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Charged and Neutral Current Production of $\Delta(1236)$

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Abstract:

Based on a hybrid quark model approach previously developed by us which employs a q^2 - continuation in terms of generalized meson dominance form factors we study the weak production of the isobar Δ (1236). First we demonstrate that our model is in agreement with the Argonne data on charged current production of the Δ . We then study neutral current Δ - production using four different gauge models, namely the standard Weinberg-Salam model, a vector-like model with six quarks, a five quark model due to Achiman, Koller and Walsh and a variant of the Gürsey - Sikivie model. We find that the results for the differential cross-section in the forward region ($|q^2| \lesssim 0.1 - 0.2 \text{ GeV}^2$) are very sensitive to the structure of the weak neutral current and suggest that measurements in this region constitute a stringent test of weak interaction models. We also calculate the density matrix elements measurable from decay correlations. The density matrix elements are not so sensitive to the models containing some axial contribution whereas the vector-like model shows a behaviour quite distinct from the others.

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Introduction

After the discovery of neutral currents in neutrino-induced reactions there has been considerable interest in the experimental determination of the structure of the hadronic weak neutral current since this structure has a direct bearing on the construction of gauge models for weak and electro magnetic interactions. The most direct test of the hadronic weak neutral current structure which requires only little additional theoretical input can be obtained through the elastic scattering processes $\nu(\bar{\nu})N \rightarrow \nu(\bar{\nu})N$. A detailed study of these elastic scattering processes has recently been undertaken in Refs. [1-3] using various representative gauge model currents [1] and general spatial current structures [2], the results of which have been compared with the recent Brookhaven data on $\nu(\bar{\nu})p \rightarrow \nu(\bar{\nu})p$ [4].

Next to the elastic scattering processes one expects also neutral current data on the quasi-elastic production of the $\Delta(1236)$. In fact, some tentative evidence for neutral current Δ -production has recently been reported in a BNL experiment with neutrinos incident on complex nuclei [5] and in an Argonne experiment with neutrinos incident on a deuterium target [6]. Although Δ -production is dynamically more complex than the elastic scattering processes one can expect that an analysis of this production process will furnish additional information on the underlying neutral current structure which will complement and strengthen the results of the neutral current elastic scattering analysis. In addition, because the Δ has isospin $\frac{3}{2}$, the $N-\Delta$ transition has the advantage that one is obtaining separate information on the $\Delta I=1$ piece of the neutral current. Furthermore, in the forward region of Δ -production only the axial current contributes so that this process affords a nice opportunity to separately study one isolated space - isospin component of the neutral weak current.

A natural framework for the formulation of the dynamics of the $N-\Delta$ transition is to use the quark model since the hadronic sector of the gauge currents is given in terms of currents of quarks. Applications of the quark model up to date have been quite successful at $q^2 = 0$ (see e.g. [7]). However, at $q^2 \neq 0$, the quark model results tend to be unreliable (see e.g. the discussions in [8,9]) and do not in general exhibit the correct structure of relativistic kinematics (see e.g. [10]). We therefore use the quark model only at $q^2 = 0$ and continue to space - like q^2 using constraint free invariant form factors as has been done successfully in the corresponding electroproduction case [11].

At $q^2 = 0$ our model is quite similar to the results of previous calculations of neutral current Δ -production including a static model calculation [12], a dispersion theoretic treatment incorporating also the $I = \frac{1}{2}$ background contribution [13] and a quark model calculation [14]. At $q^2 \neq 0$ the results of the various calculations are in general different reflecting the different q^2 -behaviour of the different dynamical schemes.

As regards the neutral current structure we shall follow the procedure of Ref. [1] and study neutral current Δ -production for several specific gauge model currents. These include the Weinberg-Salam model [15], vector-like models using 6 quarks [16], a five quark model proposed by Achiman, Koller and Walsh [17] and one variant of the Gursev - Sikivie model [18]. In view of the lack of data we feel this representative procedure to be adequate at the present time. A more general analysis that is independent of any specific gauge model should of course be attempted at a later stage when enough data becomes available.

In Sec. 2 we write down the neutral current coupling strengths corresponding

to the above four gauge models. In Sec. 3 we discuss the dynamics of the Δ - production process and give the results of our quark model approach for the $q^2 = 0$ values of invariant form factors and specify their $q^2 \neq 0$ form factor behaviour. In Sec. 4 we check our model against existing charged current data of Δ - production and compare our results with those of other model calculations. In Sec. 5 we treat the case of neutral current production of the Δ . We present results for total cross sections, differential cross sections and density matrix elements at a typical neutrino beam energy of $E = 2$ GeV using the four different sets of neutral current couplings discussed above. In Sec. 5 we present our conclusions.

2. Weak Current Structure

For the description of the hadronic coupling of the weak current we shall assume a basic $SU(2) \otimes U(1)$ gauge structure specified in terms of the weak current couplings to quarks. Whereas the charged current couplings among ordinary quarks are determined a priori the form of neutral current couplings is model dependent. We shall in the following consider 4 definite models of neutral current coupling, namely the standard Weinberg-Salam (WS) model [14], vector-like (V) models with 6 quarks [15], a 5 quark model proposed by Achiman, Koller and Walsh (AKW) [16] and one variant of the Girssey-Sikivie (GS) model [17]. The charged and neutral current couplings of the quarks have the form

$$J_{\mu}^{\pm} = \sum_L \bar{\Psi}_L \tau_{\pm} \delta_{\mu} (1 - i\gamma_5) \Psi_L + \sum_R \bar{\Psi}_R \tau_{\pm} \delta_{\mu} (1 + i\gamma_5) \Psi_R, \quad (1)$$

$$J_{\mu}^0 = \frac{1}{2} \sum_L \bar{\Psi}_L \tau_3 \delta_{\mu} (1 - i\gamma_5) \Psi_L + \frac{1}{2} \sum_R \bar{\Psi}_R \tau_3 \delta_{\mu} (1 + i\gamma_5) \Psi_R \quad (2)$$

$$- 2 \sin^2 \Theta_w J_{\mu}^{\text{em.}}$$

where the summation runs over the left-handed (L) and right-handed (R) doublet representations of the weak $SU(2)$ and τ_{\pm}, τ_3 are the usual $SU(2)$ isospin generators. The contribution of the admixed electro magnetic part in Eq. (2) is as usual

$$J_{\mu}^{\text{e.m.}} = \sum_i Q_i \bar{\Psi}_i \delta_{\mu} \Psi_i \quad (3)$$

where Q_i is the quark charge of the i^{th} quark in units of e .

It is convenient and instructive to decompose the neutral weak current into pieces transforming as $I = 1$ and $I = 0$ of $SU(2)_{\text{strong}}$. One has

$$J_{\mu}^0 = \alpha V_{\mu}^3 - \beta A_{\mu}^3 + \frac{1}{3} \delta V_{\mu}^0 - \delta A_{\mu}^0 \quad (4)$$

In this paper we are only concerned with the first two contributions of Eq. (4) transforming as $I = 1$ since the $N - \Delta$ transition is a pure $\Delta I = 1$ transition. In Table I we list the model dependent coupling values of α and β , where X_w is the usual Weinberg parameter and $X_{AKW} = \cos^2 \phi$ parametrizes the admixture of the up quark in the right-handed doublet in the AKW model.

3. Multipole Structure

Most of the recent calculations of weak Δ - production [13,14,19] exhibit similar $q^2 = 0$ multipole amplitude structures, which are quite close to the simple multipole structure of the static model calculation and the nonrelativistic quark model. Latter models predict that the vector excitation is pure M_{1+} and that the axial excitation is a definite mixture

of \mathcal{E}_{1+} and \mathcal{E}_{1+} . In terms of the invariant form factors of Ref. [20] defined by

$$\begin{aligned} \langle \Delta^+ | J_\mu^+(0) | n \rangle = & \frac{G \cos \theta_c}{\sqrt{2}} \bar{u}_\beta(p^*) \left[\frac{C_3^V}{m} (q_3 q_{3\mu} - q_{3\nu} q_{3\nu}) \delta_5^+ + \frac{C_4^V}{m^2} (p^* q_{3\mu} - q_{3\nu} p_{3\nu}^*) \delta_5^+ \right. \\ & + \frac{C_5^V}{m^2} (p^* q_{3\mu} - q_{3\nu} p_{3\nu}^*) \delta_5^+ + C_6^V q_{3\mu} \delta_5^+ + \frac{C_3^A}{m} (q_3 q_{3\mu} - q_{3\nu} q_{3\nu}) \delta_5^+ \\ & \left. + \frac{C_4^A}{m^2} (p^* q_{3\mu} - q_{3\nu} p_{3\nu}^*) + C_5^A q_{3\mu} + \frac{C_6^A}{m^2} q_{\mu\nu} p_{\nu\beta}^* \right] u_\alpha(p) \end{aligned} \quad (5)$$

these statements translate into $M C_4^V = -m C_5^V$, $C_5^A \neq 0$ and all other $C_i^{V,A} = 0$. In Eq. (5) G denotes as usual the weak interaction coupling constant and θ_c the Cabibbo angle. M and m are the masses of the Δ and the nucleon, and $q = p^* - p$.

In the relativistic quark model of ref. [2] one has (for $\nu p \rightarrow \ell^- \Delta^{++}$)

$$\begin{aligned} C_3^V(0) &= 12 \frac{Mm}{(M+m)^2} & C_3^A(0) &= 0 \\ C_4^V(0) &= -\frac{m}{M} C_3^V(0) & C_4^A(0) &= -4 \left(\frac{m}{M+m} \right)^2 Z \\ C_5^V(0) &= 0 & C_5^A(0) &= -4 \frac{M}{M+m} Z \\ C_6^V(0) &= 0 & C_6^A(0) &= 0 \end{aligned} \quad (6)$$

where Z is the renormalization of the axial quark current which is set to $Z = 1.23 \times 3/5$ in order to obtain agreement with the quark model calculation of the axial vector coupling of the nucleon.

Compared to the multipole structure of the static and nonrelativistic quark model the results Eq. (6) still imply a pure vector M_{1+} transition, however, for the axial parts one has a deviation from the simple $\mathcal{E}_{1+}, \mathcal{E}_{1+}$ structure.

The calculation of the $q^2 = 0$ values of the invariant form factors from the

quark model of Ref. [21] has been described in detail in Ref. [21] for the corresponding charm changing $\Delta C = 1$ case and need not be repeated here. In calculating the vector part of Eq. (6) we have as usual introduced a scaling factor $m/m_q \approx 3$ ($m_q =$ effective quark mass) which takes into account the different mass scales of the quark and particle magnetic moments. For the transition case one has a corresponding scaling factor $(m+m)/(m_q+m_{qf})$ which gives the same factor 3 if the effective initial and final quark masses are set to $m_q = m/3$ and $m_{qf} = M/3$ (see also Footnote [22]).

For the q^2 -dependence of our invariant form factors we shall use a power behaved generalized meson dominance q^2 -dependence in the form

$$G^{V,A}(q^2) = \prod_{n=0}^3 \left(1 - \frac{q^2}{m_{V_n,A}^2 + n\alpha'^{-1}} \right)^{-1} \quad (7)$$

where $m_{V_n,A}$ are the masses of the lowest $I = 1$ vector and axial vector mesons S and A_1 , and α' is the Regge slope determining the positions of the recurrences S', S'', \dots and A_1', A_1'', \dots . We are using $m_3^2 = 0.593 \text{ GeV}^2$ and $m_{A_1}^2 = 1.21 \text{ GeV}^2$ and an universal slope $\alpha' = 1 \text{ GeV}^{-2}$ [23, 24].

The motivation for using form factors Eq. (7) has been discussed in detail in Refs. [11, 21] and will not be repeated here. Let it suffice to remark that the vector form factor behaviour of Eq. (7) leads to an excellent agreement with data on the electro production of the Δ [1].

4. Charged Current Production of Δ

In order to check the reliability of our model we apply our results to

the charged current production of the Δ for which there already exists some data [30-32]. We shall skip the details of how to obtain cross sections, density matrix elements etc., from the invariant form factors C_i^{VA} since the necessary formulae have already been recorded in sufficient detail [19,21,33].

In Fig. 1 we show the energy dependence of Δ^{++} - production off protons [34]. The agreement with the Argonne data [31,32] is quite good, except perhaps in the energy region from threshold to $E \approx 0.9$ GeV, where our cross section prediction is somewhat larger than the data, but is still lying within the statistical errors. The cross section prediction is systematically lower than the data of the older CERN propane experiment [30]. Our cross section saturates rather quickly. Already at $E \approx 1.1$ GeV the cross section is within 10% of its asymptotic value $\sigma = 0.68 \cdot 10^{-38} \text{ cm}^2$. In Fig. 1 we have also plotted the results of the models of Adler [25] ($m_A = 0.84$ GeV), Ravndal [19], Salin [35] and Zucker [36]. The results of the quark model calculation of the Orsay group [14] practically agree with the prediction of Ravndal's model and have therefore not been included in Fig. 1. From Fig. 1 one sees that our results are closest to the predictions of the models of Adler and Ravndal. A more detailed comparison of the various models will be given at the end of this section.

In Fig. 2 we show our anti-neutrino cross section for anti-neutrinos incident on CF_3Br and compare it with the CERN data [31] and the results of the calculation of Adler ($m_A = 0.84$) [25]. The agreement is satisfactory. We have not included nuclear physics corrections due to the Pauli exclusion principle as described in Ref. [25]. These would amount to corrections of the order of $\lesssim 10\%$ [25]. One notes that the anti-neutrino cross section saturates very slowly. Returning to Fig. 1 one sees that at $E = 5$ GeV $\sigma_{\bar{\nu}p \rightarrow l^+\Delta^0}$

is still 25% below its asymptotic value given by isospin symmetry as $\frac{1}{3} \sigma_{\nu p \rightarrow l^-\Delta^{++}}$ (The difference between the neutrino and anti-neutrino cross section is given by the VA-interference term which is of the order E^{-1} relative to the VV and AA terms).

In Fig. 3 we show the results for the differential cross section where we have folded our differential cross section predictions with the neutrino flux of the Argonne experiment as quoted in Ref. [33]. Again the agreement is satisfactory. Since we are using the zero lepton mass approximation our near forward cross-section does not show the pronounced forward dip of e.g. the calculations in Ref. [33] where this approximation has not been made. This dip is due to pion exchange which produces a destructive effect in the very forward region for a small range of q^2 - values $-q_{\text{min}}^2 \lesssim q^2 \lesssim m_\pi^2$. In this range the pion propagator provides an enhancement factor sufficient to counterbalance the kinematic m_A^2 -factor (in the case of muon-neutrino scattering) in the cross-section multiplying the spin zero exchange contribution. Since we are not concentrating on the very forward region we have not attempted to include this effect in our calculation. We are not presenting results on density matrix elements since these are not very different from the results of Ravndal [19] and the simplified Adler calculation given in Ref. [33].

As has been emphasized by Llewellyn-Smith [20] a comparison of the various dynamical approaches to weak Δ - production is facilitated by listing the results in terms of the invariant form factor predictions. In Table 2 we give the $q^2 = 0$ results of some recent model calculations together with our results and, where applicable, we also list the CVC and

PCAC predictions, where the CVC values have been extracted from Ref. [29]. One should be reminded that pure $M_{1/2}$ dominance in the vector case corresponds to $M C_4^V(0) / m C_3^V(0) = -1$ and that the static model result corresponds to $C_4^A(0) = 0$.

In Table III we compare the form factor behaviour of these models. For the model of Adler we have used the convenient parametrization given in Ref. [26], which should, however, not be extended much beyond $|q^2| \approx 2 \text{ GeV}^2$. In the vector case we have included the results of the phenomenological parametrization of the N_A vector form factor of Dufner and Tsai [38] which has also been used in the analysis of Ref. [33]. There is a remarkable agreement between this parametrization and our GVDM - ansatz up to very high q^2 -values. For smaller $|q^2|$ our vector form factor lies between those of Refs. [19] and [38], whereas the form factor of Refs. [14, 25] have a somewhat faster fall-off behaviour. In the axial case again the form factor of Ref. [25] falls faster than ours, although the form factors could of course be brought closer to one another by using a higher value for m_A in Adler's calculation. We practically agree with the results of the Orsay group [14], which is quite a remarkable coincidence in view of the very elaborate form factor expressions one extracts from the q^2 -behaviour of the quark model helicity transitions given in Ref. [14], which also contain direct quark form factors. One should mention that the erratic behaviour of the C_4^A form factor in Ravndal's model (see Table III) is a result of presenting Ravndal's results in terms of invariant form factors. In terms of the c.m. helicity amplitudes which are conventionally used for the formulation of the quark model results the q^2 -dependence of Ravndal's model is not strikingly different from ours.

From Table II it is apparent that the $q^2 = 0$ coupling values $C_3^V(0), C_4^V(0)$ and $C_5^A(0)$ predicted in our quark model approach are only in approximate agreement with the predictions of CVC and PCAC. From the application of PCAC in the axial N-N coupling case (Goldberger-Treiman relation) one knows that PCAC is only approximately fulfilled, whereas CVC is believed to be an exact statement. We have calculated the sensitivity of our results to these upward adjustments and find that at $E \approx 10 \text{ GeV}$ the neutrino cross section increases by $\approx 30\%$ and $\approx 35\%$ if the CVC values for $C_3^V(0)$ and $C_4^V(0)$ and the PCAC value for $C_5^A(0)$ are substituted for our quark model values. Obviously the limited neutrino Δ -production data does not allow one at present to accurately extract the relevant $q^2 = 0$ coupling values that would allow one to test the CVC and PCAC principles.

5. Neutral Current Production of Δ

The calculation of the neutral current excitation proceeds in complete analogy to the charged current case discussed in Sec. 4. The spatial structure of the vector and axial vector current matrix elements is the same as specified in Eqs. (6). The coupling strength of the neutral vector and axial vector current can be read off from the $I = 1$ piece in Eq. (4) with the parameters α and β determined in Table I according to the 4 different $SU(2) \otimes U(1)$ gauge models that we are considering here.

All 4 sets of neutral current couplings involve the Weinberg angle as a free parameter. We shall take $X_W = \sin^2 \Theta_W \approx 0.3 - 0.4$ which lies in a range favoured by recent experiments [1]. This range also includes the value $X_W = 3/8$ favoured by grand unification schemes [18, 39]. Our

results will be given for the two values $X_W = 0.3$ and $X_W = 0.4$.

The AKW - model involves a second free parameter $\cos^2 \alpha_{AKW}$. At its extremes at $\cos^2 \alpha = 0$ and $\cos^2 \alpha = 1$ the AKW-model can be seen to coincide with the WS-model and the GS-model, respectively. We shall not present any detailed results for various values of $\cos^2 \alpha_{AKW}$ since an estimate of the dependence on this parameter can be obtained by interpolating between the extremal values represented by the WS- and GS - models.

In Fig. 4 we show the production cross sections for $\nu p \rightarrow \nu \Delta^+$. The WS-model gives the largest cross section values, and the GS- model the lowest. The results of the AKW-model lie between these extremes depending on the value of the parameter $\cos^2 \alpha_{AKW}$.

Anti-neutrino cross sections $\bar{\nu} p \rightarrow \bar{\nu} \Delta^+$ are shown in Fig. 5. Again the results for the WS- and GS-models are comparatively large. One notes that the predictions of the vector like model are quite sensitive to the value chosen for X_W . As in the charged current case the anti-neutrino cross section saturates more slowly compared to the neutrino cross section. In Figs. 6 and 7 we show differential cross-sections for $\nu p \rightarrow \nu \Delta^+$ and $\bar{\nu} p \rightarrow \bar{\nu} \Delta^+$ at a typical beam energy of $E = 2$ GeV. There is a marked difference in the shape of the differential cross sections for the WS-, GS - and V- models in the small $|q_1^2|$ - region. Due to the conserved nature of the vector current only the axial vector current contributes in the forward direction and thus the differential cross section is very sensitive to the axial current strength in this region [40]. If the differential cross-sections could be determined with some degree of accuracy in the region $|q_1^2| \lesssim 0.1$ GeV² one would have a very stringent test of the underlying weak interaction model for the

neutral current coupling. One also notes from Figs. 6 and 7 that such a test would not depend very sensitively on the Weinberg parameter X_W .

In Fig. 8 we plot the predicted behaviour of the three density matrix elements \tilde{S}_{33} , \tilde{S}_{3-1} and \tilde{S}_{31} (defined in Refs. [19,33]) which can be determined from the angular correlations of the decay products of the Δ . The indicated q^2 -range extends from the forward direction $q^2 = 0$ to the backward direction $q^2 = q_{\max}^2$. For kinematical reasons \tilde{S}_{33} , \tilde{S}_{3-1} and \tilde{S}_{31} have to vanish in the forward direction $q^2 = 0$, and \tilde{S}_{3-1} and \tilde{S}_{31} have to vanish in the backward direction $q^2 = q_{\max}^2$ [19].

The reason that \tilde{S}_{33} and \tilde{S}_{3-1} do not in fact vanish in the forward direction in the vector like model is due to the dynamical vanishing of the forward cross section in this case. Since the density matrix elements are normalized to the differential cross section the non-vanishing of \tilde{S}_{33} and \tilde{S}_{3-1} at $q^2 = 0$ is a spurious nonmeasurable result. The fact that $\tilde{S}_{31} \equiv 0$ over the whole q^2 - range for the vector-like model is due to the dynamics of the quark model, which predicts that the vector transition is purely transverse.

The difference in the predicted behaviour of the density matrix elements between the vector-like model on the one hand and the GS- and WS-models on the other hand is quite pronounced. Of course one should keep in mind that a test of the neutral current structure through the density matrix elements rather than the differential cross sections becomes meaningful only for $|q_1^2| \gtrsim 0.1$ GeV² where there would be enough event s in the case of the vector-like model.

Finally we give our results for the ratio $R_{\Delta}^{\nu(\bar{\nu})}$ of neutral and charged

current production of the Δ defined by

$$R_{\Delta}^{\nu(\bar{\nu})} = \frac{\sigma(\nu(\bar{\nu})p \rightarrow \nu(\bar{\nu})\Delta^+)}{\sigma(\nu(\bar{\nu})p \rightarrow \ell^{-(+)}\Delta^{++(\pm)})} \quad (8)$$

for two representative energy values $E = 2$ and 5 GeV. It is apparent from Table IV that the dependence of R_{Δ}^{ν} on E and X_w is not very big, whereas for the anti-neutrino case $R_{\Delta}^{\bar{\nu}}$ the WS- and vector-like models show some energy dependence. At $E = 2$ GeV the difference between the neutrino and anti-neutrino ratios R_{Δ}^{ν} and $R_{\Delta}^{\bar{\nu}}$ is quite large for the WS- and V-models, whereas there is not much difference in the GS- model.

Summary and Conclusion

We have presented a dynamical model of the weak excitation of the Δ (1236) which is both simple and which possesses no adjustable parameters.

The q^2 - dependence of the isobar excitation form factors arises from the coupling of vector and axial vector mesons and their recurrences.

We have applied the model to charged current Δ^- production and have found good agreement with the charged current neutrino and anti-neutrino cross section data.

We then applied the model to the neutral current Δ -production processes using various gauge model currents. We found that due to the vanishing of the vector current contribution in the forward direction neutral current Δ^- production in the near forward region is quite sensitive to the underlying gauge structure and is an ideal process to separately gain information on the

$I = 1$ axial vector piece of the neutral current. For the specific gauge model currents that have been considered here the Weinberg-Salam and

vector-like model show the most extreme opposite behaviour in the near forward region ($q^2 \leq 0.2 \text{ GeV}^2$). Whereas the WS-model predicts a cross section peak in the forward direction, the vector-like model shows a strong dip in this region with a cross section zero at $q^2 = 0$. The GS - model shows a somewhat weaker dip which is however pronounced enough to set it aside from the prediction of the WS- model. At larger q^2 - values the differences in the differential cross section behaviour of the various models is not so pronounced, so that experiments in this region would not be so useful. Due to the forward peaking predicted for the WS-model this model also has the largest total cross section compared to the other models. The differences of the predicted cross sections of the various models, are, however, not very large so that measurements of the total cross section with the expected large error bars of the first generation experiments will not be so useful in discriminating between the various gauge models. The density matrix elements measurable from Δ -decay are again quite sensitive to the underlying gauge structure. In particular the vector-like model shows a behaviour which is quite distinct from the behaviour of the other models.

It is clear from the above that the most useful test of the neutral current structure in the case of Δ^- -production would come from differential cross section measurements near the forward direction. Since Pauli effects and charge exchange scattering effects in heavy nuclei tend to wash out the near forward structure [4] it would be desirable to obtain such data using hydrogen or deuterium targets. Another experimental problem is the possible presence of nonnegligible background contributions under the Δ^- -mass peak as has been observed in the corresponding charged current case [32]. Background subtraction may pose a formidable experimental challenge, in particular since model calculations of the background

contribution [13,42] are themselves dependent on the structure of the weak neutral current which one is attempting to investigate. One may hope that these problems can be overcome in the near future so that optimal use can be made of neutral current Δ -production in pinning down the hadronic structure of the weak neutral current.

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22. For the initial and final quark masses involved in the current transition we use $m/3$ and $M/3$, i.e. one third of the mass of the initial nucleon and final isobar.
23. The $N-\Delta$ form factors in Eq. (7) cannot a priori be compared to the elastic nucleon form factors since they are different dynamical entities. In the dispersion theoretic model of Adler [25] the $N-\Delta$ form factor behaviour arises from a Born term contribution involving the elastic nucleon form factor. Within the context of this model one may define (q^2 - dependent!) equivalent dipole masses of $N-\Delta$ form factors. Using the explicit parametrization of Adler's results for $q^2 \approx 0$ given in Ref. [26] our form factors have equivalent dipole masses of $\hat{m}_V = 0.92$ Gev and $\hat{m}_A = 1.28$ Gev at $q^2 \approx 0$. The axial mass is close to the value $\hat{m}_A = (1.17 + 0.03)$ Gev obtained for the axial $N-\Delta$ form factor from a soft pion analysis of threshold $\Pi\Delta$ electro production at DESY [27].

- The experimental value quoted for the true axial vector form factor mass \hat{m}_A varies depending on the experimental data used and on the method of analysis (see H.H. Williams, Invited Talk at the Particles and Fields '76 Conference, Brookhaven National Laboratory 1976) and lies in the range $\hat{m}_A \approx 0.9 - 1.0$ Gev.
24. The asymptotic power behaviour of the form factors Eq. (7) is $(q^2)^{-4}$ and thus the form factors fall by one power faster than what corresponds to canonical dipole behaviour. For Δ - electroproduction this agrees with experimental observation [28,29] (see also discussion in Ref. [1]).
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Table I I = 1 neutral current parameters in four different weak interaction models

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	α	β
WS	$1 - 2X_W$	1
Vector	$2 - 2X_W$	0
GS	$\frac{3}{2} - 2X_W$	$\frac{1}{2}$
AKW	$1 - 2X_W + \frac{1}{2} X_{AKW}$	$1 - \frac{1}{2} X_{AKW}$

Table II.

$q^2 = 0$ values of form factors in different models. In the case of Adler's model we use approximate equivalent values deduced by Bijtebier [26]. The last two columns list the results of the models of Ravndal [19] and the Orsay group [14]. Results are for $\nu p \rightarrow \ell^- \Delta^{++}$.

ME	CVC (PCAC)	Adler	Ravndal	Orsay
$C_3^V(0)$	3.56	3.20	2.95	2.66
$\frac{M_{C_4^V}(0)}{m C_3^V(0)}$	-0.74	-0.63	-1	-1
$C_5^R(0)$	-2.08	-2.08	-1.68	-1.44
$\frac{C_4^R(0)}{C_5^R(0)}$	-	0.29	0.67	0.24

Table III

q^2 -dependence of (normalized) form factors $\tilde{C}_i^{V,A}$ in different models. For Adler's model we use the q^2 -parametrization deduced by Bijtebier [26]. In this parametrization $\tilde{C}_i^V(q^2) = \tilde{C}_i^A(q^2)$ if the same dipole masses are used for the vector and axial vector cases. Thus we list only $\tilde{C}_i^V(q^2)$ for Adler's model ($m_A = m_V = 0.84 \text{ GeV}$). For the models of Ravndal [19] and the Orsay group [14] suffixes refer to the behaviour of \tilde{C}_4^A and \tilde{C}_5^A . DT refers to the results of Dufner and Tsai [38].

$ q^2 $	$\tilde{C}_i^V(q^2)$				
	We	DT	Adler	Ravndal	Orsay
0	1.00	1.00	1.00	1.00	1.00
0.1	0.75	0.72	0.72	0.76	0.75
0.2	0.58	0.55	0.54	0.59	0.57
0.5	0.30	0.29	0.26	0.32	0.28
1	0.13	0.14	0.10	0.15	0.11
5	3.7×10^{-3}	4.0×10^{-3}	2.0×10^{-3}	8.1×10^{-3}	2.3×10^{-3}
10	4.2×10^{-4}	4.7×10^{-4}	-	1.6×10^{-4}	2.5×10^{-4}

$\tilde{C}_i^A(q^2)$

$ q^2 $	$\tilde{C}_i^A(q^2)$				
	We	Ravndal ₅	Ravndal ₄	Orsay ₅	Orsay ₄
0	1.00	1.00	1.00	1.00	1.00
0.1	0.84	0.90	-0.05	0.84	0.86
0.2	0.71	0.73	-0.23	0.72	0.74
0.5	0.45	0.41	-0.22	0.45	0.49
1	0.23	0.10	-0.12	0.23	0.26
5	1.1×10^{-2}	1.1×10^{-2}	-7.9×10^{-3}	9.2×10^{-3}	1.1×10^{-2}
10	1.4×10^{-3}	2.1×10^{-3}	-1.5×10^{-3}	1.2×10^{-3}	1.5×10^{-3}

Table IV Predicted neutral current ratios R_A^ν and $R_A^{\bar{\nu}}$ for various gauge models at two energies $E = 2$ GeV and 5 GeV, and for two values of the Weinberg parameter $X_W = 0.3$ and 0.4.

	E = 2 GeV						E = 5 GeV					
	R_A^ν	$R_A^{\bar{\nu}}$	R_A^ν	$R_A^{\bar{\nu}}$	R_A^ν	$R_A^{\bar{\nu}}$	R_A^ν	$R_A^{\bar{\nu}}$	R_A^ν	$R_A^{\bar{\nu}}$	R_A^ν	$R_A^{\bar{\nu}}$
$X_W = 0.3$	0.3	0.4	0.3	0.4	0.3	0.4	0.3	0.4	0.3	0.4	0.3	0.4
WS	0.21	0.18	0.91	0.99	0.22	0.20	0.74	0.72				
Vector	0.16	0.12	1.14	0.84	0.20	0.14	0.79	0.58				
GS	0.15	0.11	0.45	0.33	0.15	0.11	0.43	0.32				

Figure Captions:

- Fig. 1 Neutrino and anti-neutrino cross sections $\nu p \rightarrow \ell^- \Delta^{++}$ and $\bar{\nu} p \rightarrow \ell^+ \Delta^0$. CERN data from Ref. [30]. Argonne data from Ref. [31,32]. Our result: full line. Broken lines are results of Adler [25] ($m_\pi = 0.84$ GeV), Ravndal [19], Salin[35] and Zucker [36].
- Fig. 2 Anti-neutrino cross section on a CF_3Br - target. Data from Ref. [25,37]. Our result: full line. Broken line is the result of Adler [25].
- Fig. 3 Differential cross section $\nu p \rightarrow \ell^- \Delta^{++}$. Our differential cross section result has been folded with Argonne flux as given in Ref. [33]. Data taken from Ref. [33].
- Fig. 4 Predictions for neutral current neutrino cross section $\nu p \rightarrow \nu \Delta^+$ using neutral currents given by the models of Ref. [15] (WS), Ref. [16] (V) and Ref. [18] (GS) for two different values of the Weinberg parameter X_W .
- Fig. 5 Predictions for neutral current anti-neutrino cross section $\bar{\nu} p \rightarrow \bar{\nu} \Delta^+$. Description as in Fig. 4.
- Fig. 6 Differential cross section prediction for neutral current neutrino process $\nu p \rightarrow \nu \Delta^+$ at $E = 2$ GeV. Description as in Fig. 4.

Fig. 7
 Differential cross section prediction for neutral
 current anti-neutrino process $\bar{\nu} p \rightarrow \bar{\nu} \Delta^+$
 at $E = 2$ GeV. Description as in Fig. 4

Fig. 8
 Density matrix elements \hat{S}_{33} , \tilde{S}_{3-1} and \tilde{S}_{31}
 for neutral current reaction $\nu p \rightarrow \nu \Delta^+$ at
 $E = 2$ GeV and Weinberg parameter $X_W = 0.4$.

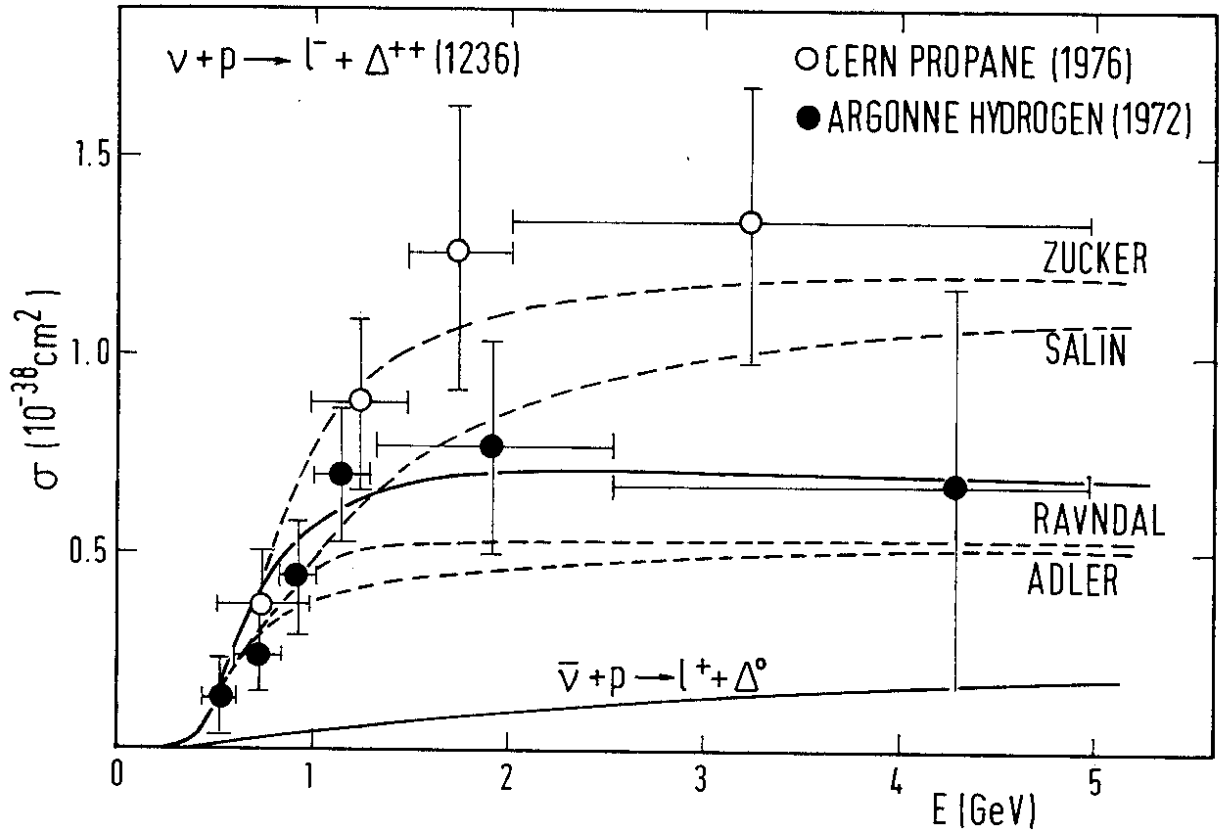


Fig.1

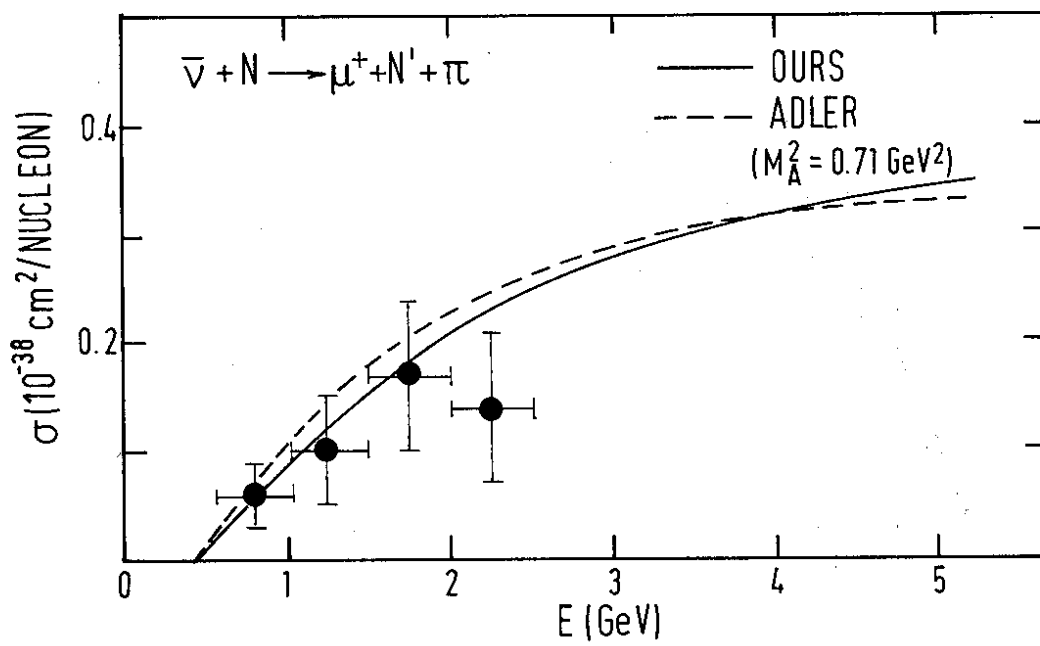


Fig. 2

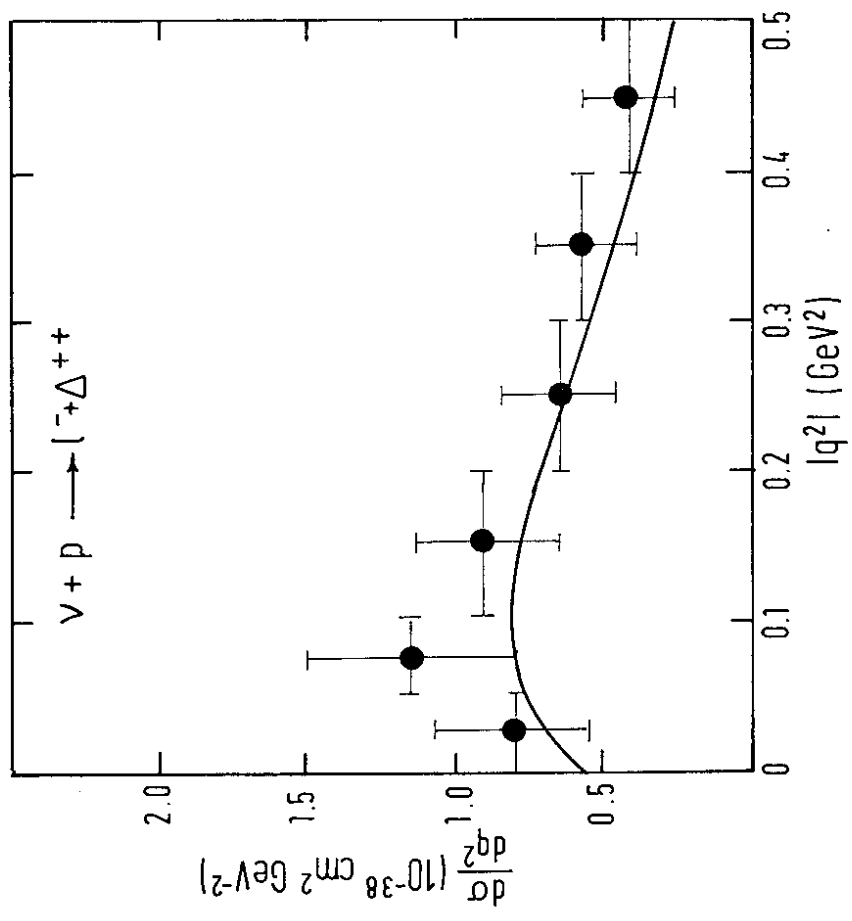


Fig. 3

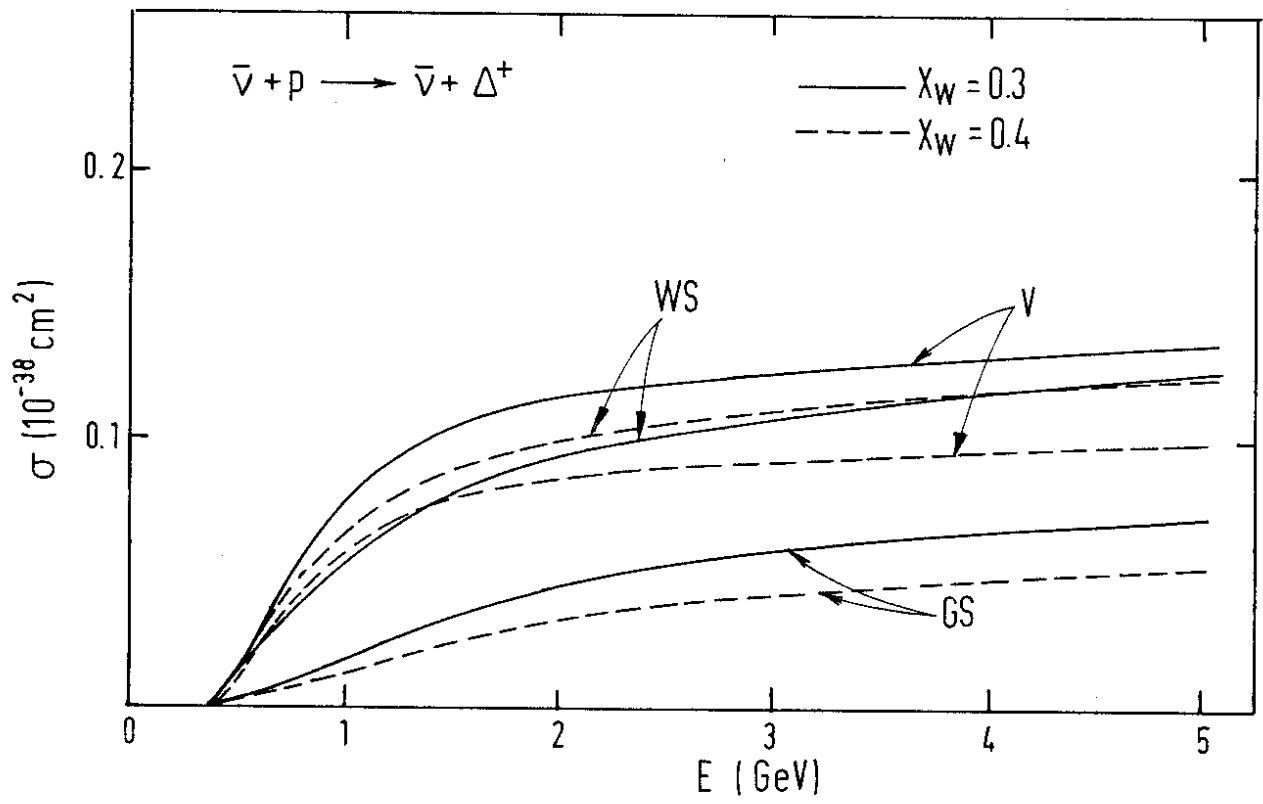


Fig.5

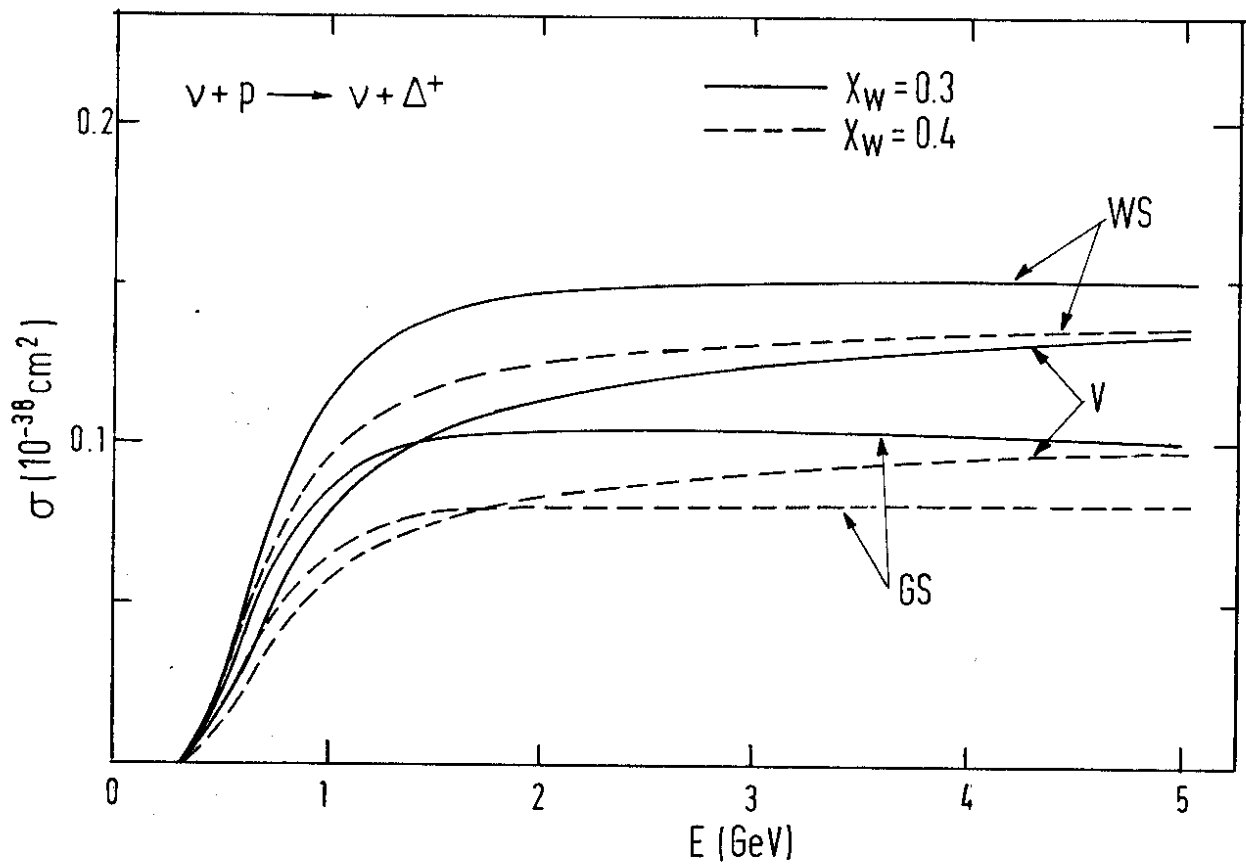


Fig.4

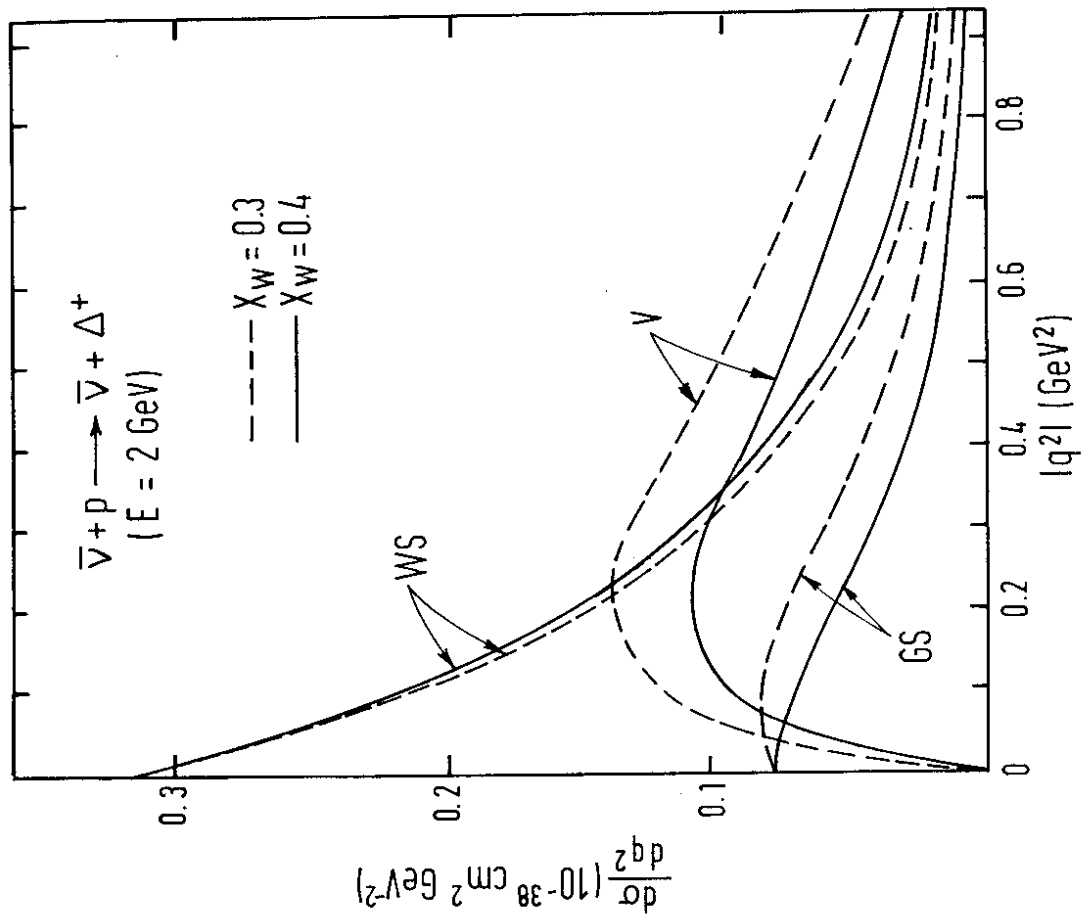


Fig. 7

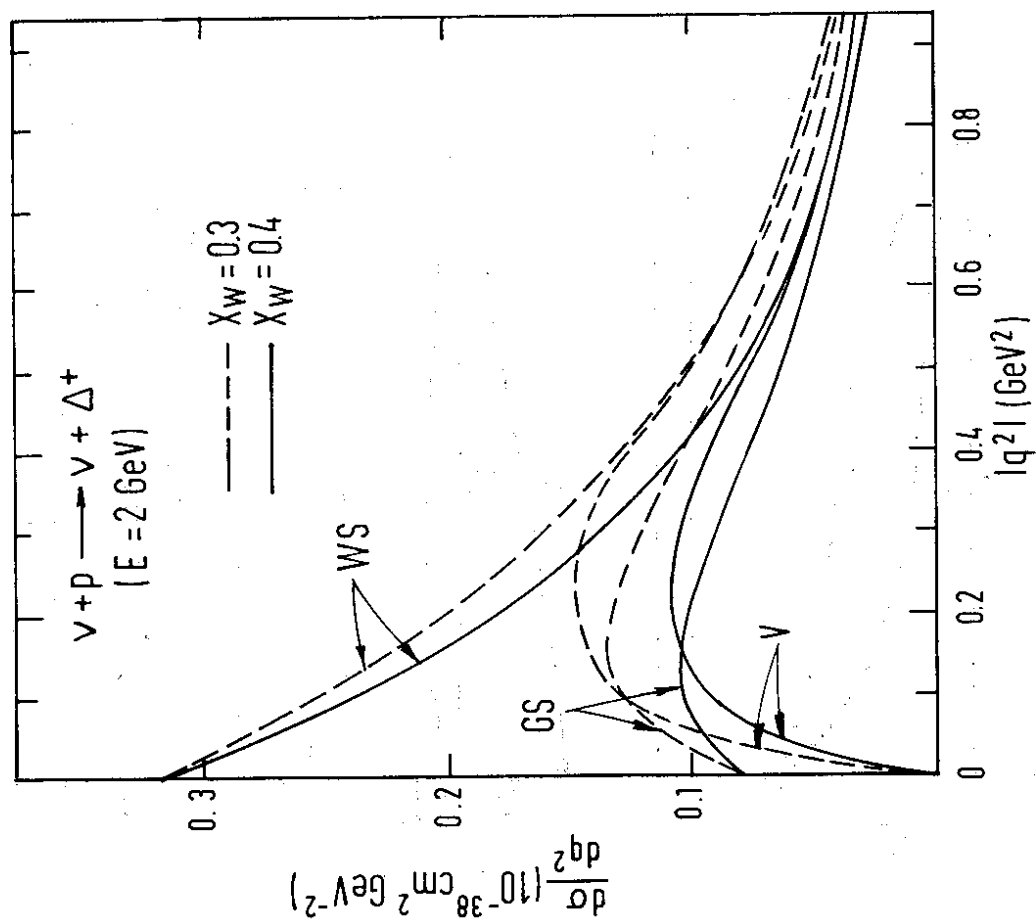


Fig. 6

$\nu + p \rightarrow \nu + \Delta^+$ ($E = 2 \text{ GeV}$)
 $X_W = 0.4$

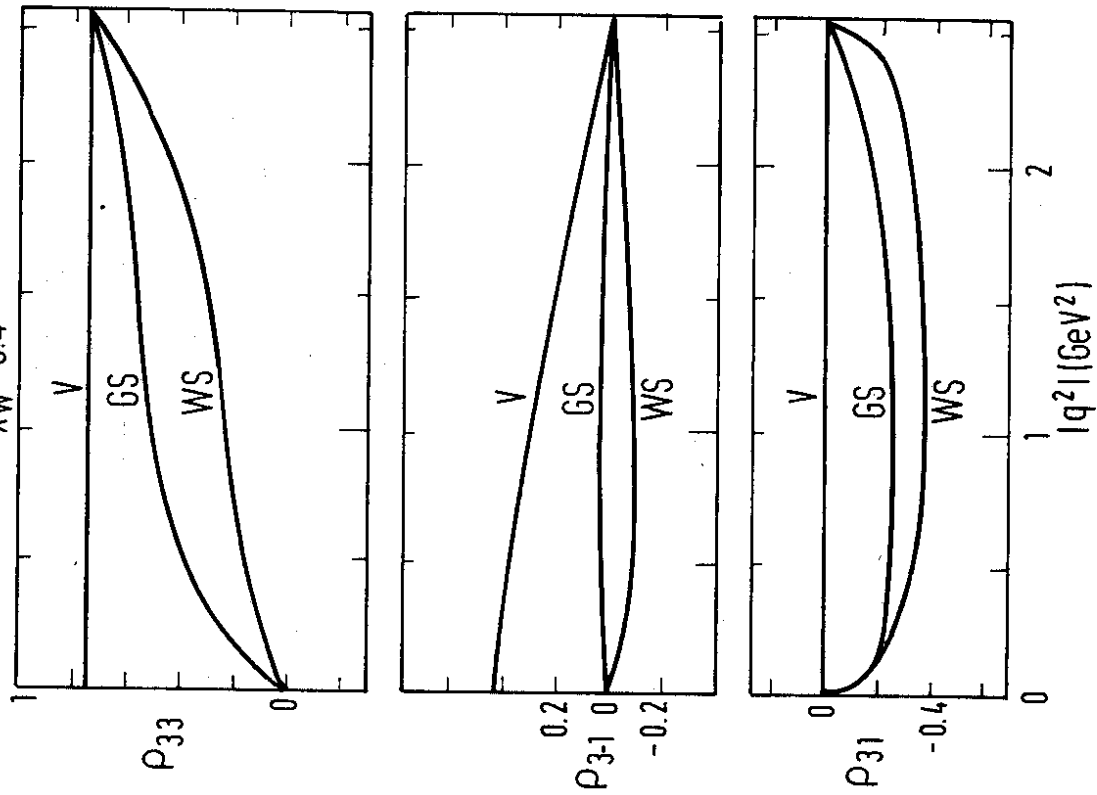


Fig. 8