(1)}D_q^B\right)\Big/\left(\Sigma'\sigma_q^{(0)}D_q^B\right). \qquad (35)$$

If the contribution of all quarks to the fragmentation process is roughly the same, as one would expect [5] for baryons composed of the light u, d and s quarks only, then (34) reduces to

$$\mathcal{P}_B = \left(2\Delta_q^{B,\frac{1}{2}} / D_q^B\right) \mathcal{P}_B^o \,, \tag{36}$$

and (35) to

$$\mathcal{P}_B^o \equiv \left(\Sigma' \sigma_q^{(1)}\right) / \left(\Sigma' \sigma_q^{(0)}\right). \tag{37}$$

This is, for $T \to 1$ (i.e., $e^+e^- \to \gamma, Z \to q\bar{q}$), precisely the result of Augustin and Renard [6], whereas (36) corresponds to Case I considered by Nieves [5]. Since the flavour dependence of $\sigma_q^{(0)}$ and $\sigma_q^{(1)}$ is the same for all up quarks (down quarks), we obtain from (37)

$$\mathcal{P}_{\Lambda^0} = \mathcal{P}_{\Sigma^0} = \left(\sigma_u^{(1)} + 2\sigma_d^{(1)}\right) / \left(\sigma_u^{(0)} + 2\sigma_d^{(0)}\right),$$

$$\mathcal{P}_{\Sigma^+} = \left(2\sigma_u^{(1)} + \sigma_d^{(1)}\right) / \left(2\sigma_u^{(0)} + \sigma_d^{(0)}\right),$$

$$\mathcal{P}_{\Sigma^-} = \left(3\sigma_d^{(1)}\right) / \left(3\sigma_d^{(0)}\right) = \mathcal{P}_d \,. \tag{38}$$

In this approximation P_{Σ^-} directly measures the polarization of the down quarks (Figs.2b, 3b and 4b). With $\sigma_q^{(0)}$ $(\sigma_q^{(1)})$ given by the numerator (denominator) in (21) for the φ-integrated case, we show P_Λ in Fig.5 as a function of s, again for $h = 0, \pm 1$ and for different values of $P^\gamma(\cos\theta, T)$. The polarization lies, at each energy $\sqrt{s}$, somewhere in between $\min(P_u(s), P_d(s))$ and $\max(P_u(s), P_d(s))$. The energy dependence of P_{Σ^+} looks similarly, as it corresponds to a slightly different weighting of $\sigma_u^{(i)}$ and $\sigma_d^{(i)}$ in (38). The low energy limit is given by the 1γ-exchange approximation result

$$\mathcal{P}_B^{[1\gamma]} \xrightarrow[i)]{} h\,\mathcal{P}^\gamma_{(\cos\theta, T)} \frac{2\Sigma' Q_q^2 \Delta_q^{B,\frac{1}{2}}}{\Sigma' Q_q^2 D_q^B} \xrightarrow[ii)]{} h\,\mathcal{P}^\gamma_{(\cos\theta, T)}, \tag{39}$$

which follows from (29) under the assumption i), then ii). The polarizations (38) can be determined experimentally, at least in principle, from the forward-backward asymmetry in the weak decays $\Lambda^0 \to p\pi^-$, $\Sigma^+ \to p\pi^0$ and $\Sigma^- \to n\pi^-$.

We finally note that the polarization $P_{\bar{B}}$ of the antibaryon $\bar{B}$ is related to P_B by

$$\mathcal{P}_{\bar{B}}(s; \varphi, \cos\theta, T, z) = -\mathcal{P}_B(s; \varphi, -\cos\theta, T, z), \tag{40}$$

where we have used (11) and charge conjugation invariance for the fragmentation functions $(D_{q\varkappa}^{B\lambda} = D_{\bar{q}\varkappa}^{\bar{B}\lambda})$.

5. Polarization of vector mesons.

In this case the density matrix (30) becomes

$$\hat{\rho}^V = \frac{1}{\sigma_V^{(0)}} \,\mathrm{diag}\left[\sigma_{V,1}^{(0)} + \sigma_{V,1}^{(1)},\ \sigma_{V,0}^{(0)},\ \sigma_{V,1}^{(0)} - \sigma_{V,1}^{(1)}\right] \tag{41}$$

and can be expanded in terms of multipole moments q_M^L [16]

$$\hat{\rho}^V = \frac{1}{3}\left(\mathbb{1} + q_0^1 Q_0^1 + q_0^2 Q_0^2\right), \tag{42}$$

where $\mathbb{1}$ is the 3 x 3 unit matrix and

$$Q_0^1 = \sqrt{\tfrac{3}{2}}\, S_z = \sqrt{\tfrac{3}{2}}\,\mathrm{diag}\,[1, 0, -1],$$

$$Q_0^2 = \frac{1}{\sqrt{2}}\left(3S_z^2 - 2\right) = \frac{1}{\sqrt{2}}\,\mathrm{diag}\,[1, -2, 1]. \tag{43}$$

The polarization vector of the meson is then given by

$$\mathcal{P}_V = \frac{1}{\sqrt{2}} q_0^1 = \sqrt{3}\, \frac{\sigma_{V,1}^{(1)}}{\sigma_V^{(0)}}$$

and the alignment (tensor polarization) by

$$q_0^2 = \frac{1}{\sqrt{2}}\left(1 - 3\,\frac{\sigma^{(0)}_{V,0}}{\sigma^{(0)}_V}\right) = \sqrt{2}\left(3\,\frac{\sigma^{(0)}_{V,1}}{\sigma^{(0)}_V} - 1\right). \tag{44}$$

For vector mesons $V(Q,\bar{q})$ containing a quark Q and an antiquark $\bar{q}$ the assumptions of i) valence quark dominance and ii) helicity conservation in the fragmentation process, i.e.

$$D^{V,-1}_{Q,\frac{1}{2}} = D^{V,1}_{Q,-\frac{1}{2}} = D^{V,0}_{Q,\pm\frac{1}{2}} = 0\ , \tag{45}$$

lead to

$$\sigma^{(0)}_{V,0} = 0 \quad \longrightarrow \quad q_0^2 = \frac{1}{\sqrt{2}}\ ,$$

$$P_V = \frac{\sqrt{3}}{2}\,\frac{\sigma^{(1)}_Q D^V_Q - \sigma^{(1)}_{\bar{q}} D^V_{\bar{q}}}{\sigma^{(0)}_Q D^V_Q + \sigma^{(0)}_{\bar{q}} D^V_{\bar{q}}}\ . \tag{46}$$

Due to the constant alignment $q_0^2 = 1/\sqrt{2}$, P_V is bounded by $|P_V| < \sqrt{3}/2$. If the contribution of Q [or $\bar{q}$] to the fragmentation process is dominant, then P_V reduces to $P_V = (\sqrt{3}/2)\ P_Q$ [or $P_V = (\sqrt{3}/2)P_{\bar{q}}$]. This case may apply to mesons composed of a heavy quark [antiquark] and a light antiquark [quark]. For self-conjugate mesons $V(q,\bar{q}) = \rho^o, \omega, \phi, \psi, \ldots$ we have $D^V_q = D^V_{\bar{q}}$ by charge conjugation and therefore

$$P_V = \frac{\sqrt{3}}{2}\,\frac{\sigma^{(1)}_q - \sigma^{(1)}_{\bar{q}}}{\sigma^{(0)}_q + \sigma^{(0)}_{\bar{q}}} = \frac{\sqrt{3}}{2}\,\frac{G^q_{D1}}{G^q_{A1}}\,P^{\gamma}(\cos\theta, T)\ , \tag{47}$$

which reduces to $(\pm\sqrt{3}/2)P^{\gamma}$ for $h = \pm 1$. Although the polarization of vector mesons turns out to be rather large, there is no hope to measure it experimentally, since the dominant decay modes of these particles are parity conserving.

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Appendix A

To make this paper self-contained we collect here some of the relevant formulas already given in [9]. Thus the functions $g^f_{rr'}$ appearing in (6) are given by I(2.7):

$$\begin{aligned}
g^f_{11} &= Q_f^2 - 2Q_f v v_f \,\mathrm{Re}\,\chi_Z + (v^2+a^2)(v_f^2+a_f^2)\,|\chi_Z|^2\ ,\\
g^f_{14} &= 2Q_f v a_f \,\mathrm{Re}\,\chi_Z - 2(v^2+a^2)\,v_f a_f\,|\chi_Z|^2\ ,\\
g^f_{41} &= 2Q_f a v_f \,\mathrm{Re}\,\chi_Z - 2va\,(v_f^2+a_f^2)\,|\chi_Z|^2\ ,\\
g^f_{44} &= -2Q_f a a_f \,\mathrm{Re}\,\chi_Z + 4va\,v_f a_f\,|\chi_Z|^2\ ,\\
g^f_{21} &= Q_f^2 - 2Q_f v v_f \,\mathrm{Re}\,\chi_Z + (v^2-a^2)(v_f^2+a_f^2)\,|\chi_Z|^2\ ,\\
g^f_{24} &= 2Q_f v a_f \,\mathrm{Re}\,\chi_Z - 2(v^2-a^2)\,v_f a_f\,|\chi_Z|^2\ ,\\
g^f_{31} &= -2Q_f a v_f \,\mathrm{Im}\,\chi_Z\ ,\\
g^f_{34} &= 2Q_f a a_f \,\mathrm{Im}\,\chi_Z\ ,
\end{aligned} \tag{A.1}$$

where Q_f is the electric charge of the quark q_f; v(a) and $v_f(a_f)$ is the vector (axial vector) coupling at the e^-e^+ Z- and $Zq_f\bar{q}_f$-vertex, respectively; $\chi_Z(s) = gM_Z^2 s(s-M_Z^2+iM_Z\Gamma_Z)^{-1}$, with $g = 4.49\times 10^{-5}\ \mathrm{GeV}^{-2}$, $M_Z(\Gamma_Z)$ the mass (width) of the Z^o and $s = (p_- + p_+)^2 = (2E_{beam})^2$ the total center of mass energy squared. In the Weinberg-Salam model (with Θ_W the Weinberg angle) we have: $v = -1 + 4\sin^2\Theta_W$, $a = -1$; $v_f = 1-(8/3)\sin^2\Theta_W$, $a = 1$ for $Q_f = 2/3$; $v_f = -1 + (4/3)\sin^2\Theta_W$, $a_f = -1$ for $Q_f = -1/3$; $M_Z\sqrt{g} = (2\sin 2\Theta_W)^{-1}$. For numerical computations we have taken $\sin^2\Theta_W = 0.23$ and $\Gamma_Z = 2.49$ GeV (corresponding to 3 generations of fundamental leptons and coloured quarks).

From (A.1) we define (compare I(2.19))

$$G^f_{A\tau} = (1-h_-h_+)\,g^f_{1\tau} + (h_- - h_+)\,g^f_{4\tau}\,,$$
$$G^f_{D\tau} = (1-h_-h_+)\,g^f_{4\tau} + (h_- - h_+)\,g^f_{1\tau}\,, \qquad \tau = 1,4 \tag{A.2}$$

where $h_\pm = \vec{\xi}^\pm \cdot \hat{p}_\pm$ is the longitudinal polarization of the $e^\pm$-beam. The transverse beam polarization is taken into account by

$$X(\varphi) = (\xi^-_{x'}\xi^+_{x'} - \xi^-_{y'}\xi^+_{y'})\cos 2\varphi + (\xi^-_{x'}\xi^+_{y'} + \xi^-_{y'}\xi^+_{x'})\sin 2\varphi\,,$$

$$Y(\varphi) = -(\xi^-_{x'}\xi^+_{x'} - \xi^-_{y'}\xi^+_{y'})\sin 2\varphi + (\xi^-_{x'}\xi^+_{y'} + \xi^-_{y'}\xi^+_{x'})\cos 2\varphi\,. \tag{A.3}$$

where the components of $\vec{\xi}^\pm$ refer to the lab frame Ox'y'z' with Oz' along $\hat{p}_-$ and Ox' along the Lorentz force. For natural transverse beam polarization we then have $\vec{\xi}^\pm = (0, \pm P, 0)$, so that $X(\varphi) = P^2\cos 2\varphi$ and $Y(\varphi) = -P^2\sin 2\varphi$.

Appendix B

The simplicity of the expressions (12) and (21) for the quark polarization is due mainly to the vector nature of the radiated gluon and to our neglect of the quark masses, implying quark helicity conservation at the $q\bar{q}g$-vertex. If quark masses are included or the emission of scalar gluons (even for $m_q = 0$) is considered, helicity is no longer conserved and the expression for $\vec{P}_q$ becomes more involved. Its general structure, however, can be easily found by the method developed in [9]. For a polarized quark the hadron tensors are of the form

$$\mathcal{H}^{\mu\nu}_\tau = \frac{1}{2}\,\mathrm{Tr}\left[Q^{\mu\nu}_\tau(\not{p}_1 + m_q)(1+\gamma_5 \not{S}_1)\right], \tag{B.1}$$

where $Q^{\mu\nu}_\tau$ are known quantities following from the Feynman rules. Since S^λ_1 is linearly related to the rest frame polarization vector $\vec{\xi}_1$

$$S^\lambda_1 = \left\{\beta_1\gamma_1(\vec{\xi}_1\cdot\hat{p}_1)\,;\ \vec{\xi}_1 + (\gamma_1 - 1)(\vec{\xi}_1\cdot\hat{p}_1)\hat{p}_1\right\} \equiv \sum_{i=1}^{3} L^\lambda{}_i\,\xi^i_1 \tag{B.2}$$

(with $\hat{p} = \vec{p}/|\vec{p}|$, $\beta = |\vec{p}|/p^0$, $\gamma^{-2} = 1-\beta^2$), we have $\not{S} = \gamma^\lambda S_\lambda = \gamma^\lambda L_{\lambda i}\xi^i = \not{L}_i\,\xi^i = -\vec{\not{L}}\cdot\vec{\xi}$ and thus

$$\mathcal{H}^{\mu\nu}_\tau = \frac{1}{2}\left(H^{\mu\nu}_\tau + \vec{\xi}_1\cdot\vec{H}^{\mu\nu}_{\tau 5}\right), \tag{B.3}$$

where

$$H^{\mu\nu}_\tau = \mathrm{Tr}\left[Q^{\mu\nu}_\tau(\not{p}_1 + m_q)\right],$$
$$\vec{H}^{\mu\nu}_{\tau 5} = \mathrm{Tr}\left[Q^{\mu\nu}_\tau(\not{p}_1 + m_q)\vec{\not{L}}\gamma_5\right]. \tag{B.4}$$

A decomposition analogous to (B.3) then holds for the quantities $S^q_{K\alpha}$ $(K = A,B,C,D;\ \alpha = 1,2,\ldots,9)$ defined in I(2.20)

$$S^q_{K\alpha}(\mathcal{H}) = \sum_{\tau=1}^{4} G^q_{K\tau}\,\sigma_\alpha(\mathcal{H}_\tau)$$
$$= \frac{1}{2}\left[S^q_{K\alpha}(H) + \sum_{i=1}^{3}\xi^i_1\,S^q_{K\alpha}(H^i_5)\right], \tag{B.5}$$

where the $\sigma_\alpha(\mathcal{H}_\tau)$'s are given in terms of $\mathcal{H}^{\mu\nu}_\tau$ by I(B.3) with index U,L,T for $\alpha = 1,2,3$. Introducing now (B.3) into I(2.21) we obtain

$$\frac{(2\pi)^2}{\sigma_{pt}}\frac{d^5\sigma_q(\vec{\zeta}_1)}{d\varphi\, d\cos\theta\, d\chi\, dx_1 dx_2} = \frac{1}{2}\Big[\sigma_q^{(0)}(H) + \sum_{i=1}^{3}\zeta_1^i\,\sigma_q^{(1)}(H_5^i)\Big], \qquad (B.6)$$

where $\sigma_q^{(0)}(H)$ and $\sigma_q^{(1)}(H_5^i)$ are given by the r.h.s. of I(2.21) evaluated for $S_{K\alpha}^q(H)$ and $S_{K\alpha}^q(H_5^i)$, respectively. The polarization vector of the quark is now given by

$$P_q^i = \sigma_q^{(1)}(H_5^i)\,/\,\sigma_q^{(0)}(H)\,, \qquad i = 1,2,3. \qquad (B.7)$$

Integrating over φ and χ we find explicitly

$$P_q^i = \frac{[S_{AU}^q(H_5^i)+2S_{AL}^q(H_5^i)] + [S_{AU}^q(H_5^i)-2S_{AL}^q(H_5^i)]\cos^2\theta + 2S_{D7}^q(H_5^i)\cos\theta}{[S_{AU}^q(H)+2S_{AL}^q(H)] + [S_{AU}^q(H)-2S_{AL}^q(H)]\cos^2\theta + 2S_{D7}^q(H)\cos\theta}\,. \qquad (B.8)$$

In the case of massless quarks the vectors $\vec{H}_{r5}^{\mu\nu}$ become parallel to the quark momentum

$$\vec{H}_{r5}^{\mu\nu} = H_{r5}^{\mu\nu}\,\hat{p}_1\,, \qquad (B.9)$$

so that

$$\mathcal{H}_r^{\mu\nu} = \frac{1}{2}\left(H_r^{\mu\nu} + h_1 H_{r5}^{\mu\nu}\right), \qquad h_1 = \vec{\zeta}_1\cdot\hat{p}_1\,, \qquad (B.10)$$

where $H_{r5}^{\mu\nu} = Tr(Q_r^{\mu\nu}\not{p}_1\gamma_5)$ if $H_r^{\mu\nu} = Tr(Q_r^{\mu\nu}\not{p}_1)$. From (B.7) and (B.9) it now follows that the quark is longitudinally **polarized**

$$\vec{P}_q = \left[\sigma_q^{(1)}(H_5)/\sigma_q^{(0)}(H)\right]\hat{p}_1\,. \qquad (B.11)$$

For vector gluons (QCD) we find from (B.10) and (2):

$$H_{15}^{\mu\nu} = H_4^{\mu\nu}, \qquad H_{25}^{\mu\nu} = H_{35}^{\mu\nu} = 0, \qquad H_{45}^{\mu\nu} = H_1^{\mu\nu} \qquad (B.12)$$

so that, for example, $S_{AU}(H_5) = \sum_r G_{Ar}\sigma_U(H_{r5}) = G_{A4}\sigma_{U1}$.

since $H_{15}^{\mu\nu}$ being antisymmetric does'nt contribute to σ_U. This way (B.8), if integrated over x_2 for $x_1 = T$, reduces to (21). In the case of scalar gluons (B.9) and (B.10) still hold, but now there are no simplifying relations between $H_{r5}^{\mu\nu}$ and $H_r^{\mu\nu}$ like those in (B.12); in addition, there are contributions also from $\mathcal{H}_r^{\mu\nu}$ for r = 2,3. Thus the scalar gluon case is more complicated than the vector gluon case. Note that the general formulas of this Appendix apply also to quarks of an arbitrary final state (e.g.multi-jet event), with $\sigma^{(0)}$ and $\sigma^{(1)}$ being the appropriate cross sections.

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<u>Figure Captions</u>

Fig.1: $P^{\gamma}(\cos\Theta,T) = -P^{\gamma}(-\cos\Theta,T)$ as a function of $\cos\Theta$ for thrust $T = 1$ (——), 0.9 (---), 0.8 (-•-•-) and $T = 0.7$ (-••-).

Fig.2: Energy dependence of quark and antiquark polarization for $h = +1$ and $P^{\gamma}(\cos\Theta,T) = 0.8$(...), 0.4 (-•-), 0.0(——), -0.4 (---), -0.8 (-••-): a) up-quark; b) down-quark; c) anti-up-quark; d) anti-down-quark.

Fig.3: Energy dependence of quark and antiquark polarization for $h = 0$ and $P^{\gamma}(\cos\Theta,T) = 0.8$(...), 0.4 (-•-), 0.0(——), -0.4 (---), -0.8 (-••-): a) up-quark; b) down-quark; c) anti-up-quark; d) anti-down-quark.

Fig.4: Energy dependence of quark and antiquark polarization for $h = -1$ and $P^{\gamma}(\cos\Theta,T) = 0.8$(...), 0.4 (-•-), 0.0(——), -0.4 (---), -0.8 (-••-): a) up-quark; b) down-quark; c) anti-up-quark; d) anti-down-quark.

Fig.5: Energy dependence of the polarization of the Λ^{0} hyperon for $P^{\gamma}(\cos\Theta,T) \doteq 0.8$ (...), 0.4 (-•-), 0.0 (——), -0.4(---), -0.8 (-••-) and for a) $h = +1$; b) $h = 0$; c) $h = -1$.

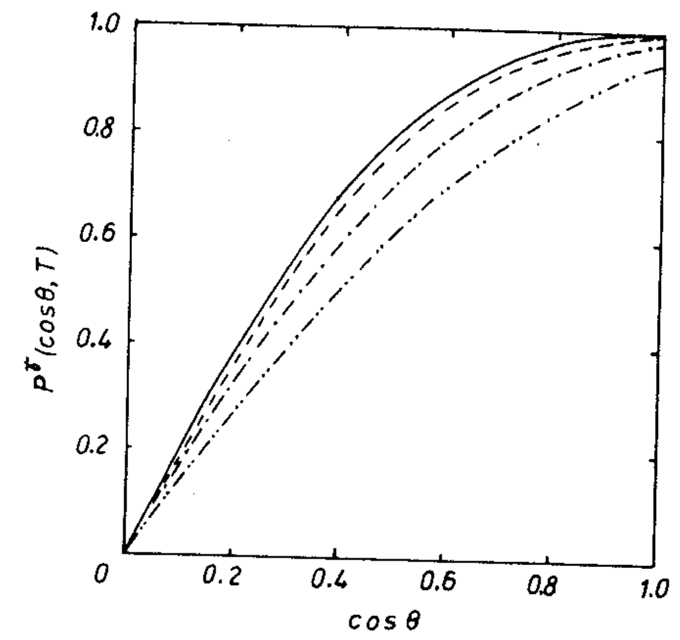


Fig.1

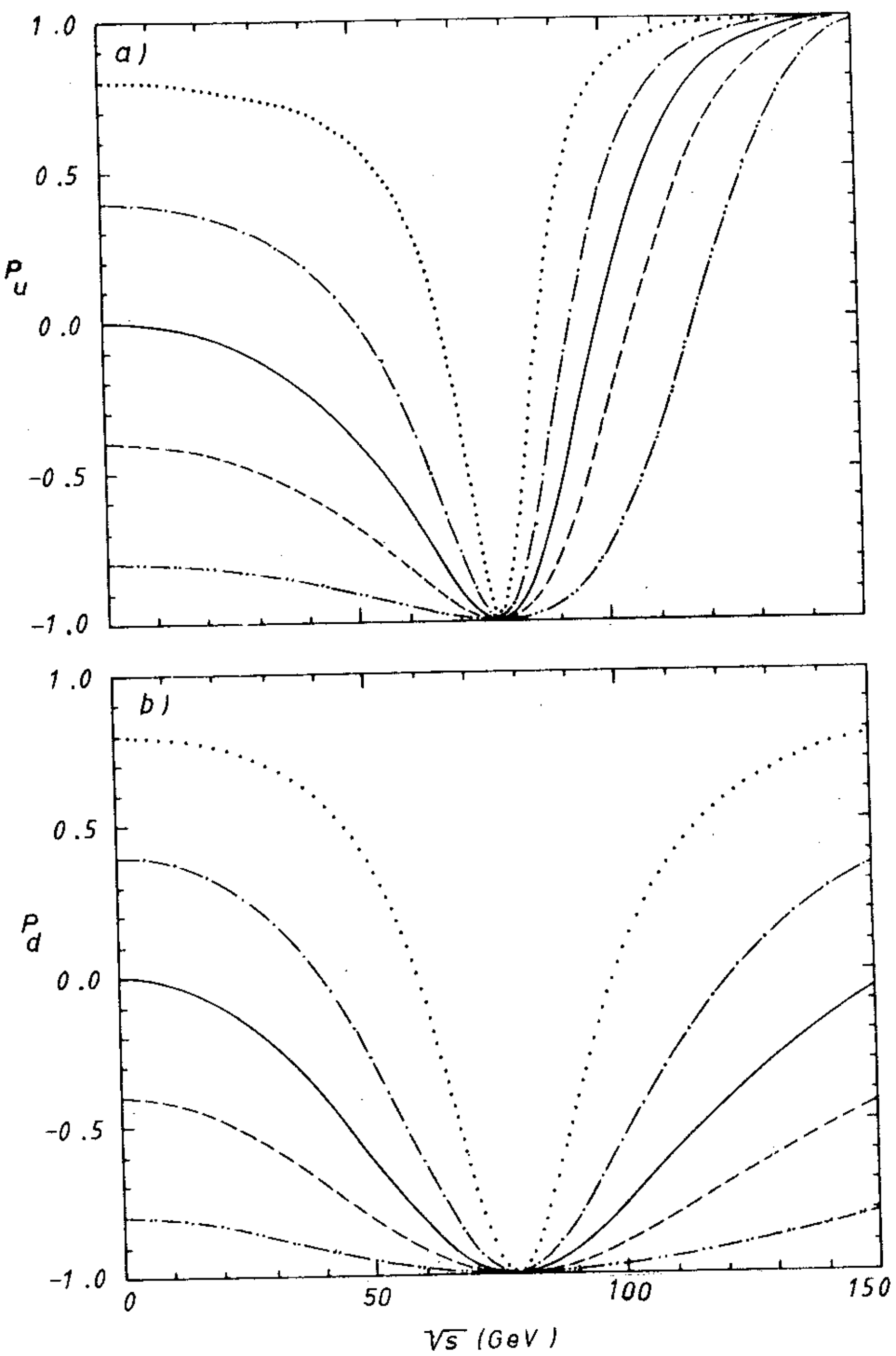

1.0
0.5
0.0
-0.5
-1.0
P_u
a)
b)
P_d
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.2

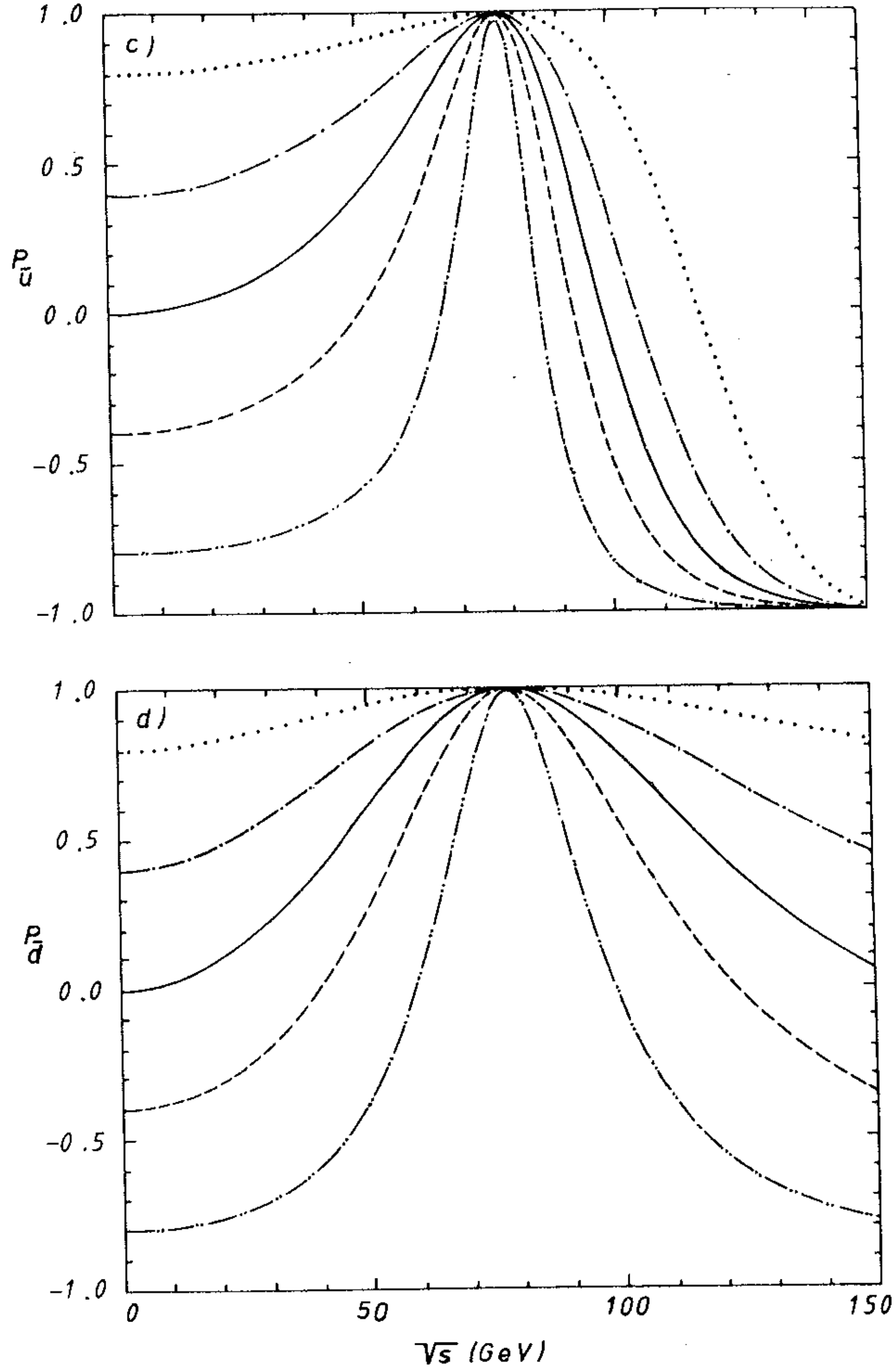

1.0
0.5
0.0
-0.5
-1.0
$P_{\bar{u}}$
c)
d)
$P_{\bar{d}}$
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.2

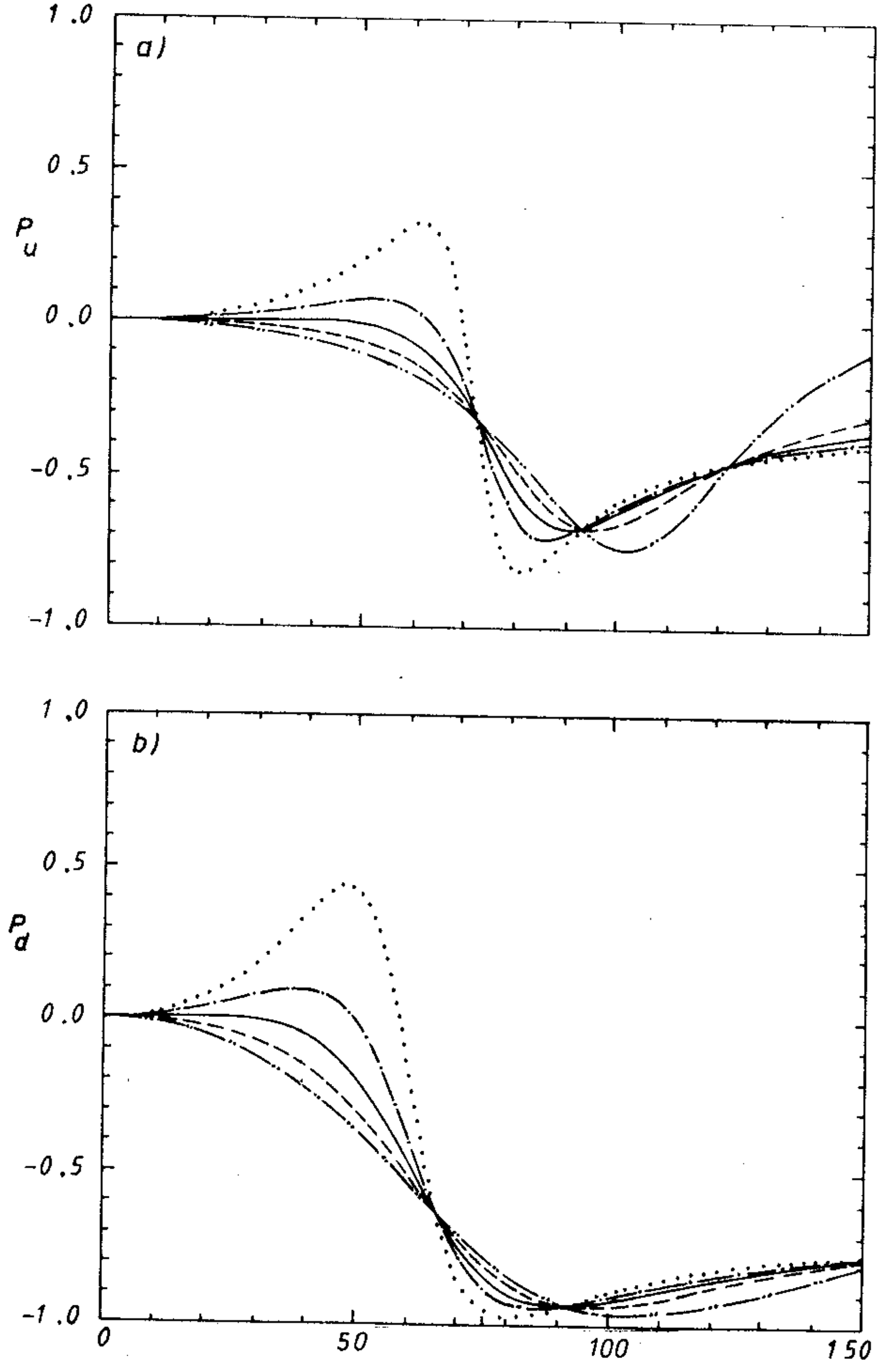

a)
b)
1.0
0.5
0.0
-0.5
-1.0
P_u
P_d
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.3

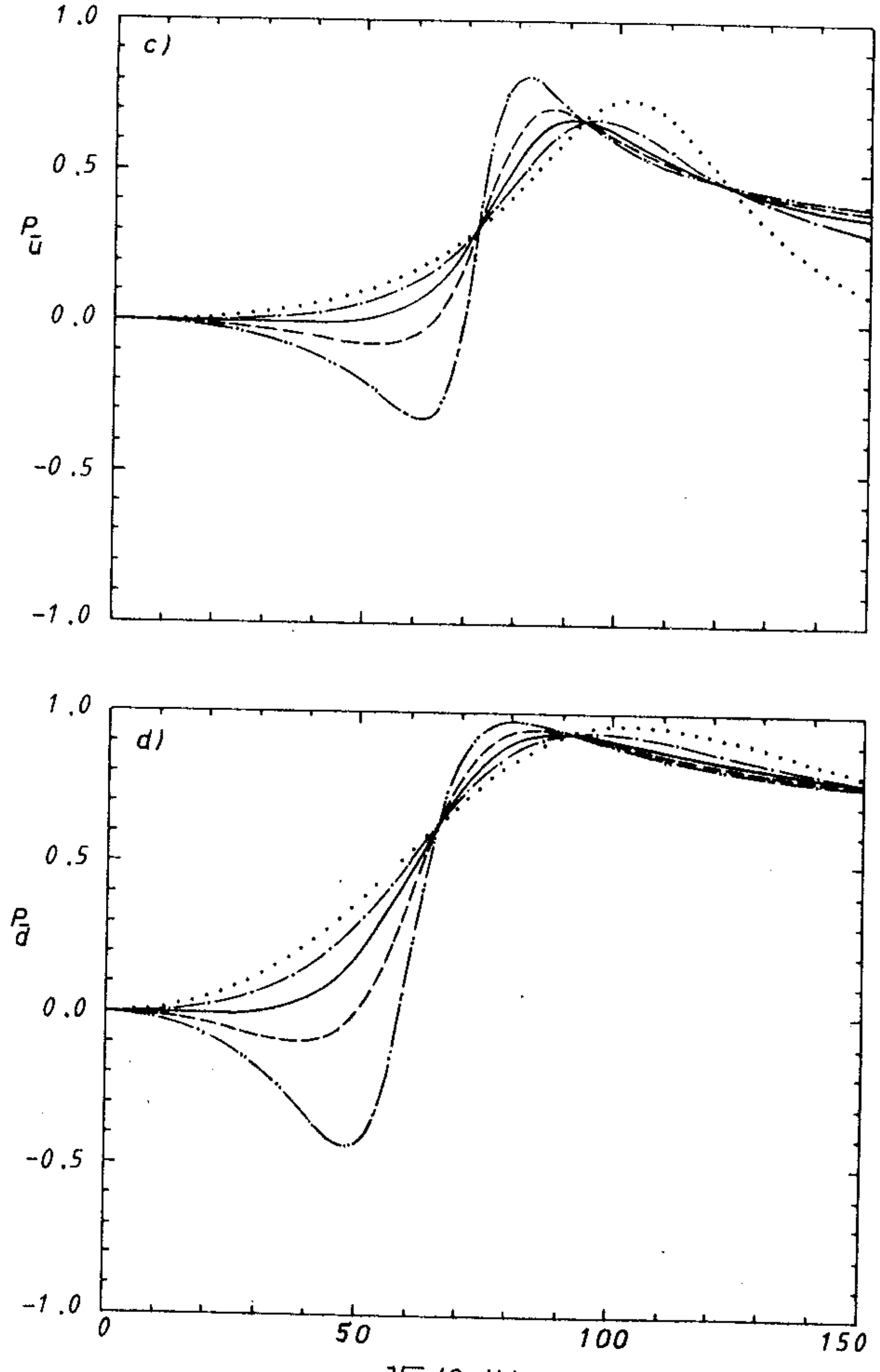
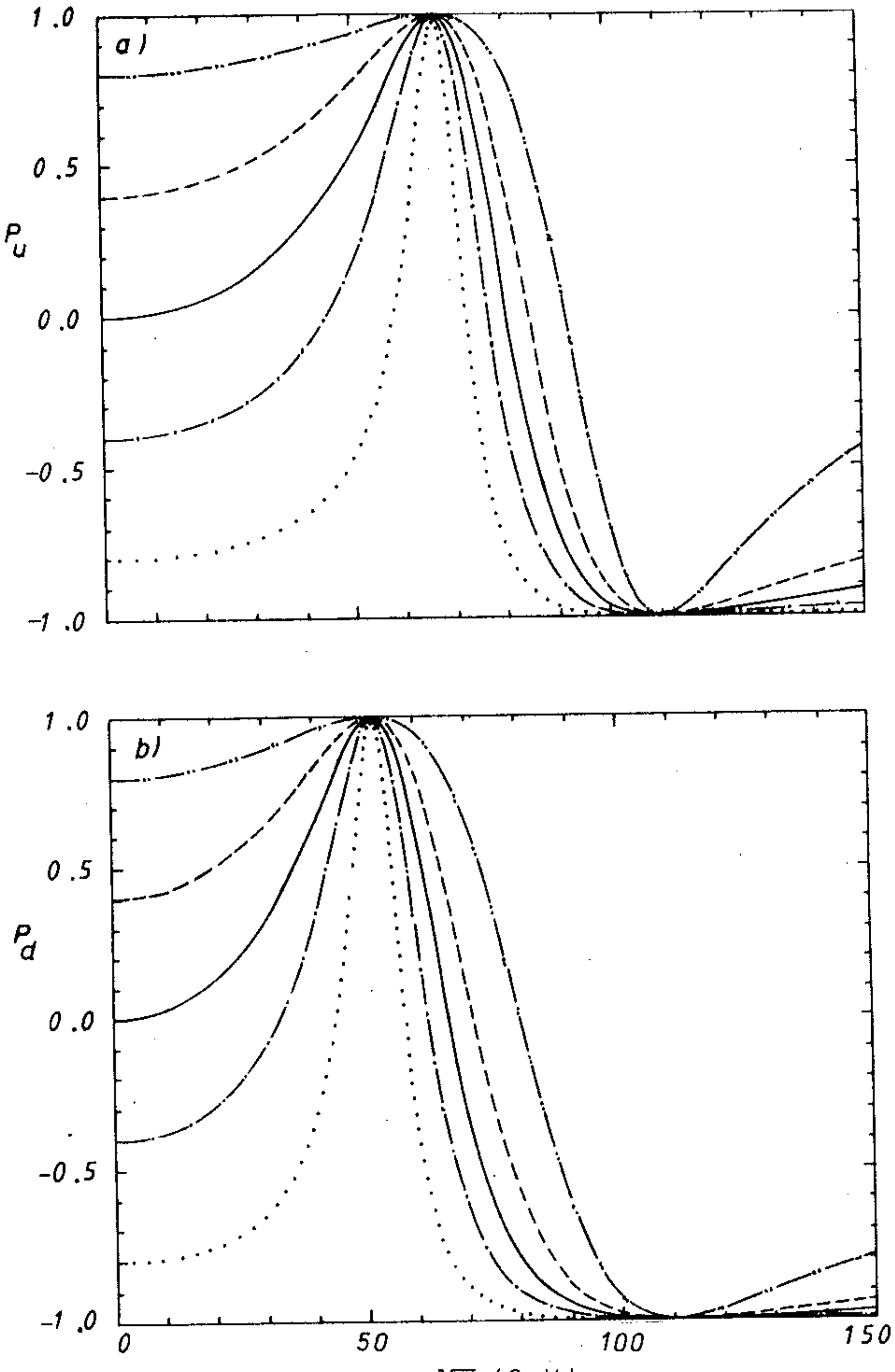
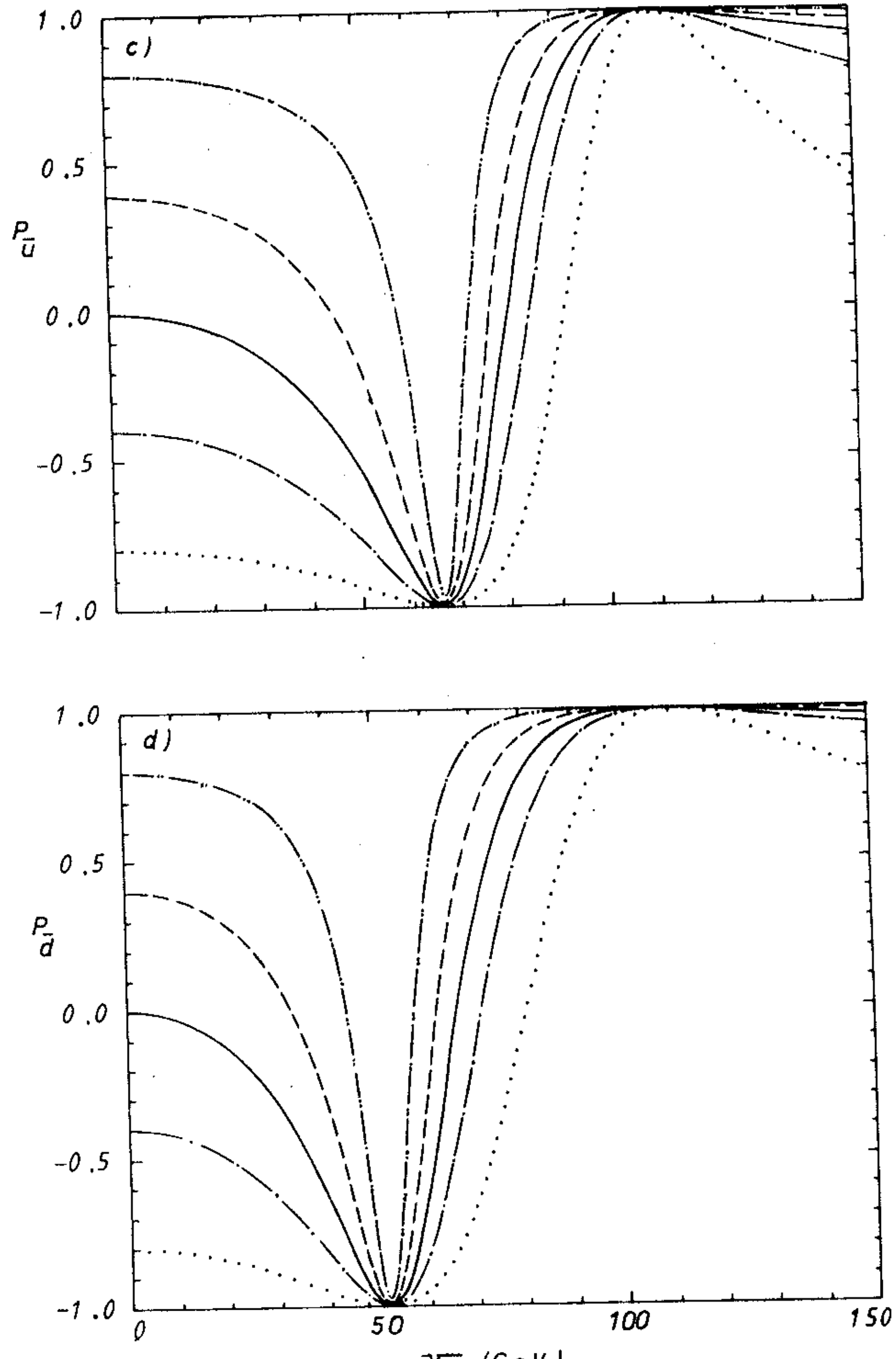

c)
d)
1.0
0.5
0.0
-0.5
-1.0
$P_{\bar{u}}$
$P_{\bar{d}}$
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.3



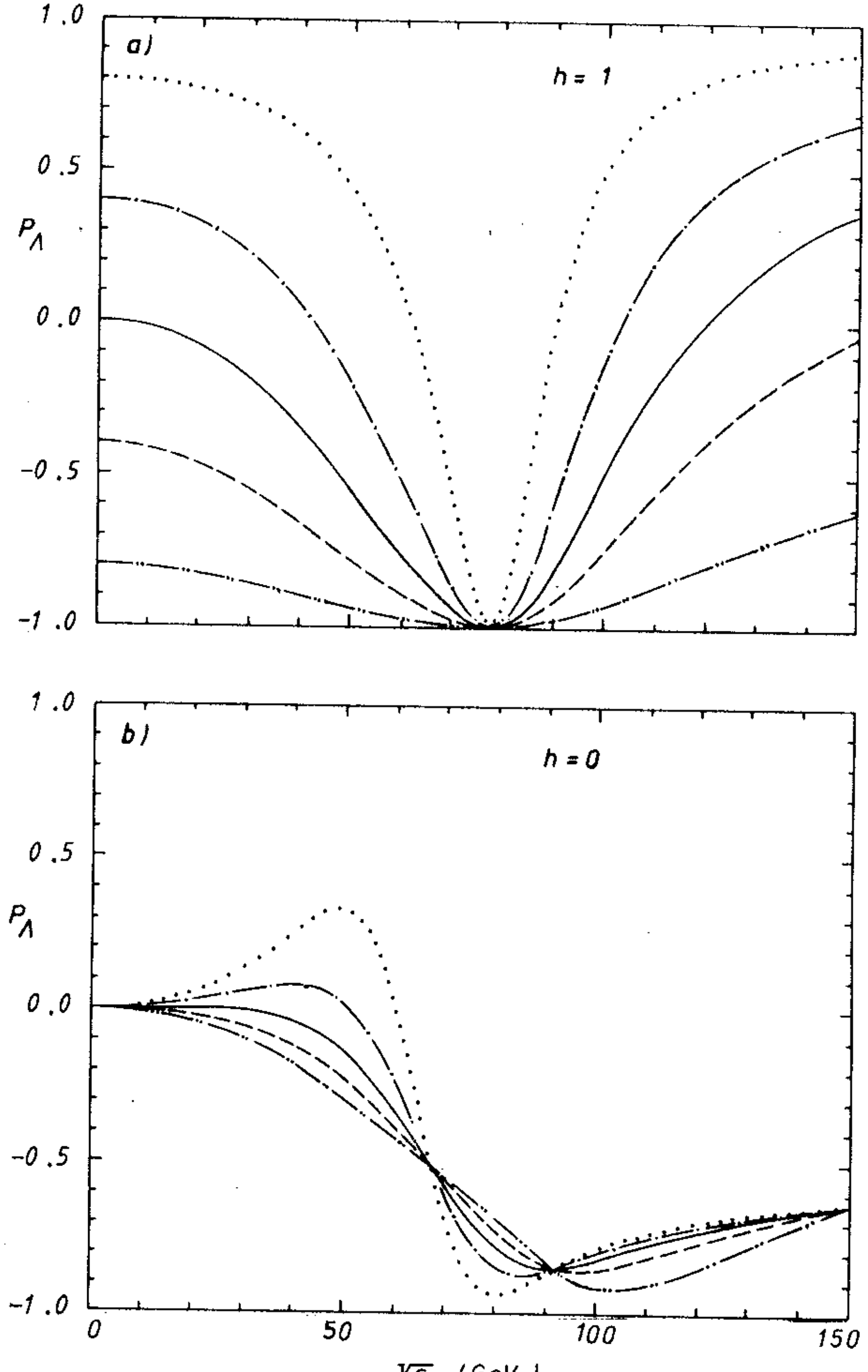
a)
b)
1.0
0.5
0.0
-0.5
-1.0
P_u
P_d
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.4



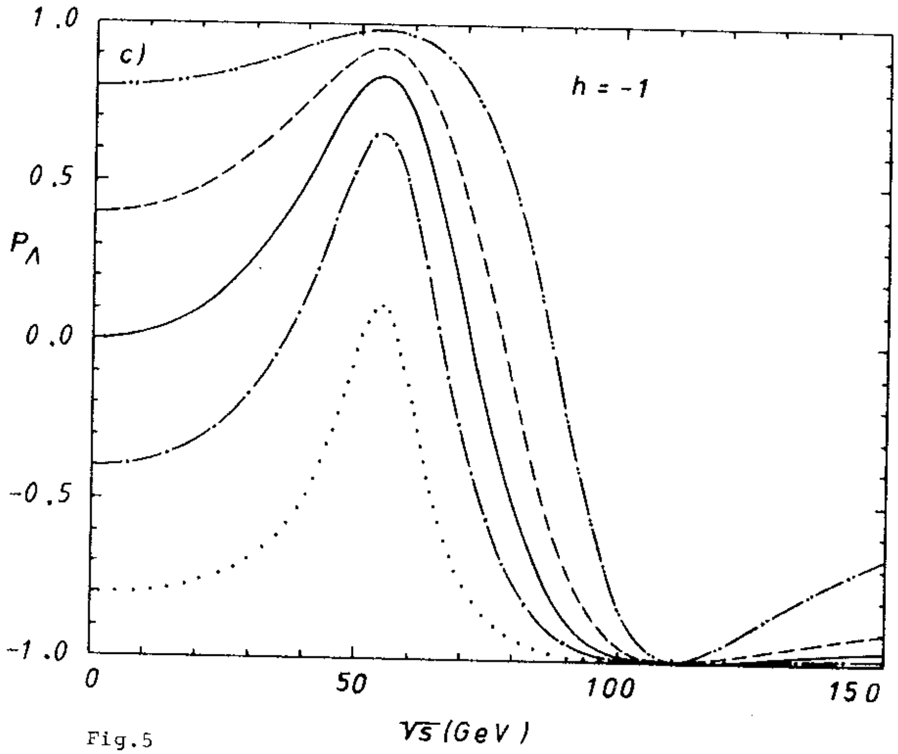
c)
d)
1.0
0.5
0.0
-0.5
-1.0
$P_{\bar{u}}$
$P_{\bar{d}}$
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.4



a)
h = 1
b)
h = 0
1.0
0.5
0.0
-0.5
-1.0
P_Λ
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.5



c)
h = -1
1.0
0.5
0.0
-0.5
-1.0
P_Λ
0
50
100
150
$\sqrt{s}$ (GeV)

Fig.5